The Language of Symmetry

Benedict Rattigan, Denis Noble, and Afiq Hatta (Eds.)

A Review by

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This is a very interesting, but rather curious, book. The eight chapters at its core take less than 90 pages, and even with a *Preface*, *Introduction*, and *Editors Comment* plus an Appendix, the whole is less than 150 pages. Yet, it manages to cover an astonishing range of topics, from Art and Architecture through Astronomy, Physics, and Biology, to Mathematics, Music, and Philosophy. The link between these disparate subjects is, as the title suggests, symmetry, rather generously defined. There is a second theme, the relationship between order and disorder – the first question posed by Noble in the *Preface* is "Is our world ruled by order or disorder?" Order is (loosely) related to symmetry, but what about disorder? Again, as Noble states in the Preface, "But there is one sense of symmetry that stands out from the variety of forms: a fundamental symmetry is that between order and disorder." This is a very bold claim, and one that the rest of the book aims, if not to justify, at least to make plausible.

Rattigan's *Introduction* starts with a discussion about the relationship between creativity and science, correctly identifying that scientific discovery is a creative act embedded in the scientific method (the interplay of hypothesis and experiment). The quest (at least as far as the physicist is concerned) is to find the simplest explanation for the phenomena that are observed – Occam's Razor is their most powerful instrument. Eventually, it would be good to have a "Theory of Everything", a single (simple?) equation from which all other physical laws, applicable in limited domains, could be rigorously derived. In this spirit, then, I become uncomfortable with statements such as "Modern physics is full of contradictions like this" and "At its most fundamental, the world of physics is brimming with paradoxes". In the context, it is the incompatibility between General Relativity and Quantum Mechanics, but there are also references to other systems (particle-wave duality, for example). The essential point here is the assumption that the Universe is rational implies that it is free of paradoxes; apparent paradoxes arise because of incomplete understanding. For example, particle-wave duality is *not* a paradox because there is a rational basis for choosing the appropriate description in any situation. (A photon or an electron starts as a "particle", passes through a slit as a “wave”, and is detected as a “particle”, all described by the evolution through time of the state function as it interacts with its environment.)

The rest of the *Introduction* presents an overview of symmetry through the ages and across domains, using the British Museum’s collection for illustration. The attractions of symmetry in, for example, pottery, art, and architecture, have been evident for thousands of years, sometimes accompanied by a deliberate measure of asymmetry. Order was preferred to chaos, harmony to discord, balance to imbalance. Many of these came with religious overtones. The *Introduction* finishes with an interesting set of observations about the Museum Café, its clientele, and the “paradox of choice”.

Hatta’s *Editors’ Note* gives a brief overview of the contents of the book and draws parallels between the interplay of order and disorder in the different scientific fields. It also introduces important concepts used in the later chapters, like entropy, the Poincaré group, and Noether’s Theorem.

Each chapter starts with an *Editors’ Preface* that introduces the author and topic and sets the context and finishes with an *Editors’ Commentary* reviewing the material. The chapters themselves usually start with a didactic presentation of the topic under discussion, followed by an, often challenging, exploration of order (equated with symmetry) and chaos or disorder.

The first essay is by Caroline Terquem on *Planetary Systems: From Symmetry to Chaos*. The special characteristic of gravitation, that it is always attractive, inevitably leads to the complex structures that are observed today at all scales in the Universe. Whatever the starting conditions, any inhomogeneity will expand as more and more matter is attracted. Above some mass, the objects will become approximately spherical and (unless that inflow is exactly radial) acquire angular momentum. Some of the material may have sufficient momentum to stay in orbit around the central object (for example, planets around a star or moons around a planet). While this might initially be chaotic, tidal forces between objects, though small, can reinforce until there is a resonance; once established, there is a phase stability that maintains the configuration, at least until disturbed by an external event. However, a different system, with more objects, might evolve chaotically.

Dimitra Rigopoulou next discusses *Entropy and Symmetry in the Universe*. Entropy is, loosely, a measure of the degree of disorder in a system; in the Boltzmann formulation, it is given by $k\ln(W)$

where W is the number of possible states in a system. The Second Law of Thermodynamics states that the entropy of an isolated system cannot decrease, and the direction in which the entropy increases leads to the concept of the (thermodynamic) arrow of time. The rich structure that is observed in the Universe today is a consequence of its evolution from the initial low-entropy state of the Big Bang. This evolution has been constrained by the fundamental symmetries and (through Noether's Theorem) conservation laws. Most of the symmetries of the quantum universe have their counterpart in the macroscopic universe but there are three rather special symmetries – Parity (P), Charge Conjugation (C) and Time Reversal (T) – that have no macroscopic analogue. Parity is like a normal mirror, except that all three spatial components are reversed, Charge Conjugation interchanges matter and anti-matter (switching the signs of all scalar charges), and Time Reversal changes the direction of time, with the added twist of changing the transition amplitude into its complex conjugate. These assume their importance due to a theorem of Pauli and Lűders, and of Bell, who showed that relativistic quantum field theories (of which the 'Standard Model' of particle interactions is an example) must be invariant under the combined operations of C, P and T taken in any order. While the strong and electromagnetic forces conserve all three separately, the weak interaction does not, maximally violating C and P, with T (or equivalently the combination CP) being violated to a small extent. However, this effect, while small and in a way not yet understood, leads to the gross structure of the Universe today, where matter predominates over antimatter, in contrast to the initial state of the Big Bang where they were equally present.

Darkness may be defined as the absence of Light, but Light is not defined as the absence of Darkness; Light is a substance, made of photons. True Darkness, in this sense, is the Darkness of the empty Universe – the Darkness of the quantum vacuum. However, as Alan Barr points out in his essay *Darkness, Light, and How Symmetry Might Relate Them*, Darkness might also be a substance. He discusses three examples. Firstly, and most familiar, is the darkness of black objects like soot, that absorb all wavelengths of visible light without re-emitting it in the visible range (at least at low temperatures); this is a rather trivial darkness. The second and third forms of dark substance share a characteristic – they do not interact with light and are thus, while existing, "invisible". However, they do interact gravitationally and so their influence is felt throughout the Universe and (through the weak interaction) their presence might be detected. The first of these is the *neutrino* (probably the second most abundant particle in the Universe), essential for driving the nuclear interactions in the Sun, from which billions pass through our bodies each second without our knowledge. However, we know that they are there; occasionally, in huge detectors, a neutrino from the Sun will initiate the inverse reaction to that which created it and be detected with sensitive instruments. The second, *dark matter*, is more speculative. From astronomical observations, it is known that there is approximately five times as much matter controlling galactic dynamics than can be accounted for with ordinary matter (stars, planets, gas, dust, and neutrinos) but which does not "shine" – its presence is inferred from its gravitational influence. Given that the known particles (quarks, leptons, gauge bosons like the photon, gluon, and W and Z particles, and the Higgs boson) are related through the symmetries of the Standard Model (technically $SU(3) \bigotimes SU(2) \bigotimes U(1)$), it is natural to examine whether there are *other* symmetries extending the Standard Model that might include Dark Matter. The search for evidence continues, man-made at the Large Hadron Collider for example or by direct detection of the cosmic component. Finally, and not discussed by Barr but mentioned by Noble later, there is Dark Energy, whose influence is around three times that of the matter component of the Universe but whose origins are still mysterious. Perhaps resolving this will also resolve the tension between General Relativity and Quantum Mechanics.

Changing from the real to the abstract world, Joel David Hamkins discusses *Self-Similar Self-Similarity.* The chapter starts with a gentle introduction to groups and symmetries, using the letters of the alphabet, simple geometric shapes, and the system of complex numbers as examples. Generalising, any mathematical structure will have a symmetry group associated with it. Now, this group is also a mathematical structure (!) and so has its own symmetry group, which is an *automorphism* of the original group. But why stop there; the process can be continued, building an *automorphism tower*. What the analysis shows is that, at some limiting point, perhaps surprisingly, there is a complete group which is not only an automorphism of its predecessor but is also an inner automorphism of itself; this is a perfect self-similar self-similarity. This is mind-expanding stuff. In the Editor's Commentary, this is compared with *Indra's Net* (from Hindu mythology), which is an infinite network of silken strands, with a perfect diamond at each crossing point; looking at any one of these diamonds reveals an infinity of reflections.

The fifth chapter moves back, slightly, towards the real world with Robert Quinney's essay on *The Language of Symmetry in Music*. As Quinney notes, this discussion is entirely contained within the western tradition of classical music, where the symmetries are, often deeply, hidden. Music, unlike the previous examples, is not visual or mathematical (despite the Pythagorean scheme of octaves and intervals) but *aural*. Using examples across seven centuries, Quinney examines the use concepts from symmetry (reflection, inversion, similarity, translation, circularity…) in inventive ways. The challenge is to find ways to resolve musically the disharmonies that might arise when blindly following a mechanical procedure. The music ranges from choral works by de Machaut, and Tallis and Byrd, where different voices use reflection, inversion, timing shifts to weave complex harmonies, through the intricacies of Bach's Goldberg Variations to more modern composers like Schoenberg, Webern, Berg, Boulez, and Reich.

The final section of the book starts with an interesting and important essay by Denis Noble: *The Interdependence of Order and Disorder: How Complexity Arises in the Living and the Inanimate Universe*. The chapter starts with a succinct description of classical Darwinian evolution; random mutations are slowly filtered through natural selection to create successful organisms (Dawkins' *Blind Watchmaker*). The organism that participates in, and benefits from, this process plays no active part in it. However, there are some situations where organisms intervene to give natural selection a helping hand. For example, "when an organism encounters a new virus, bacterium, or any other foreign body, they tell a very specific part of their genomes, the part that could make a new antibody to tackle the invader, to produce literally millions of new DNA sequences." Those that are effective reproduce. Noble then goes on to speculate whether this process might be used in other situations, and, whether this might contribute to resolving the issue of free will, through having "neural processes that generate that repertoire [of stochastic responses] can then mesh with your social interactions to generate a 'logical' response..." The general issue is to explain how order can emerge naturally from disorder which, of course, seems to violate the Second Law of Thermodynamics. However, in these cases, order (complexity) emerges from disorder in only *part* of the system, with the energy required leading to increasing disorder (entropy) elsewhere. The common feature is that purely random fluctuations in the disordered state create the conditions whereby a self-reinforcing interaction (like gravity or natural selection) amplifies it until it becomes an entity with its own structure and properties.

Immediately following this essay is a critical response from Anthony Kenny: *A Philosopher's Perspective on the Harnessing of Stochasticity*. Kenny focuses on two issues: the harnessing of stochasticity and the possibility of free agency. On the former, he is broadly in agreement with Noble. There is a debate about *how* the process is described – Noble writes that the immune system "work[ing] out" and "tell[ing]" the successful cells to reproduce which, according to Kenny, mixes the psychological with the physiological, and suggests "harnessing" as a more appropriately neutral descriptor. However, on the second issue there is debate. He uses the selection of a particular book from his library as an illustration. The act of picking up the book is a physiological process, driven by a decision made elsewhere, in the psychological domain. That decision is not an isolated random event but the culmination of a chain of decisions, directed towards a particular goal. While at some point in the past there might have been a choice that was in part settled by chance (*shall I mow the lawn or finish the essay*), once that choice has been made, subsequent actions are "determined". As Kenny points out, "the verb 'determine' is ambiguous: it may mean 'constrain' or 'control'." Actions are constrained by physical and physiological considerations but controlled by the desire to reach the goal.

Before discussing the final chapter, it is probably useful to discuss the Appendix by Anant Parekh, Frederick Parekh-Glitsch and Daniel Balowski: *A Response to Professors Noble's Paper: Ordered Disorder to Drive Physiology*. This analyses a specific example, the Ca^{2+} ion channel used by most cells. The channels are either open or closed and respond to external stimuli to move from closed to open. The efficiency of any single channel is quite low (less than 10%), presumably to reduce the noise. However, the cell requires high efficiency, and achieves this by having many channels available; even if the probability of a single channel is only around 1%, 1,000 channels will mean that typically ten will be open, while the probability that several channels will be randomly open at the same time is low. This is a fine example of *ordered disorder*. Cells also use this principle again by clustering channels tuned to different pathways, so that if any of these channels is open, the relevant pathway is stimulated.

The last chapter is *A Dialogue between Denis Noble and Benedict Rattigan*. It is an amiable ramble through some of the main themes of the book. If the book itself sometimes reads like an edited transcript of High Table conversation at one of the more cerebral colleges, this feels like a post-prandial discussion while passing the port. They discuss some of the issues raised in the essays, drawing comparisons and conclusions, examining the evidence to support the idea that there is a "symmetry" between order and disorder. But there are also new insights. Noble observes that "we aggressively attack the cancerous growth by zapping it with radiotherapy or chemotherapy. What happens? The cancerous cells realise they're being attacked, and they start randomly accumulating different forms of themselves to see whether they can find a solution." (This is an interesting illustration of Nietzsche's famous dictum "that which does not kill you makes you stronger;" unfortunately, in this case, it is the cancer and not the host that is strengthened.)

So, in the end, what have we learnt? Noble comments that “The more I’ve thought this through, the more it seems that [the symmetry of order and disorder] is the fundamental symmetry of the universe. […] And if that is so, then symmetry itself *must be* the fundamental principle of Nature.”

Let us take an example. At low temperatures, the properties of a perfect crystal are defined by the lattice cell (a few lengths and a symmetry, for example, body-centred cubic), except near the surface, where there are discontinuities in refractive index, conductivities, etc. As we raise the temperature, the description becomes more complex, passing through two phase transitions. Depending upon the rate of heating, the motions might become chaotic through turbulence or cavitation, but eventually it will become a gas. At that point, over a large range of conditions, its behaviour is described by three related parameters (Temperature, Pressure, and Volume) and the Maxwell-Boltzmann distribution. If we look at the *complexity*, it starts simple, becomes complex, and returns to simplicity. But there is no symmetry operator that transforms the simplicity of the beginning into the simplicity at the end.

Nevertheless, we have learnt a lot. Explicitly, we have learnt that *disorder* does not imply *disorganisation* – quite the opposite! Now, at this point, I might expect Kenny to object that in introducing the concept of organisation I have implicitly introduced the concept of an organiser. However, as we have seen, stochastic processes combined with specific interactions can create self-assembling (and impressive) structures.

This is, without doubt, a stimulating and challenging book. It presents novel (even outrageous) concepts and produces evidence to support them. It challenges some of our mostly deeply embedded notions of how nature works at all levels. If it does not quite succeed in establishing its central hypothesis beyond reasonable doubt, this is largely because our knowledge is incomplete – Rattigan’s “paradoxes”. But it is a brave attempt to elevate the ideas of order, disorder, and stochasticity to the level that they deserve. This is a book to be read and re-read slowly and to be thought about deeply.