The Language of Symmetry

Benedict Rattigan, Denis Noble, and Afiq Hatta (Eds.)

A Review by

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"The desire for symmetry, for balance, for rhythm in form as well as in sound, is one of the most inveterate of human instincts."

So writes Edith Wharton in her 1897 book "The Decoration of Houses". She was writing about the aesthetics of interior design, of course, but such a sentiment applies much more universally to express our fascination with the concept of symmetry and the consequent emergence of order from chaos in so many situations and contexts. These topics run throughout the recently published book "The Language of Symmetry" (hereafter referred to as LoS), an eclectic collection of essays by a distinguished group of (mainly) Oxford academics working across a broad range of disciplines embracing the arts, the physical and biological sciences and mathematics. As reviewed extensively by Benedict Rattigan in his introductory essay, the contrast between symmetry and disorder has inspired the philosophical and cultural outlook of civilisations from the ancient Egyptians, Mesopotamians and Greeks onwards to the present day, though attitudes to this contrast have differed widely. The Mesopotamians were evidently fascinated by the cyclic repetitions of nature, such as the seasonal cycles or the cycles of birth, life and death, while the Greeks (among others) were convinced of a balance between order and chaos in the Universe. The Egyptians, however, seemed to view the emergence of chaos and disorder as the natural tendency of all things, against which civilisation was impelled to resist and to impose order and balance.

Such a battle against the natural tendency of the universe towards chaos and disorder seems to a physicist as an allegory for the 2nd law of thermodynamics, which expresses the tendency of entropy, a thermodynamic state variable whose magnitude is associated with the degree of disorder in a system, either to remain unchanged during processes which are perfectly reversible, or to increase as time passes. As discussed in Dimitra Rigopoulou's essay, this tendency implies a preferred direction for the passage of time, thus breaking an important symmetry in Physics, namely, the symmetry of time reversal.

As noted long ago by Aristotle, symmetry plays a particularly important role in the mathematical and physical sciences, although in this context symmetry has a very specific definition. As helpfully explained in one of the editorial notes by Afiq Hatta in the LoS, in Mathematics and Physics symmetries are associated with particular operations or transformations, acting on a system described in mathematical terms, that leaves the system unchanged. Examples of such operations might include a simple displacement or translation in space or time, a mirror reflection or rotation about an axis. Such symmetries can also be perceived in familiar terms, such as associated with the perception of beauty in a face that exhibits left-right mirror symmetry. But since the work of Emmy Noether in her first theorem, published in 1918, the existence of a continuous symmetry in the mathematical equations describing a system is seen to imply a conservation principle. Thus, for example, where it can be shown that a system behaves identically in different, arbitrarily chosen, places and at different times, this indicates that energy and linear momentum are conserved. Conservation laws or principles are part of the "bread and butter" of physics, since so many things can be deduced from an application of such conservation laws.

This and other roles of symmetry in Mathematics and Physics are well represented in the LoS in the essays by Caroline Terquem, Dimitra Rigopoulou, Alan Barr and Joel David Hamkins. As mentioned above, Rigopoulou's essay deals directly with the role of entropy in determining the "arrow of time" as an extreme asymmetry of the universe, but notes that it is only a statistical law based on finding the most likely outcome of an event or process. Macroscopic states of a system that can be composed indistinguishably from a number of distinct configurations of its microscopic components, for which the one with maximum number consistent with the macroscopic state is most likely to be observed. When the component parts of a macroscopic system are at the molecular level then the astronomical numbers of components of the system make the optimum state overwhelmingly likely and alternative states almost impossible to observe. Both Rigopoulou and Barr explore the role of other mathematical symmetries on physical laws at the atomic and sub-atomic level, underpinning the standard model particle physics but also suggesting possible explanations for the nature of dark matter and dark energy which is currently so topical among astrophysicists. Both authors note, however, that some of these symmetries turn out to be somewhat fragile and not universal, raising the possibility of potential flaws in well established theories.

Many of the essays focus on the dichotomy between order and disorder or chaos, the transition between being interpreted in some cases as a breaking of a particular kind of symmetry. Caroline

Terquem's essay, for example, discusses the potential emergence of order and regularity in the relationship between planetary orbits from the initial chaos of the formation of a planetary system from an amorphous, self-gravitating cloud of gas and dust. Once formed, each planet can influence its neighbours through gravitational perturbations which, through the action of tidal friction and other complex interactions, may allow orbits to change over time until they reach more stable configurations. These turn out to have orbital periods related to that of their neighbours in ratios of simple whole numbers. The fact that such ratios are encountered so frequently in astronomy, including among planetary bodies within our own Solar System (as noted as early as the 18th century e.g. in the Titius-Bode law of planetary orbits), suggests that the emergence of such order is inevitable. But such ordered states are not found everywhere, with more chaotic motions being found in other parts of the Solar System such as in the asteroid belts.

The juxtaposition of ordered and chaotic states and transitions between them resonates with the situation in many other systems governed by dynamical laws. The transition from smooth, laminar flows towards fully developed, highly chaotic turbulence in fluid dynamics is a context familiar in my own work. But even in such a rich and complex nonlinear system as a fluid, the dichotomy between order and chaos is much more complicated than a binary transition from perfect order to complete chaos. Depending upon the context, the transition from a steady, highly ordered flow towards more complex states as external forcing is ramped up may well involve the breaking of symmetries, but this may be more incremental than an immediate and sudden breakdown to chaos.

In the classic Taylor-Couette experiment, for example, first explored in the 1920s by Geoffrey Ingram Taylor in Cambridge, a fluid is placed in the annular space between two upright, coaxial cylinders (see Figure 1). The outer cylinder is typically held fixed while the inner is made to rotate. At slow rotations the fluid forms a uniform shear flow in the gap between the two cylinders, with the flow varying smoothly from the stationary outer cylinder to the rotating inner. The flow looks the same from every angle and at every point along the cylinder at a particular radius. So it effectively possesses a broad set of continuous symmetries, including translational (along the axis of the cylinders), rotational (about the rotation axis) and is steady in time. Taylor discovered that, as the rotation speed of the inner cylinder was increased up to a certain critical value, the flow changed spontaneously to a pattern of identical toroidal roll vortices, each stacked one above the other (see Fig. 1). This can be seen as a transition that breaks just one of the original symmetries, namely, the continuous translation parallel to the axis becomes discrete, but leaving all the other symmetries unbroken. As the rotation speed continues to increase, another critical point is reached when each toroidal roll becomes wavy in the azimuthal direction around the cylinder, such that the rotational symmetry changes from continuous to discrete. Further increases in the rotation speed eventually lead to a breakdown of the flow into fully developed turbulence, but *en route* the flow may also pass through stages when it oscillates periodically in time (with a period-doubling cascade) before becoming completely and uniformly chaotic, paradoxically restoring a kind of continuous symmetry, at least in a statistical average sense.

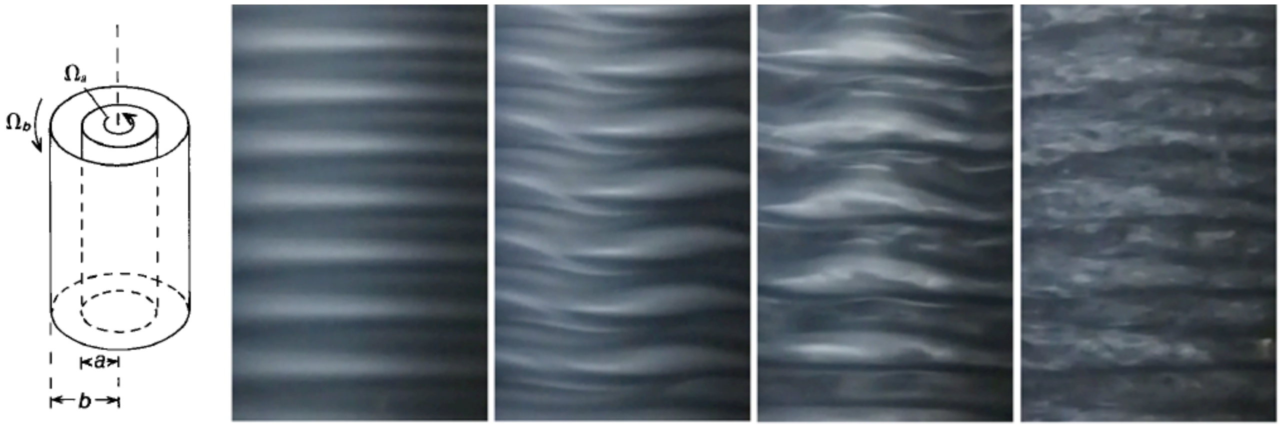

Figure 1. (Left) Schematic diagram of the Taylor-Couette apparatus, with differentially rotating cylinders; (Right) sequence of Taylor vortex instabilities, successively breaking continuous symmetries of the flow as the cylinder rotation speed increases from left to right. *(Images courtesy of D. Borrero-Echeverry, Willamette University, USA).*

Even more intriguing is that, at much higher rotation speeds than at the onset of turbulence, the turbulent Taylor-Couette flow itself may begin to organise itself spontaneously into toroidal roll-like or spiral structures that are filled with small-scale, highly chaotic turbulent eddies. Thus, highly chaotic turbulence can be seen to coexist with more ordered, persistent and symmetrical structures. While this sequence of partially broken symmetries and coexisting order and chaos is particularly prominent in an experiment like the Taylor-Couette cylinders, where the flow takes place in a container that itself possesses continuous spatial symmetries, it is by no means unknown in nature even on the grandest scales.

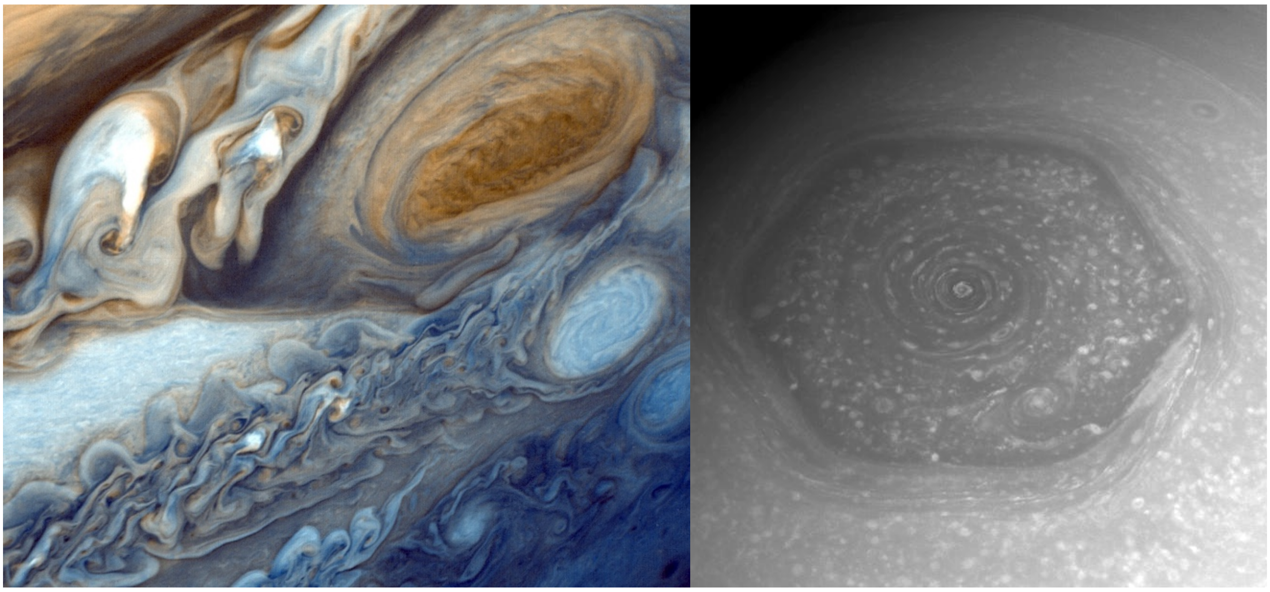

Figure 2. Order within chaos in the atmospheres of Jupiter (left) and Saturn (right), showing Jupiter's Great Red Spot and White Oval, and Saturn's North Polar Hexagon wave. Images are from the Voyager 1 and Cassini missions respectively *(Credit: NASA/JPL).*

The atmospheres of Jupiter and Saturn, for example, are filled with highly chaotic and turbulent motions. Yet within these deep chaotic oceans are found surprisingly ordered and persistent features, such as Jupiter's Great Red Spot, a huge oval vortex that is large enough to swallow three Earth-sized planets. Despite being surrounded and battered by energetic eddies that come and go on a daily basis, the Great Red Spot has been observed to persist more or less unscathed since its discovery by Robert Hooke in the 1660s (see Figure 2 left). In a similar fashion, a remarkable jet stream in Saturn's north polar regions was discovered in the 1980s, in images from the Voyager 1 spacecraft, that takes the shape of a regular, six-sided hexagon (see Fig, 2 right). Like the Great Red Spot, the highly symmetrical North Polar Hexagon is also persistent and long-lived, having been observed again some 30 years later by NASA's Cassini spacecraft. It also persists despite being surrounded by a chaotic sea of smaller eddies and vortices.

Even more intriguing in light of the order vs chaos theme of the LoS is that some of the turbulent eddies surrounding Saturn's North Polar Hexagon have been shown actually to contribute to maintaining the jet stream on which it rides, by concentrating momentum into the jet. This seems to apply to almost all of the major log-lived jet streams that encircle the planet on both Jupiter and Saturn. Thus, even in the most complex and turbulent of atmospheres there is a constructive symbiosis between chaotic and ordered structures.

It was fascinating, then, to find in Denis Noble's essay that such a constructive synergy between order and disorder occurs in living organisms, which have actually evolved to exploit such a synergy to their advantage. The ability of the immune system to induce stochastic changes in DNA structure at a molecular level in order to evolve ways to survive environmental stresses and new threats from infection seems incredible and awesome. But this clearly underpins the tenacity of life and its ability to survive some of the most hostile environments and stresses. Noble even ventures to suggest that such a constructive synergy between order and disorder might play a role in determining the extent to which we are able to exercise free will against forces that constrain us towards determinism. A note of caution in this interpretation is sounded by Sir Anthony Kenny in his brief response, however, giving us an insight into a long-standing debate between these two distinguished opponents.

Returning to an aesthetic interpretation of symmetry, Robert Quinney explores the role of a constructive dialogue between order and chaos (interpreted in one context as assonance and dissonance) in musical composition. Such a dialogue seems essential to the creative process, leading to music that both challenges and satisfies the discerning ear. But as Quinney discusses, you can have too much of a good thing (symmetry that is!). He draws attention to several composers from the Renaissance and Baroque eras who composed pieces that consciously invoke higher symmetries such as reflection or inversion, by cleverly weaving musical phrases and their reflected or inverted forms in complex fugues and canons. That such an exercise can lead to sounds that also make pleasing musical sense leaves the listener awestruck by the level of craftsmanship and imagination. But this technique is not common among composers, perhaps for the obvious reason that it is very difficult to achieve successfully, and is certainly not necessary to produce great music.

As can be seen from all of this, the LoS takes the reader on a fascinating interdisciplinary tour of the various different ways in which both classical symmetry and the order-chaos dichotomy are understood to apply in hugely diverse contexts. Speaking as someone steeped in both classical symmetry (also as an amateur musician) and in theoretical physics, treating the transition from order to chaos as a natural and direct application of symmetry breaking took some initial adjustment, which other readers might also wish to take into account. Going from a fully ordered state to full-blown chaos seems an extreme transition for which a binary interpretation as a single symmetry-breaking transition seems too simplistic. The theory of nonlinear dynamical systems, for example, deals with routes from a trivial (steady) solution to chaos via ordered states, for which even chaotic systems still preserve differing degrees of symmetry on the way towards complete disorder; a state, incidentally, which is almost never reached in many systems. Mathematical symmetries in the governing equations can also influence how the transition from order to chaos takes place. This kind of subtle complexity is not really reflected in the essays in LoS as presented, and should perhaps be pursued further in a future publication. But the book does make abundantly clear, and in an engaging manner, that a synergy and even an interdependency between order and chaos underlies many aspects of the Universe, confirming that the Greeks knew a thing or two……!