Geometry Numerics I: Preliminary Results on the Dynamic-Symmetry Band in a Quantum-Stochastic de Sitter Model

Introduction

The Geometry programme has developed, in a sequence of exploratory compositions, a curved-spacetime toy effective field theory for dynamic symmetry, a homogeneous de Sitter reduction of that framework, a coarse-grained quantum-stochastic interpretation of the noise sector, and a qualitative account of how the dynamic-symmetry band depends on noise amplitude, memory time and smoothing scale. The next natural step is numerical. The purpose of the present paper is to offer the first explicit computational illustration of the central claim that the homogeneous Geometry model contains three qualitatively distinct regimes: rigid order, dynamic symmetry and disorder.

This paper is deliberately modest in scope. It does not yet attempt the full parameter sweep envisaged in the Geometry Numerics I design note, nor does it claim a precision numerical determination of phase boundaries. Instead, it studies three anchor-point cases in parameter space at a mesoscopic coarse-graining scale and asks whether they display the expected behaviours. In that restricted sense, the paper serves as a first numerical proof of concept for the Geometry programme.

The model

The system under study is the homogeneous de Sitter Geometry model. The background spacetime is taken to have scale factor

$$a(t) = \mathrm{e}^{Ht},$$

with constant Hubble parameter H. The order field $\phi(t)$ evolves according to an overdamped Langevin-type equation,

$$\dot{\phi}(t) = -\frac{1}{3H} f(\phi(t)) + \frac{\gamma}{3H}\,\xi(t),$$

where $f(\phi)$ is the restoring drift generated by the deterministic sector, γ is the coupling of the stochastic forcing to the order field, and $\xi(t)$ is a centred Gaussian process with finite memory.

For the purposes of this first numerical exploration, the deterministic sector is fixed in simple dimensionless units:

$$H = 1, \mu^2 = 1, \lambda = 1, \beta = 1.$$

The effective potential is taken in the familiar double-well form

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4,$$

with an edge-regulating interaction term understood in the same simplified spirit as the earlier Geometry notes. The stochastic forcing is represented by an Ornstein–Uhlenbeck process with covariance

$$\mathbb{E}[\xi(t)\xi(t')] = \sigma^2(\Delta)\exp\left(-\frac{|t-t'|}{\tau(\Delta)}\right).$$

The present paper focuses on a mesoscopic coarse-graining scale $\Delta = 1.0$ and treats the effective quantities $\gamma\sigma$ and τ as the main control parameters.

Numerical setup

The numerical experiment studies three anchor points in the $(\gamma\sigma, \tau)$ plane, chosen to represent the three qualitative regimes that the Geometry programme has long treated as central.

- Rigid candidate: $\gamma\sigma = 0.1$,
- Dynamic-symmetry candidate: $\gamma\sigma = 0.5$,
- Disorder candidate: $\gamma\sigma = 2.0$,

These values are not intended to be physically canonical. They are illustrative anchor points chosen to test whether the numerical model can make the three regimes visible in practice.

The simulations are run for sufficiently long times to permit the decay of initial transients and the extraction of stationary behaviour. The principal observables are the form of the trajectory $\phi(t)$, the width of the stationary distribution, and the overall character of the field's fluctuations. At this stage, the emphasis is on clarity of regime rather than on exhaustive numerical optimisation.

Preliminary numerical findings

The three anchor-point cases show the expected qualitative distinctions.

Rigid regime

In the low-amplitude, medium-memory case, the order field remains tightly confined near its preferred minimum. The resulting trajectories show only small excursions away from the ordered baseline. The stationary distribution is correspondingly narrow, indicating that the system has settled into a stable but brittle state. This is the numerical realisation of rigid order within the Geometry framework. The model therefore behaves as expected when effective stochastic forcing is too weak to sustain a broader adaptive band.

Dynamic-symmetry regime

At intermediate effective noise amplitude with the same memory time, the behaviour changes substantially. The order field now explores a noticeably wider region of state space, but the motion remains bounded and structured rather than diffusive. The stationary distribution broadens in a controlled way, and the trajectories display sustained mesoscopic activity rather than collapse or dispersion. This is the clearest numerical candidate for the dynamic-symmetry band. It corresponds to the regime in which order and fluctuation remain jointly active, neither freezing into rigidity nor dissolving into noise.

Disorder regime

When the effective noise amplitude is increased further and the memory time reduced, the order field experiences strong, frequent and weakly correlated stochastic impulses. In this regime, the trajectories show large and irregular excursions, and the stationary distribution becomes clearly broader than in the rigid case. The resulting behaviour illustrates the onset of a disorder-dominated phase. Motion persists, but the coherent bounded structure characteristic of the dynamic-symmetry regime is weakened or lost. This provides the numerical contrast needed to distinguish the adaptive middle regime from the upper disorder sector of the phase portrait.

Interpretation

These first results are significant not because they determine an exact phase diagram, but because they show that the Geometry programme's three-regime language is numerically meaningful. The model does not simply produce random variation in the width of trajectories. It exhibits a clear qualitative ordering of behaviours as effective noise strength and persistence are varied.

At low effective forcing, order is preserved at the cost of adaptability. At high effective forcing with short memory, the system becomes noisy and structurally unstable. Between them lies a bounded regime in which fluctuation is strong enough to prevent brittle collapse but not so strong as to destroy coherence. This is precisely the kind of mesoscopic band that Dynamic Symmetry Theory has claimed should exist.

The numerical significance of this result is limited but genuine. It shows that the Geometry framework has progressed beyond a purely conceptual model. Even at this first stage, it now supports concrete simulation outputs that distinguish the intended regimes.

Relation to the wider Geometry programme

The wider importance of these preliminary findings lies in what they enable next. They justify the expansion from a small set of anchor points to the full parameter grid outlined in the Geometry Numerics I design paper. Once that larger sweep is performed, the dynamic-symmetry band can be located more systematically and its movement under changes in coarse-graining scale can be displayed explicitly.

The present paper also supports the broader claim that the curved-spacetime branch of Dynamic Symmetry Theory is becoming computationally tractable. The stochastic sector is no longer merely interpreted in conceptual terms; it can be simulated in a way that yields identifiable regimes and testable observables. That is an important threshold for the programme.

Limits

Several limitations should be stated clearly. The deterministic sector is fixed in highly simplified units, the interaction term is used in a stylised form, and only one coarse-graining scale has been explored directly. The paper therefore does not establish universal phase boundaries, realistic cosmological phenomenology or a definitive quantitative relation between quantum coarse-graining and effective vacuum structure.

These limitations are not weaknesses so much as part of the design of the study. The intention here is not to overclaim, but to publish the first clean numerical evidence that the dynamic-symmetry band is not merely rhetorical within the de Sitter Geometry model.

Next phase

The natural continuation of the present work is already clear. The next numerical paper should extend the analysis to the full $3 \times 3 \times 3$ grid in $(\gamma\sigma, \tau, \Delta)$, compute short numerical summaries for each point, and produce explicit phase portraits showing how the dynamic-symmetry band shifts with coarse-graining scale. Only after that wider mapping will it make sense to pursue more ambitious extensions, such as quasi-de Sitter backgrounds or structural comparisons with cosmological data.

The present paper therefore stands as a first result rather than as a final treatment. Its contribution is to establish that the numerical project is viable and that the Geometry programme has now crossed from formal construction into computational demonstration.

Conclusion

This paper has presented the first numerical illustrations of the dynamic-symmetry band in the homogeneous quantum-stochastic de Sitter Geometry model. By examining three anchor points in parameter space, it has shown that the model displays a rigid regime at low effective forcing, a disorder regime at high forcing with short memory, and a bounded intermediate regime that corresponds naturally to dynamic symmetry.

The significance of this result is it shows that the key distinctions claimed by the Geometry framework can already be seen in explicit simulations. In that sense, the paper marks the transition from conceptual possibility to first numerical evidence.