Coarse-Graining de Sitter Fluctuations into Geometry Noise

Introduction

Dynamic Symmetry Theory and the Geometry framework treat stochasticity not as arbitrary disturbance but as a disciplined representation of microscopic fluctuation viewed from a mesoscopic vantage point. In the curved-spacetime toy models, this role is played by the field $\xi(t)$, which drives the order field through a structured probability band rather than leaving it either to freeze into rigidity or dissolve into noise. The purpose of the present note is to explain, at a conceptual but mathematically clean level, how such a stochastic process may be obtained by coarse-graining vacuum fluctuations of a light scalar quantum field on a de Sitter background.

This note is not intended to provide a full treatment of quantum field theory in curved spacetime, nor to solve the problem of quantum gravity. Its aim is more modest and more precise. It seeks to show how a known quantum structure, namely the two-point fluctuations of a light scalar field in de Sitter space, may be converted into a smooth and controllable stochastic kernel suitable for use in the Geometry programme. In doing so, it strengthens the connection between order, fluctuation and spacetime without overreaching the limited but useful scope of the existing toy models.

Light scalars on a de Sitter background

Consider a free light scalar field $\hat{\chi}(x)$ on the spatially flat Friedmann–Robertson–Walker form of de Sitter spacetime, with scale factor $a(t) = \mathrm{e}^{Ht}$ and constant Hubble parameter H. Let the quantum state $|\Omega\rangle$ be a de Sitter-invariant Gaussian vacuum state, so that the basic structure of vacuum fluctuation is captured by the Wightman function

$$G^{+}(t, t'; \mathbf{x} - \mathbf{x}') = \langle \hat{\chi}(t, \mathbf{x}) \hat{\chi}(t', \mathbf{x}') \rangle_{\Omega}.$$

In the homogeneous versions of the Geometry framework, spatial structure is suppressed and attention is restricted to a single effective degree of freedom. In practice, this means that the relevant object is not the full spacetime two-point function but its spatially averaged or homogeneous projection, which may be represented schematically as a time-dependent function $G^{+}(t, t')$. At the level relevant to the present note, it is enough to assume that this reduced two-point function is symmetric in its arguments, approximately stationary over the timescales of interest, and decays with increasing time separation for a sufficiently light field.

The point of departure is therefore clear. The Geometry programme needs a mesoscopic stochastic process $\xi(t)$. Quantum field theory on curved spacetime supplies a microscopic two-point function $G^+(t,t')$. The task is to construct a mathematically intelligible bridge between them.

Mesoscopic coarse-graining

Dynamic symmetry is meant to operate neither at the fully microscopic level of quantum oscillation nor at the fully macroscopic level of cosmological description. It is a mesoscopic concept, concerned with the structured band in which order and fluctuation remain jointly active. To encode that idea mathematically, introduce a Gaussian coarse-graining window in cosmic time,

$$W_\Delta(s) = \frac{1}{\sqrt{2\pi}\,\Delta} \exp\left(-\frac{s^2}{2\Delta^2}\right),$$

where Δ is the coarse-graining timescale. Physically, Δ represents the temporal resolution at which the system is observed. It is large enough to smooth over rapid microscopic oscillations, yet small enough to preserve the effective band within which dynamic symmetry is expressed.

The coarse-grained covariance kernel is then defined by double convolution:

$$C_\Delta(t,t') = \int \; ds\, ds'\, W_\Delta(t-s)\, G^+(s,s')\, W_\Delta(t'-s').$$

This construction does several things at once. It preserves symmetry in the time arguments, because both the underlying two-point function and the Gaussian window are symmetric. It suppresses short-scale irregularities and ultraviolet structure, turning the singular or rapidly varying microscopic two-point function into a smooth and finite mesoscopic kernel. It also preserves approximate stationarity whenever the underlying quantum state has the corresponding invariance, since the window depends only on time differences and does not privilege any absolute origin of time.

The conceptual significance of this step should be emphasised. The kernel $C_\Delta(t,t')$ is not introduced by hand as an arbitrary stochastic input. It is obtained from a quantum field-theoretic object by smoothing over short timescales in a controlled way. The stochastic sector of the Geometry framework therefore becomes an effective description of quantum-origin fluctuations rather than a mere rhetorical placeholder for noise.

Effective Ornstein–Uhlenbeck description

For practical use in the homogeneous Geometry models, it is neither necessary nor desirable to retain the full detail of the coarse-grained kernel. What matters is the dominant structure of its decay with respect to time separation. The role of the Gaussian window is precisely to suppress rapid oscillatory features and leave behind a slowly varying envelope that captures the effective memory of the fluctuations.

Under these conditions, the coarse-grained kernel may be approximated by the covariance of an Ornstein–Uhlenbeck process,

$$C_\Delta(t,t') \approx \sigma^2(\Delta)\exp\left(-\frac{|t-t'|}{\tau(\Delta)}\right),$$

where $\sigma^2(\Delta)$ is an effective variance and $\tau(\Delta)$ an effective correlation time. These quantities are not fundamental constants. They are mesoscopic summaries of the strength and persistence of vacuum fluctuations once those fluctuations have been smoothed to the timescale Δ.

At zero time separation, the kernel gives the variance,

$$\sigma^2(\Delta) = C_\Delta(0).$$

The correlation time $\tau(\Delta)$ may be understood as the characteristic rate at which the coarse-grained kernel decays with increasing $|t-t'|$. In principle, both quantities could be estimated numerically once a specific light scalar field and state are chosen. For the present purposes, however, their exact analytic form is not required. What matters is that they emerge from a physically motivated smoothing procedure and can therefore be treated as meaningful parameters of the effective stochastic description.

This is the point at which the stochastic field $\xi(t)$ used in the Geometry models acquires a precise interpretation. It is no longer merely an abstract Gaussian process. It becomes the Ornstein–Uhlenbeck surrogate for a coarse-grained homogeneous projection of a light quantum field on a de Sitter background.

Role in the Geometry framework

Once the field $\xi(t)$ is understood in this way, the homogeneous Geometry models acquire a more clearly articulated quantum side. The order field is no longer driven by an unspecified noise source, but by a stochastic process whose variance and memory encode the coarse-grained imprint of vacuum fluctuations. This strengthens the claim that the Geometry framework is not simply borrowing the language of curved spacetime, but is beginning to connect in an intelligible way to the structures of quantum field theory on that background.

The role of the coarse-graining scale Δ is especially important. By varying Δ, one changes the degree to which the microscopic fluctuations are resolved or suppressed. This in turn changes the effective parameters $\sigma(\Delta)$ and $\tau(\Delta)$, and therefore changes the way the stochastic sector acts on the order field. In the language of dynamic symmetry, the mesoscopic band in which order and fluctuation remain productively coupled should depend on the level at which the underlying quantum structure is being observed. The coarse-graining scale is therefore not a technical afterthought but part of the conceptual architecture of the theory.

This perspective also clarifies why the Geometry programme has chosen to work, at least initially, with toy models rather than with a full quantum-gravitational formalism. The present note does not claim that spacetime itself has been quantised, nor that the stochastic kernel captures every relevant quantum effect. It demonstrates something narrower but still important: that there is a technically responsible way to move from a recognised quantum structure in curved spacetime to a stochastic description simple enough to support further work on order, curvature and adaptive stability.

Toward the next phase

The immediate value of the present construction is methodological. It provides a clear rationale for the Ornstein–Uhlenbeck noise used in the homogeneous de Sitter Geometry models, and it suggests how the parameters of that noise may be tied to a physically interpretable coarse-graining timescale. This, in turn, opens the way to the next stage of the programme: a small but meaningful exploration of how the dynamic-symmetry band depends on the coarse-graining scale, on the effective noise amplitude, and on the memory time of the fluctuations.

Such a study would not yet amount to a unified theory of quantum mechanics and general relativity. It would, however, push the Geometry programme a step closer to a unified picture of order, fluctuation and spacetime by showing in explicit terms how quantum-origin noise can be smoothed into a mesoscopic stochastic kernel, and how that kernel can support the structured regime in which adaptive order persists on a curved background.

The ambition of the programme remains controlled. The aim is not to replace existing physical theory with a single dramatic synthesis. It is to articulate a series of disciplined bridges. The present note offers one such bridge: from de Sitter vacuum fluctuation to Geometry noise, from microscopic two-point structure to mesoscopic stochastic dynamics, and from abstract fluctuation to a mathematically intelligible ingredient in the wider language of dynamic symmetry.

Conclusion

The Geometry framework requires a stochastic sector that is neither arbitrary nor merely metaphorical. By coarse-graining the Wightman function of a light scalar field on a de Sitter background with a Gaussian time window, one obtains a smooth covariance kernel that can be interpreted as the mesoscopic imprint of quantum fluctuations. Approximating that kernel by an Ornstein–Uhlenbeck form then yields an effective stochastic field $\xi(t)$ whose variance and memory are tied to a definite coarse-graining scale.

This does not complete the Geometry programme, but it gives one of its central ingredients a firmer foundation. The stochastic sector can now be seen as a disciplined simplification of a known quantum structure rather than as a free-floating addition to the model. In that sense, the present note advances the broader aim of the programme: to bring order, fluctuation and spacetime into a single, coherent descriptive frame while remaining honest about the provisional and exploratory nature of the theory.