

Appendix: Dynamic Symmetry on a de Sitter Background

This technical appendix specialises the curved-spacetime toy effective field theory for dynamic symmetry to a simple expanding cosmological background. The purpose is not to provide a full cosmological model, but to show in explicit terms how the central ingredients of the Geometry framework may be written in a concrete setting. By working with a de Sitter-like spacetime and a homogeneous order field, the model becomes simple enough to analyse while retaining the key themes of curvature, stochasticity and critical balance.

de Sitter background

A natural background for this exercise is the spatially flat Friedmann-Robertson-Walker form of de Sitter spacetime,

$$ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2, a(t) = e^{Ht},$$

where H is a constant Hubble parameter. In this geometry, the metric is homogeneous and isotropic, and the curvature enters the dynamics through the cosmic expansion rate. The determinant factor in the action is $\sqrt{-g} = a(t)^3$, so the expansion of spacetime directly modifies the effective evolution of the fields.

Homogeneous order field

To obtain a tractable first model, let the order field be spatially homogeneous,

$$\phi = \phi(t).$$

Spatial gradients are then neglected, and the order sector simplifies to a single time-dependent degree of freedom evolving on the expanding background. Using the symmetry-breaking potential

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4,$$

the order field still has the same basic interpretation as in the main Geometry note: it acts as a proxy for macroscopic structural coherence, with the potential favouring a non-trivial ordered regime.

Stochastic sector

The stochastic field is likewise reduced to a homogeneous temporal process,

$$\xi = \xi(t).$$

For a first approximation, it is useful to treat $\xi(t)$ as a centred Gaussian process,

$$\mathbb{E}[\xi(t)] = 0,$$

with a chosen covariance that reflects an effective coarse-graining of quantum fluctuations on the de Sitter background. In this reduced setting, the stochastic sector does not attempt to capture the full structure of quantum field theory in curved spacetime. Instead, it supplies a disciplined representation of time-dependent fluctuation that can be coupled to the order field.

Effective homogeneous dynamics

Dropping spatial gradients and suppressing the gauge-field sector for simplicity, the relevant part of the toy model reduces to an effective Lagrangian density of the form

$$\mathcal{L}_{\text{hom}} = \frac{1}{2}\dot{\phi}^2 - V(\phi) - \alpha\phi^2 + \gamma\xi(t) - \beta(V(\phi) - V_0)^2.$$

The corresponding action is weighted by the de Sitter volume factor,

$$S = \int dt a(t)^3 \mathcal{L}_{\text{hom}}.$$

Varying this action leads schematically to an equation of motion with Hubble friction,

$$\ddot{\phi} + 3H\dot{\phi} + f(\phi) = \gamma\eta(t),$$

where $\eta(t)$ is a Gaussian noise process and $f(\phi)$ is the deterministic drift term arising from the symmetry-breaking potential together with the edge-penalty term. In the long-time regime, it is often useful to work in an overdamped approximation, giving the simpler Langevin form

$$\dot{\phi}(t) = -\frac{1}{3H}f(\phi(t)) + \frac{\gamma}{3H}\eta(t).$$

This equation expresses the central dynamic-symmetry idea in explicit cosmological form: ordered structure is neither left to settle into a rigid minimum nor abandoned to unconstrained fluctuation, but is continually driven within a curved expanding background by a balance of drift and stochastic forcing.

Phase behaviour

Even in this reduced model, three broad regimes can be distinguished. If the stochastic forcing is too strong relative to the restoring drift, the order field is pushed erratically across configuration space and no stable

large-scale structure is maintained. If the stochastic forcing is too weak and the penalty term too strong, the field collapses too tightly around a minimum and loses adaptive responsiveness. Between these extremes lies a regime in which the field fluctuates within a controlled band around a preferred baseline. That intermediate regime is the de Sitter analogue of dynamic symmetry.

In this picture, the edge of chaos is not a single point but a probability band. The field does not remain fixed at one value; rather, its stationary distribution is concentrated in a structured region of configuration space where coherence and variability coexist. The width of that band is controlled by the competition between the Hubble scale H , the noise strength γ , and the edge-penalty parameter β .

Effective vacuum energy

The de Sitter background also makes it natural to revisit the vacuum-energy question in a reduced setting. Because the stochastic field drives the order field through a stationary probability distribution rather than a single static configuration, the relevant object is not simply $V(\phi)$ evaluated at one minimum, but an averaged quantity such as $\langle V(\phi) \rangle$, together with the mean contribution of the interaction sector.

If those expectation values remain approximately homogeneous in the long-time regime, they act like an effective vacuum energy density for the toy model. This does not solve the cosmological-constant problem, but it gives a concrete framework in which the interaction between ordered structure and constrained fluctuation may contribute to a curvature-supporting energy scale without being inserted purely by hand.

Scope

This appendix remains highly idealised. It assumes spatial homogeneity, suppresses the detailed gauge dynamics, and treats stochasticity in an effective rather than first-principles manner. Even so, it serves an important role: it shows that the curved-spacetime toy effective field theory can be specialised to a recognisable cosmological background and reduced to equations that are simple enough to analyse numerically or qualitatively.

As such, the appendix does not attempt to complete the theory. It demonstrates that the Geometry framework can be pushed one step further into explicit dynamical form, and that the language of dynamic symmetry remains meaningful when transferred from an abstract covariant Lagrangian to an expanding de Sitter universe.