Assessing K-8 Preservice Teachers' Numeracy Knowledge in a Foundations of Numbers and Operations Course

Tuyin An, Gregory Chamblee, Ha Nguyen, & Logan Delgado

Georgia Southern University

Background

- The Georgia Numeracy Project (GNP) is a free, optional, evidence-based resource provided by the Georgia Department of Education and used by K-8 schools and districts to help assess students and provide resources to support the building of students' number sense knowledge and skills.

- Based on the New Zealand Numeracy Project materials (https://nzmaths.co.nz/what-numeracy-project), the New Zealand Numeracy Project aims to develop students' understanding of numbers and their ability to use numbers to solve problems (number sense).

- Number sense refers to “a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations" (Mcintosh et al., 1992, p. 3).

- Number sense allows students to make connections among mathematical relationships, principles, and procedures, and is considered as a foundation of formal mathematics learning such as algebra (Gersten et al., 2005; Howden, 1989; National Mathematics Advisory Panel, 2008).

- The *Principles and Standards for School Mathematics (PSSM)* posits all students need to "understand numbers, ways of representing numbers, relationships among numbers, and number systems; understand meanings of operations; and compute fluently and make reasonable estimates" (National Council of Teachers of Mathematics, 2000, p. 32).

- The *Standards for Preparing Teachers of Mathematics (SPTM)* posit K-8 pre-service teachers (PSTs) need to gain a deep understanding of K-8 mathematics including number sense related topics, such as counting and cardinality, number and operations in base ten, multiplicative structures, fractions and decimals, and the number system (Association of Mathematics Teacher Educators, 2017).

- Students and teachers have encountered challenges in learning and teaching number sense knowledge and skills across nations.
  - An international comparison study shows K-8 students' consistent low performance on number sense items across four participating countries: Australia, Sweden, Taiwan, and the United States (Reys et al., 1999).
  - Research also shows that elementary PSTs in the United States possess limited number sense (Thanheiser et al., 2014; Whitacre & Rumsey, 2020), but it is possible to improve PSTs' number sense knowledge and skills through various teaching interventions (Whitacre, 2017; Whitacre & Nickerson, 2016; Yaman, 2015).

Research Question

The overarching purpose of our pilot study was to determine K-8 PSTs understanding of numbers and their ability to use numbers to solve problems who were enrolled in a mathematics course that focused on number and operation concepts. Our research question was:

What is K-8 PSTs' number sense content knowledge in a Foundations of Numbers and Operations course?

Methods - Research Setting

- Data were collected fall 2021 at a public, 4-year comprehensive university in the southeastern United States.

- PSTS were enrolled in a *Foundations of Numbers and Operations* course
  - Three semester hour course
  - First required course for students seeking teacher certification (Elementary, Middle Grades, Elementary/Special Education, Special Education)
  - Course content focused on: sets and logic, numeration systems, number theory, and operations

Methods - GNP Instruments

Uses two instruments to assess students:

- Global Strategy Stage (GloSS) for grades K-8
  - Individual assessment between teacher and student [Interview]

.

- Individual Knowledge Assessment of Number (IKAN) for K-8
  - Counting Interview or Written Instrument (8 stages of number knowledge assessed)
  - The Counting Interview assesses numeracy concepts students should master by Grade 2 (Stages 0 - 3)
  - Written Instrument: **For grades 4-8 [our project] (Stages 4-8)**

IKAN Written Instrument

- Assesses four content domains: Number Sequence and Order, Fractions, Place Value, and Basic Facts.
- Three of the four domains are divided into five stages (Stage 4 - 8). Number Sequence & Order domain does not have Stage 8.
- Each stage corresponds to a grade level.
- Each stage consists of eight questions [shown via a video].
- Each question is scored as correct/incorrect using an answer key.
- Domain stage score is determined as having correct answers for each stage domain question even if they answer questions in higher stages correctly.
- For example, if a PST missed one question in Fractions Stage 5 then the PST is classified as a Stage 4 learner.

Methods - Data Collection and Analysis

- Data were collected using the IKAN Written instrument fall 2021 in five sections of Numbers and Foundations course.
- Pre-test data were collected during one of the first four class meetings using the New Zealand Project IKAN 1 video.
- Post-test data were collected during one of the final two class meetings using the New Zealand Project IKAN 2 video.
- The pre- and post-test data were compared. Changes in stages of PSTs' number sense knowledge domains were identified.

Findings

- 129 participants of the total 161 participants completed IKAN 1 (80% response rate).
- 77 participants completed the IKAN 2 at the end of their course completion (48% response rate.
- There were 66 participants (41%) with paired data.
- One reason for the low participation rates is participants consenting “no” to participate in the study and thus, analysis was not conducted for these participants.

Pre-Test Count (count/129)

Domain	Pre-Stage 4	Stage 4	Stage 5	Stage 6	Stage 7	Stage 8
Number Sequence and Order	4 (3%)	3 (2%)	58 (45%)	4 (3%)	60 (47%)	N/A
Fractions	5 (4%)	76 (59%)	18 (14%)	18 (14%)	4 (3%)	8 (6%)
Place Value	17 (13%)	74 (57%)	30 (23%)	1 (1%)	2 (2%)	5 (4%)
Basic Facts	12 (9%)	15 (12%)	15 (12%)	31 (24%)	54 (42%)	2 (2%)

Post-Test Count (count/77)

Domain	Pre-Stage 4	Stage 4	Stage 5	Stage 6	Stage 7	Stage 8
Number Sequence and Order	1 (1%)	0	15 (19%)	18 (25%)	42 (55%)	N/A
Fractions	6 (8%)	12 (16%)	25 (32%)	21 (27%)	7 (9%)	6 (8%)
Place Value	6 (8%)	38 (49%)	24 (31%)	0	4 (5%)	5 (6%)
Basic Facts	6 (8%)	8 (10%)	10 (13%)	14 (18%)	37 (48%)	2 (3%)

IKAN Number Sequence and Order Stage Change (N = 66)

Pre/Post	Pre-Stage 4	Stage 4	Stage 5	Stage 6	Stage 7
Pre-Stage 4 (N = 2)	0 (0%)	0 (0%)	1 (1.5%)	1 (1.5%)	0 (0%)
Stage 4 (N = 1)	0 (0%)	0 (0%)	1(2%)	0 (0%)	0 (0%)
Stage 5 (N = 26)	0 (0%)	0 (0%)	6 (9%)	8(12%)	11 (18%)
Stage 6 (N = 3)	0 (0%)	0 (0%)	0 (0%)	1 (1.5%)	2 (3%)
Stage 7 (N = 34)	1 (1.5%)	0 (0%)	2 (3%)	6 (9%)	25 (38%)

IKAN Fractions Stage Change (N = 66)

Pre/Post	Pre-Stage 4	Stage 4	Stage 5	Stage 6	Stage 7	Stage 8
Pre-Stage 4 (N = 1)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	1 (1.5%)
Stage 4 (N = 42)	4 (6%)	6 (9%)	17 (26%)	8 (12%)	5 (8%)	2 (3%)
Stage 5 (N = 8)	0 (0%)	1 (1.5%)	3 (5%)	3 (5%)	0 (0%)	1 (1.5%)
Stage 6 (N = 10)	0 (0%)	2 (3%)	0 (0%)	4 (6%)	2 (3%)	2 (3%)
Stage 7 (N = 2)	0 (0%)	0 (0%)	1 (1.5%)	1 (1.5%)	0 (0%)	0 (0%)
Stage 8 (N = 3)	1 (1.5%)	0 (0%)	0 (0%)	2 (3%)	0 (0%)	0 (0%)

IKAN Place Value Stage Change (N = 66)

Pre/Post	Pre-Stage 4	Stage 4	Stage 5	Stage 6	Stage 7	Stage 8
Pre-Stage 4 (N = 7)	2 (3%)	1 (1.5%)	4 (6%)	0 (0%)	0 (0%)	0 (0%)
Stage 4 (N = 38)	4 (6%)	22 (33%)	10 (15%)	0 (0%)	2 (3%)	0 (0%)
Stage 5 (N = 16)	0 (0%)	4 (6%)	8 (12%)	0 (0%)	1 (1.5%)	3 (5%)
Stage 6 (N = 0)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)
Stage 7 (N = 2)	0 (0%)	1 (1.5%)	1 (1.5%)	0 (0%)	0 (0%)	0 (0%)
Stage 8 (N = 3)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	1 (2%)	2 (3%)

IKAN Basic Facts Stage Change (N = 66)

Pre/Post	Pre-Stage 4	Stage 4	Stage 5	Stage 6	Stage 7	Stage 8
Pre-Stage 4 (N = 7)	2 (3%)	1 (1.5%)	1 (1.5%)	1 (1.5%)	2 (3%)	0 (0%)
Stage 4 (N = 7)	1 (1.5%)	1 (1.5%)	2 (3%)	0 (0%)	3 (5%)	0 (0%)
Stage 5 (N = 6)	0 (0%)	0 (0%)	3 (5%)	3 (5%)	0 (0%)	0 (0%)
Stage 6 (N = 15)	1 (1.5%)	3 (5%)	1 (1.5%)	2 (3%)	8 (12%)	0 (0%)
Stage 7 (N = 30)	0 (0%)	1 (2%)	2 (3%)	4 (6%)	22 (33%)	1(1.5%)
Stage 8 (N = 1)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)	1(1.5%)

Discussion

- PSTs highest pre-test domain stage scores were Basic Facts (Stage 7) [42%] and Number Sequence and Order (Stage 7) [47%]. [e.g., 6 x 7 = ?; What number is one more 439 999?]
- PSTs lowest pre-test domain stage scores were Fractions (Stage 4) [59%] and Place Value (Stage 4) [57%]. These domains required PSTs to use number sense ideas to determine relationships between or among values. [Write 4 and 1/5 as a fraction & How many tenths are in all of the number, 5.8?]
- PSTs highest post-test domain stage scores continued to be Basic Facts (Stage 7) [48%] and Number Sequence and Order (Stage 7) [55%].
- PSTs lowest post-test domain stage score was Place Value (Stage 4) [49%] and Fraction (Stage 5) [32%].
- PSTs domain stage scores overall did increase especially those at lower stages. However, few students moved to the highest stage domain scores.

Discussion

- Most of Pre-Stage 4 PSTs progressed to higher level stages. Only a small portion of PSTs stayed at Pre-Stage 4 level (2 out of 7 PSTs in Basic Facts and Place Value, respectively).
- Most of Stage 4 PSTs progressed to higher level stages, except for Place Value, in which 22 out of 38 PSTs stayed in Stage 4 and 4 out of 38 PSTs dropped to Pre-Stage 4.
- Most PSTs at Stages 5 - 8 progressed one or more stages, but some stayed or dropped to a lower stage. For example,
    - Number Sequence and Order: Only 25 out of 34 PSTs stayed at Stage 7.
    - Fractions: All Stage 7 and Stage 8 dropped to lower stages.

Recommendations/Future Directions

- This pilot data does support Yamen's (2015) finding that a content course can impact PSTs number sense.
- Additional GNP data needs to be collected to verify results (GLoSS).
- PSTs content courses do need to focus on computation/algorithm knowledge/fluency [not all PSTS at highest Basic Facts domain stage]. These courses also need to expand how PSTS gain a deep understanding of K-8 mathematics (AMTE, 2017).
- Additional research is needed to verify a timed test is a valid and reliable measurement of numeracy (number sense) knowledge as defined by GNP for PSTS and K-8 students in general.
- Since place value remained as the lowest-score content knowledge domain across the pre/post tests, research targeting the relevant instructional designs and/or teaching and learning strategies is needed.

Contact Information

Tuyin An	tan@georgiasouthern.edu
Gregory Chamblee	gchamblee@georgiasouthern.edu
Ha Nguyen	hnguyen@georgiasouthern.edu
Logan Delgado	ld12086@georgiasouthern.edu

References

Association of Mathematics Teacher Educators. (2017). *Standards for preparing teachers of mathematics.* https://amte.net/sites/default/files/SPTM.pdf

Georgia Department of Education, & Woods, R. (2019). *Developing foundational numeracy using THE GEORGIA NUMERACY PROJECT effectively assessing numeracy to address students' needs for acceleration, remediation & intervention*. https://www.gadoe.org/Curriculum-Instruction-and-Assessment/Curriculum-and-Instruction/Documents/Fall%20Curriculum%20Leaders'/Developing_Foundational_Numeracy_Using_GA%20Numeracy_Project.pdf#search=numeracy

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities*, *38*(4), 293–304.

Howden, H. (1989). Teaching number sense. *Arithmetic Teacher*, *36*(6), 6–11. https://eric.ed.gov/?id=EJ389479

Mcintosh, A., Reys, B. J., & Reys, R. E. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics*, *12*(3), 2–44.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. National Council of Teachers of Mathematics.

National Mathematics Advisory Panel. (2008). *Foundations for success:The final report of the National Mathematics Advisory Panel*. Washing-ton, DC: U.S. Department of Education.

Reys, R., Reys, B., Emanuelsson, G., Johansson, B., McIntosh, A., & Yang, D. C. (1999). Assessing number sense of students in Australia, Sweden, Taiwan, and the United States. *School Science and Mathematics, 99*(2), 61–70. https://doi.org/10.1111/j.1949-8594.1999.tb17449.x

Thanheiser, E., Browning, C., Edson, A. J., Lo, J.-J., Whitacre, I., Olanoff, D., & Morton, C. (2014). Prospective elementary mathematics teacher content knowledge: What do we know, what do we not know, and where do we go? *The Mathematics Enthusiast, 11*(2), 433–448.

Whitacre, I. (2018). Prospective elementary teachers learning to reason flexibly with sums and differences: Number sense development viewed through the lens of collective activity. *Cognition and Instruction, 36*(1), 56–82.

Whitacre, I., & Nickerson, S. D. (2016). Investigating the improvement of prospective elementary teachers' number sense in reasoning about fraction magnitude. *Journal of Mathematics Teacher Education, 19*(1), 57–77.

Whitacre, I., & Rumsey, C. (2020). The roles of tools and models in a prospective elementary teachers' developing understanding of multidigit multiplication. *The Journal of Mathematical Behavior, 60*, 100816.

Yaman, H. (2015). The mathematics education I and II courses' effect on teacher candidates' development of number sense. *Educational Sciences: Theory & Practice*, *15*(4). https://doi.org/10.12738/estp.2015.4.2322