Common Errors In Preservice Middle-Grades Mathematics Teachers' Proof Writing

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Background

Reasoning and proof are important to mathematics (e.g., NCTM, 2000, 2020), but there are gaps in K-12 and teacher preparation standards (NGA & CCSSO, 2010; AMTE, 2017).

Most K-12 research on reasoning and proof has been conducted at the high school level (Herbst, 2002).

Research shows that middle school students are poorly prepared to engage with high school-level reasoning and proving (Mansi, 2003).

Teacher education programs for elementary and middle grades mathematics teachers provide inconsistent opportunities to learn about and engage in reasoning and proving (Buchbinder & McCrone, 2020).

This leads to inadequacies in preservice teachers' understanding of reasoning and proof, as well as an inability to identify the importance of proof within mathematics (Oflaz et al., 2016)

Background

Practicing teachers can also experience difficulty with reasoning and proving (Bieda, 2010):

- Insufficient feedback to students to sustain discussions about conjectures and/or justifications
- Using empirically justified reasoning instead of deductive reasoning

There is little research that examines preservice middle-grades mathematics teachers’ (PMMTs’) errors in deductive reasoning and formal proof writing.

Research question:

What are the common errors in PMMTs’ proof writing in a geometry content course of PMMTs?

Theoretical Perspective

We use Duval's (2007) framework of the cognitive functioning of proof to

a) Deconstruct and examine PMMTs' proofs, and
b) Identify and interpret PMMTs' difficulties with deductive reasoning

In this framework, deductive reasoning involves two different levels of discursive organization

a) Organizing several propositions in one deductive step, and
b) Organizing several steps into a proof

Each proposition has three dimensions

a) Semantic (content),
b) Knowledge (epistemic value), and
c) Logic (true or false)

Methods - Site & Participants

The study was conducted at a large public doctoral university with high undergraduate enrollment in the Southeastern United States.

Participants were 20 PMMTs in a geometry course for PMMTs.

The course is a required mathematics content course for preservice K-8 mathematics teachers.

Two prerequisite course: Number & Operations, and Data & Geometry

Textbook: Musser et al. (2008)

One course/textbook goal: “Expose students to the axiomatic method of synthetic Euclidean geometry.”

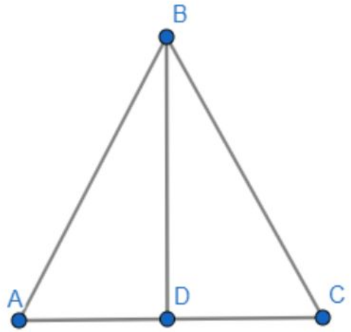
Methods - Data Collection

Data includes PMMTs' solutions of two geometry proof problems, Q12 and Q19.

Q12. In △ABC, $\overline{BD}$ is an altitude of △ABC and also the bisector of ∠B. Prove that $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$.

Q19. Given △ABC, where $\overline{BD}$ is an altitude and AD = CD. Prove △ADB ≅ △CDB.

(The figure below works for both problems.)



Methods - Data Analysis

The error taxonomy developing process includes iterative alternations of inductive and deductive analyses.

- The research team first analyzed the Q19 data set individually (for creating initial categories) and collaboratively (for resolving discrepancies) for multiple rounds to develop an initial coding system.
- The initial coding system was then used to analyze the second set of data (Q12) following the same individual-collaborative working process.

Basic quantitative analysis was performed to understand the proportion and frequency of the errors in PMMTs' proof writing.

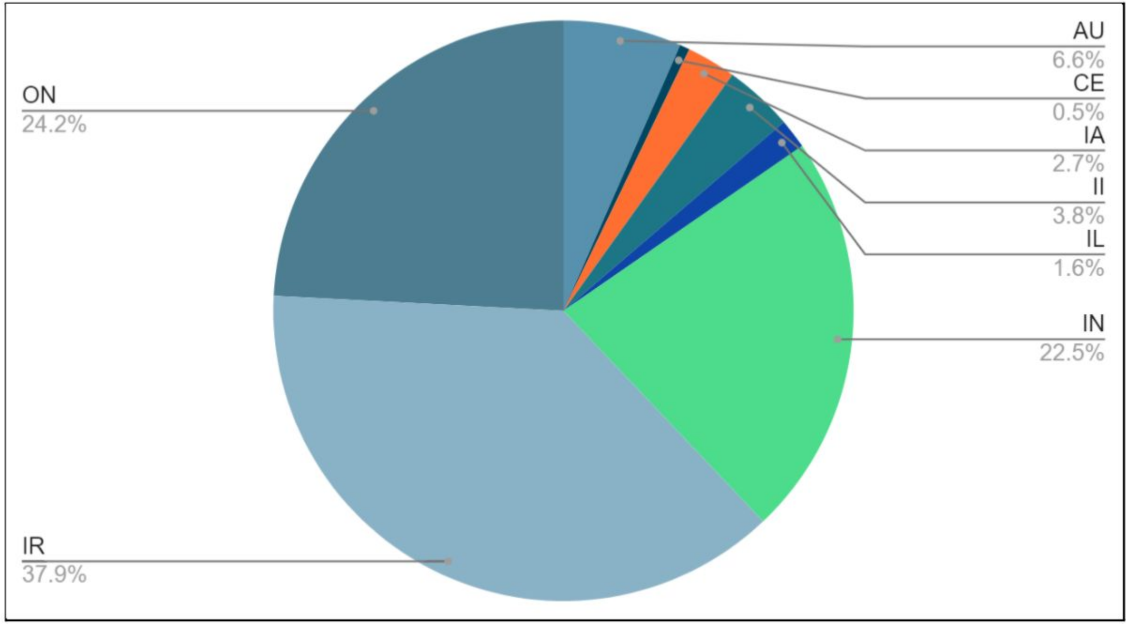
Findings

A total of eight error categories are identified from PMMT's proof writing, one of which contains four subcategories.

1. Adding Unnecessary Proof Steps (AU)
2. Completely Empty (CE)
3. Incorrect Assumptions (IA)
4. Informal/Incomplete Language (II)
5. Incomprehensible Language (IL)
6. Incorrect Notations (IN)
7. Incorrect Reasons (IR)
    a. Circular Reasoning (IR-CR)
    b. Missing the Reason (IR-MR)
    c. Misapplying Theorems/Axioms/Definitions (IR-MT)
    d. Only Listing Given/Proved Conditions as the Reason (IR-OL)
8. Omission of Necessary Proof Steps (ON)

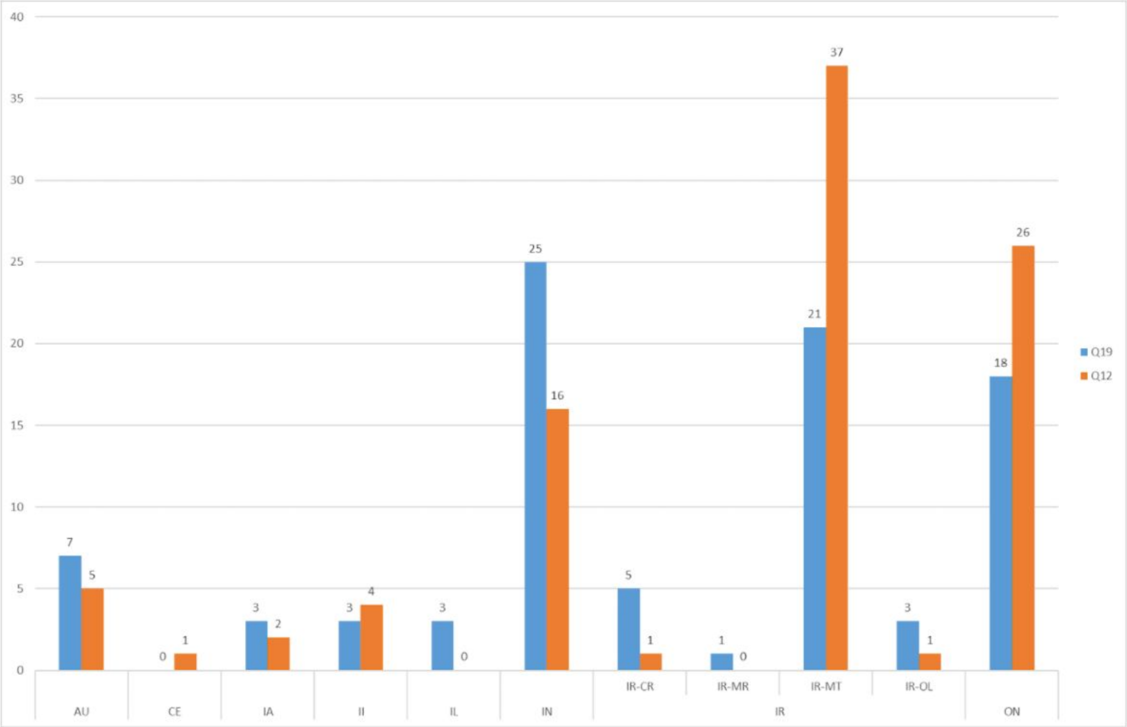
Findings

The pie chart below shows the percentages of all eight categories of errors identified in the two problems.



Findings

Comparison of categories of the two problems shows similar patterns.



Discussion

Two levels of challenges in PMMTs' proof construction according to Duval's (2007) framework of cognitive functioning of proof.

First-level: organizing the given/proved conditions and applicable theorems/axioms/definitions within one deductive step (e.g., *IA, IR-MR, IR-MT,* and *IR-OL*).

Second-level: linking deductive steps into a proof (e.g., *AU, IR-CR, ON*).

Three-stage teaching strategy suggested by Duval (2007).

Another challenge is the lack of rigor in PMMTs' use of mathematical language and notations (e.g., *II, IL,* and *IN*).

Lack of emphasis on deductive reasoning and axiomatic methods in middle-level curriculum and teacher preparation standards (AMTE, 2017; CBMS, 2012; NCTM, 2020).

Limitations: limited number and context of the problems that might cause limited scope of the emerged error categories, possible misinterpretation of errors.

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