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Preservice Mathematics Teachers' Learning of Geometric Sequence with Applications in Finance

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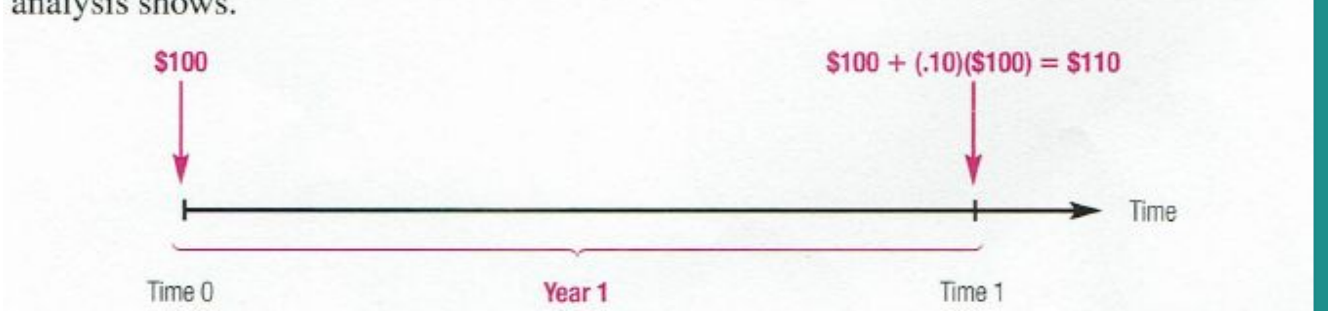
Geometric Sequence and Compound Interest

- *Geometric Sequence* is a number sequence in which the ratio of every pair of successive terms is constant. The constant ratio is called the *common ratio (r)*.
  - This is a geometric sequence with r = 3.

  - We usually use a set of notations to represent a generic sequence: $a_1, a_2, a_3, a_4, a_5, \dots, a_n, \dots$ Any term in the sequence can be found using the *explicit formula* below:

$$a_n = a_1 * r^{n-1}$$

- The concept of the Geometric Sequence is applied widely in everyday life, including in finance and accounting. The most common example is *Compound Interest* (Hilton & Platt, 2014).
  - **Compound Interest** Suppose you invest $100 today (time 0) at 10 percent interest for one year. How much will you have after one year? The answer is $110, as the following analysis shows.

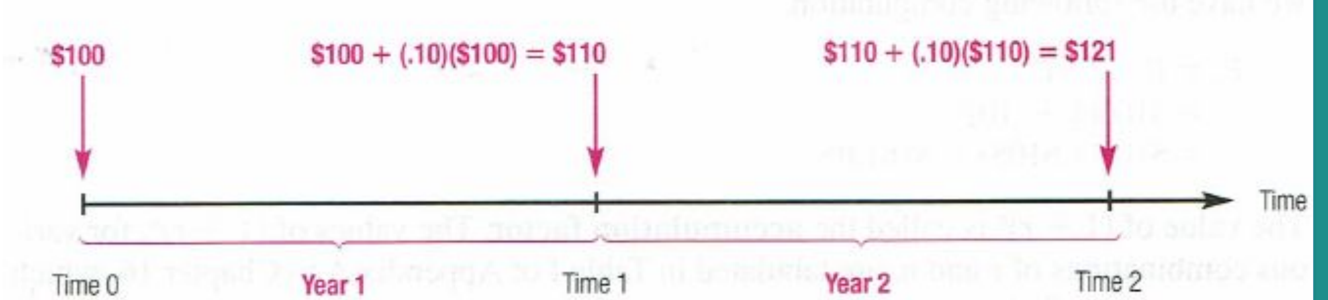


The $110 at time 1 (end of one year) is composed of two parts, as shown below.

Principal, time 0 amount	$100
Interest earned during year 1 (.10 × $100)	10
Amount at time 1	$110

Thus, the $110 at time 1 consists of the $100 at time 0, called the **principal,** plus the $10 of interest earned during the year.

Now suppose you leave your $110 invested during the second year. How much will you have at the end of two years? As the following analysis shows, the answer is $121.



We can break down the $121 at time 2 into two parts as follows:

Amount at time 1	$110
Interest earned during year 2 (.10 × $110)	11
Amount at time 2	$121

Interest earned on prior periods' interest is called **compound interest**.

- An efficient way to find the investment value in a future year is to view the yearly investment values as terms of a geometric sequence and apply its explicit formula.
  - $$F_n = P(1 + r)^n \quad (1)$$

    where *P* denotes principal
    r denotes interest rate per year
    n denotes number of years

    Using formula (1) to compute the future value after five years of your $100 investment, we have the following computation.

    $$F_n = P(1 + r)^n = \$100(1 + .10)^5 = \$100(1.6105) = \$161.05$$

  - Note: Students need to be aware of the different meanings of "r" and "n" used in the explicit formula and the compound interest formula, respectively.

K-8 Preservice Mathematics Teachers' Learning of Sequence

- Mathematical sequences play an important role in K-8 preservice mathematics teachers' (PMTs) mathematical content knowledge (AMTE, 2017), and learning of sequences lays a foundation for PMTs' learning of functions.
- PMTs often are challenged by real world applications of sequence.
  - E.g., converting real-world contexts to mathematical models, differentiating the geometric sequence and arithmetic sequence, and applying the explicit formula of the geometric sequence.
- Compound Interest serves perfectly as a real-world context for exploring geometric sequence problems (matching mathematical concept, practical meaning to PMTs' life).
- Integrating technology and creating a classroom environment that is inquiry-based and real-world solution-driven is crucial to engage students and foster genuine learning (Wang, Kinzie, McGuire, & Pan, 2010).
  - A free interactive online tool, GeoGebra, was incorporated in the task design of the lesson to create an equitable and engaging learning environment.
  - Example task: https://www.geogebra.org/m/gvdn5bkz

Research Goal and Design

- Include one of the most commonly applied mathematical models in finance and accounting, Compound Interest, in the task design of a mathematics content course for PMTs and examine the potential effectiveness of such instructional design.
- Instructional design that supports PMTs' learning of geometric sequence:
  - Inquiry-based real-world applications
  - Interactive online tool
  - Active learning activity: group presentations
- Treatment- and control-group design to examine the effectiveness of the instruction
  - Solutions of one compound interest type of problem from each group.

Findings

- Correctness rate of PMTs' solutions
  - Control Group (N=58): 50%
  - Treatment Group (N=51): 47%
- Percentage of successful use of the geometric sequence explicit formula (out of the total correct solutions)
  - Control Group: 21%
  - Treatment Group: 71%
- Percentage of attempts of using the geometric sequence explicit formula (out of the total incorrect solutions)
  - Control Group: 14%
  - Treatment Group: 44%

Discussion

- Overall the correctness rates are close. The slight drop in the treatment group could be caused by the change of teaching mode of the course (from face-to-face to hybrid) due to the Covid pandemic.
- Far higher percentage of PMTs in the treatment group successfuly applied (and attempted) the geometric sequence explicit formula, compared to the control group, which indicates the effectiveness of the designed instructionl activites.
- Limitations: PMTs' varied entry-level knowledge, changes in teaching and learning caused by the pandemic.

References

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