ONLINE CLASSES

Classical Mechanics

Lecture 1

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DATED:- 12TH OCTOBER, 2021

TIME:- 7:30PM TO 10:30PM

A	20 (Questions to be attempted: 15)	30
B	25 (Questions to be attempted: 20)	70
C	30 (Questions to be attempted: 20)	100
Total	75 (Questions to be attempted: 55)	200

278.5 marks

CSIR NET Physical Science Paper Parts	Marks Allotted for Each Question	Marks Deducted for Each Question
A	+2	-0.5
B	+3.5	-0.875
C	+5	-1.25

ALSO GATE TIFR,IIT JAM

Syllabus

II. Classical Mechanics

Newton's laws. Dynamical systems, Phase space dynamics, stability analysis. Central force motions. Two body Collisions - scattering in laboratory and Centre of mass frames. Rigid body dynamics-moment of inertia tensor. Non-inertial frames and pseudoforces. Variational principle. Generalized coordinates. Lagrangian and Hamiltonian formalism and equations of motion. Conservation laws and cyclic coordinates. Periodic motion: small oscillations, normal modes. Special theory of relativity-Lorentz transformations, relativistic kinematics and mass–energy equivalence.

II. Classical Mechanics

Dynamical systems, Phase space dynamics, stability analysis. Poisson brackets and canonical transformations. Symmetry, invariance and Noether's theorem. Hamilton-Jacobi theory.

Reference Books

1. Concept of Physics (Vol I): **H.C. Verma** (12th Stnd.)
2. An Introduction to Mechanics: **D. Kleppner and R. Kolenkow**
3. Classical Mechanics: **J.C. Upadhyay** / Gupta - Kumar
4. Classical Dynamics of Particles and Systems: **Thornton and Marion**, (Cengage Learning, Singapore, 2004)

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1- Copy
2- Pen
3- Material

Old year papers (NET, JEST, GATE, TIFR)

Assignments.

Let's Start

Picture is always move around problems

The true test of understanding something

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Strategies for solving problems

- ❑ Draw a diagram, if appropriate
  - ❑ Write what you know, and what you are trying to find
    - ❑ Solve things symbolically
      - ❑ Check units/dimensions
        - ❑ Check limiting/special cases
          - ❑ Check order of magnitude

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Units, dimensional analysis

It won't tell you that
your answer is definitely correct,
But
It might tell you that
your answer is definitely incorrect.

Let us discuss some problems

If the task of a given problem is to find a certain time, which of the following quantities could be the answer?

$\downarrow [T]$

~~(a)~~ a/t ~~(b)~~ mv/l ~~(c)~~ v^2/a (d) $\sqrt{l/a}$ ✓

~~(e)~~ $\sqrt{v/a}$

Sol^n

$$\left[\frac{a}{t}\right] = \frac{LT^{-2}}{T} = [LT^{-3}]$$

$$\left[\frac{mv}{l}\right] = \left[\frac{MLT^{-1}}{L}\right] = [MT^{-1}]$$

$$\left[\frac{v^2}{a}\right] = \left(\frac{L^2T^{-2}}{LT^{-2}}\right) = [L]$$

$$\left[\sqrt{\frac{l}{a}}\right] = \left[\frac{L}{LT^{-2}}\right]^{1/2} = [T] \checkmark$$

$$\left[\sqrt{\frac{v}{a}}\right] = \left[\frac{LT^{-1}}{LT^{-2}}\right]^{1/2} = [\sqrt{T}]$$

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~~Prob~~ How does the speed of waves in a fluid depend on its density, ρ, and "bulk modulus," B (which has units of pressure, which is force per area) ?

<u>Sol^n</u>

$$v \propto \rho^{a} \; B^{b}$$

$$[LT^{-1}] = \left[\frac{M}{L^3}\right]^a \left[\frac{M\cancel{L}T^{-2}}{L^{\cancel{2}}}\right]^b$$

Compare powers,

$$a + b = 0 \qquad a = -\tfrac{1}{2}$$

$$-3a - b = 1$$

$$-2b = -1 \Rightarrow b = \frac{1}{2}$$

$$v \propto \rho^{-1/2} B^{1/2} \Rightarrow \boxed{v \propto \sqrt{\frac{B}{\rho}}} \text{ or } \boxed{v = K\sqrt{\frac{B}{\rho}}}$$

where K is dim. constant.

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Prob. The van der Waals' equation of state for a gas is given by

$$\left(P+\frac{a}{V^2}\right)(V-b)=RT$$

where P, V and T represent the pressure, volume and temperature respectively, and a and b are constant parameters. At the critical point, where all the roots of the above cubic equation are degenerate, the volume is given by (CSIR-NET 2014)

Thermal Physics

(a) $a/9b$ (b) $a/27b^2$ (c) $8a/27bR$ (d) $3b$

Sol^n:

$$[P] = \left[\frac{a}{V^2}\right]$$ Here [ ] represents Dim. Fm.

$$[a] = [PV^2] = [ML^{-1}T^{-2}.L^6]$$

$$[b] = [V] = [L^3]$$

Prob Using dimensional analysis, Planck defined a characteristic temperature T_P from powers of the gravitational constant G, Planck's constant h, Boltzmann constant k_B and the speed of light c in vacuum. The expression for T_P is proportional to

(a) $\sqrt{\frac{hc^5}{k_B^2 G}}$ ✓ (b) $\sqrt{\frac{hc^3}{k_B^2 G}}$ (c) $\sqrt{\frac{G}{hc^4 k_B^2}}$ (d) $\sqrt{\frac{hk_B^2}{Gc^3}}$

Sol^n:

$$T_P \propto G^{\alpha} h^{\beta} k_B^{\gamma} c^{\delta}$$

$$[\theta] = [\quad]^{\alpha} [\quad]^{\beta} [\quad]^{\gamma} [\quad]^{\delta}$$

$F = G\frac{m_1 m_2}{r^2}$

$E = h\nu$

$E = \frac{3}{2} k_B T$

Compare powers,

$$\alpha = -\frac{1}{2}, \beta = \frac{1}{2}, \gamma = -1, \delta = \frac{5}{2}$$

$$\sqrt{\frac{h\,c^5}{K_B^2\,G}} = \sqrt{\frac{E\cdot c^5}{\nu\cdot\frac{E^2}{T^2}\cdot\frac{F r^2}{m^2}}} = \sqrt{\frac{t\cdot c^5\cdot m^2}{E\cdot F r^2}} \quad T$$

$$= \sqrt{\frac{t\,c^5\,m^2}{F^2\,r^3}}$$

Prob

The pressure P of a system of N particles contained in a volume V at a temperature T is given by $P = nk_BT - \frac{1}{2}an^2 + \frac{1}{6}bn^3$, where n is the number density and a and b are temperature independent constants. If the system exhibits a gas – liquid transition, the critical temperature is

(Thermodynamics)

1. $\frac{a}{bk_B}$ ✗
2. $\frac{a}{2b^2k_B}$ ✗
3. $\frac{a^2}{2bk_B}$ ✓
4. $\frac{a^2}{b^2k_B}$ ✗

Sol^n:-

$$P = nk_BT - \frac{1}{2}an^2 + \frac{1}{6}bn^3$$

$$[n] = [L^{-3}] = \left[\frac{1}{V}\right]$$

$$[a] = \left[\frac{P}{n^2}\right] = [PV^2]$$

$$[b] = \left[\frac{P}{n^3}\right] = [PV^3]$$

$$\left[\frac{a^2}{b}\right] = \left[\frac{P^2V^4}{PV^3}\right] = [PV] = [E]$$

$$\left[\frac{a^2}{b}\right] = E$$

$$\frac{\left[\frac{a^2}{b}\right]}{[K_B]} = \left[\frac{E}{K_B}\right] = [T]$$

option (c) is correct.

Drawbacks of dimensional analysis

It won't tell you that

your answer is definitely correct,

But

It might tell you that

your answer is definitely incorrect.

Checking units won't tell you that your answer is definitely CORRECT,

Drawbacks of dimensional analysis

- It fails while using it to derive a relation among physical quantities, if there are more than 3 unknown variables on which a given physical quantity depends
- It does not tell whether a given Physical quantity is a scalar or a vector.
- It does not tell us the value of constants involved
- It does not always tell us the exact FORM of a relation
- It cannot be used for deriving logarithmic, trigonometric or exponential relations $y = a \sin(\omega t - kx)$
- A dimensionally correct equation may not always be the correct relation. (Because there are more than one physical quantity having the same dimensions)

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Check limiting/special cases making approximations

- If you're having trouble figuring out how a given system/problem behaves, then you can figure out
- Having convinced yourself
- Modifying the various parameters and observing the effects on the system can lead to an enormous amount of information

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Let's see some examples

Prob

A particle of mass m moves in a central potential $V(\mathrm{r}) = -\frac{k}{r}$ in an elliptic orbit $r(\theta) = \frac{a(1-e^2)}{1+e\cos\theta'}$ where $0 \le \theta < 2\pi$ and a and e denote the semi – major axis and eccentricity, respectively. If its total energy is $E = -\frac{k}{2a}$, the maximum kinetic energy is

1. $E(1-e^2)$ 2. $E\frac{(e+1)}{(e-1)}$ 3. $E/(1-e^2)$ 4. $E\frac{(1-e)}{(1+e)}$

Soln:

$$\left.\begin{array}{l} E = -ve \\ 0 < e < 1 \end{array}\right\}$$

K.E. = +ve

$e-1 = -ve$

$E = -ve$

$$\boxed{E\left(\frac{e+1}{e-1}\right) = +ve}$$

A body of mass m falls from rest at a height 'h' under gravity (acceleration due to gravity g) through a dense medium which provides a resistive force $F = -kv^2$, where k is a constant and v is the speed. It will hit the ground with a kinetic energy

(a) $\frac{m^2 g}{2k}\exp\left(-\frac{2kh}{m}\right)$ (b) $\frac{m^2 g}{2k}\tanh\frac{2kh}{m}$

(c) $\frac{m^2 g}{2k}\left\{1+\exp\left(-\frac{2kh}{m}\right)\right\}$ (d) $\frac{m^2 g}{2k}\left\{1-\exp\left(-\frac{2kh}{m}\right)\right\}$

Sol^n if $K \to 0$, $K.E. \to mgh$

(c) $\frac{m^2 g}{2k}\cdot\left[1+1-\frac{2kh}{m}\right]$

$\left(e^x = 1+x+\frac{x^2}{2!}\right)$

d) $\frac{m^{\cancel{2}} g}{\cancel{2k}}\cdot\left(\cancel{1}-\cancel{1}+\cancel{2}\frac{\cancel{k}h}{\cancel{m}}\right) = mgh$

Prob

A small raindrop of mass m experiences a viscous drag force $F_d = bv$, proportional to its instantaneous speed 'v'. If it starts from rest at a height h, its speed after a time 't' is

(a) $\upsilon(t) = \frac{mg}{b}\tanh\left(\frac{bt}{m}\right)$

(b) $\upsilon(t) = \frac{mg}{b}e^{-bt/m}$

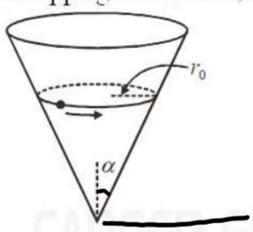
(c) $\upsilon(t) = \frac{mg}{2b}(1-e^{-2bt/m})$

(d) $\upsilon(t) = \frac{mg}{b}(1-e^{-bt/m})$

$\boxed{\text{Home - Work}}$

Prob A particle slides on the inside surface of a frictionless cone. The cone is fixed with its tip on the ground and its axis vertical, as shown in the figure on the right. The semi-vertex angle of the cone is α. If the particle moves in a circle of radius r_0, without slipping downwards, the angular frequency ω of this motion will be

(JEST)



$\tan 0 = 0$

$\cot 0 = \infty$

$\tan 90 = \infty$

~~(a)~~ $\sqrt{\dfrac{g}{r_0 \cos\alpha}}$ ~~(b)~~ $\sqrt{\dfrac{g}{r_0 \sin\alpha}}$ ~~(c)~~ $\sqrt{\dfrac{g}{r_0 \cot\alpha}}$ ✓(d) $\sqrt{\dfrac{g}{r_0 \tan\alpha}}$

Sol^n: $\Rightarrow \alpha \to 0$ $\quad w$ should increase

$\Rightarrow \alpha \to 90^\circ$ $\quad w \to 0$

Option (d) is correct.

Prob A very long rod rotates about a pivot with a constant angular velocity ω. A bead is constrained to slide along the rod without friction. At time $t = 0$, the bead is at rest a distance d away from the pivot. Its distance $r(t)$ from the pivot at time t is

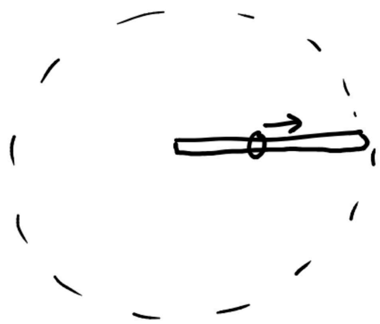
~~(a) $d\sinh(\omega t)$~~ ~~(b) $d\sin(\omega t)$~~ (c) $d\cosh(\omega t)$ ✓ ~~(d) $d\cos(\omega t)$~~

Sol^n

$t=0$, $v=0$, $r(0)=d$

$r(t) = \ldots$

as time passes, $r \uparrow$.



Thank you