ELECTRO DYNAMICS

PART 1

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SHORT NOTES

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Electromagnetic Theory :

★ Motion of charge particle in uniform Magnetic field :

When a charge particle is thrown into a uniform Magnetic field such that $\vec{v} \perp \vec{B}$, the path is circle.

Radius = $\boxed{r = \frac{mV}{qB}}$ or $\boxed{r = \frac{p}{qB}}$

Time of Revolution $\boxed{T = \frac{2\pi m}{qB}}$ (independent of speed)

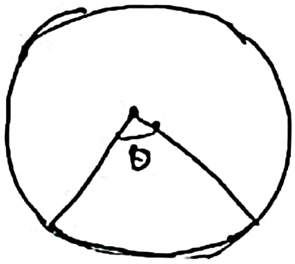
$V_x = V_0 \cos\left(\frac{qB}{m} \cdot t\right)$

$V_y = V_0 \sin\left(\frac{qB}{m} \cdot t\right)$

$V_t = \sqrt{V_x^2 + V_y^2} = V_0$

⇒ if path is arc of circle with θ angle subtended, then

$\boxed{t = \frac{\theta m}{qB}}$.



Case I : if $\vec{V} \parallel \vec{B}$ or $\vec{V} \parallel -\vec{B}$

∴ path is straight line & $\vec{v}$ = Constant.

Case II : $\vec{V}_i$ makes some Angle with $\vec{B}$.

∴ Resultant path is helical / (helix)

Radius of helix $\boxed{r = \frac{mV_\perp}{qB}}$

Pitch is dist moved ∥ to B during one revolution

$\boxed{l = V_{\parallel} T}$

Time of one Revolution : $\boxed{T = \frac{2\pi m}{qB}}$

If a charge particle is accelerated through Pot. diff V, Then gain in KE = qV

$\boxed{\text{gain in K.E} = |qV|}$ $\boxed{K = |qV|}$.

→ In helical path, if particle starts from a line then it Reaches Returns to that Line at distance l, $2l$, $3l$, ... , therefore if a target is kept on that Line then target will be hit by particle if distance of target is from Initial position of particle is l, $2l$, $3l$, ... etc.

★ **Motion of charge particle in uniform Electric field:**

(1) When a charge particle is thrown into a uniform Electric Field ($\vec{E}$) such that initial velocity v is not parallel or Antiparallel to $\vec{E}$, then path is parabola (Similar to projectile Near Earth)

(2) if $\vec{v} \parallel \vec{E}$ or $\vec{v} \parallel -\vec{E}$, then path is straight Line.

★ **Motion of charge particle in uniform Electric & uniform Magnetic field:**

→ When a charge particle is placed in uniform $\perp$ $\vec{E}$ & $\vec{B}$, then Path is cycloid and Motion takes place in plane defined by vectors $\vec{E}$ & $(\vec{E}\times\vec{B})$

eqn. of cycloid $$\boxed{\left(x-\frac{mE}{qB}\right)^2+\left(y+\frac{E}{B}t\right)^2=\left(\frac{mE}{qB^2}\right)^2}\quad [y = y(t)]$$

↳ Eqn of Moving Circle (cycloid)

here, Centre of circle is moving with speed $\frac{E}{B}$ in dirn $(\vec{E}\times\vec{B})$

with Centre $\left(\frac{mE}{qB^2} \quad -\frac{E}{B}t\right)$.

★ **Case I:** $\vec{E}\parallel\vec{B}$ or $\vec{E}\parallel-\vec{B}$: [& $\vec{v}_i = 0$]

∴ path is straight line

★ **Case II:** $\vec{v}_i$ makes some angle θ with fields ($\vec{E}\parallel B$ or $\vec{E}\parallel-\vec{B}$)

path is helical with Constant Radius & Varying pitch (l)

* If a particle is Moving in a Circle in uniform Magnetic field and $\vec{E}$ is applied ⊥ to the plane of Circle then path becomes helical

* If a particle is Moving in a Circle in a uniform Magnetic field and then a uniform Electric field is Applied ∥ to plane of Circle (ie $\vec{E} \perp \vec{B}$), then path will be a Cycloid.

* If there is $\vec{E}$ & $\vec{B}$ in a Region and particle moves undeviated, Net force must be Zero. ($F_{net}=0$)

$$\therefore \left[qE + q(\vec{v}\times\vec{B}) = 0\right]$$

Note: To Solve Any Question of Circle in Magnetic field, use property of Circle, (Do not use EOM)

Basic Concepts:

charge: $\boxed{Q = ne}$ $e = 1.6\times10^{-19}\,C$ | $n = 0, \pm1, \pm2, \ldots$

charge is always quantised for an Isolated System.

→ Total charge of a System is always Conserved.

charge Density:

(1) Linear charge Density:

$$\lambda = \frac{\text{charge}}{\text{Length}} = \frac{dQ}{dl} \quad \left[\text{q is distributed on line}\right]$$

(2) Surface charge Density

$$\sigma = \frac{\text{charge}}{\text{Area}} = \frac{dQ}{dA} \quad \left[\text{q is distributed on Surface}\right]$$

(3) Volume charge Density.

$$\rho = \frac{\text{charge}}{\text{Volume}} = \frac{dQ}{d\tau} \text{ or } \frac{dQ}{dV} \quad \left[\text{q is distributed on a Volume}\right]$$

If distribution of charge is uniform

$$\lambda = \frac{dQ}{dl} = \frac{Q}{l}$$

$$\sigma = \frac{dQ}{dA} = \frac{Q}{A}$$

$$\rho = \frac{dQ}{dV} = \frac{Q}{V}$$

★ **Current :** [Flow of charge]

Current through a point or through a cross section is defined as Rate of flow of charge through that point / Cross Section.

$$\boxed{I = \frac{dQ}{dt}}$$

★ **Current Density / Volume Current density :**

it is defined when a current flows through a Volume.

$$\boxed{J = \frac{dI}{dA_\perp}} \quad \text{or} \quad \boxed{I = \int \vec{J} \cdot d\vec{A}} \quad \text{or} \quad \left(I = \int \vec{J} \cdot d\vec{s} \right) \quad \boxed{dA = ds}$$

★ **Surface Current Density :**

It is defined when current flows on a surface

$$\boxed{K = \frac{dI}{dl_\perp}} \qquad (dl_\perp = \text{length} \perp \text{to flow of current})$$

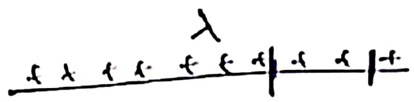
if distribution of current is uniform, then

$$\left[J = \frac{dI}{dA_\perp} = \frac{I}{A_\perp} \right] \quad \& \quad \left[K = \frac{dI}{dl_\perp} = \frac{I}{l_\perp} \right]$$

→ **Note :** Ratio is used in uniform case only.

★ **Relation b/w charge / charge density & Current / Current Density For a Moving charged object :**

(1) **Moving Line charge :**

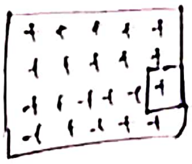


$$\therefore \quad \boxed{I = \lambda v}$$

it can be used in any case where line charge moves ∥ to length

(2) Surface charge Moving ∥ to its Surface:

$$\boxed{K = \sigma v} \quad \text{or} \quad \boxed{\vec{K} = \sigma \vec{v}}$$



This Can be used in Any Case Where Surface charge Moves parallel to its Surface.

(3) Moving Volume charge:

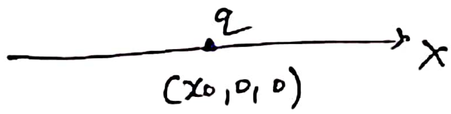
Similarly $\boxed{\vec{J} = \rho \vec{v}}$ → Can be used in Any Case Where Volume charge moves in any direction.

→ Imp) When a sphere (R) having uniform Surface charge density σ is Rotated about its diameter.

Imp $\boxed{\vec{K} = \sigma \omega R \sin\theta \, \hat{\phi}}$

$$\left[\begin{array}{l} K = \sigma v \\ K = \sigma \vec{\omega} \times \vec{r} \end{array} \right]$$

(4) Charge density in Space Where there are Some point charges.



$$\rho = \begin{cases} 0 & \forall \; x \neq x_0, \; y \neq 0, \; z \neq 0 \\ \infty & x = x_0, \; y = 0, \; z = 0 \end{cases}$$

$$\left[\rho = \frac{q}{V} = \frac{q}{0} \Rightarrow \infty \right]$$

or $\rho = q \, \delta(x - x_0) \, \delta(y - 0) \, \delta(z - 0)$

or $\boxed{\rho = q \, \delta(x - x_0)}$

→ Note: If Boundary is Not parallel to x or y axis, then limit is Variable. either x limit depends on y or y limit depends on x.

Concepts of Vector Calculus:

Coordinate System:

Cartesian – (x, y, z) | Spherical polar – (r, θ, ϕ) | cylindrical polar – (r, ϕ, z)

$h_1 h_2 h_3 \to$ $(1,1,1)$ | $(1, r, r\sin\theta)$ | $(1, r, 1)$

Volume Element:

$$\boxed{d\tau = h_1 h_2 h_3 \, dq_1 \, dq_2 \, dq_3}$$

In Cartesian:

$$d\tau = dx \, dy \, dz$$

In Spherical polar

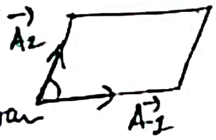
$$\boxed{d\tau = r^2 \sin\theta \, dr \, d\theta \, d\phi}$$

In cylindrical polar

$$\boxed{d\tau = r \, dr \, d\phi \, dz}$$

Area Element:

Cross product of two adjacent sides of a parallelogram gives Area of parallelogram.



General Line Element:

$$\boxed{\vec{dl} = h_1 dq_1 \hat{e_1} + h_2 dq_2 \hat{e_2} + h_3 dq_3 \hat{e_3}}$$

In Cartesian

$$\vec{dl} = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$

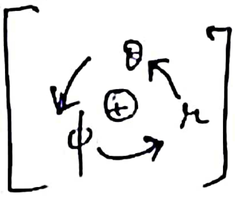
Take Cross product of Any two Components to get Area Vector

$\therefore$ Area Vector elements.

$$dx\,dy\,\hat{z}, \quad dx\,dz\,\hat{y}, \quad dy\,dz\,\hat{x}$$

In Spherical polar:

$$\boxed{\vec{dl} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}}$$



$\therefore$ Area Elements.

$$r\,dr\,d\theta\,\hat{\phi}, \quad r\sin\theta\,dr\,d\phi\,\hat{\theta}, \quad r^2\sin\theta\,d\theta\,d\phi\,\hat{r}$$

[on Surface of sphere, dA or $ds = r^2 \sin\theta \, d\theta \, d\phi$]

[on Slanting Surface of Cone $ds = r\sin\theta \, dr \, d\phi$]

In Cylindrical polar: (r, ϕ, z)

$$\vec{dl} = dr\,\hat{r} + r\,d\phi\,\hat{\phi} + dz\,\hat{z}$$

$\therefore$ area elements are

$$r\,dr\,d\phi\,\hat{z}, \quad r\,d\phi\,dz\,\hat{r}, \quad dr\,dz\,\hat{\phi}$$

area Elements on Cylinder

on Curved Surface of Cylinder, $ds = r\,d\phi\,dz$

on Cross Section of Cylinder, $ds = r\,dr\,d\phi$

on Circular Surface, We can use $\boxed{ds = r\,dr\,d\phi \text{ or } ds = r\,dr\,d\theta}$

★ Line Element on Arc of Circle:

$$\boxed{dl = r\,d\theta}$$

$\left[d\theta = \frac{\text{arc}}{\text{radius}} = \frac{dl}{r}\right]$

★ Line Element on Ellipse:

$$\boxed{\vec{dl} = dr\,\hat{r} + r\,d\phi\,\hat{\phi}}$$

or $\left[\vec{dl} = dx\,\hat{x} + dy\,\hat{y}\right]$

★ Gradient:

→ Spatial Rate of change of function.

→ it gives dirn of Normal to a Surface on which function is Constant

$$\boxed{\vec{\nabla} f = \frac{\hat{e}_1}{h_1}\frac{\partial f}{\partial q_1} + \frac{\hat{e}_2}{h_2}\frac{\partial f}{\partial q_2} + \frac{\hat{e}_3}{h_3}\frac{\partial f}{\partial q_3}}$$

f is Scalar

$$\boxed{\hat{n} = \frac{\vec{\nabla} f}{|\vec{\nabla} f|}}$$

general formula of Normal to a Surface Whose Equation is $f = \text{Constant}$.

Divergence :

→ It is defined for a Vector

→ It Represents Convergence or divergence of a Vector at a point.

$$\boxed{\operatorname{div}(A) = \vec{\nabla}\cdot\vec{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial q_1}(h_2 h_3 A_1) + \frac{\partial}{\partial q_2}(h_1 h_3 A_2) + \frac{\partial}{\partial q_3}(h_1 h_2 A_3)\right]}$$

divergence of a vector at a point is flux of Vector through Small Area Around given point per unit Volume.

$$\boxed{\vec{\nabla}\cdot\vec{A} = \lim_{\Delta V \to 0} \frac{\oint \vec{A}\cdot d\vec{s}}{\Delta V}}$$

Curl of a Vector :

→ it Represents either Rotation of Vector or Rotational effect of Vector.

→ it is defined as Line Integration of Vector around a point per unit Area.

$$\boxed{\vec{\nabla}\times\vec{A} = \lim_{\Delta S \to 0} \frac{\oint \vec{A}\cdot d\vec{l}}{\Delta S}}$$

$$\boxed{\vec{\nabla}\times\vec{A} = \frac{1}{h_1 h_2 h_3}\begin{bmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \partial/\partial q_1 & \partial/\partial q_2 & \partial/\partial q_3 \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{bmatrix}}$$

↓

General formula of Curl.

* Laplacian operator :

$$\boxed{\nabla^2 f = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial q_1}\left(\frac{h_1h_2h_3}{h_1^2}\frac{\partial f}{\partial q_1}\right) + \frac{\partial}{\partial q_2}\left(\frac{h_1h_2h_3}{h_2^2}\frac{\partial f}{\partial q_2}\right) + \frac{\partial}{\partial q_3}\left(\frac{h_1h_2h_3}{h_3^3}\frac{\partial f}{\partial q_3}\right)\right]}$$

* Limits in different Co-ordinate System :

(1) In Cartesian : [IN 2D]

if Boundary is not parallel to Co-ordinate axes, then limit is Variable.

y limit x dependent or x limit y-dependent (on xy plane)

[IN 3D]

if No Boundary is parallel to Any Co-ordinate axes,

z limit x & y dependent, Then y limit x dependent full limit for x.

(2) In spherical polar (r, θ, ϕ) :

$$\boxed{\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}}$$

θ = Angle with z axis

ϕ = angle b/w x & z.

→ Limit for sphere :

Sphere Can be generated by Rotating a half disc

∴ For full sphere

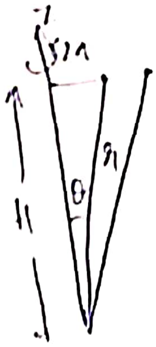
$$\boxed{\begin{aligned} \phi &= 0 \text{ to } 2\pi \\ \theta &= 0 \text{ to } \pi \\ r &= 0 \text{ to } R \end{aligned}}$$

for half Disc :

$$\boxed{\begin{aligned} \phi &= 0 \text{ to } 2\pi \\ \theta &= 0 \text{ to } \pi/2 \\ r &= 0 \text{ to } R \end{aligned}}$$

→ Limit for Cone:

Cone can be generated by Rotating a triangle



$\therefore$

$$\boxed{\begin{aligned} \phi &: 0 \text{ to } 2\pi \\ \theta &: 0 \text{ to } \alpha \\ r &: 0 \text{ to } \frac{H}{\cos\theta} \end{aligned}}$$

→ points on Boundary are at different distances from origin, So limit is variable.

(3) In Cylindrical Polar Co-ordinates: $[r, \phi, z]$

$$\boxed{\begin{aligned} x &= r\cos\phi \\ y &= r\sin\phi \\ z &= z \end{aligned}}$$

→ Limit for Cylinder:

Cylinder can be generated by Rotating a Rectangle.

$\therefore$

$$\boxed{\begin{aligned} \phi &= 0 \text{ to } 2\pi \\ r &= 0 \text{ to } R \\ z &= 0 \text{ to } H \end{aligned}}$$

→ Limit for Cone:

$$\boxed{\begin{aligned} \phi &= 0 \text{ to } 2\pi \\ r &= 0 \text{ to } \frac{R}{H}z \\ z &= 0 \text{ to } H \end{aligned}}$$

another limit

$$\left[\begin{aligned} \phi &= 0 \text{ to } 2\pi \\ z &= \frac{H}{R}r \text{ to } H \end{aligned}\right]$$

★ Plane polar co-ordinate : $[(r,\theta)$ or $(r,\phi)]$

Mostly used on a plane whose Boundary is Circular.

$$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta}$$

$$dA = r\,dr\,d\theta$$

Limit for Circular object when origin is at Centre (Disc)

r - 0 to R

θ : 0 to 2π

Limit, when object origin is at periphery (for Disc)

r : 0 to $2R\cos\theta$

θ : $-\frac{\pi}{2}$ to $\frac{\pi}{2}$

For, Arc

Limit for θ

θ : $\frac{\pi}{2} - \frac{\alpha}{2}$ to $\frac{\pi}{2} + \frac{\alpha}{2}$

or

θ : $-\frac{\alpha}{2}$ to $\frac{\alpha}{2}$

★ Some useful Integrations :

① $\displaystyle\int_0^{2\pi} \cos^2\theta\, d\theta = \int_0^{2\pi} \frac{1+\cos 2\theta}{2}\, d\theta = \pi$

② $\displaystyle\int_0^{2\pi} \sin^2\theta\, d\theta = \pi$

③ $\displaystyle\int_0^{2\pi} \cos\theta\, d\theta = 0$

(A) $\displaystyle\int_0^{\pi} \sin^3\theta\, d\theta = \frac{3}{4}$

④ $\displaystyle\int_0^{2\pi} \sin\theta\, d\theta = 0$

⑤ $\displaystyle\int_0^{2\pi} \cos m\theta \sin n\theta = 0$

Whenever $\sin\theta$ has odd power, put $\cos\theta = x$

Whenever $\cos\theta$ has odd power, put $\sin\theta = x$

eg. $\displaystyle\int_0^{\pi} \sin^3\theta\, d\theta = \int_0^{\pi} \sin^2\theta \sin\theta\, d\theta$ ∴ Put $\cos\theta = x$

★ Basic Concept of Vector :

① Resultant of two Vectors

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

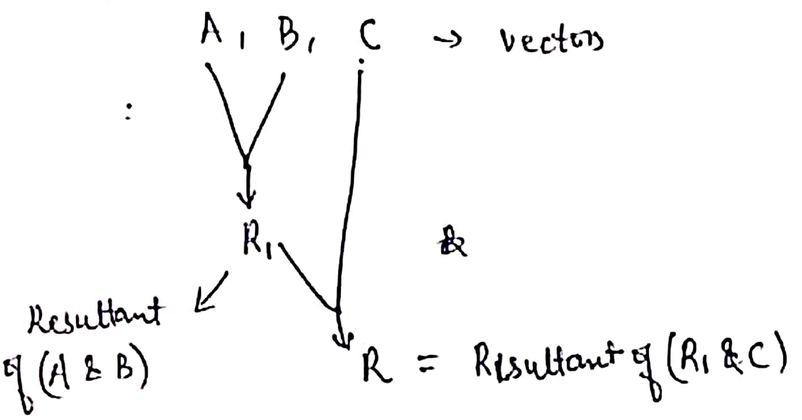
But if both vectors are Equal, i.e $A = B$

$\therefore$ ★ Imp $\boxed{R = 2A \cos\frac{\theta}{2}}$ Resultant of two Equal vectors

$\sin 37° = \frac{3}{5}$

$\cos 37° = \frac{4}{5}$

② If there are three Vectors, A then



③ General Method :

First, write all Vectors in Component form and add them, Then take Magnitude of Resultant.

★ Electric Dipolemoment and Qudrupole Moment :

(1) Electric Dipolemoment :

For discrete System $\boxed{\vec{p} = \sum_i q_i \vec{r}_i}$

For Continuous charge distribution $\boxed{\vec{p} = \int dq\, \vec{r}}$

here $\vec{r}_i \rightarrow$ position vector of q_i

$\vec{r} \rightarrow$ position vector of dq

* Some Important Points Related to Electric dipolemoment ($\vec{p}$):

(1) If total charge of the System is Zero, then $\vec{p}$ is independent of choice of origin.

i.e $\vec{p}$ doesn't change if origin is changed.

(2) if origin is Symmetric point, then $\vec{p}=0$.

[origin will be Symmetric if charges at $\vec{r}$ & $-\vec{r}$ are equal]

(3) Concept of Centre of charge (Similar to Centre of Mass) Can be used for dipolemoment Calculation

[only in ~~COC~~ dipolemoment Calculation]

Nowhere else in EMT.

$$\vec{r}_{com} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

Similar $\sum q_i \vec{r}_i$

* Electric Quadrupole Moment :

For Any charge distribution, Quadrupole Moment (Q_{ij}) is defined as

$$\boxed{Q_{ij} = \sum q\,(3x_i x_j - r^2 \delta_{ij})}$$

$i = 1, 2, 3$

$j = 1, 2, 3$

For continuous System

$$\boxed{Q_{ij} = \int dq\,(3x_i x_j - r^2 \delta_{ij})}$$

→ There are 9 terms of Quadrupole moments

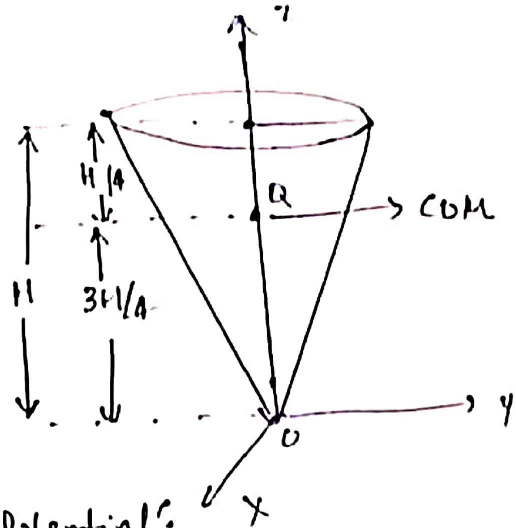
→ if all Nine terms are Zero, then Quadrupole Moment is Zero.

→ if Any one of Nine terms are Non-Zero, then We say that Quadrupole Moment is Non-Zero.

→ if charge distribution is spherically Symmetric, then Q_{ij} is definitely Zero.

★ Centre of Mass of uniform Cone:

Centre of charge is at $3H/4$ dist from origin



★ Asympototic Variation of Electric Field & Potential: [E & V]

Variation at large distance from a finite object.

$$\boxed{① \text{ if } Q_{total} \neq 0, \quad E \propto \frac{1}{r^2}, \quad V \propto \frac{1}{r}}$$

$$\boxed{② \text{ if } Q_{tot} = 0, \quad \vec{P}_{tot} \neq 0, \Rightarrow E \propto \frac{1}{r^3}, \quad V \propto \frac{1}{r^2}}$$

$$\boxed{③ \text{ if } Q_{tot} = 0, \quad \vec{P}_{tot} = 0, \quad Q_{ij} \neq 0 \Rightarrow E \propto \frac{1}{r^4}, \quad V \propto \frac{1}{r^3}}$$

$$\boxed{④ \text{ if } Q_{tot} = 0, \quad \vec{P}_{tot} = 0, \quad Q_{ij} = 0, \text{ then } E \propto \frac{1}{r^5}, \quad V \propto \frac{1}{r^4}}$$

→ if options do not contain $E \propto \frac{1}{r^5}$, $V \propto \frac{1}{r^4}$, then No Need to check Quadrupole Moment.

→ if there is a negative and a +ve charge, then form a dipole to check whether $\vec{P}$ is zero or not.

* Magnetostatics:

Study of Magnetic effects due to Constant Current or Slowly Varying Current

* Biot-Savart Law (BSL)

Magnetic field produced by a Static/Constant or Slowly Varying Current is given as

BSL

$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l} \times \hat{r}}{r^2}$	(due to wire current)
$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{(\vec{K} \times \hat{r})\, ds}{r^2}$	(due to surface current)
$B = \frac{\mu_0}{4\pi}\int \frac{(\vec{J} \times \hat{r})\, d\tau}{r^2}$	(due to Volume current)

here $\hat{r}$: unit vector from Element to the Field point → The point at which B is to be Calculated.

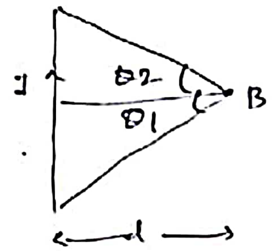
r: Dist of field point from Element.

→ These 3 formulas Can be used to Calculate Magnetic field due to Any object.

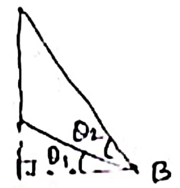
* Magnetic field due to Some standard objects:

(1) Straight Wire:

$$\boxed{B = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)}$$



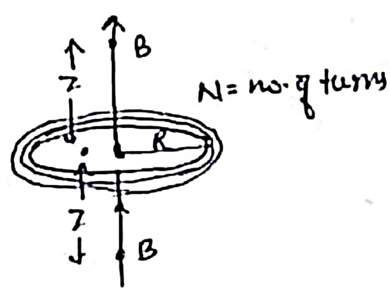
$$\boxed{B = \frac{\mu_0 I}{4\pi d}(\sin\theta_2 - \sin\theta_1)}$$



(2) Magnetic field at axial point of a Circular Loop/Wire:

$$\boxed{B = \frac{N\mu_0 I R^2}{2(R^2+Z^2)^{3/2}}}$$

B at axial dist Z from Centre.



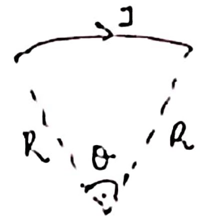
Imp *

$$\boxed{B = \frac{N\mu_0 I}{2R}}$$

B at Centre (Z=0)

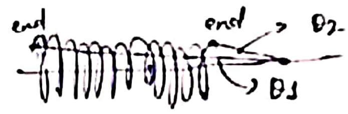
(3) Magnetic field at the Centre of an Arc of Circle:

$$\boxed{B = \frac{\mu_0 I}{4\pi R}\,\theta}$$



(4) Magnetic field at Axial point of a Solenoid:

$$\boxed{B = \frac{\mu_0 n I}{2}\left[\cos\theta_1 - \cos\theta_2\right]}$$



$nI = \frac{NI}{l} = K$ = Surface current density.

$n = \frac{\text{No. of turns}}{\text{Length}}$ $\left(n = \frac{N}{l}\right)$

(5) B due to Infinite thin sheet:

$$\boxed{\vec{B} = \frac{\mu_0\, \vec{K} \times \hat{n}}{2}}$$

$\vec{K}$ = Surface element on the sheet.

$\hat{n}$ = Normal on sheet toward field point

* Direction of Magnetic field:

$\hat{B}$ $\boxed{\hat{B} = \hat{I} \times \hat{r}}$

$\hat{I}$ = dirn of current

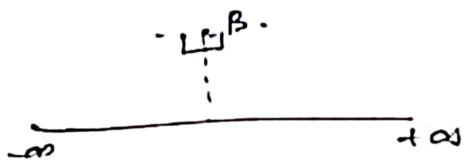
$\hat{r}$ = unit vector from element to field point

Conventionally

Right hand thumb Rule.

* Magnetic field for some standard case:

(1) B Near Centre of ∞ Wire.



$$\therefore \boxed{B = \frac{\mu_0 I}{2\pi d}} \quad (\text{bcoz } \theta_1 = \theta_2 = 90^\circ)$$

(2) B field near end of ∞ wire.

$$\boxed{B = \frac{\mu_0 I}{4\pi d}} \quad \therefore \left(\text{bcoz } \theta_1 = 0,\ \theta_2 = 90^\circ\right)$$

(2) B due to infinite Solenoid Near its Mid point:

$\boxed{B = \mu_0 n I}$

or $\boxed{B = \mu_0 K}$ } Imp

(bcoz $\theta_1 = 0°$, $\theta_2 = 180°$)

(3) B due to Infinite Solenoid Near End point:

$$\boxed{B = \frac{\mu_0 n I}{2}}$$

$$\therefore \boxed{B_{end} = \frac{B_{centre}}{2}}$$

* Properties of static Magnetic field:

① Div. of B is always Zero. ∴ Magnetic field is a Solenoidal vector

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0} \quad — ①$$

② Magnetic flux through a closed Surface is always Zero.

$$\boxed{\oint \vec{B} \cdot d\vec{s} = 0} \quad — ②$$

$$\left[\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enclosed}}{\varepsilon_0} \right]$$

From Above two statements, we conclude that Magnetic Monopole doesnot Exist.

③ Curl of B is given as

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}} \quad — ③$$

That means, Magnetic field is not Conservative field.

$\therefore \vec{\nabla} \times \vec{B} = 0$ at all points.

④ using stokes theorem, above Equation can be written as

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 J_{enclosed}} \quad — ④$$

Eq. ① & ② → gauss's Law for B

Eq. ③ & ④ → Ampere's Circuital Law (ACL) for B

Eq. ① & ③ → Local Equations applied at a point.

★ Imp ★ Vector Potential : [It No physical Significance]

it is Introduced Mathematically bcoz divergence of Magnetic field is always Zero

★ Imp ∴ $\boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$ $\vec{A}$ = Vector potential

→ Vector potential is not uniquely defined bcoz if We add a Quantity Whose Curl is Zero to $\vec{A}$ then $\vec{A}$ will change But $\vec{B}$ doesn't change

$$\boxed{\vec{A}' = \vec{A} \pm \vec{\nabla} f(x,y,z)} \qquad \therefore \boxed{\vec{\nabla} \times \vec{\nabla} f = 0}$$

→ using BSL,

$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \int \frac{I\,d\vec{l}}{r}}$ → for Wire Current

$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{K}\,ds}{r}}$ → for Surface Current

$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}\,d\tau}{r}}$ → for Volume Current

Not used for Infinite objects.

→ Direction of $\vec{A}$ (Vector potential) due to a current Carrying object is Same as direction of Current. Sometimes it May be opposite to direction of Current.

Stokes theorem
$\oint \vec{v} \cdot d\vec{l} = \int \vec{\nabla} \times \vec{v} \cdot d\vec{s}$
gauss-div. theorem
$\oint \vec{v} \cdot d\vec{s} = \int \vec{\nabla} \cdot \vec{v}\, d\tau$

★ Magnetic flux : [ϕ_B]

$$\boxed{\phi_B = \int \vec{B} \cdot d\vec{s}}$$

using stokes theorem

$$\boxed{\phi_B = \oint \vec{A} \cdot d\vec{l}}$$

∴ Comparing Above two Equations

Imp $\boxed{\oint \vec{A} \cdot d\vec{l} = \int \vec{B} \cdot d\vec{s}}$ → used to find $\vec{A}$ for Infinite objects.

* Conclusion:

For uniform $\vec{B}$, $\vec{A}$ Can be Written as

$$\boxed{\vec{A} = \frac{\vec{B}\times\vec{r}}{2}}$$ ← one possible choice for $\vec{A}$ if $\vec{B}$ is uniform [Becoz $\vec{A}$ is not unique]

* Few Imp. points:

→ If $\vec{B}$ is given and $\vec{A}$ is to be found then put Values of $\vec{A}$ from options in formula $(\vec{B} = \vec{\nabla}\times\vec{A})$, and check whether it gives Magnetic field Equal to given field or not.

→ if $\vec{B} = B\hat{k}$ = uniform, then Vector potential $\vec{A} = \frac{\vec{B}\times\vec{r}}{2}$

$$\therefore \boxed{\vec{A} = \frac{B}{2}(x\hat{j} - y\hat{i})}$$

Whenever uniform Magnetic field Exists in +z direction, then in $\vec{A}$

Co-efficients of $\hat{i}$ is $-y$ or Zero
Co-efficient of $\hat{j}$ is x or Zero

$$\boxed{\vec{A} = \frac{B}{2}(x\hat{j} - y\hat{i})}$$

→ Integration of Vector $(d\vec{l})$ is Zero

→ But if dl is not a Vector, then its Integration is finite (eg $2\pi r$)

For Vector $d\vec{l}$ $\quad \oint d\vec{l} = 0$

But $\quad \oint dl = 2\pi r$

* Vector potential due to Long Solenoid Near its Mid point:

We will use the formula $\oint \vec{A}\cdot d\vec{l} = \oint \vec{B}\cdot d\vec{s}$

(I) Inside (at distance r from axis $(r<R)$)

$$\boxed{A = \frac{\mu_0 n I r}{2}} \quad (r<R)$$

or

$$\boxed{\vec{A}_{inside} = \frac{\mu_0 n I}{2}[\hat{z}\times\vec{r}]}$$

(or)

$$\boxed{\vec{A}_{inside} = \frac{\mu_0 n I}{2}(x\hat{j} - y\hat{i})}$$

outside: at distance r from axis $(r > R)$

$$\boxed{A = \frac{\mu_0 n I R^2}{2r}} \left(A \propto \frac{1}{r}\right)$$ and $\phi_B = (\mu_0 n I)(\pi R^2)$

$$\therefore \boxed{\vec{A}_{outside} = \frac{\phi_B}{2\pi r}\hat{\phi}}$$

$\left(\text{But } \hat{\phi} = \hat{K} \times \hat{r}\right)$

or
$$\boxed{\vec{A}_{outside} = \frac{\phi_B}{2\pi}\left[\frac{x}{x^2+y^2}\hat{j} - \frac{y}{x^2+y^2}\hat{i}\right]}$$

★ Vector Potential Due to Long straight Wire Carrying steady Current I:

$$\boxed{\vec{A} = \frac{-\mu_0 I}{2\pi}\ln r + C}$$

Vector potential of Infinite thin sheet carrying a surface current $\vec{K}$.

$$\boxed{A = -\frac{\mu_0 K z}{2}}$$ (sheet is lying in xy plane)

★ Ampere's Circuital Law:
(Application for B Calculation)

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}}$$

B calculation of B is possible only in following symmetric cases:

(1) Cylindrical Symmetry

Infinite cylinder $\vec{K} = \text{Const.}\,\hat{z}$

$\vec{J} = \text{Const}\,\hat{z}$

or $\vec{J} = f(r)\hat{z}$

(2) Solenoidal symmetry:

Infinite cylinder $\vec{K} = \text{Const}\,\hat{\phi}$

$\vec{J} = \text{Const}\,\hat{\phi}$

$\vec{J} = f(r)\hat{\phi}$

(3) Infinite plane

$\vec{K} = \text{Constant}$, $\vec{J} = \text{Constant}$

(4) Torroid

In these four cases, B calculation is possible using ACL.

★ How to Apply ACL :

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$$

Mathematical loop (Not a current loop) → $I_{enclosed}$ → Current enclosed by Mathematical loop.

★ Mathematical field Calculation for cylindrical symmetry :

★ Cylindrical symmetry [Magnetic field Calculation]

Imp. $$\boxed{B = \frac{\mu_0 I_{enclosed}}{2\pi r}}$$

R=

Inside solid cylinder :

$$\boxed{I_{enclosed} = \int_0^r J r\,dr\,2\pi}$$

(R = Radius of cylinder)

outside solid cylinder :

$$\boxed{I_{enclosed} = \int_0^R J r\,dr\,2\pi}$$

★ Limit for thick hollow cylinder :

$R_1 < r < R_2$

$$\boxed{I_{encl} = \int_{R_1}^{r} J r\,dr\,2\pi}$$

$r > R_2$

$$\boxed{I_{encl} = \int_{R_1}^{R_2} J r\,dr\,2\pi}$$

$$\left\{ \begin{array}{l} B_{inside} \propto r^{n+1} \\ B_{outside} \propto \frac{1}{r} \end{array} \right\}$$

★ Magnetic field due to Long Cable [Cylinder carrying uniform current Density (J)]:
(of Radius R)

Inside $r < R$

$$\boxed{B = \frac{\mu_0 J r}{2}} \quad B \propto r \quad \text{or} \quad \boxed{\vec{B} = \frac{\mu_0}{2}[\vec{J} \times \vec{r}]}$$

outside ($r > R$)

$$\boxed{B = \frac{\mu_0 J R^2}{2r}} \quad \left(B \propto \frac{1}{r}\right)$$

→ When two cylinders having $\vec{J}$ & $-\vec{J}$ are merged, then in common Region, B is uniform (same for Electric field with ρ & $-\rho$)

★ Plane Symmetry: [infinite plane] [$\vec{K}$ = const, $\vec{J}$ = const]

★ Magnetic field due to infinite thin sheet:

$$\boxed{B = \frac{\mu_0 K}{2}} \quad \text{or} \quad \boxed{\vec{B} = \frac{\mu_0 \vec{K} \times \hat{n}}{2}}$$

★ Magnetic field due to thick infinite sheet: [thickness = 2a]

inside Region

$$\boxed{B = \mu_0 J Z}$$

outside

$$\boxed{B = \mu_0 J a} = \text{constant}$$

a = half of total thickness

★ B due to two thin parallel sheets:

$$\boxed{B_1 = B_2 = \frac{\mu_0 K}{2}}$$

★ B due to Torroid:

$$\boxed{B = \frac{\mu_0 n I}{2\pi r}}$$

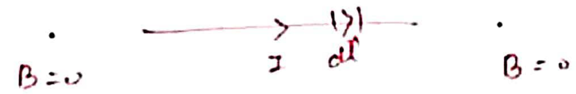
r = central Radius

R_1 = inner Radius $R_1 < r < R_2$

R_2 = outer Radius

B = 0 in other Regions

Note :

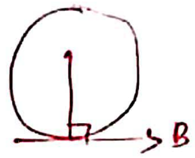


$\left[\begin{array}{c} \text{because } (d\vec{l} \times \vec{r} = 0) \\ \therefore B = 0 \end{array}\right]$

B on front & behind of straight line is zero

→ In Mathematics, Any Vector or Scalar defined in space is called field

→ If → $\vec{B}$ is always ⊥ to the Radius of position Vector



A. Magnetic force :

Experienced by moving charge

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

When a wire carrying current is placed in External B, then

$$\boxed{\vec{F} = \int I d\vec{l} \times \vec{B}}$$

if current flows on a surface

$$\boxed{\vec{F} = \int (\vec{K} \times \vec{B})\, ds}$$

if current flows in Volume.

$$\boxed{\vec{F} = \int (\vec{J} \times \vec{B})\, d\tau}$$

Special Case:

if $\vec{B}$ = uniform, then force on wire $\underline{\vec{F} = I\vec{l} \times \vec{B}}$

where $\vec{l}$ is a vector from first to 2^{nd} end of wire.

→ Note: if a closed loop is placed in uniform Magnetic field, Then Magnetic force on the Loop is zero.

$\left(\vec{F}_{loop} = 0 \quad \text{if} \quad \vec{B} = \text{Uniform}\right)$

* Torque on a Current loop:

$$\boxed{\vec{\tau} = \int I \, d\vec{s} \times \vec{B}}$$

$\Rightarrow$ $d\vec{s}$: area Element of the loop.

* Magnetic Moment of the Current loop:

$\vec{m}$ = (Current)(area Vector)

$$\therefore \boxed{\vec{m} = I \cdot \vec{s}} \quad \text{or} \quad \boxed{\vec{m} = \sum I_i \vec{s_i}}.$$

* Rotating charged object:

$$\boxed{\frac{\text{Magnetic Moment}}{\text{Angular Momentum}} = \frac{\text{charge}}{2 \times \text{Mass}}}$$

$\rightarrow$ For Any classical Case (Not for Q.M case)

or

$$\boxed{\frac{|\vec{m}|}{|\vec{\ell}|} = \frac{q}{2 \times M}}$$

$\rightarrow$ This Ratio is called gyromagnetic Ratio.

$$\boxed{|\vec{\ell}| = \text{Moment of Inertia} \times \omega} \quad \text{or} \quad \boxed{|\vec{\ell}| = I\omega^2}$$

* Force b/w two parallel Wires:

$$\boxed{\vec{F}_{21} = \frac{\mu_0 I_1 I_2 \ell}{2\pi d} (-\hat{i})}$$

(d = dist b/w two wires)

(ℓ is the length of Second wire)

* Parallel Currents attract.
* Anti-parallel Currents Repel.

* Force b/w two perpendicular Wires:

$$\boxed{\vec{F}_{21} = \frac{\mu_0 I_1 I_2}{2\pi} \ln\left(\frac{d+\ell}{d}\right) (\hat{j})}$$

(d = dist b/w two $\perp$ wires)

(ℓ = length of Second wire)

($\vec{F}_{21}$ = Force on 2 bcoz of 1)

★ Force b/w Long Wire and Square loop :

$$\boxed{F_{loop} = \frac{\mu_0 I_1 I_2 a^2}{2\pi s(s+a)}}$$

s = dist of the loop from wire
I_1 = primary current in wire
I_2 = Induced current in loop

★ Tension developed in a wire loop placed in uniform Magnetic field :

$\vec{F}_m$ on ~~Cir~~ Circular loop

$$\boxed{F_m = 2IRB}$$

$\therefore\ 2T = F_m$

$$\therefore\ \boxed{T = IRB}$$

★ Magnetic force per unit Area b/w two Large parallel sheets :

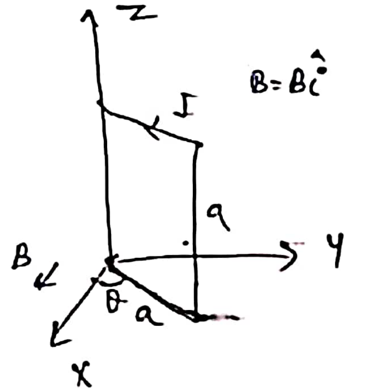
$$\boxed{\frac{d\vec{F}_{21}}{ds} = \frac{\mu_0 K_1 K_2}{2}(-\hat{n})}$$

$\hat{n}$ = area vector or Normal unit vector

★ Torque on a Square loop placed in a uniform Magnetic field as shown [~~Axis like in~~ spherical polar]

$$\boxed{\vec{\tau} = I a^2 B \cos\theta\,(-\hat{k})}$$

($\hat{k}$ due to $\vec{s}\times\vec{B} = (\hat{j}\times\hat{i}) = -\hat{k}$)



★ Torque on Straight current carrying Wire hinged at one end due to uniform Magnetic Field :

$$\boxed{\vec{\tau} = \frac{1}{2} I B l^2 (-\hat{k})}$$

($\hat{k}$ is direction of B)

★ Angular acceleration of Rod :

$$\boxed{\alpha = \frac{3IB}{2M}}$$

Concept ($\tau = I_0\omega$) (I_0 = Moment of Inertia)

* Electromagnetic Induction:

* Faraday's Law of EMI:

$$\boxed{\text{Emf} = \frac{-d\phi_B}{dt}}$$ (used for Any loop) → Neuman's formula.

here $\phi_B = \int \vec{B} \cdot d\vec{s}$

* charge flown in the loop due to EMI:

$$\boxed{\Delta Q = \frac{\Delta \phi_B}{R}}$$

* Imp.

* Sign of Emf:

→ if flux ϕ_B increase, $\boxed{emf = -ve}$

→ if ϕ_B decreases $\boxed{emf = +ve}$

* Lenz Law:

direction of induced current in a loop is always such that it opposes the cause of EMI.

How To find the direction of Induced Current

if ϕ_B increasing,	⇒ thumb → opposite to $\vec{B}$ ∴ Curled finger → Induced
if ϕ_B decreasing	⇒ thumb → in direction of $\vec{B}$ ∴ Curled finger → Induced

Note: When Smaller & Bigger loops are connected, then direction of Current is according to bigger loop.

* EMF induced in a Rectangular ~~loop~~ loop Entering in a $\vec{B}$ region:

$$\boxed{Emf = Bl v}$$

l = breadth of Rectangular loop
v = velocity of " "

$$I_{induced} = \frac{emf}{R} \quad \therefore \quad \boxed{I_{induced} = \frac{Blv}{R}}$$

also $$\boxed{\vec{F}_m = \frac{B^2 l^2 v}{R}(-\hat{i})}$$ → opposite to velocity

* Emf induced in a Rectangular loop translating ⊥ to long Current Carrying wire

$$\boxed{emf = \frac{-\mu_0 I a b v}{2\pi s(s+a)}}$$

a & b are length & breadth of Rectangular loop
s = dist of loop from wire

* Emf induced in a loop flipping/Rotating with uniform Angular Velocity ω in a uniform $\vec{B}$

$$\boxed{emf = NBS\omega \sin(\omega t + \theta_0)}$$

S = Area of loop
N = no of turns

* Cause of EMI:

it arises due to Electric or Magnetic Force on Electrons of Conductors

* EMI due to Magnetic force / Motional emf :

When a conductor is moved in a Magnetic field, the Electrons of conductor starts Moving in a certain direction due to Magnetic force. This leads to flow of current or development of emf in conductor.

Imp. * $\boxed{\vec{E} = -\vec{v} \times \vec{B}}$ $\vec{E}$ developed inside a conductor when it is moved in a Magnetic field.

But Potential Diff. (PD) = $\int -\vec{E} \cdot d\vec{l}$

$$\therefore \boxed{\text{P.D/emf} = \left| \int_i^f (\vec{v} \times \vec{B}) \cdot d\vec{l} \right|}$$

emf developed b/w any two points of conductor which is being Moved in Magnetic field.

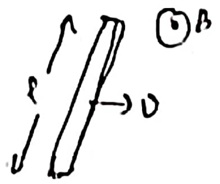
(Translation / Rotation)

(Applicable for both, Loop / Conductor)

$\boxed{\rightarrow \text{+ve End of the Conductor will be toward the direction of } \vec{v} \times \vec{B}}$

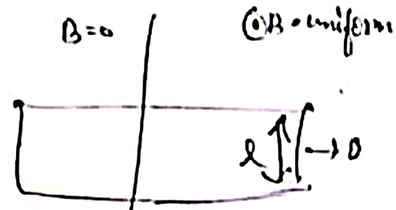
* Straight Wire translating in uniform $\vec{B}$ $\therefore$ Emf b/w two ends

$$\boxed{\text{emf} = Blv \sin\theta}$$



* Rectangular loop Entering into Magnetic field :

$$\boxed{\text{Total emf in loop} = Blv}$$



* Arbitary shaped conducting Wire Translating in uniform $\vec{B}$:

$$\text{emf} = \left| (\vec{v} \times \vec{B}) \cdot \vec{l} \right|$$

$\vec{l}$ = dist. vector from first end to last end.

✱ Straight Conducting Rod / or Wire Rotating about one end in uniform ⊥ $\vec{B}$

$$\boxed{emf = \frac{1}{2} B\omega l^2}$$

→ emf b/w fixed end & free end

ω = angular velocity

l = length of Rod.

emf b/w fixed end & Centre:

$$\boxed{emf = \frac{1}{8} B\omega l^2}$$

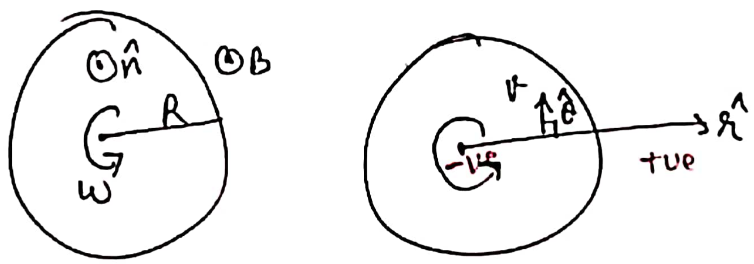
emf b/w Centre & free end

$$\boxed{emf = \frac{3}{8} B\omega l^2}$$

✱ Circular conducting disc Rotating in uniform perpendicular Magnetic field:

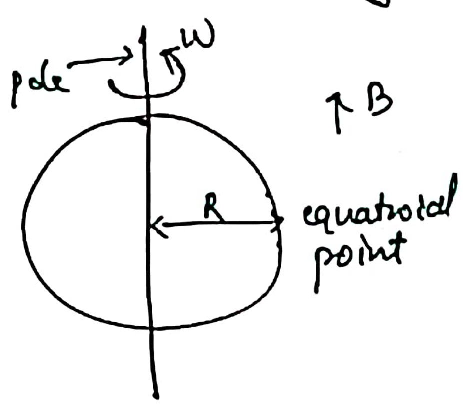
✱ Potential Diff / EMF b/w Centre and periphery

$$\boxed{emf = \frac{1}{2} B\omega R^2}$$



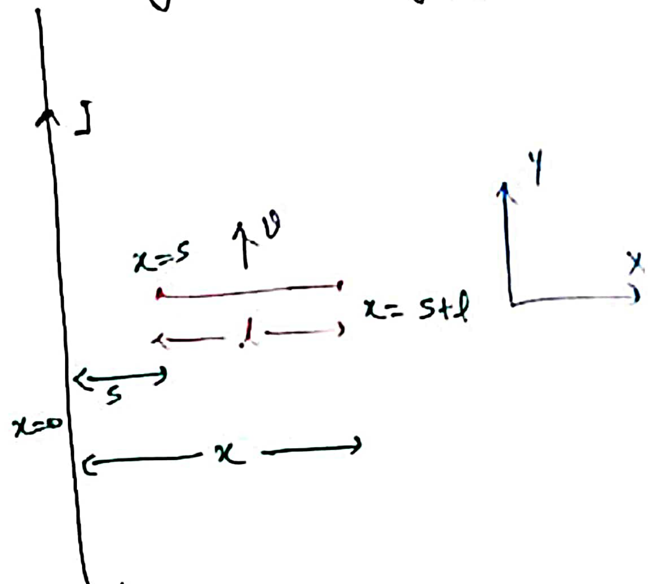
✱ Potential diff b/w a pole and Equatorial point of a Conducting Sphere Rotating in Magnetic field.

$$\boxed{emf = \frac{1}{2} B\omega R^2}$$



* Straight Conducting Wire translating Near a Long Current Carrying Wire:

$$\boxed{emf = \frac{\mu_0 I v}{2\pi} \ln\left(\frac{s+l}{s}\right)}$$



* Electromagnetic Induction due to Electric field:

(Concept of Induced Electric field)

Time Varying Magnetic field Creates an $\vec{E}$. This $\vec{E}$ applies force on e^-, due to which e^- move and produce Current in the loop.

$$\vec{B}(t) \longrightarrow \vec{E}$$

→ Induced $\vec{E}$ field lines Must form a closed loops.

→ Induced $\vec{E}$ field must be Non conservative.

$$\vec{\nabla} \times \vec{E} \neq 0 , \quad \oint \vec{E} \cdot d\vec{l} \neq 0 , \quad \int_i^f \vec{E} \cdot d\vec{l} = \text{path dependent.}$$

$$\boxed{\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \phi_B}{\partial t}} \quad (1)$$

→ Relation b/w Induced $\vec{E}$ & $\vec{B}(t)$

→ also faraday's law of (Integral form)

or

$$\boxed{\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad (2)$$

→ diff Eqn of faradays law

↓

Local eqn. (Applicable at Any point)

(Eqn. (1) & (2) are Called Maxwells Equation)

→ Sense of Induced $\vec{E}$ is Same as Sense of Induced Current.

→ If $\vec{B}$ Varies with time in a Region then, Induced $\vec{E}$ develops both Inside and outside in that Region.

→ if $\vec{B} \to$ straight $\therefore \vec{E} =$ loops

& if $\vec{B} \to$ loops $\therefore \vec{E} =$ ~~Grow~~ straight

★ **Induced $\vec{E}$ Inside and outside a long Solenoid Carrying Variable Current.**

Note: Results of Induced $\vec{E}$ are Same as Results of Vector potential.

(I) Inside

$$\boxed{E = -\frac{\mu_0 n r}{2}\frac{\partial I}{\partial t}} \quad (E \propto r)$$

(II) outside

$$\boxed{E = -\frac{\mu_0 n R^2}{2r}\frac{\partial I}{\partial t}} \quad \left(E \propto \frac{1}{r}\right)$$

★ **Some useful Concept:**

In Symmetric Cases, Results of Magnetic field/potential match with the Result of Electric field/potential.

$\vec{B} \to \vec{E}$

$\vec{A} \to \phi$

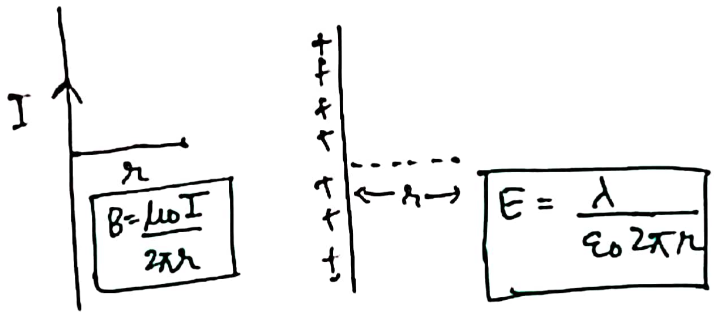
$\mu_0 \leftrightarrow \frac{1}{\epsilon_0}$

$I \leftrightarrow \lambda$

$K \leftrightarrow \sigma$

$J \leftrightarrow \rho$

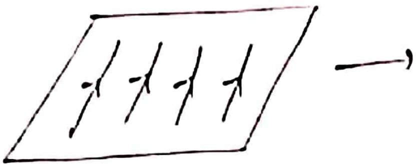
eg: ① Long Wire



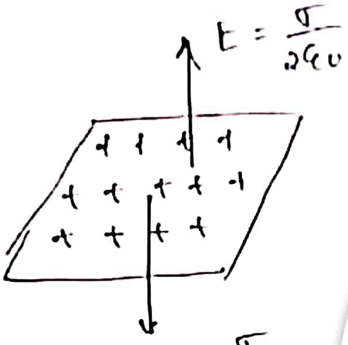
$I \to \lambda$,

$\mu_0 \to \frac{1}{\epsilon_0}$

(4) Infinite sheet.

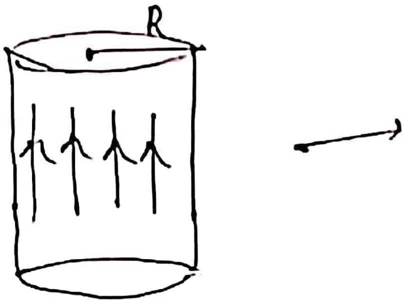


$$B = \frac{\mu_0 K}{2}$$



$$E = \frac{\sigma}{2\epsilon_0}$$

(5) Long Cable (uniform J)



$$\boxed{B_{in} = \frac{\mu_0 J r}{2}}$$

$$\boxed{B_{out} = \frac{\mu_0 J R^2}{2r}}$$

$$\boxed{E_{in} = \frac{\rho r}{2\epsilon_0}}, \quad \boxed{E_{out} = \frac{\rho R^2}{2\epsilon_0 r^2}}$$

* Equation of Continuity :

it is a Mathematical Statement of Conservation of charge.

$$\boxed{\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}}$$ → Eqn. of Continuity (always Correct | true)

* KCL

$I_1 + I_2 = I_3 \Rightarrow \rho =$ Const. at junction

or

$\therefore \boxed{\vec{\nabla} \cdot \vec{J} = 0}$ → Another form of KCL