FUTURE OF PHYSICS :FORUM FOR IIT JAM,JEST,TIFR,NET,GATE

ELECTRO DYNAMICS

PART 2

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SHORT NOTES

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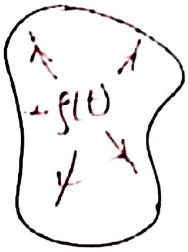
* Variation of charge density in a Material due to outward flow of charge:

$$\boxed{\rho = \rho_0 e^{-\frac{\sigma}{\varepsilon_0} t}}$$

→ This Represents a Variation of charge density in a material due to outward flow of charges



↓(or)

$$\boxed{\rho = \rho_0 e^{-\frac{t}{\tau}}}$$

$\tau = \frac{\varepsilon_0}{\sigma}$ = relaxation time (time Constant)

τ gives estimation of time in which all charges flow to the surface

$$\left(\tau \propto \frac{1}{\sigma}\right) \text{ or } \left[\text{time Constant} \propto \frac{1}{\text{Conductivity}}\right]$$

* Value of τ for a good conductor:

eg for Cu,

$\boxed{\tau = 10^{-17} \text{ sec}}$ fs

τ is Very Small, so in good Conductor, all charges quickly flow to the Surface, so We can say that charge inside a good Conductor is Zero.

* If Material has dielectric property also, then

$$\boxed{\rho = \rho_0 e^{-\frac{\sigma}{\varepsilon_0 \varepsilon_r} t}}$$

ε_r = dielectric Constant, relative permeability = (dimensionless

ε_0 = permittivity of free space

ε = permittivity of Medium

$$\boxed{\varepsilon_0 \varepsilon_r = \varepsilon}$$

★ Modification of Ampere's Circuital law :

ACL in Integral form

$$\boxed{\oint \vec{B}\ \vec{dl} = \mu_0 I_{encl}}$$

ACL in differential form

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}}$$

∴ Acc. to eqn. of continuity, $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ is not correct

∴ Modified ACL is

$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{d\vec{E}}{\partial t} \right)}$$

→ Maxwell's 4th Eqn.

→ (differential form of modified ACL)

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \cancel{\mu_0}\, \mu_0 \left(I_{encl} + \epsilon_0 \frac{d}{dt} \phi_E \right)}$$

ϕ_E = electric flux

→ (Integral form of Modified ACL)

★ Conclusion from Maxwell's 4th Eqn :

★ Imp

Time Variation of Electric field $\vec{E}$ produces Magnetic field $\vec{B}$

When $J=0$

∴

$$\boxed{\begin{aligned} \vec{\nabla} \times \vec{B} &= \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \\ \oint \vec{B} \cdot \vec{dl} &= \mu_0 \epsilon_0 \frac{\partial \phi_E}{\partial t} \end{aligned}}$$

→ These Eqns or Relations are used to find $\vec{B}$ produced ~~b~~ due to ~~to~~ time Varying $\vec{E}$.

These Eqns have similar Mathematical structure as Faraday's law or Vector potential Relations

$$\boxed{\begin{aligned} \oint \vec{E} \cdot \vec{dl} &= \frac{-\partial \phi_B}{dt} \\ \vec{\nabla} \times \vec{E} &= \frac{-\partial B}{\partial t} \end{aligned}}$$

↓

Faraday's Law

$$\boxed{\begin{aligned} \oint \vec{A} \cdot \vec{dl} &= \phi_B \\ \vec{\nabla} \times \vec{A} &= B \end{aligned}}$$

↓

Vector potential Relations

→ If $\vec{E}(t)$ lines are straight line then Induced Magnetic field lines will be Circle & Vice Versa

→ If all derivative of Initial Magnetic field | Current Exist, then Interconversion of $\vec{E}$ & $\vec{B}$ will Never stop, So this Leads to ε formation of EM wave.

$$I(t) \Rightarrow B(t) \rightarrow E_1(t) \rightarrow B_1(t)$$

direction of B_1

B → (straight line) → E_1 (Circle) → B_1 (straight line)

★ ~~Dis-place-curr~~ ment

★ Displacement current density:

We define a displacement current in those Region where $\vec{E}$ Varies with time

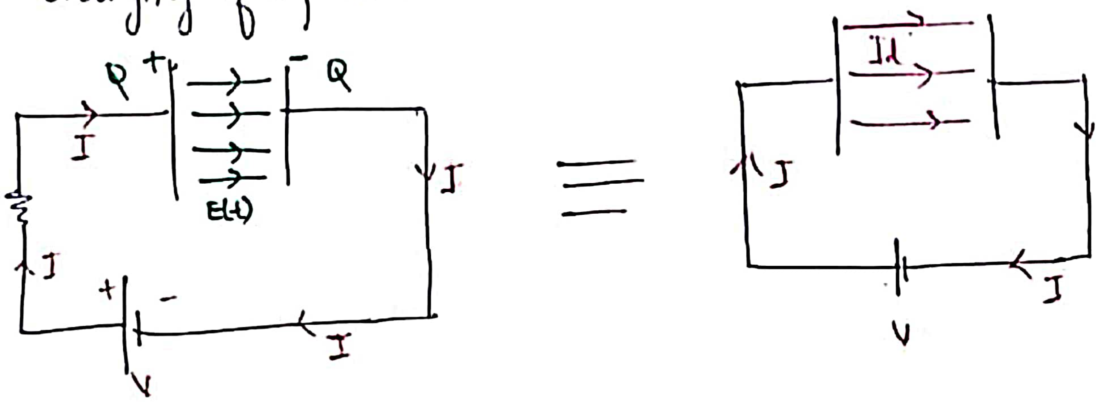
displacement current density $\boxed{\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}$ displacement current $\boxed{I_d = \varepsilon_0 \frac{\partial \phi_E}{\partial t}}$

* $\vec{J}_d \parallel \vec{E}$ if $\vec{E}$ increases with time

* $\vec{J}_d \parallel -\vec{E}$ if $\vec{E}$ decreases with time.

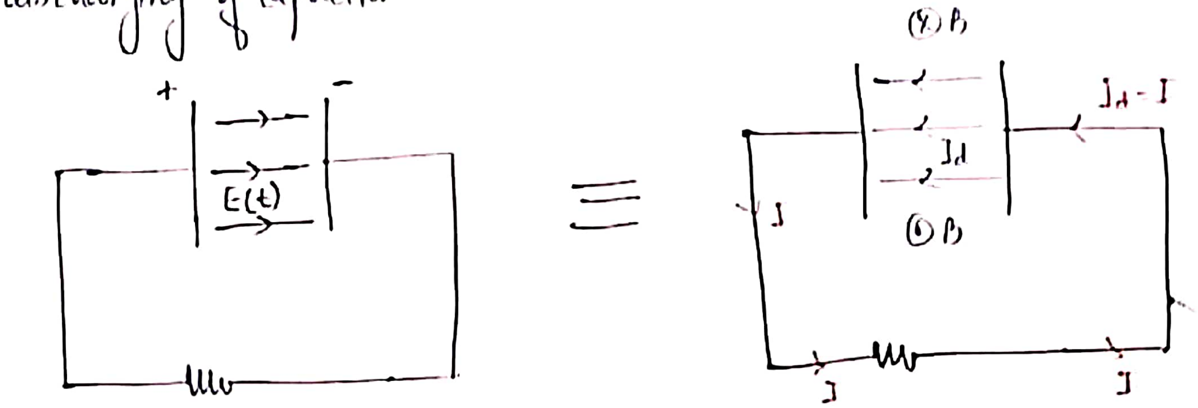
→ When we Introduce J_d / I_d in a Region, then it Establishes Continuity of Current if Continuity was Not present Earlier.

eg. charging of Capacitor



★ $\boxed{I_d = I}$

discharging of Capacitor:



* B/w the plates of capacitor, displacement Current is always Equal to Current in the Wire

A· Magnetic field b/w plates of a Capacitor having Circular plates:

Inside

$$\boxed{B = \frac{\mu_0 I}{2\pi R^2} \cdot r} \quad (B \propto r)$$

outside

$$\boxed{B = \frac{\mu_0 I}{2\pi r}} \quad \left(B \propto \frac{1}{r}\right)$$

R = Radius of circular plate of capacitor

Electronics

In Case of charging of Capacitor

$$\boxed{Q = CV\left[1 - e^{-\frac{t}{RC}}\right]} \qquad \boxed{I = \frac{V}{R} e^{-\frac{t}{RC}}}$$

In Case of discharging of Capacitor

$$\boxed{Q = Q_0 e^{-\frac{t}{RC}}}$$

★ Maxwell's Equation:

These are basic Eqns of Electromagnetic theory which describe all electromagnetic phenomenon (Except Superconductivity & Meisner effect)

$$\boxed{(1)\quad \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}}$$

$$\boxed{(2)\quad \vec{\nabla}\cdot\vec{B} = 0}$$

$$\boxed{(3)\quad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

$$\boxed{(4)\quad \vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \epsilon_0\frac{\partial\vec{E}}{\partial t}\right)}$$

used Everywhere
(Vaccum & Medium)
(there are total 8 E qns,)
(first 2 eqns is already contained in last 2 Eqns)

There are only six Independent Equations which are sufficient for finding six Components of electric field and magnetic field!

★ Integral form of Maxwell's Eqns.

$$\boxed{1)\quad \oint \vec{E}\cdot d\vec{s} = \frac{Q_{enclosed}}{\epsilon_0}}$$ → gauss law for $\vec{E}$

$$\boxed{(2)\quad \oint \vec{B}\cdot d\vec{s} = 0}$$ → gauss law for $\vec{B}$

$$\boxed{(3)\quad \oint \vec{E}\cdot d\vec{l} = \frac{-\partial\phi_B}{\partial t}}$$ → Faraday's law

$$\boxed{(4)\quad \oint \vec{B}\cdot d\vec{l} = \mu_0\left(I_{enclosed} + \epsilon_0\frac{\partial\phi_E}{\partial t}\right)}$$ → Modified ACL.

If Magnetic charge is discovered, then Maxwell's second and third Equation will have to be modified

* Electromagnetic Potential

* Imp. The quantities in terms of which EM fields can be expressed are called EM potential (Scalar and Vector potential)

$$\boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$$ ($\vec{A}$ = vector potential)

* Imp $$\boxed{\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}}$$ (ϕ = scalar potential) $[\phi \leftrightarrow V]$

$(\phi, \vec{A})$ = Electromagnetic Potential

$(\phi, \vec{A})$ = four potential (Relativity point of View)

Any quantity Which has four Components is called four Vector.

(four Vector is a set of four Quantities Which transform according to Lorentz trans)

$$\left[\frac{\phi}{c}\right] = [A]$$ (to make dim. of both quantity Equal, divide ϕ by c)

* Gauze transformation of EM Potential :

it is Such a transformation of EM Potential, due to Which $\vec{E}$ & $\vec{B}$ do not change

$A \to A'$, $\phi \to \phi'$

$$\boxed{\vec{A}' = A \pm \vec{\nabla} f(r,t)} \quad , \quad \boxed{\phi' = \phi \mp \frac{\partial f}{\partial t}(r,t)}$$

* Maxwell's Equation in terms of EM Potential:

$\boxed{1^{st} \text{ Eqn}}$ g. Putting $\vec{\nabla}\cdot\vec{A} = 0$ gives

$$\boxed{\nabla^2\phi = \frac{-\rho}{\varepsilon_0}}$$ → Poisson's Eqn.

2 Condition $\boxed{\vec{\nabla}\cdot\vec{A} = 0}$ is Called Coulomb gauze Condition.

(2nd & 3rd Eqn does not give Any Result)

$\boxed{\text{4th Eqn}}$ if we put $\vec\nabla\cdot\vec{A} + \mu_0\epsilon_0 \frac{\partial\phi}{\partial t} = 0$, 4th eqn becomes

$$\boxed{\nabla^2\vec{A} = -\mu_0\vec{J} + \mu_0\epsilon_0 \frac{\partial^2\vec{A}}{\partial t^2}} \rightarrow \text{This is Wave Eqn in Vaccum}$$

& the condition

$$\boxed{\vec\nabla\cdot\vec{A} + \epsilon_0\mu_0 \frac{\partial\phi}{\partial t} = 0} \rightarrow \text{Lorrentz-gauze Condition}$$

if in any eqn, one side contains double derivative of space co-ordinate and another side contain double derivative of time, that is a Equation of Wave in Non dispersive medium.

★ Gauze transformation and Coulomb gauge Condition Applied together:

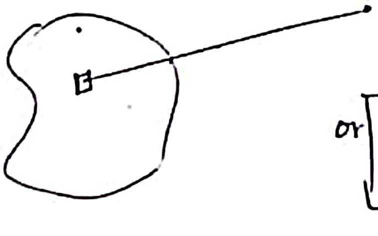
$$\boxed{\nabla^2 f = \mu_0\epsilon_0 \frac{\partial^2 f}{\partial t^2}} \qquad (f = \text{gauge function})$$

G.T & Lorentz gauge Condition can be simultaneously used if gauge function satisfies Wave Eqn.

Trick:- If solutions of Partial diff. Eqn. (PDE) is asked, then check the options

A ~~Retarded Potential~~:

★ Electrostatic Potential due to a charge:



$$V = \int \frac{dq}{4\pi\epsilon_0 r}$$

or $$\boxed{V = \int \frac{\rho\, d\tau}{4\pi\epsilon_0 r}}$$

Vector potential in Magnetostatis

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l}}{r}$$

$$\vec{A}' = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\, d\tau}{r}$$

★ Retarded potential:

Electric or Magnetic potential written by considering the time taken by Information to travel from Source to a given point is called Retarded Potential

We assume that Information travels with speed of light (c), ∴ if distance b/w source point (charge/current) and field point is r, then change in Potential will occur after time $\frac{r}{c}$

$\hookrightarrow$ (with a time lag of $\frac{r}{c}$)

★ General formula of Retarded Potential:

$$\boxed{V(t) = \int \frac{\rho(t-\frac{r}{c})}{4\pi\varepsilon_0 r}\, d\tau}$$

$$\boxed{\vec{A}(t) = \frac{\mu_0}{4\pi}\int \frac{I(t-\frac{r}{c})}{r}\, dl}$$

r = dist. b/w element and given point, where potential is to be found.

★ Special theory of Relativity :- (S.T.R)

→ This theory was developed by Analysis of maxwell's electrodynamics

→ This theory discusses about Concept / laws of physics w.r.t different Inertial frame of Reference.

★ Frame of Reference:

Place of observation.

★ Inertial frame of Reference:

two frames of References are said to be Inertial, if they move with constant velocity w.r.t Each other.

★ Postulates of STR: [Postulate means assumption]

1) Laws of physics Must be Invariant (Same) w.r.t all Inertial frame of Reference

2) Speed of Light in Vaccum is Same w.r.t. all Inertial frame of Reference.

V_{lo} = Speed of light w.r.t any object (O) = c

$\boxed{V_{ol} = -c}$ → speed of Any object w.r.t light (photon) is equal to speed of light But in opposite direction.

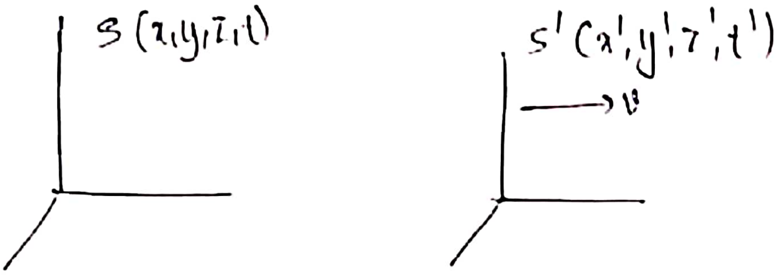
★ Event:

Happening of Something.

* Lorentz transformation (L.T)

→ it Relates time and co-ordinates of an Event as observed from two Inertial frame of Reference.

→ L.T is Relation b/w Co-ordinate and time of an Event Measured from two Inertial frame of Reference (IFR).



⊙ if S' moves in x direction w.r.t S

$$\boxed{\begin{aligned} x' &= (x - vt)\gamma \\ y' &= y, \\ z' &= z \\ t' &= \left(t - \frac{xv}{c^2}\right)\gamma \end{aligned}}$$

⊙ if S' moves in y direction

$$\boxed{\begin{aligned} y' &= (y - vt)\gamma \\ x' &= x \\ z' &= z \\ t' &= \left(t - \frac{yv}{c^2}\right)\gamma \end{aligned}}$$

here $\gamma = \dfrac{1}{\sqrt{1 - \dfrac{v^2}{c^2}}}$ ⇒ Lorentz factor.

*Imp

If there are two Events, then difference in Co-ordinates and time of two Events is written as

$$\boxed{\begin{aligned} \Delta x' &= (\Delta x - v\Delta t)\gamma, & \Delta y' &= \Delta y \\ \Delta t' &= \left(\Delta t - \Delta x \cdot \frac{v}{c^2}\right)\gamma, & \Delta z' &= \Delta z \end{aligned}}$$

→ These transformation Relations Can be used to calculati Co-ordinate and time in S' frame when Values in S frame are known and Vice-Versa.

* Inverse lorrentz transformation :

$$x = (x' + vt')\gamma$$

$$t = \left(t' + \frac{x'v}{c^2}\right)\gamma$$

$$y = y'$$

$$z = z'$$

* Properties of Lorrentz transformation :

1) two Successive L.T's are Equivalent to a single LT.

② * Imp. $\boxed{x'^2 + y'^2 + z'^2 - c^2t'^2 = x^2 + y^2 + z^2 - c^2t^2}$ Valid for all direction of motion of s'.

3) $(\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - c^2(\Delta t')^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - c^2(\Delta t)^2$

4) If we do two Successive L.T with Velocities v_1 and v_2, Then it will be Equivalent to a single L.T with velocity $\dfrac{v_1 + v_2}{1 - \dfrac{v_1 v_2}{c^2}}$

(5) Property ③ Can also be written as

* $\boxed{(\Delta r)^2 - c^2(\Delta t)^2 = (\Delta r')^2 - c^2(\Delta t')^2}$

$\downarrow$

We prefer using this formula in those Questions in which v is not given

(5) Simultaneous event in S frame $\Delta t = 0$

Simultaneous event in S' frame $\Delta t' = 0$

(7) proper time interval in S frame $\Delta x = \Delta y = \Delta z = 0$

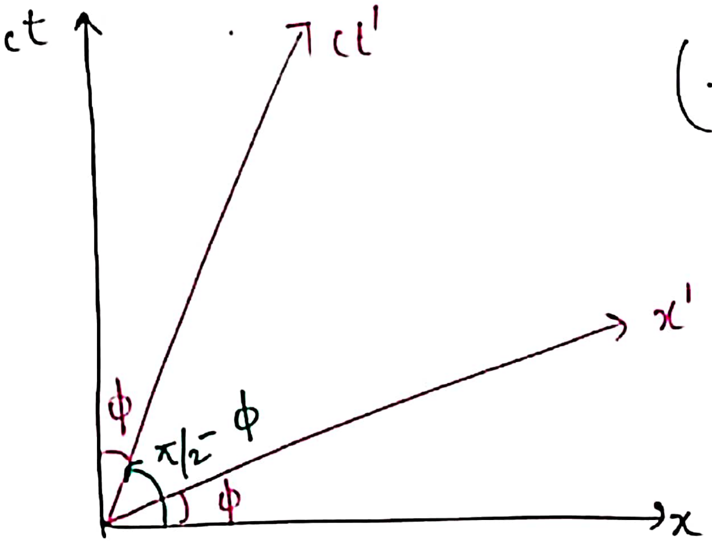
proper time interval in S' frame $\Delta x' = \Delta y' = \Delta z' = 0$

Time interval b/w two Events is proper, if the two Events take place at the Same location.

If Interval is not proper in one frame, it can be proper in Another frame.

(8) L.T can be seen as Rotation of Co-ordinate axes.

$$x' = (x - vt)\gamma, \quad \& \quad t' = \left(t - \frac{xv}{c^2}\right)\gamma, \quad ct' = \left(ct - \frac{xv}{c}\right)\gamma$$



$$\left(\tan\phi = \frac{v}{c}\right)$$

$$\left(\theta = \frac{\pi}{2} - \phi\right)$$

Imp ☆ ★ Velocity transformation / Addition formula: (used in Every other topic)

These are relations b/w Components of Velocity as Measured from two Inertial frame of Reference.

$$\boxed{v'_x = \frac{v_x - v}{1 - \frac{v_x v}{c^2}}} \quad \boxed{v'_y = \frac{v_y\sqrt{1 - \frac{v^2}{c^2}}}{\left(1 - \frac{v_x v}{c^2}\right)}} \quad \boxed{v'_z = \frac{v_z\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v_x v}{c^2}}}$$

if S' moves in x direction w.r.t S (denominator of all 3 are Same)

* Inverse transformation formula for velocity: ($s' \rightarrow s$)

$$\boxed{v_x = \frac{v_x' + v}{1 + \frac{v_x' v}{c^2}}} \quad \boxed{v_y = \frac{v_y'\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v_x' v}{c^2}}} \quad \boxed{v_z = \frac{v_z'\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{v_x' v}{c^2}}}$$

Similarly if s' moves in y direction, then velocity transformation

$$\boxed{v_x' = \frac{v_x\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v_y \cdot v}{c^2}}} \quad \boxed{v_y' = \frac{v_y - v}{1 - \frac{v_y v}{c^2}}} \quad \boxed{v_z' = \frac{v_z\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v_y v}{c^2}}}$$

& so on

→ All questions Related to velocity transformation can be solved using these formulae.

* 1-D Motion:

* $$\boxed{V_{AB} = \frac{V_A - V_B}{1 - \frac{V_A V_B}{c^2}}}$$

or if three as I.F.R are Involved (A,B,C), then

* $$\boxed{V_{AB} = \frac{V_{AB} - V_{BC}}{1 - \frac{V_{AC} V_{BC}}{c^2}}}$$

as we know V_{xy} = velocity of x w.r.t y

frequently used symbols }

w.r.t ≡ in ≡ from
w.r.t a frame
in a frame
from a fram

$V_{lo} = c$

$V_{ol} = -c$

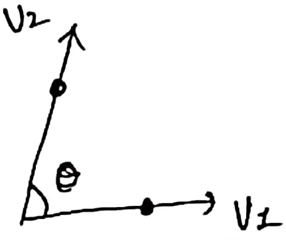
that means

$V_{AB} = -V_{BA}$

* We can calculate Relative Velocity b/w two of objects if velocities of both are given w.r.t same frame.

✯ Relative velocity of two particles Moving at some angle : [Relativistic Case]

$$\boxed{v_{21} = \frac{\sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos\theta - \frac{v_1^2 v_2^2 \sin^2\theta}{c^2}}}{1 - \frac{v_1 v_2 \cos\theta}{c^2}}}$$



v_1, v_2 → must be known w.r.t same frame

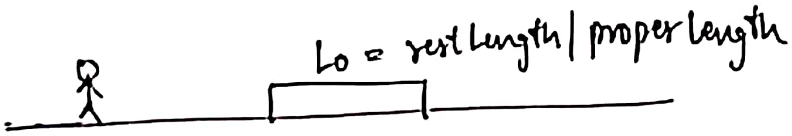
For Non-Relativistic Case.

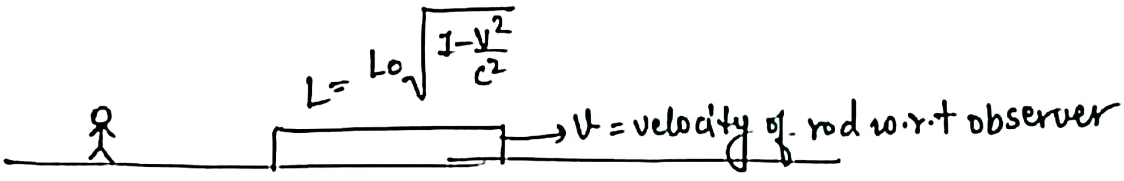
$$V_{12} = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos\theta}$$

$$\boxed{\left.\begin{array}{l}\text{Speed of Light in glass/medium of}\\ \text{Refractive index } n \text{ is}\end{array}\right\} = \frac{c}{n}}$$

✯ Conclusions drawn from L.T :-

1) Length Contraction:





☆ Imp.

$$L = l_0\sqrt{1 - \frac{v_{21}^2}{c^2}}$$

~~velocity of~~ · v_1, v_2 → w.r.t ground

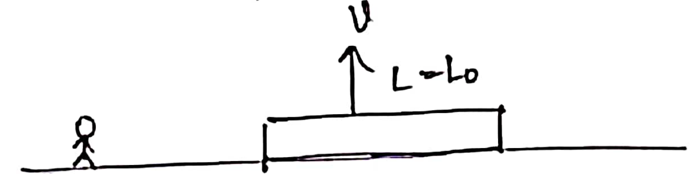
$$V_{21} = \frac{V_2 - V_1}{1 - \frac{V_2 V_1}{c^2}}$$

If we put V_{21}, then we get

$$\boxed{L = \frac{L_0\sqrt{1 - \frac{V_1^2}{c^2}}\sqrt{1 - \frac{V_2^2}{c^2}}}{1 - \frac{V_1 V_2}{c^2}}}$$

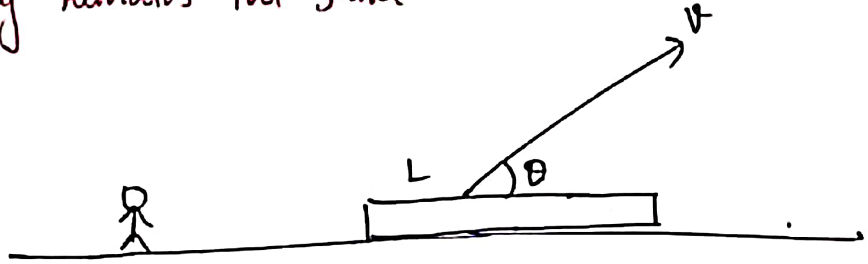
or Simply put V_{21} in L_0

Note: To calculate Length of Moving rod, we must use velocity of rod w.r.t that frame from where Length of rod is being Measured.



[if rod is moving ⊥ to its Length, then its length will Remains Same i.e L = Lo]

→ Length Contraction does not applies to objects moving ⊥ to their Length. They Remains the Same.



$$\boxed{L = L_0\sqrt{1 - \frac{(v\cos\theta)^2}{c^2}}}$$

Note: If we do measurement from Some place then that place appears to be at Rest.

✱ Area Contraction of 2D or planar object:

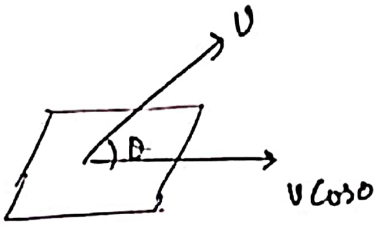
Let A_0 = rest/proper area, when object is moved in Any direction parallel to its plane, then its area becomes

$$\boxed{A = A_0\sqrt{1-\frac{v^2}{c^2}}}$$

→ If object is moved ⊥ to its plane, then its area doesn't change.

→ if object is moved at some Angle (θ) with its plane

$$\boxed{A = A_0\sqrt{1-\frac{(v\cos\theta)^2}{c^2}}}$$



✱ Volume Contraction:

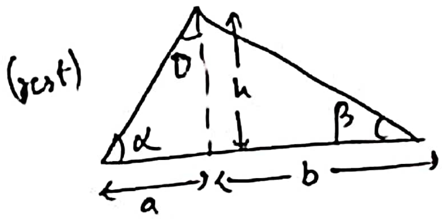
When a 3D object is moved in any direction, then its Volume becomes

$$\boxed{V = V_0\sqrt{1-\frac{v^2}{c^2}}}$$

⇒ V = Volume in motion

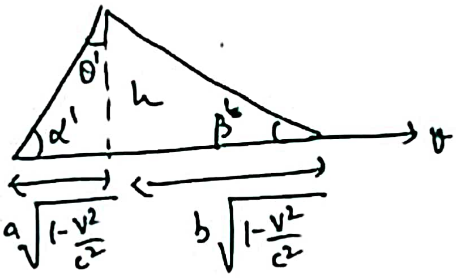
V_0 = Volume at Rest.

✱ Transformation of Geometrical Angle due to motion:



$\tan\alpha = \frac{h}{a}$, $\tan\beta = \frac{h}{b}$, $\tan\theta = \frac{a}{h}$.

When Angle is taken from line of Velocity, then tan (Angle) is multiplied by r, & if Angle is taken from a line ⊥ to line of velocity, then tan (angle) gets divided by r.



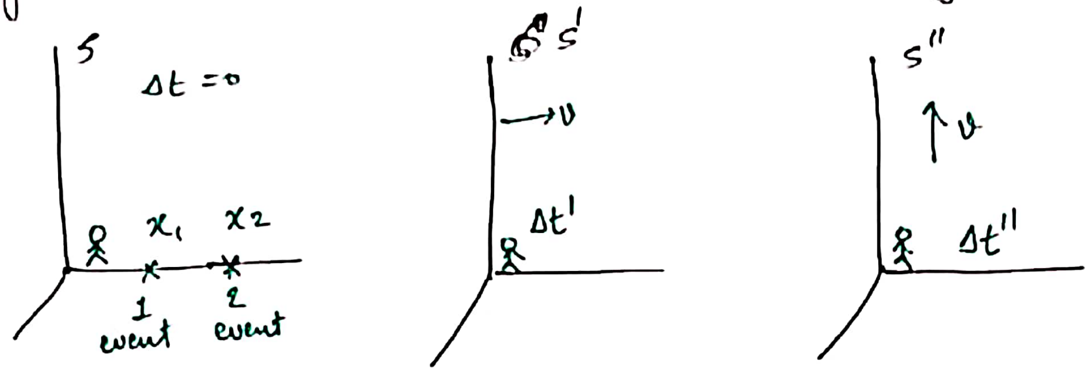
$$\boxed{\tan\alpha' = r\tan\alpha}$$

$$\boxed{\tan\beta = r\tan\beta}$$

$$\boxed{\tan\theta' = \frac{\tan\theta}{r}}$$

* Relativity of Simultaneity of two Events:

If two Events are simultaneous in a frame then they will be simultaneous only in that frame which move ⊥ to line joining the two Events.



in S' $\Delta t' \neq 0$, In S'' $\Delta t'' = 0$

* Time dilation:

According to S.T.R, all process slow down in Motion due to which Completion time of process increases this is called Time dilation.

* Formula for time dilation:

$$\boxed{\Delta t = \frac{\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}} \xrightarrow{(or)} \boxed{t = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$\Delta t'$ or τ → time in Rest frame (proper time)

Δt or t → time Measured by observer w.r.t to which sample is moving

v → velocity of Sample w.r.t observer.

Note: τ or $\Delta t'$ is not directly used in Calculations of something for Moving object. τ or $\Delta t'$ is used for Calculation of Δt or t, & then Δt or t is used for other Calculations.

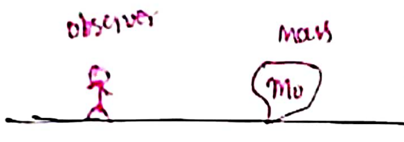
Note:

if 

$$\boxed{t = \frac{\tau}{\sqrt{1 - \frac{v_{12}^2}{c^2}}}}$$ → if both are moving.

✯ Relativistic Dynamics $[v \sim c,\ p \gtrsim m_0 c,\ KE \gtrsim m_0 c^2]$

✯) Variation of Mass with Speed:



m_0 → rest mass

$$\boxed{m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

m = dynamic mass

$$\therefore \boxed{m = \frac{m_0}{\sqrt{1 - \frac{v_{21}^2}{c^2}}}}$$

v_{21} → speed of 2 w.r.t 1

∴ Speed in terms of mass

$$\boxed{v = c\sqrt{1 - \left(\frac{m_0}{m}\right)^2}}$$

1st order Relativistic Correction

$$m = m_0\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$$\rightarrow m = m_0 + \frac{m_0 v^2}{2c^2} + \cdots$$

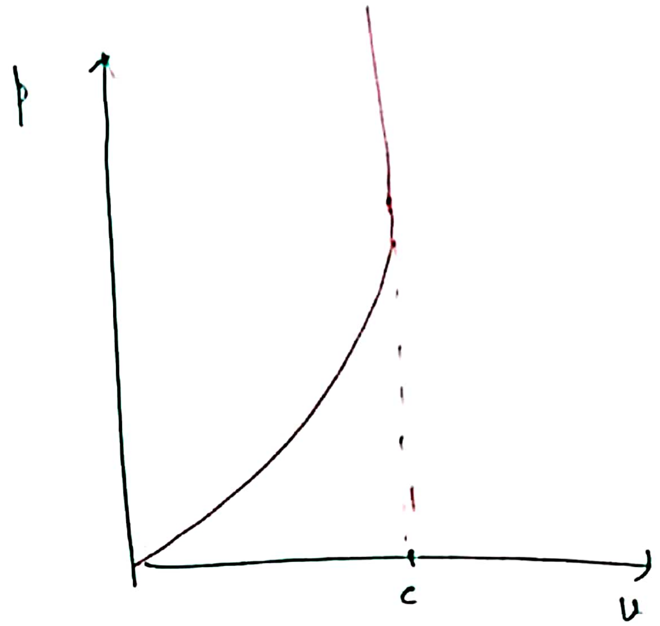
1st order Relativistic Correction to mass $\frac{m_0 v^2}{2c^2}$

Note: If Question states that mass of object is m, then we take it to be Rest mass.

pc:

* Relativistic Momentum:

$$\boxed{p = mv = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}}$$



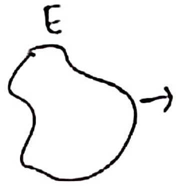
v in terms of p (Invert above Relnl.)

$$\boxed{v = \frac{pc}{\sqrt{m_0^2 c^2 + p^2}}}$$

* Rest Mass Energy:

$$\boxed{\Delta m_0 = \frac{\Delta E_0}{c^2}}$$ → Mass Energy Equivalence

* Energy of Moving object: (free particle)



→ v = velocity of object w.r.t observer.

+J.

$$\boxed{E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Invert.

$$\boxed{v = c\sqrt{1 - \left(\frac{m_0 c^2}{E}\right)^2}}$$

* Relativistic Kinetic Energy:

$$T = \boxed{E - E_0} \Rightarrow \boxed{mc^2 - m_0c^2} \Rightarrow \boxed{\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2}$$

1st order Correction to K.E :

$$T = \underset{\substack{\uparrow \\ \text{Non Relativistic formula}}}{\frac{1}{2}m_0v^2} + \underbrace{\frac{3}{8}m_0\frac{v^4}{c^2}}_{\substack{\downarrow \\ \text{first order Correction term}}}$$

formula used

$$[(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2$$

* Kinetic Energy, Momentum Relation:

* Imp

$$\boxed{p = \frac{1}{c}\sqrt{T(T+2m_0c^2)}}$$

* Energy Momentum Relation for free particle:

$$\boxed{E = \sqrt{p^2c^2 + m_0^2c^4}}$$

if there is force acting on particle, then add P.E.

* Kinetic Energy Momentum Relation:

$$T = E - E_0 \Rightarrow \boxed{T = \sqrt{p^2c^2 + m_0^2c^4} - m_0c^2}$$

1st order Correction

$$\boxed{T = \frac{p^2}{2m_0} - \frac{p^4}{8m_0^3c^2} + \cdots}$$

1st order Correction term $= \frac{-p^4}{8m_0^3c^2}$

used as Relativistic Correction in perturbation theory.

★ Lorrentz transformation of Energy and Momentum:

if S' moves in x- direction.

$$\left.\begin{aligned} E' &= (E - v p_x)\gamma \\ p_x' &= \left(p_x - \frac{Ev}{c^2}\right)\gamma \\ p_y' &= p_y \\ p_z' &= p_z \end{aligned}\right\}$$

$\downarrow$

If S' moves in y. direction

$$\left.\begin{aligned} E' &= (E - v p_y)\gamma \\ p_y' &= \left(p_y - \frac{Ev}{c^2}\right)\gamma \\ p_x' &= p_x \\ p_z' &= p_z \end{aligned}\right\}$$

E formula same as x, & p formula same as t.

Note: There is only one formula for massless particle

$$E = \sqrt{p^2c^2 + \underset{\overset{\downarrow}{0}}{m_0^2}\, c^2} \quad \therefore \quad \boxed{E = pc}$$

★ Relativistic Doppler effect: (Doppler effect of Light)

change in frequency of light due to Relative motion b/w Source and observer is called Doppler effect of Light.

★ Imp

$$\boxed{\nu = \frac{\nu_0\sqrt{1 - \frac{V^2}{c^2}}}{1 - \frac{V}{c}\cos\theta}}$$

$\downarrow$

General formula of Doppler effect.

V = Velocity. of Source w.r.t observer

θ = Angle b/w Source Velocity and line Connecting Source & obser

ν = Apparent frequency.

ν_0 = actual frequency.

* Special Cases of Doppler effect :

(1) Source moving toward observer (i.e $\theta = 0$)

$$\boxed{\nu = \frac{\nu_0 \sqrt{\left(1+\left|\frac{v}{c}\right|\right)}}{\sqrt{\left(1-\left|\frac{v}{c}\right|\right)}}}$$

$\nu > \nu_0$

i.e Apparent frequency is greater than actual frequency.

↓

(when seperation is decreasing)

(2) Source Moving away from observer: (i.e $\theta = 180$)

$$\boxed{\nu = \nu_0 \sqrt{\frac{1-\left|\frac{v}{c}\right|}{1+\left|\frac{v}{c}\right|}}}$$

$\nu < \nu_0$

actual frequency is greater than apparent freq.

or apparant freq is smaller than actual freq.

↓

(used ~~or~~ when Seperation is increasing).

Note: In Case of Doppler effect of Light, it does not Matter, whether source ~~of~~ or observer is Moving.

only Relative velocity b/w the two is Important.

* In terms of Wave length (Doppler effect)

(1) Seperation decreasing

$$\boxed{\lambda = \lambda_0 \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}}$$

$\boxed{\lambda < \lambda_0}$

(2) Seperation Increasing

$$\boxed{\lambda = \lambda_0 \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}}$$

$\boxed{\lambda > \lambda_0}$

→ Apparent Wavelength Increases when seperation Increases and decreases when seperation decreased.

if $v << c$ (in Doppler effect)

$$\boxed{\frac{\nu - \nu_0}{\nu_0} = \frac{v}{c}} \Rightarrow \boxed{\frac{\Delta\nu}{\nu_0} = \frac{v}{c}}$$

Similarly $$\boxed{\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}}$$

Note : $\boxed{|eV| = \text{change in K.E}}$ V = potential diff.
$C = 1.6 \times 10^{-19} C$

Relativistic Dynamics :

*** Questions Related to force :**

* Newton's Eqn.

$F = \frac{dp}{dt}$. if force is constant or function of time
$\int dp = \int F\, dt$

Integrate to of obtain p and then use standard formulae of STR.

* Work - Energy theorem

Work done = change in K.E

$$\boxed{\int \vec{F} \cdot d\vec{l} = KE_f - KE_i}$$

* Force Acceleration Relation.

$$\vec{F} = m_0 \left[\frac{\vec{a}}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{(\vec{v} \cdot \vec{a}) \frac{v^2}{c^2}}{\left(1 - \frac{v^2}{c^2}\right)^{3/2}} \right]$$

→ Special Case of $\vec{F}$ & $\vec{a}$ relation

1) if $\vec{v}$ & $\vec{a}$ are in Same direction or a Straight line motion

$$\boxed{F = \frac{m_0 a}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}} \quad \text{or} \quad \boxed{F = \frac{ma}{\left(1-\frac{v^2}{c^2}\right)}}$$

↓ if Rest mass (Mo) is given

↓ if Dynamic mass (m) is given

2) Uniform Circular Motion:

When $\vec{v}$ = Constant in uniform Circular motion

∴ there is only one acceleration i.e Radial acceleration towards the Centre.

$$\therefore \boxed{\vec{F} = m\vec{a}}$$

↓

$$m = \frac{M_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

In Case of Circular motion, Newton's Eqn for ~~Relativistic~~ Relativistic Case matches with Non-Relativistic formula, only difference is that mass is Replaced by dynamic mass.

★ Relativistic Collision / Breaking:

If in Question of Collision or Breaking, following words are used, then it will be Relativistic Case.

Rest mass, change in mass, initial/Final mass, Zero mass, c, massless particle.

To solve all Such Question, use Conservation of Momentum.

(1) Conservation of Momentum.

$$\Sigma p_i = \Sigma p_f \quad (1\,d)$$

$$(\Sigma p_i)_x = (\Sigma p_f)_x \,, \quad (\Sigma p_i)_y = (\Sigma p_f)_y \quad (2\,d \text{ Case})$$

or Apply Conservation of Momentum In Vector form.

(2) Conservation of Energy

$$\sum E_i = \sum E_f$$

* use appropriate formula for E & p

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{V^2}{c^2}}}, \quad E = \sqrt{p^2 c^2 + m_0^2 c^4}, \quad E = T + m_0 c^2$$

$$p = \frac{m_0 V}{\sqrt{1 - \frac{V^2}{c^2}}}, \quad p = \frac{1}{c}\sqrt{T(T + 2m_0 c^2)}, \quad p = \frac{1}{c}\sqrt{E^2 - M_0^2 c^4},$$

* if Masses Collide and stick:

$$(M_0)_{final} > \sum (m_0)_{initial}$$

[M_0 will be always greater than $\sum m_0$]

if two particles are of same mass m_0,

$$M_0 > 2m_0$$

* If a particle breaks into two or more particle then

$$(M_0)_{initial} > \sum (m_0)_{final}$$

When Internal Energy of a System increases, its mass Increases

* Relativistically Invariant Quantities or Equations:

The physical Quantities (Equations), whose Value (form) Remains Same in all I.F.R are called Relativistically Invariant Quantities (Equations).

The physical Quantities (Equations), whose Value (form) doesn't change under L.T are called Relativistically invariant quantities (Equations).

*List of Invariant quantities / Equations :-

* $\underbrace{c,\ q,\ l_0,\ m_0,\ \tau}_{\text{expt. fact.}}$

* $\boxed{E^2 - c^2p^2,\quad \frac{d^3p}{E} \quad \text{or} \quad \frac{dp_x\, dp_y\, dp_z}{E}}$

* $\boxed{E^2 - c^2B^2}$. $\boxed{\vec{E}\cdot\vec{B}}$,

* phase of wave $(\vec{k}\cdot\vec{r} - \omega t)$

* $\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$, $\xrightarrow{\square^2}$ D. Alembertian

* $x^2+y^2+z^2 - ct$,

* Maxwell's eqn.

* Wave Eqn.

* Current = $\frac{\text{charge}\,(q)}{\text{time}\,(t)}$ $\overset{\text{Invariant}}{\leftarrow}$ (charge), $\leftarrow$ not Invariant (time)

∴ Current is not Relativistically Invariant.

* charge density = $\frac{\text{charge}}{\text{Volume}}$ $\rightarrow$ Invariant (charge), $\rightarrow$ not-Invariant (Volume)

∴ ρ is not Invariant

$$\boxed{\rho = \frac{\rho_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$