NUCLEAR & PARTICLE PHYSICS 2

FUTURE OF PHYSICS

LECTURE 2

# FUTURE OF PHYSICS

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# Ground state static properties of Nuclei

## Masses

Measured with great accurary with mass spectrograph (First made by Aston in 1919)

They can be expressed in different units

- Kilogram (kg)
- Atomic mass unit (amu) or u (unified Atomic mass unit)
- Relativistic unit ($MeV/c^2$)



## Relation between various units

### Atomic mass unit (amu)

Prior to 1961

$1 \text{ amu} = \frac{1}{16}$ times the mass of one Oxygen atom

But after 1961

$1 \text{ amu} = \frac{1}{12}$ times the mass of one carbon atom

We Know 1 mole of $C^{12}$ has mass equal to mass number if expressed in grams

i.e. 1 mole of $C^{12}$ has mass = 12 gm

Also 1 mole has $6.02205 \times 10^{23}$ atoms of $C^{12}$

Thus $6.02205 \times 10^{23}$ atoms of $C^{12}$ has mass = 12 gm

Thus mass of one Carbon $C^{12} = \dfrac{12 \times 10^{-3}\,kg}{6.022 \times 10^{23}}$

Thus $1\,amu = \dfrac{1}{12} \times \dfrac{12 \times 10^{-3}}{6.022 \times 10^{23}} = 1.66 \times 10^{-27}\,kg$

$$\boxed{1\,amu = 1.66 \times 10^{-27}\,kg}$$

In energy units

Energy = mass $\times c^2$

$\Rightarrow$ $1 \text{ amu} \times c^2 = 1.66 \times 10^{-27} \times c^2$ Joule

and $1 \text{ Joule} = \frac{1}{1.6 \times 10^{-13}} \text{MeV}$ $\left[ \begin{array}{l} \text{i.e. } 1\text{eV} = 1.6 \times 10^{-19} \text{J} \\ 1 \text{MeV} = 1.6 \times 10^{-13} \text{J} \end{array} \right]$

$$\Rightarrow 1 \text{amu} \times c^2 = \frac{1.66 \times 10^{-27} \times (3 \times 10^8)^2}{1.6 \times 10^{-13}} \text{MeV}$$

$\Rightarrow$ $\boxed{1 \text{amu} \times c^2 = 931.5 \text{ MeV}}$ 1amu of mass is equivalent to 931.5 MeV of energy

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Rest mass of electron

$$\boxed{m_e = 9.10953 \times 10^{-31}\,\text{Kg}}$$

$$m_e = \frac{9.1 \times 10^{-31}}{1.66 \times 10^{-27}}\,\text{amu} \qquad [\because 1\,\text{amu} = 1.66 \times 10^{-27}\,\text{Kg}]$$

$$\Rightarrow \boxed{m_e = 0.000548\,\text{amu}}$$

To convert into $\text{MeV}/c^2$

$$m_e = \frac{9.1 \times 10^{-31}}{1.66 \times 10^{-27}} \times 931.5 \qquad [\because 1\,\text{amu} \times c^2 = 931.5\,\text{MeV}]$$

$$\Rightarrow \boxed{m_e = 0.511003\,\text{MeV}/c^2}$$

Rest mass of proton

$$\boxed{\begin{aligned} m_p &= 1.67265 \times 10^{-27}\,kg \\ m_p &= 1.00727647\ amu \\ m_p &= 938.2805\ MeV/c^2 \end{aligned}}$$

Mass of neutron

$$\boxed{\begin{aligned} m_n &= 1.67495 \times 10^{-27}\,kg \\ m_n &= 1.008665\ amu \\ m_n &= 939.57077\ MeV/c^2 \end{aligned}}$$

$$\boxed{m_p \simeq m_n \simeq 1\,amu}$$

$m_p$ = mass of H nucleus

mass of H Atom = 1.007825 amu

Atomic mass $(M_A)$ and Nuclear mass $(M_N)$

Atom has Nucleus + Z electrons

Thus $M_A = M_N + Z\, m_e$ − Binding energy of electrons (which is negligible → eV)

Thus $\boxed{M_A = M_N + Z\, m_e}$

## Atomic Mass and Atomic Weight

- Mass spectrograph data gives Atomic masses.
- Atomic mass are approximately equal to their mass numbers.
- Each element has various isotopes having different Atomic masses.
- But each isotope has different % of relative abundance in nature.

e.g. Relative abundance of H isotopes in %

| Symbol | Atomic Mass | Relative Abundance (%) | Atomic Weight |
| --- | --- | --- | --- |
| $^1H$ | 1.0078 amu | > 99.98 | 1.0078 amu (same for all isotopes) |
| $^2H$ | 2.014 amu | 0.0026 to 0.0184 | |
| $^3H$ | 3.016 amu | very small traces | |

Atomic Weight → Weighted Average of the mass of all atoms of an element in accordance to their relative abundance in nature.

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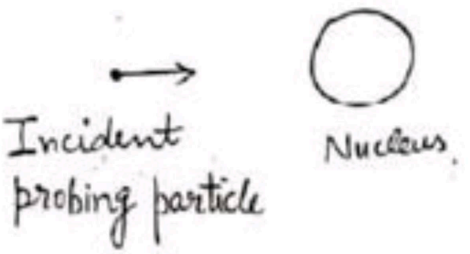
## Nuclear Size (Radius)

Very small size; thus very difficult to measure, various scattering experiments were performed.




The de Broglie wavelength of the probing particle must be less than or equal to the size of the object being studied.

NOTE

① $\underline{\bar{e}}$ → The high energy $e^-$s ($\simeq$ 100 MeV) are suitable to study radius and charge distribution because electrons do not take part in strong interactions.

② Proton → Protons would be used and its beam is readily available. Its disadvantage is that both EM and strong interactions are present in p-n scattering and the results are complex to analyse.

③ Neutron → Neutron as a probe are better than Proton (No EM force). But it is more difficult to generate high energy neutron (thermally accelerated) and measurements are also difficult (take part in nuclear force)

④ Photons → Use of photon as probe is not practical as high energy photons used to interact with nuclei and complicate the problem.

- Atomic dimensions of the order of $10^{-10}$ m
- Nuclear dimensions of the order of $10^{-15}$ to $10^{-14}$ m
- Nuclear dimensions expressed in femtometer (fm) or fermi where $1\,fm = 10^{-15}\,m$

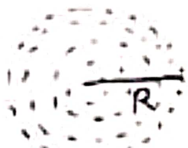
[ mm ($10^{-3}$), μm ($10^{-6}$), nm ($10^{-9}$), pm ($10^{-12}$), fm ($10^{-15}$) ]

## Nuclear Radius

Since there are no sharp boundaries for the nucleus, therefore to determine the radius we define nuclear density

$\rho(r)$ = Very High if $r < R$

= very Low if $r > R$
(Almost zero)




Considering spherical shape only, it is experimentally observed that the density inside the nucleus is constant i.e. $\rho$ = constant

Thus $\dfrac{\text{Mass}}{\text{Volume}}$ = constant

Thus $\frac{\text{Mass}}{\text{Volume}}$ = constant

$\Rightarrow$ Volume $\propto$ Mass [where Mass = mass of one nucleon $\times$ total no. of nucleons $= m \times A$]

$\Rightarrow \frac{4}{3}\pi R^3 \propto m A$

(Let $m_p \simeq m_n \simeq m$)

$\Rightarrow R \propto A^{1/3}$

$\Rightarrow \boxed{R = R_0 A^{1/3}}$

where $R_0$ is constant

NOTE $\rightarrow$ $R_0 = 1.1$ to $1.5$ fm but for numerical if $R_0$ is not given use $R_0 = 1.2$ fm

Page 1/Q3 $V \propto A$ (mass number) $R \propto A^{1/3}$ $V = \frac{4}{3}\pi R^3$

Page 2/Q2 $A_1 = 216$

$A_2 = 64$

$$\frac{R_1}{R_2} = \frac{R_0 (A_1)^{1/3}}{R_0 (A_2)^{1/3}} = \left(\frac{216}{64}\right)^{1/3} = \frac{6}{4} = 1.5$$

(b)

Page 16/Q6 ${}_8O^{16}$ Volume $V = \frac{4}{3}\pi R^3$

$V_{Xe} = \frac{4}{3}\pi R_{Xe}^3$ Divide $\frac{V_{Xe}}{V} = \frac{R_{Xe}^3}{R^3} = \frac{\cancel{R_0^3} A_{Xe}}{\cancel{R_0^3} A_0} = \frac{\cancel{128}^{32}}{\cancel{16}_{4}}$ $(R_0 A_{Xe}^{1/3})^3$

$V_0 = V = \frac{4}{3}\pi R^3$

$= 8$

$V_{Xe} = 8V$ (a)

Page 17/Q1 (A)

Page 19/Q1

## Nuclear density

Nuclear desity is constant (except near the surface of the nuclei)

Mass density, Number density and charge density all three are constant

Mass density ($\rho_M$)

$$\rho_M = \frac{\text{Mass}}{\text{Volume}}$$

If $m_n \simeq m_p = m = 1.67 \times 10^{-27}$ Kg

Thus mass of nucleus $= mA$

Volume $= \frac{4}{3}\pi R^3$ (Spherical shape)

and $R = R_0 A^{1/3}$

$$= \frac{mA}{\frac{4}{3}\pi R^3}$$

$$\Rightarrow \rho_M = \frac{mA}{\frac{4}{3}\pi R_0^3 A}$$

$$\Rightarrow \rho_M = \frac{3m}{4\pi R_0^3}$$

(constant → independent of A)

($R_0 = 1.2$ fm $= 1.2 \times 10^{-15}$ m)

$$= \frac{3 \times 1.67 \times 10^{-27}}{4 \times 3.14 \times (1.2 \times 10^{-15})^3}$$

$$\rho_M = 2.3 \times 10^{17}\ \text{Kg/m}^3$$

$$\Rightarrow \boxed{\rho_M \simeq 10^{17}\ \text{Kg/m}^3}$$

Magnitude is same for different nuclei and is independent of A.

Number density ($\rho_N$)

$$\rho_N = \frac{\text{No. of nucleons}}{\text{Volume}} = \frac{A}{\frac{4}{3}\pi R^3} = \frac{3\cancel{A}}{4\pi R_0^3 \cancel{A}} = \frac{3}{4\pi R_0^3}$$

Putting values $\boxed{\rho_N \simeq 10^{44} \text{ Nucleons}/m^3}$ same for all nuclei.

Thus $\rho_M = \frac{3m}{4\pi R_0^3}$ & $\rho_N = \frac{3}{4\pi R_0^3}$

$\Rightarrow \rho_M = \rho_N \times m$

or Mass density = No. density $\times$ mass of one nucleon

$$10^{17} \simeq 10^{44} \times 10^{-27}$$

Q A nucleus with $A = 235$ splits into two fragments whose mass numbers are in the ratio 3:2. Find the seperation between the fragments at the moment of splitting.

Q If earth would conist of only nuclear matter, then what will be its radius if the mass of the earth is $6 \times 10^{24}$ Kg.

Mean square radius- In spherical polar coordinates if density is given

Q Find mean square radius of a nucleus if charge density is given as

$$\rho(r) = \rho_0 \text{ (constant)}, \quad r \leq R$$
$$= 0, \quad r > R$$

where R is radius & r is distance from centre of nucleus.

Soln: As In Quantum Root mean Square radius

Move from back to front

Thus RMS radius $= \sqrt{\langle r^2 \rangle}$

$$\langle r^2 \rangle = \frac{\int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} r^2\, \rho(r) \,.\, r^2 \sin\theta \, dr\, d\theta\, d\phi}{\int_{r=0}^{\infty} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \rho(r) \,.\, r^2 \sin\theta \, dr\, d\theta\, d\phi}$$

$$= \frac{\cancel{4\pi}\left[\int_{r=0}^{R} \rho_0 \, r^4 \, dr + \int_{r=R}^{\infty} \times 0 \times r^4 \, dr\right]}{\cancel{4\pi}\left[\int_{r=0}^{R} \rho_0 \, r^2 \, dr + 0\right]}$$

$$= \frac{\cancel{\rho_0}}{\cancel{\rho_0}} \frac{\int_{r=0}^{R} r^4 \, dr}{\int_{r=0}^{R} r^2 \, dr} = \left| \frac{r^5/5}{r^3/3} \right|_0^R = \frac{3}{5} R^2$$

Density = constant

$$\boxed{\langle r^2 \rangle = \frac{3}{5} R^2}$$

Q Assuming nuclear radius to given given by $R = R_0 A^{1/3}$, find the number of neutron density in the nucleus with the number of neutrons equal to number of Protons.

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## Radial dependence of Nuclear charge density

By scattering experiments we get form factor $F(q)$ as

$$|F(q)|^2 = \frac{(d\sigma/d\Omega)_{\text{finite size}}}{(d\sigma/d\Omega)_{\text{point charge}}}$$

Form factor is Fourier transform of charge distribution $\rho(r)$.

We can obtain charge distribution by inverse fourier transform of Form factor. This is possible if measurements are made at a sufficiently large number of angles. When this is not possible, a form of density distribution has to be assumed which best fits the experimental data.

Different types of distributions are Uniform, Exponential, Gaussian, Yukawa, Fermi type (Wood-Saxon), Harmonic well etc.

- For ., $^{12}C$, $^{16}O$ best fitted distribution of charge density is Harmonic type.
- For He → Gaussian
- For Medium and heavy nuclei → best fitted distribution is Fermi (Wood-Saxon)

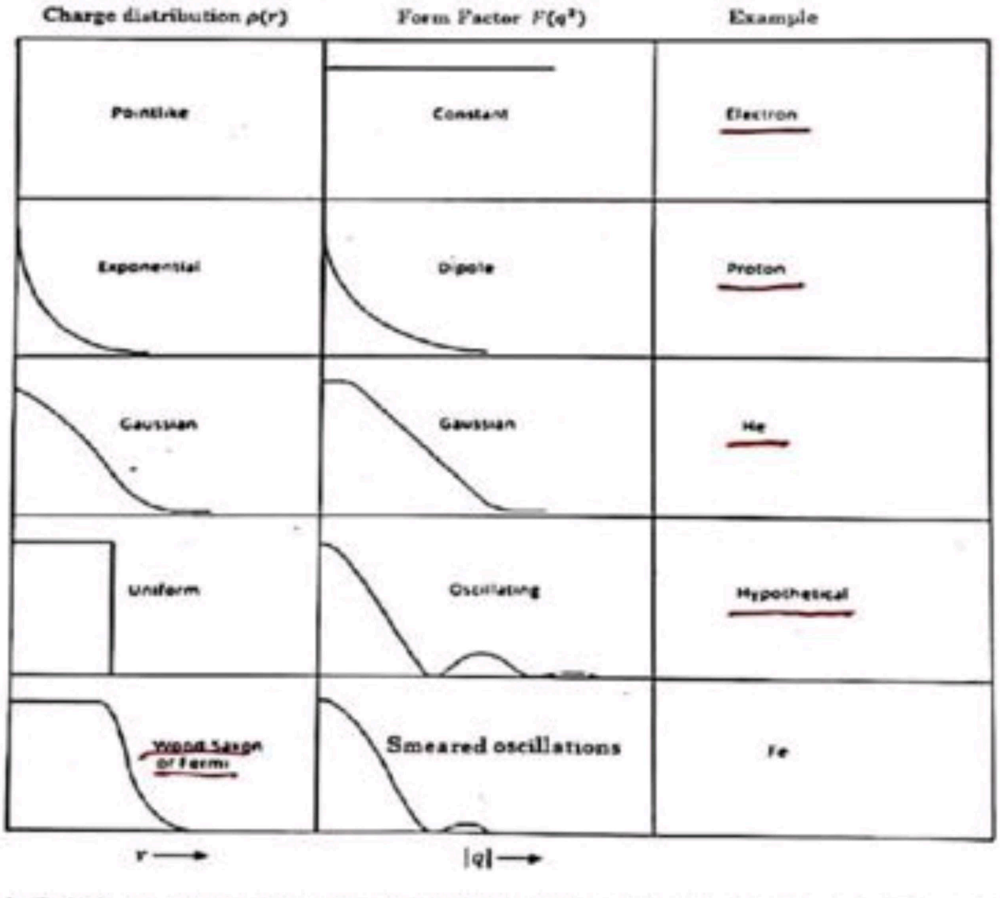
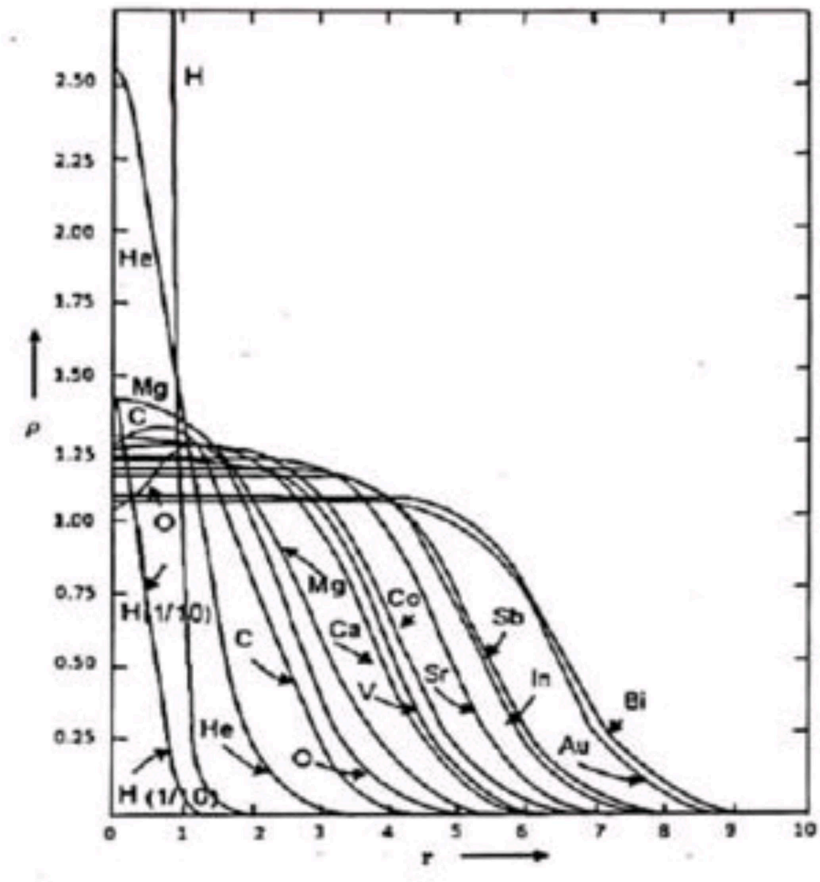



**Fig. 4.4** Relation between the radial charge distribution and the corresponding form factor on Born approximation

**Fig. 4.5** Nuclear charge density as a function of distance from the center of the nucleus found by electron scattering methods. Ordinate unit: $10^{19}$ C cm$^{-3}$ [8]



$A$ and tends to the limiting value of 0.17 nucleon/fm$^3$.

$\rho_{charge}$ decreases as Z increases

Approximately
$\rho(r) = \left(\frac{A}{Z}\right) \rho_{charge}(r)$

$\frac{A}{Z} \uparrow$ as $Z \uparrow$
$\rho_{charge}(r) \downarrow$ as $Z \uparrow$
Thus product remain constant

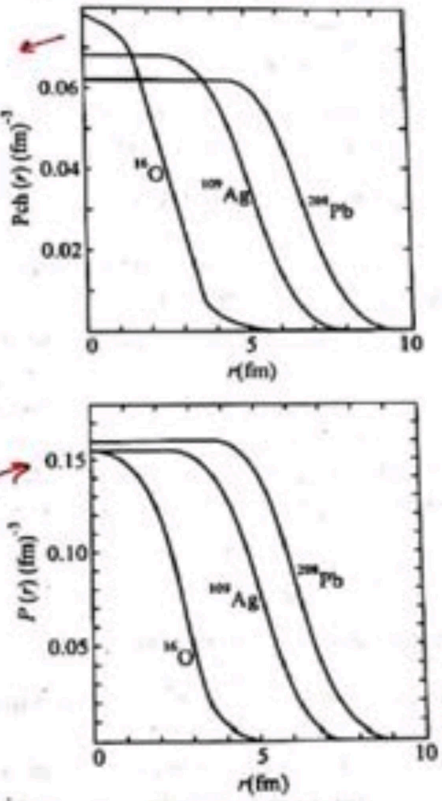



**Fig. 2.6.** Experimentally determined (*a*) nuclear charge and (*b*) nucleon distributions.

Fermi (or Wood-Saxon) distribution - Radial dependence

Suitable for medium and heavy nuclei. (not for lighter nuclei)

$$\boxed{\rho(r) = \frac{\rho_0}{1 + e^{(r-c)/a}}}$$

r is distance from centre.

① Charge density in nuclei is constant within nuclear interior & falls rapidly to zero at nuclear surface.

② c → is half way radius, where density falls to half its value that at centre.

At the centre $r=0$, then

$$\rho(r) = \frac{\rho_0}{1 + e^{-c/a}}$$ but $e^{-c/a}$ is very small as compared to 1 (as $c >> a$)

Thus $\rho(r) \simeq \rho_0$ at $r=0$ (centre of the nucleus)

Now for c

If $r = c$

then $\rho(r) = \frac{\rho_0}{1+e^0} = \frac{\rho_0}{2}$

Thus c is distance from centre where density falls to half than at centre

This half value radius $c \propto A^{1/3} \Rightarrow \boxed{c = 1.07\,A^{1/3}\ \text{fm}}$

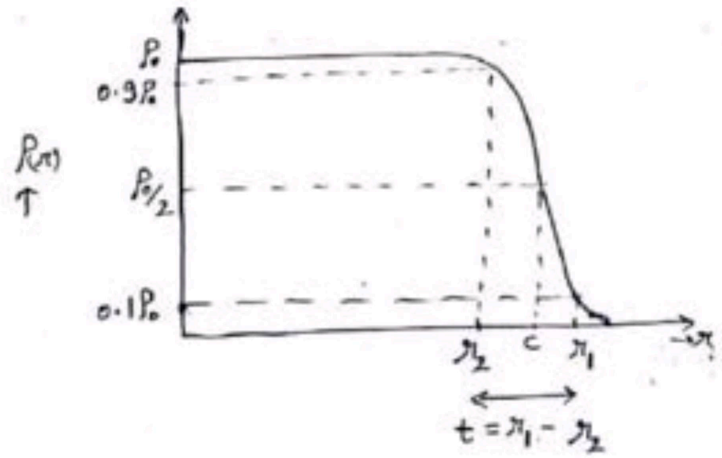
Thus c is different for different nuclei

(3) a → is a constant and is called diffuseness. It is same (constant) for all nuclei. a has a relation with thickness t.

Thickness t → of a nuclear surface is defined as the distance between the points at 10% and 90% of the maximum density $\rho_0$ at the centre.

This thickness is also same for all nuclei.

$\boxed{t \simeq 4.4\,a}$ & $\boxed{a = 0.55\ \text{fm}} \Rightarrow \boxed{t = 2.42\ \text{fm}}$ for all nuclei



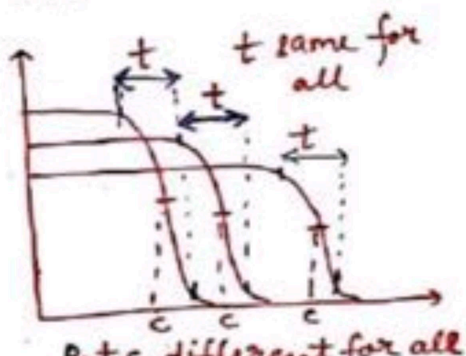

- Note the values of a & t calculated here are not for lighter nuclei as their radius is less than 2.4 fm.
- Also Wood Saxon distribution is for medium and heavy nuclei

At $r = r_1$ $\rho(r) = 10\%$ of $\rho_0 = 0.1\rho_0$ Putting value in $\rho(r)$

$$\Rightarrow \rho(r) = \frac{\rho_0}{1 + e^{(r-c)/a}} \Rightarrow 0.1\rho_0 = \frac{\rho_0}{1 + e^{(r_1-c)/a}}$$

$$\Rightarrow 1 + e^{(r_1-c)/a} = \frac{1}{0.1} = 10$$

$$\Rightarrow e^{(r_1-c)/a} = 9 \quad \text{—} ①$$



At $r = r_2$ $\quad \rho(r) = 90\%$ of $\rho_0 = 0.9\rho_0$

Thus $\quad 0.9\rho_0 = \dfrac{\rho_0}{1+e^{(r_2-c)/a}}$

$$\Rightarrow \quad 1+e^{(r_2-c)/a} = \frac{1}{0.9} = \frac{10}{9} \Rightarrow e^{(r_2-c)/a} = \frac{1}{9} \quad \text{--(2)}$$

Dividing (1) by (2), we get

$\exp\left(\dfrac{r_1-r_2}{a}\right) = 9^2$ $\quad$ Taking natural log (base e)

$$\Rightarrow \left(\frac{r_1-r_2}{a}\right) = 2\ln 9 = 2\ln(3^2) = 4\ln 3$$

Thus thickness $t = r_1 - r_2 = 4a\ln 3$ $\qquad (\ln 3 = 1.0986)$

$\Rightarrow t = 4.394a \Rightarrow \boxed{t = 4.4a}$

Experiment value of $t$ from distribution is $\underline{t = 2.4\text{ fm}}$ for all nuclei.

Thus $\underline{a = 0.55\text{ fm for all nuclei}}$

**Q** The possible isotopes of Boron are $_5B^{10}$ and $_5B^{11}$. The Atomic weight of this mixture is 10.82 amu. The percentage of each isotope present in nature are

(a) 50% ($_5B^{10}$), 50% ($_5B^{11}$) (b) 18% ($_5B^{10}$), 82% ($_5B^{11}$)

(c) 32% ($_5B^{10}$), 68% ($_5B^{11}$) (d) 26% ($_5B^{10}$), 74% ($_5B^{11}$)

1. If the nuclear radius of $Al^{27}$ is 3.6 fermi, the approximate nuclear radius of $Cu^{64}$ (in fermi) is
(a) 4.8 (b) 3.6 (c) 2.4 (d) 1.2

2. Assuming the nuclear radius R for an atomic nucleus of mass A, to be given as $R = R_0 A^{1/3}$, where $R_0 = 1.2 \times 10^{-15}$ m. The number of nucleons per unit volume is equal to
(a) $1.38 \times 10^{22}$ nucleons/$m^3$ (b) $1.38 \times 10^{12}$ nucleons/$m^3$
(c) $1.38 \times 10^{11}$ nucleons/$m^3$ (d) $1.38 \times 10^{44}$ nucleons/$m^3$

3. If the charge density of the nucleus is given as following: $P(r) = P_0 e^{-r/a}$. The r.m.s. charge radius of the nucleus is
(a) $a$ (b) $\sqrt{2}\,a$ (c) $\sqrt{3}\,a$ (d) $2\sqrt{3}\,a$

4. If the radius of the nucleus is 3.46 fm, then the mass number A of the nucleus is
(a) 23 (b) 24 (c) 25 (d) 26

5. The nuclear density of $U^{235}$ is ($R_0$ = 1.2 fm, mass of 1 nucleon = $1.66 \times 10^{-27}$kg)
(a) $2.29 \times 10^{17}$ kg/$m^3$ (b) $1.29 \times 10^{17}$ kg/$m^3$
(c) $3.24 \times 10^{17}$ kg/$m^3$ (d) $5.31 \times 10^{17}$ kg/$m^3$

6. The atomic mass of deuteron is 2.014103 amu. The packing fraction of deuteron is
(a) 0.00608 (b) 0.00705 (c) –0.00705 (d) –0.00608

7. The binding energy per nucleon is given by

[Here f → packing fraction, $M_H$ → Mass of hydrogen atom, $M_N$ → Mass of neutron]



(a) $\frac{Z}{A}(M_H - M_N) + M_N + (1 + f)$ amu (b) $\frac{Z}{A}(M_H - M_N) + M_H + (1 + f)$ amu

(c) $\frac{Z}{A}(M_H - M_N) + M_H - (1 + f)$ amu (d) $\frac{Z}{A}(M_H - M_N) + M_N - (1 + f)$ amu

8. If the charge density of the nucleus is given as following

$\rho(r) = \rho_0$ for $r \le R$

$= 0$ for $r > R$, where $R$ is radius of the nucleus. The mean square charge radius of the nucleus is

(a) $0.4\ R^2$ (b) $0.6\ R^2$ (c) $0.8\ R^2$ (d) $R^2$

9. Consider the following statements

P) If the nucleus radius is doubled, then the mass number is increased by 4 times

Q) The mass number of an element whose radius is 2.71 fm ($r_0$ = 1.3 fm) is 9

R) If the radius of $^{165}$Ho nucleus is 7.731 fm, the radius of $^4$He is 2.23 fm

Which of the following statements is/are CORRECT?

(a) P and Q (b) Q and R (c) P and R (d) P, Q and R