LECTURE 3 NUCLEAR & PARTICLE PHYSICS

FOR FUTURE OF PHYSICS

IIT JAM,TIFR,JEST

FUTURE OF PHYSICS

FORUM FOR IIT JAM,GATE,JEST,TIFR,NET,DU,JNU

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Radial dependence of Nuclear charge density

By scattering experiments we get form factor $F(q)$ as

$$|F(q)|^2 = \frac{(d\sigma/d\Omega)_{\text{finite size}}}{(d\sigma/d\Omega)_{\text{point charge}}}$$

Form factor is Fourier transform of charge distribution $\rho(r)$.

we can obtain Charge distribution by inverse fourier transform of Form factor. This is possible if measurements are made at a sufficiently large number of angles. When this is not possible, a form of density distribution has to be assumed which best fits the experimental data.

Different types of distributions are Uniform, Exponential, Gaussian, Yukawa, Fermi type (Wood-Saxon), Harmonic well etc.

- For, ^{12}C, ^{16}O bet fitted distribution of charge density is Harmonic type.
- For He → Gaussian
- For Medium and heavy nuclei → best fitted distribution is Fermi (Wood-Saxon)

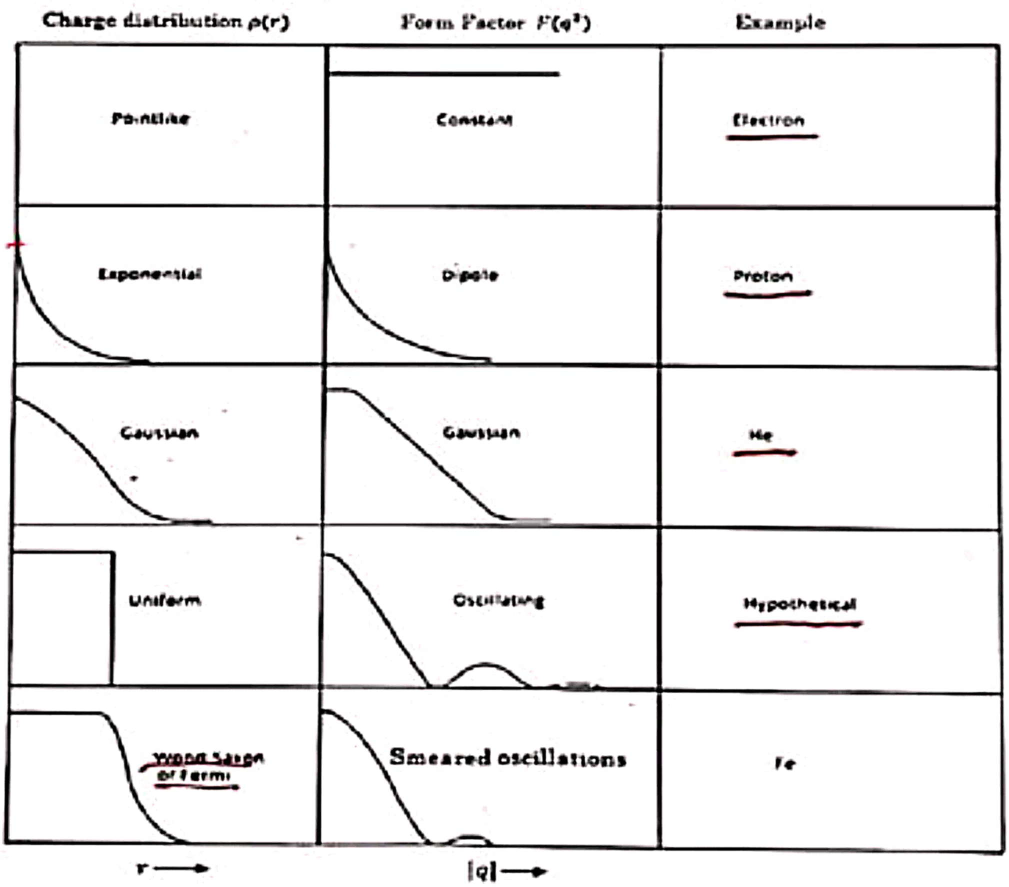
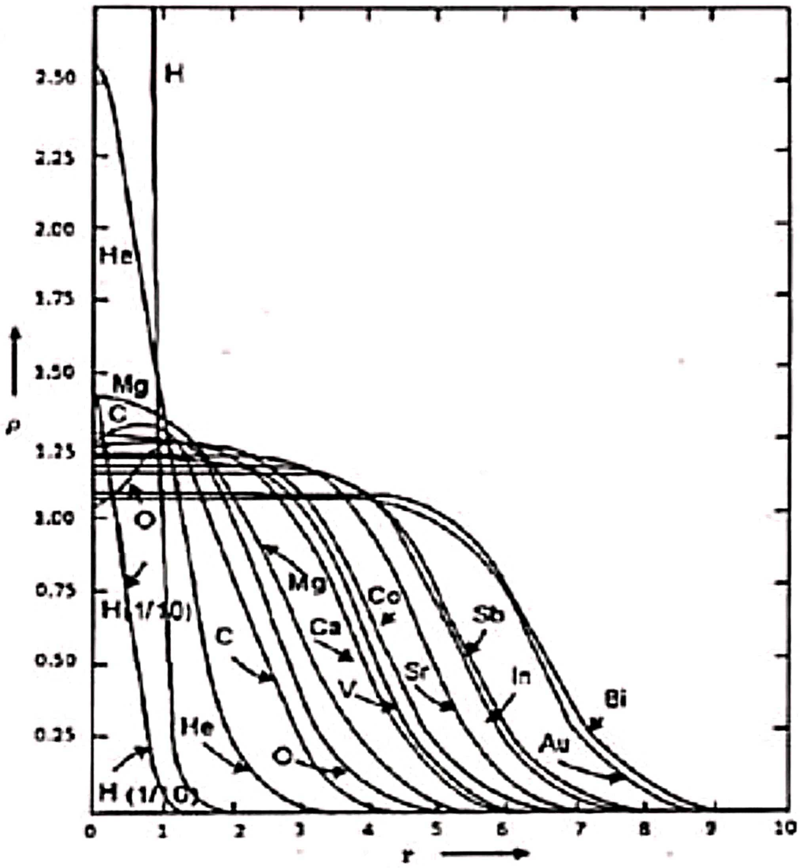


Fig. 4.4 Relation between the radial charge distribution and the corresponding form factor on Born approximation

Fig. 4.5 Nuclear charge density as a function of distance from the center of the nucleus found by electron scattering methods. Ordinate unit: 10^{19} $\mathrm{C\,cm^{-3}}$ [8]



A and tends to the limiting value of 0.17 nucleon/fm³.

ρ_{charge} decreases as Z increases

Approximately
$\rho(r) = \left(\frac{A}{Z}\right) \rho_{charge}(r)$

$\frac{A}{Z}$ ↑ as Z ↑
$\rho_{charge}(r)$ ↓ as Z ↑
Thus product remain constant

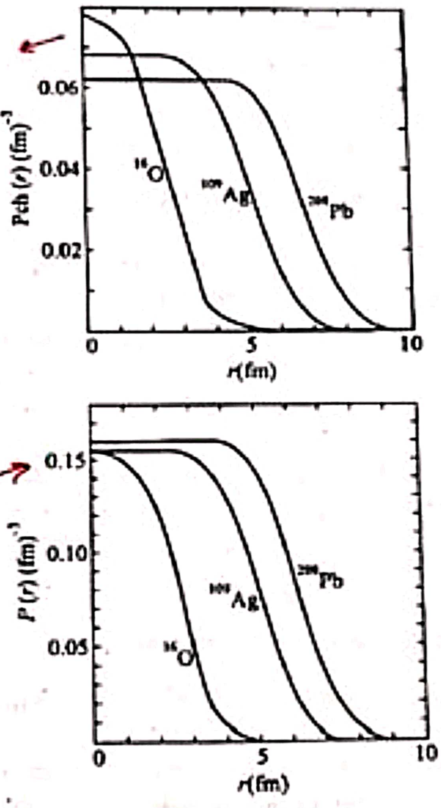


Fig. 2.6. Experimentally determined (*a*) nuclear charge and (*b*) nucleon distributions.

Fermi (or Wood-Saxon) distribution – Radial dependence

Suitable for medium and heavy nuclei. (not for lighter nuclei)

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-c)/a}}$$

r is distance from centre.

① Charge density in nuclei is constant within nuclear interior & falls rapidly to zero at nuclear surface.

② $c \rightarrow$ is half way radius, where density falls to half its value that at centre.

At the centre $r = 0$, then

$$\rho(r) = \frac{\rho_0}{1 + e^{-c/a}}$$

but $e^{-c/a}$ is very small as compared to 1 (as $c >> a$)

Thus $\rho(r) \simeq \rho_0$ at $r = 0$ (centre of the nucleus)

Now for c

If $r = c$

then $\rho(r) = \frac{\rho_0}{1+e^0} = \frac{\rho_0}{2}$

Thus c is distance from centre where density falls to half than at centre

This half value radius $c \propto A^{1/3} \Rightarrow$ $\boxed{c = 1.07\, A^{1/3}\ fm}$

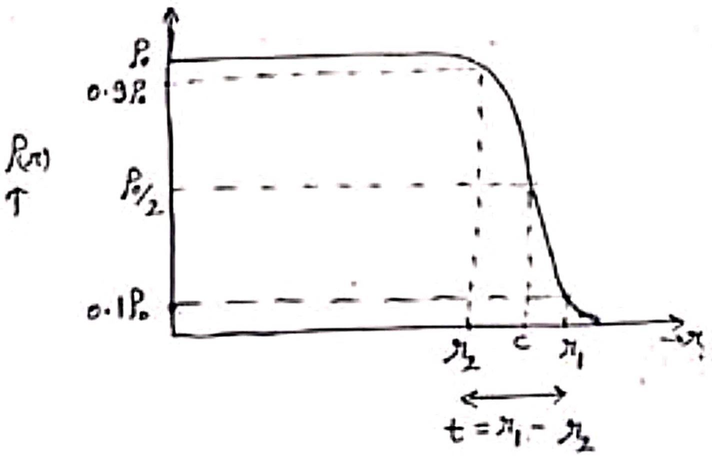
Thus c is different for different nuclei

③ a → is a constant and is called diffuseness. It is same (constant) for all nuclei. a has a relation with thickness t.

Thickness t → of a nuclear surface is defined as the distance between the points at 10% and 90% of the maximum density ρ_0 at the centre.

This thickness is also same for all nuclei.

$\boxed{t \simeq 4.4\, a}$ & $\boxed{a = 0.55\ fm} \Rightarrow \boxed{t = 2.42\ fm}$ for all nuclei



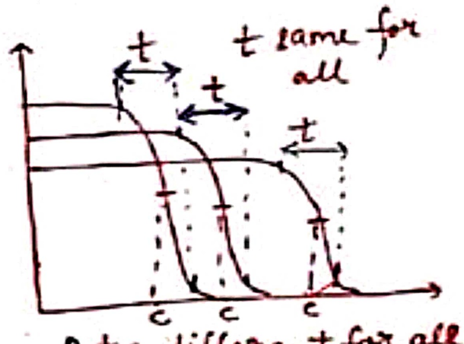
- Note the values of a & t calculated here are not for lighter nuclei as their radius is less than 2·4 fm.
- Also Wood Saxon distribution is for medium and heavy nuclei

At $r = r_1$ $\quad \rho(r) = 10\%$ of $\rho_0 = 0.1\rho_0$ $\quad$ Putting value in $\rho(r)$

$$\Rightarrow \rho(r) = \frac{\rho_0}{1 + e^{(r-c)/a}} \Rightarrow 0.1\rho_0 = \frac{\rho_0}{1 + e^{(r_1-c)/a}}$$

$$\Rightarrow 1 + e^{(r_1-c)/a} = \frac{1}{0.1} = 10$$

$$\Rightarrow e^{(r_1-c)/a} = 9 \quad \text{—} \textcircled{1}$$



At $r = r_2$ $\qquad \rho(r) = 90\%$ of $\rho_0 = 0.9\,\rho_0$

Thus $\quad 0.9\,\rho_0 = \dfrac{\rho_0}{1+e^{(r_2-c)/a}}$

$$\Rightarrow \quad 1+e^{(r_2-c)/a} = \frac{1}{0.9} = \frac{10}{9} \quad \Rightarrow e^{(r_2-c)/a} = \frac{1}{9} \quad \text{--- (2)}$$

Dividing (1) by (2), we get

$$\exp\left(\frac{r_1-r_2}{a}\right) = 9^2$$

Taking natural log (base e)

$$\Rightarrow \left(\frac{r_1-r_2}{a}\right) = 2\ln 9 = 2\ln(3^2) = 4\ln 3$$

Thus thickness $t = r_1 - r_2 = 4a\ln 3$ $\qquad (\ln 3 = 1.0986)$

$\Rightarrow t = 4.394a \Rightarrow \boxed{t = 4.4a}$

Experiment value of t from distribution is $t = 2.4$ fm for all nuclei.

Thus $a = 0.55$ fm for all nuclei

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$$F(q^2) = \exp\left[-\frac{q^2}{2Q^2}\right], \quad F(q^2) = \frac{4\pi}{q}\int_0^\infty r\,dr\,\rho(r)\sin(qr)$$

and $\int \rho(r)\, d^3r = 1$

$\sqrt{\langle (r)^2 \rangle} = ?$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$$

$$\frac{1}{2!}\left(\frac{q^2}{2Q^2}\right)^2$$

$$F(q^2) = 1 - \frac{q^2}{2Q^2} + \frac{q^4}{8Q^4} + \cdots \quad (1)$$

$$\sin(qr) = qr - \frac{(qr)^3}{3!} + \cdots$$

$$F(q^2) = \frac{4\pi}{q}\left[\int_0^\infty r\,\rho(r)\,(qr)\,dr - \int_0^\infty r\,\rho(r)\frac{(qr)^3}{3!}\,dr + \cdots\right]$$

$$= \frac{1}{q}\left[\int_V q\,\rho(r)\,\underbrace{r^2 \sin\theta\, dr\, d\theta\, d\phi}_{d^3r} - \int_V \frac{r^2\,\rho(r)\,q^3\cdot r^2 \sin\theta\, dr\, d\theta\, d\phi}{6} + \cdots\right]$$

$$\langle r^2 \rangle = \frac{\int_V r^2 \rho(r)\, d^3r}{\int_V \rho(r)\, d^3r}$$

$$F(q^2) = \int_V \rho(r)\, d^3r - \frac{q^2}{6}\int_V r^2 \rho(r)\, d^3r + \cdots$$

$$= 1 - \frac{q^2}{6}\langle r^2 \rangle + \cdots \quad (2)$$

Equating coefficient of $q^2 \Rightarrow -\frac{1}{2Q^2} = -\frac{\langle r^2 \rangle}{6} \Rightarrow \langle r^2 \rangle = \frac{3}{Q^2}$

$$\sqrt{\langle r^2 \rangle} = \sqrt{3}/Q$$

Useful constants in Nuclear Physics

Value of $\hbar c$:

$$\hbar c = \frac{hc}{2\pi} = \frac{6.626 \times 10^{-34}\ \text{J-sec} \times 3\times 10^{8}\ \text{m/sec}}{2\times 3.14} = 3.1855\times 10^{-26}\ \text{J-m}$$

$$= \frac{3.1855\times 10^{-26}}{1.6\times 10^{-13}} \times 10^{15}\ \text{MeV-fm} = 197.3\ \text{MeV-fm}$$

imp ⇒ $\boxed{\hbar c = 197.3\ \text{MeV-fm}}$

Value of $e^2/4\pi\varepsilon_0$

bt $\boxed{\dfrac{e^2}{4\pi\varepsilon_0} = 1.44\ \text{MeV-fm}}$

In esu units

$\frac{1}{4\pi\varepsilon_0} = 1 \Rightarrow e^2 = 1.44\ \text{MeV-fm}$

Since Potential enery (Unit of Joule)

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \Rightarrow \frac{1}{4\pi\varepsilon_0} q_1 q_2 = U\cdot r\ \text{(Joule-m)}$$

$$\Rightarrow \frac{e^2}{4\pi\varepsilon_0} = 9\times 10^{9} \times (1.6\times 10^{-19})^2\ \text{J-m}$$

$$= \frac{9\times 10^{9} \times (1.6\times 10^{-19})^2}{1.6\times 10^{-13}} \times 10^{15}\ \text{MeV-fm}$$

$$= 1.44\ \text{MeV-fm}$$

Q. What should be the minimum kinetic energy of the electrons and protons to probe the size of $^{40}_{20}Ca$ nucleus?

$D = 2R = 8\ fm$

$R_{Ca} = R_0 A^{1/3} = 1.2\ fm \times (40)^{1/3} = 4\ fm$

$\lambda = \frac{h}{p} \simeq 8\ fm$

$p = \frac{h}{\lambda}$

Total e^- $\boxed{E = mc^2 = m_0c^2 + K}$

$E = \sqrt{p^2c^2 + m_0^2c^4} \Rightarrow E^2 = p^2c^2 + m_0^2c^4$

$E^2 = \left(\frac{h}{\lambda}\right)^2 c^2 + (m_ec^2)^2 = \frac{(hc)^2}{\lambda^2} + (m_ec^2)^2$ $\quad m_ec^2 = 0.511\ MeV$

$\hbar c = 197.3\ MeV\text{-}fm \Rightarrow hc = 197.3 \times 2 \times 3.14 = 1239\ MeV\text{-}fm$

$\lambda = 8\ fm$

$E^2 = \frac{(1239\ MeV\text{-}fm)^2}{(8\ fm)^2} + (0.511)^2 = 23986.5267\ MeV^2$

$E = 154.87\ MeV \Rightarrow KE = E - m_ec^2 = 154.87 - 0.511 = 154.36\ MeV$

Aston Rule and Packing Fraction

1919 → developed Mass Spectrograph to calculate accurate Atomic masses.

Atomic masses are very close to the whole numbers which are actually the mass numbers of the atoms when expressed in amu units.

For $A < 20$ & $A > 180$ } Atomic masses are slightly greater than A (mass number)

For $20 < A < 180$ (intermediate nuclei), Atomic masses are slightly smaller than A.

This departure is called mass defect by Aston.

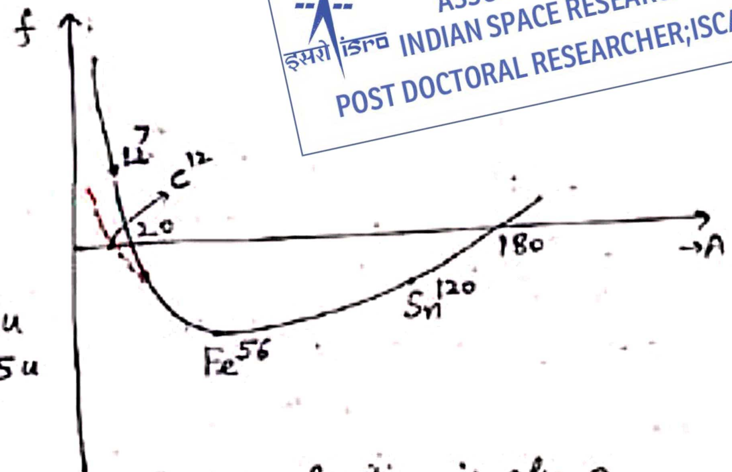
$$\boxed{\Delta M = M_A - A}$$

where M_A is Atomic mass
A is mass number

Paking fraction (f)

$$f = \frac{\Delta M}{A} \Rightarrow \boxed{f = \frac{M_A - A}{A}} \quad \text{OR} \quad f = \frac{M_A}{A} - 1$$

$$\Rightarrow \boxed{M_A = A(1+f)}$$



For $A < 20$ & $A > 180$
f is +ve as $M_A > A$

For $20 < A < 180$
f is −ve as $M_A < A$
with some exceptions

e.g. At. masses of

$H^1 = 1.0078$ amu
$He^4 = 4.0026$ u
$Li^7 = 7.016$ u
$C^{12} = 12$ (by definition)
$N^{14} = 14.003$ u
$O^{16} = 15.994$ u (exception)
$Mg^{24} = 23.985$ u
$Al^{27} = 26.981$ u

$Cl^{35} = 34.96$ u
$Fe^{56} = 55.85$ u

NOTE → Packing fraction is also a measurement of stability and has relation with binding energy.

Binding Energy of a Nuclei

Binding energy is the energy required to break the nucleus into its consituent nucleons.

(p n) Nucleus more stable	+ B.E. supplied	→	o o o o o o P and n seperated
⇓			⇓
More stable mean Less energy			Less stable
⇓			⇓
Means less equivalent mass			More energy
			⇓
			Means more equivalent mass

o o o o o → O + B.E released

n & P → nucleus

Thus combined nuclear mass is less than the sum of the masses of individual nucleons

Binding energy (B) in terms of nuclear mass M_N

Binding energy = mass defect $\times c^2$

Here mass defect = $Z m_p + (A-Z) m_n - M_N$

Thus $$\boxed{B = [Z m_p + (A-Z) m_n - M_N] c^2}$$

m_p - mass of Proton, m_n - mass of neutron, M_N - nuclear mass

Binding energy (B) in terms of atomic mass M_A

Atomic mass $M_A = M_N + Z m_e \Rightarrow M_N = M_A - Z m_e$

neglecting B.E. of electrons. Putting above

$$B = [Z m_p + (A-Z) m_n - (M_A - Z m_e)] c^2$$

$$B = [Z (m_p + m_e) + (A-Z) m_n - M_A] c^2$$

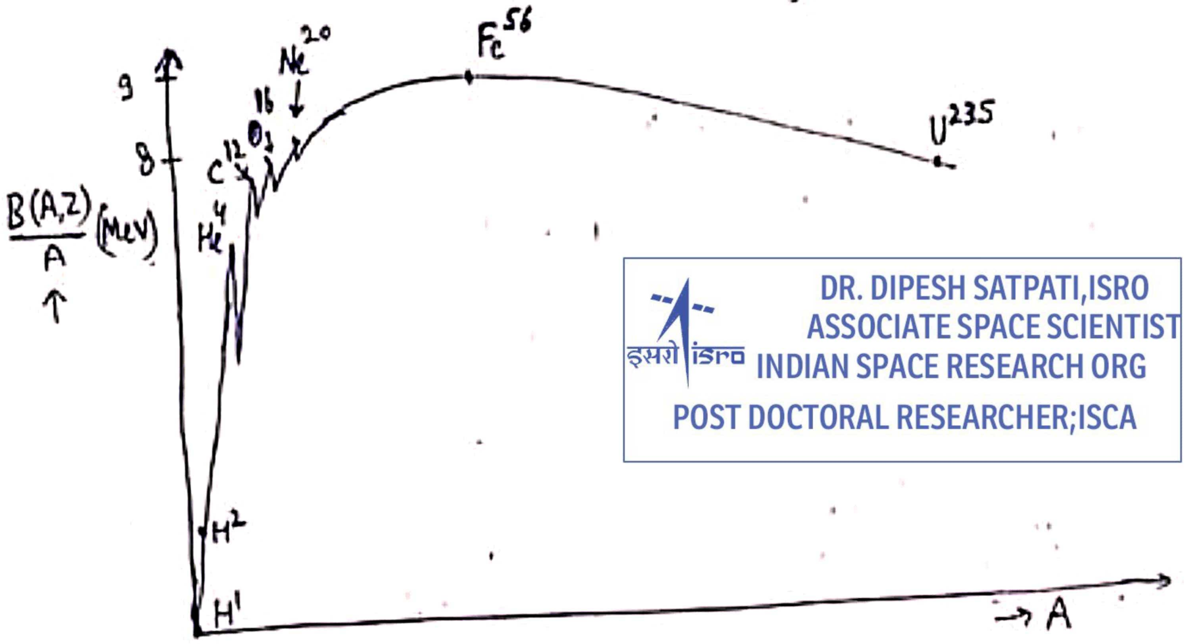
Here $m_p = m_H$ nucleus & $m_p + m_e = m_{H_{Atom}}$

$$\Rightarrow \boxed{B = [Z m_{H_{Atom}} + (A-Z) m_n - M_A] c^2}$$

Mass spectrograph calculates Atomic masses. In numerical generally Atomic masses are given.

Binding energy per nucleon (or Average binding energy)

$$\text{B.E. per nucleon} = \frac{\text{B.E.}}{\text{Total number of nucleons}} = \frac{B(A,Z)}{A}$$



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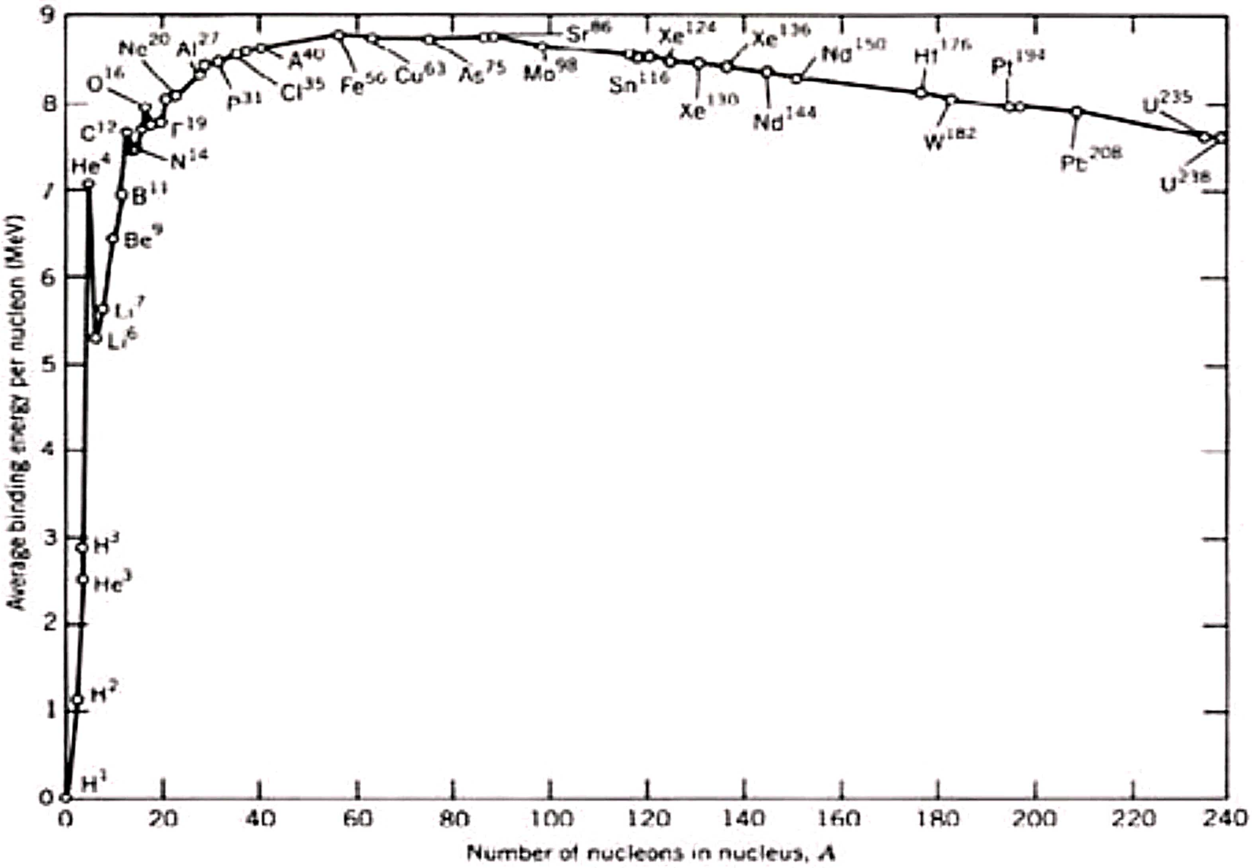

Average binding energy per nucleon (MeV)
Number of nucleons in nucleus, A
H1
H2
He3
H3
Li6
Li7
Be9
B11
He4
C12
N14
F19
O16
Ne20
Al27
P31
Cl35
A40
Fe56
Cu63
As75
Sr86
Mo98
Sn116
Xe124
Xe130
Xe136
Nd144
Nd150
Hf176
W182
Pt194
Pt208
U235
U238

- B(A,Z) per nucleon is almost 8 MeV except some lighter nuclei.
- B.E. per nucleon is maximum for Fe^{56} equal to 8.7 MeV/nucleon.
- For heavy nuclei B.E./A decreases with increasing A.
- For heaviest nuclei, B.E./A is about 7.5 MeV/nucleon.
- For lighter nuclei, there are rapid fluctuations in B.E./A.
- For Even-Even nuclei ^4He, ^8Be, ^{12}C, ^{16}O, ^{20}N etc for which A = 4n where n is integer, peaks are observed. Thus these nuclei are more stable than their neighbours.
- Similar, but less prominent peaks are observed at the values of Z or N equal to 20, 28, 50, 82 and 126. These are magic numbers. (2, 8 already covered for lighter nuclei).

- This curve also explains Nuclear fusion and Nuclear fission.
- Since Nuclear Force are short range, they will act on its neighbours & thus $B.E. \propto A$ and $\frac{B.E.}{A} \simeq$ constant.

  If nuclear force were long range → Then nuclear force act on all nucleons i.e. ${}^{N}C_2 = \frac{N(N-1)}{2} \approx \frac{N^2}{2}$ for large N.

  Then $B.E. \propto A^2$ & $\frac{B.E.}{A} \propto A$
- 13 stable isotopes having p=n → ${}_1H^2, {}_2He^4, {}_3Li^6, {}_5B^{10}, {}_6C^{12}, {}_7N^{14}, {}_8O^{16}, {}_{10}Ne^{20}$ | ${}_{12}Mg^{24}, {}_{14}Si^{28}, {}_{16}S^{32}, {}_{18}Ar^{36}, {}_{20}Ca^{40}$
- Only ${}_1H^1$ and ${}_2He^3$ are stable having p>n.

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Q Calculate B.E. per nucleon of deuteron, H^2 nucleus.

$m_H = 1.007825\ u$, $m_n = 1.008665\ u$, $m_d = 2.014102\ u$ (Atomic mass)

1p
1n

$$B = [Zm_H + (A-Z)m_n - M_A]c^2$$

$$= [1 \times 1.007825 + 1 \times 1.008665 - 2.014102] \times 931.5$$

amu

$$= 2.2244\ MeV$$

$$\frac{B}{A} = \frac{2.2244}{2} = 1.112\ MeV$$

Q Calculate binding energy per nucleon for α particle ($_2He^4$)

Atomic mass of $_2He^4 = 4.002603$ amu, $m_H = 1.007825$ amu,

$m_n = 1.008665$ amu.

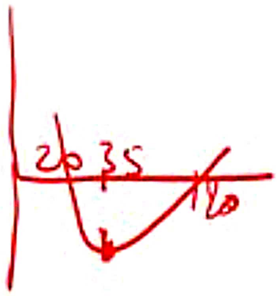
Q The binding energy of $_{17}Cl^{35}$ nucleus is 298 MeV. Find its Atomic mass. The mass of H Atom = 1·007825 amu & m_n = 1·008665 amu. Given 1 amu = 931·5 MeV

$$B.E. = [Z M_H + (A-Z) m_n - M_A] c^2$$

$$298\ \text{MeV} = [17 \times 1{\cdot}007825\ \text{amu} + 18 \times 1{\cdot}008665 - M_A\ (\text{amu})] \times 931{\cdot}5$$

$$\boxed{M_A = 34{\cdot}97\ \text{amu}} < 35$$

$$f = \frac{M_A - A}{A}$$



7. The binding energy per nucleon is given by

[Here f → packing fraction, M_H → Mass of hydrogen atom, M_N → Mass of neutron]

(a) $\frac{Z}{A}(M_H - M_N) + M_N + (1 + f)$ amu (b) $\frac{Z}{A}(M_H - M_N) + M_H + (1 + f)$ amu

(c) $\frac{Z}{A}(M_H - M_N) + M_H - (1 + f)$ amu (d) $\frac{Z}{A}(M_H - M_N) + M_N - (1 + f)$ amu

CAREER ENDEAVOUR

$$f = \frac{M_A - A}{A} = \frac{M_A}{A} - 1 \Rightarrow M_A = A(1+f)$$

$$B = [Z M_H + (A-Z)M_N - M_A] \text{ amu}$$

$$= [Z M_H + (A-Z)M_N - A(1+f)]$$

$$\frac{B}{A} = \frac{Z}{A} M_H + M_N - \frac{Z}{A} M_N - (1+f)$$

$$= \frac{Z}{A}(M_H - M_N) + M_N - (1+f)$$

Q The possible isotopes of Boron are $_5B^{10}$ and $_5B^{11}$. The Atomic weight of this mixture is 10.82 amu. The percentage of each isotope present in nature are

(a) 50% ($_5B^{10}$), 50% ($_5B^{11}$) (b) 18% ($_5B^{10}$), 82% ($_5B^{11}$)

(c) 32% ($_5B^{10}$), 68% ($_5B^{11}$) (d) 26% ($_5B^{10}$), 74% ($_5B^{11}$)

Solve from booklet
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1. If the nuclear radius of Al^{27} is 3.6 fermi, the approximate nuclear radius of Cu^{64} (in fermi) is

   (a) 4.8 (b) 3.6 (c) 2.4 (d) 1.2

2. Assuming the nuclear radius R for an atomic nucleus of mass A, to be given as $R = R_0 A^{1/3}$, where $R_0 = 1.2 \times 10^{-15}$ m. The number of nucleons per unit volume is equal to

   (a) 1.38×10^{22} nucleons/m^3 (b) 1.38×10^{12} nucleons/m^3

   (c) 1.38×10^{11} nucleons/m^3 (d) 1.38×10^{44} nucleons/m^3

3. If the charge density of the nucleus is given as following: $P(r) = P_0 e^{-r/a}$. The r.m.s. charge radius of the nucleus is

   (a) a (b) $\sqrt{2}\,a$ (c) $\sqrt{3}\,a$ (d) $2\sqrt{3}\,a$

4. If the radius of the nucleus is 3.46 fm, then the mass number A of the nucleus is

   (a) 23 (b) 24 (c) 25 (d) 26

5. The nuclear density of U^{235} is ($R_0 = 1.2$ fm, mass of 1 nucleon $= 1.66 \times 10^{-27}$kg)

   (a) 2.29×10^{17} kg/m^3 (b) 1.29×10^{17} kg/m^3

   (c) 3.24×10^{17} kg/m^3 (d) 5.31×10^{17} kg/m^3

6. The atomic mass of deuteron is 2.014103 amu. The packing fraction of deuteron is

   (a) 0.00608 (b) 0.00705 (c) −0.00705 (d) −0.00608

7. The binding energy per nucleon is given by

[Here f → packing fraction, M_H → Mass of hydrogen atom, M_N → Mass of neutron]

(a) $\frac{Z}{A}(M_H - M_N) + M_N + (1 + f)$ amu (b) $\frac{Z}{A}(M_H - M_N) + M_N + (1 + f)$ amu

(c) $\frac{Z}{A}(M_H - M_N) + M_H - (1 + f)$ amu (d) $\frac{Z}{A}(M_H - M_N) + M_N - (1 + f)$ amu

8. If the charge density of the nucleus is given as following

$\rho(r) = \rho_0$ for $r \leq R$

$= 0$ for $r > R$, where R is radius of the nucleus. The mean square charge radius of the nucleus is

(a) $0.4\ R^2$ (b) $0.6\ R^2$ (c) $0.8\ R^2$ (d) R^2

9. Consider the following statements

P) If the nucleus radius is doubled, then the mass number is increased by 4 times

Q) The mass number of an element whose radius is 2.71 fm (r_0 = 1.3 fm) is 9

R) If the radius of ^{165}Ho nucleus is 7.731 fm, the radius of ^{4}He is 2.23 fm

Which of the following statements is/are CORRECT?

(a) P and Q (b) Q and R (c) P and R (d) P, Q and R