CAPSULE PROGRAMME MOCK TEST

TIFR,GATE,IIT JAM

Maximum Marks: 90

INSTRUCTIONS FOR CANDIDATES

1. All questions are compulsory.
2. For each question, one and only one of the four choices given is the correct answer.
3. Each correct answer will be given +3 marks.
4. Each wrong answer will be given -1 mark.
5. Use of calculator is permitted.

Q1. The general solution of the differential equation

$$x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$$

in terms of two arbitrary constants A and B is

(a) $e^{1/x}\left(A\cos\left(\frac{1}{x}\right)+B\sin\left(\frac{1}{x}\right)\right)$ (b) $Ax+\frac{B}{x}$

(c) $Ax+Bxe^{x}$ (d) $Ax+Bx^2$

Q2. If a, b and c are non-zero real numbers not equal to 1, $\log_a c$ can be expressed as

(a) $\log_b c/\log_b a$ (b) $\log_b a/\log_b c$ (c) $\log_c a/\log_b a$ (d) $\log_c b/\log_a b$

Q3. A homogeneous linear transformation takes the point (1, 1) in the xy-plane to the point (3, 3) and keeps the point (1, -1) fixed (i.e., it remains (1, -1) after the transformation). The matrix corresponding to this transformation is

(a) $\begin{pmatrix}1 & 2\\ 2 & 1\end{pmatrix}$ (b) $\begin{pmatrix}3 & 0\\ 0 & 2\end{pmatrix}$ (c) $\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}$ (d) $\begin{pmatrix}2 & -1\\ -1 & 2\end{pmatrix}$

Q4. The function $\dfrac{1}{\cosh x}$ may be expressed around the point $x = 0$ as a power series as

(a) $1 - \dfrac{1}{2}x^2 + \dfrac{1}{24}x^4 - \dfrac{1}{720}x^6 + \ldots\ldots$

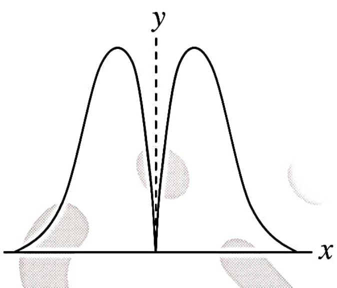
(b) $1 - \dfrac{1}{2}x^2 + \dfrac{5}{24}x^4 - \dfrac{61}{720}x^6 + \ldots\ldots$

(c) $1 - \dfrac{1}{2}x^2 + \dfrac{11}{24}x^4 - \dfrac{331}{720}x^6 + \ldots\ldots$

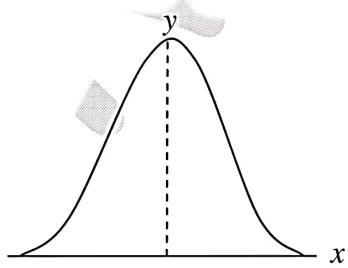
(d) $1 - \dfrac{1}{2x^2} + \dfrac{1}{24x^4} - \dfrac{1}{720x^6} + \ldots\ldots$

Q5. Which of the following graphs gives the best representation of the real-valued function $y = x^2 e^{-x^2}$?

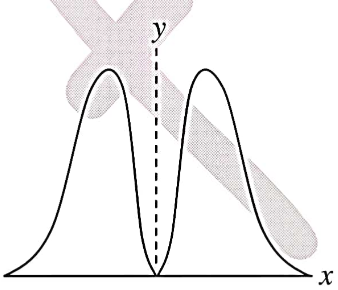
(a)



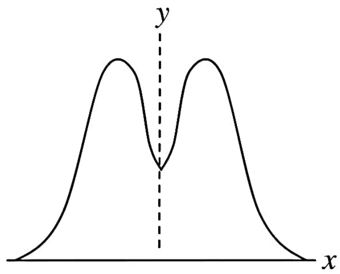
(b)



(c)



(d)



Q6. An observer O uses the coordinate system (x, t) to describe non-relativistic motion in one dimension. Another observer O', moving with respect to O with a uniform velocity v (much smaller than the speed of light c) along the positive x-direction, uses (x', t'), such that at t = 0, t' = 0 and that instant x and x' coincide. Then

(a) $x' = x - vt, t' = t, \frac{\partial}{\partial x'} = \frac{\partial}{\partial x} - \frac{1}{v}\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial t'} = \frac{\partial}{\partial t}$

(b) $x' = x - vt, t' = t, \frac{\partial}{\partial x'} = \frac{\partial}{\partial x}$ and $\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + v\frac{\partial}{\partial x}$

(c) $x' = x + vt, t' = t, \frac{\partial}{\partial x'} = \frac{\partial}{\partial x} + \frac{1}{v}\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial t'} = \frac{\partial}{\partial t}$

(d) $x' = x + vt, t' = t, \frac{\partial}{\partial x'} = \frac{\partial}{\partial x}$ and $\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} - v\frac{\partial}{\partial x}$

Q7. A ball dropped from a height h can only attain the height $4h/5$ after bouncing off the floor. If the ball is dropped from a height of 1 m, the time it will take to come to rest is, approximately

[Ignore air resistance and the finite radius of the ball.]

(a) 1.9 s (b) 3.8 s (c) 8.0 s (d) 4.1 s

Q8. A small raindrop of mass m experiences a viscous drag force $F_d = bv$, proportional to its instantaneous speed v, If it starts from rest at a height h, its speed after a time t is

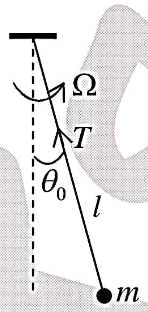
(a) $v(t) = \frac{mg}{b}\tanh\left(\frac{bt}{m}\right)$ (b) $v(t) = \frac{mg}{b}e^{-bt/m}$

(c) $v(t) = \frac{mg}{b}\left(1 - e^{-2bt/m}\right)$ (d) $v(t) = \frac{mg}{b}\left(1 - e^{-bt/m}\right)$

Q9. The nature of flow in a viscous liquid is characterised by the dimensionless Reynolds' number Re proportional to v (the flow velocity): $Re \propto v$. Given that Re also depends on (i) the density ρ of the fluid, (ii) the dynamical viscosity η and (iii) a characteristic length l, of the flow. By dimensional analysis, we find that

(a) $\mathrm{Re} = \frac{\eta l v}{\rho}$ (b) $\mathrm{Re} = \frac{\rho l v}{\eta}$ (c) $\mathrm{Re} = \frac{\rho \eta v}{l}$ (d) $\mathrm{Re} = \frac{\rho v}{\eta l}$

Q10. A ball of mass m is hung from a support by a massless wire of length l. The support is rotated with an angular speed $\Omega > \sqrt{g/l}$ around a vertical axis through the point of suspension as shown in the figure. The ball rests in equilibrium at an angle θ_0. Which of the following statements concerning θ_0 and the tension T, is true?



(a) $\theta_0 = 0$ and $T = mg$

(b) $\theta_0 = \tan^{-1}\left(\frac{q}{\Omega^2 l}\right)$ and $T < mg\cos\theta_0$

(c) $\theta_0 = \sin^{-1}\left(\frac{q}{\Omega^2 l}\right)$ and $T > mg\cos\theta_0$

(d) $\theta_0 = \cos^{-1}\left(\frac{q}{\Omega^2 l}\right)$ and $T > mg\cos\theta_0$

Q11. In a wire loop of resistance R and inductance L, an e.m.f. ξ is switched on at $t = 0$. The magnetic flux through the loop if given by

(a) $\frac{L\xi}{R}\left(1 - e^{-tR/L}\right)$ (b) $\frac{L\xi}{R}e^{-tR/L}$ (c) $\frac{L\xi}{R}\left(1 - \frac{L}{tR}\right)$ (d) $\frac{L\xi}{R}$

Q12. The electric and magnetic fields of electromagnetic fields of an electromagnetic wave in vacuum are given by $\vec{E} = \hat{i}E_0\sin(kz - \omega t)$ and $\vec{B} = \hat{j}B_0\sin(kz - \omega t)$ respectively. Which of the following relations is correct?

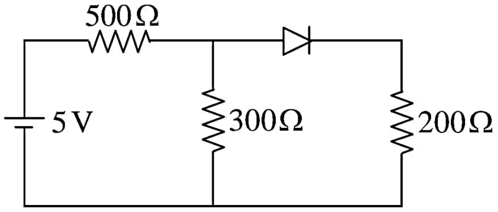
(a) $k^2E_0 = \omega^2 B_0$ (b) $\omega E_0 = kB_0$ (c) $kE_0 = \omega B_0$ (d) $E_0B_0 = \omega k$

Q13. The radius of the nucleus of the Ra atom, which carries an electric charge +88e, is 7.0×10^{-15} m. What should roughly be the speed of a proton, if it has to reach as close as 1.0×10^{-14} m from the centre of the nucleus? [The radius of the cloud of orbital electrons of the Ra atom is approximately 5.0×10^{-11} m.]

(a) 6.7×10^9 m/s (b) 3.1×10^8 m/s

(c) 1.4×10^5 m/s (d) 4.9×10^7 m/s

Q14. In the circuit shown below, the diode is non-ideal and has a voltage drop of 0.7 V. What is the value of the diode current?



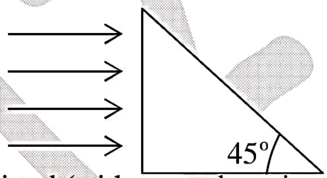
(a) 4.84 mA (b) 8.06 mA (c) 3.03 mA (d) 6.25 mA

Q15. The Doppler width $\Delta\lambda$ of the orange line (for which $\lambda = 6058$ Å) of Kr is 0.0055 Å. What is the spread in frequency of this spectral line?

(a) 2.7×10^7 Hz (b) 2.7×10^9 Hz

(c) 4.5×10^6 Hz (d) 4.5×10^8 Hz

Q16. A beam of light, consisting of red (R), green (G) and blue (B) colours, is incident normal to a face on a right-angled prism (see figure). The refractive indices of the material of the prism for R, G and B wavelengths are 1.39, 1.44 and 1.47 respectively. Then



(a) R, G and B get transmitted (without undergoing total internal reflection)

(b) R and G undergo total internal reflection and B gets transmitted

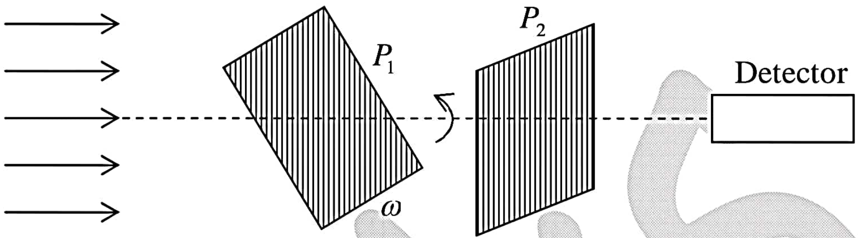
(c) R gets transmitted, while G and B undergo total internal reflection

(d) All of R, G and B undergo total internal reflection

Q17. The two slits in a Young's double-slit experiment are of unequal width, one being four time wider than the other. If I_{max} and I_{min} denote the intensities at a neighbouring maximum and a minimum, then the ratio I_{min}/I_{max} is

(a) $\frac{1}{9}$ (b) $\frac{1}{4}$ (c) $\frac{3}{5}$ (d) 0

Q18. A linear beam of unpolarised light passes through two plane polarisers, the planes of which are perpendicular to the direction of propagation of the beam. The first polariser rotates around this direction with an angular velocity of 20 π radians per second. If the initial intensity of the light beam is I_0, then the intensity when it leaves the second polariser



(a) is periodic with frequency of 20 Hz and maximum of $I_0/4$

(b) is periodic with frequency of 20 Hz and maximum of $I_0/2$

(c) is periodic with frequency of 10 Hz and maximum of $I_0/4$

(d) is periodic with frequency of 10 Hz and maximum of $I_0/2$

Q19. The Boolean expression $B\cdot(A+B)+A\cdot(A+\overline{B})$ can be realised using a minimum number of

(a) 1 OR gate (b) 1 AND gate (c) 2 OR gate (d) 2 AND gate

Q20. An ideal diatomic gas (of $\gamma = 5/3$) is expanded adiabatically so that its volume is doubled. By what ratio is its temperature reduced in this process?

(a) 1/2 (b) $1/2^{1/3}$ (c) $1/2^{2/3}$ (d) $1/2^{5/3}$

Q21. Two buckets B_1 and B_2, each containing 25 litres of water, are initially at temperatures T_1 and T_2, respectively. Now take 1 litre of water from B_1, put it in B_2 and allow thermal equilibrium to be established. Then take 1 litre of water from B_2, put it back in B_1 and again allow it to come to thermal equilibrium. At the end of this cycle the amount of water in each bucket does not change, but their temperatures will change. When this process is repeated, the difference in temperature reduces by the same factor after each cycle. If $|T_1 - T_2|$ was 40° C to begin with, what would be its value after 5 cycle?

(a) 27° C (b) 10° C (c) 19° C (d) 35° C

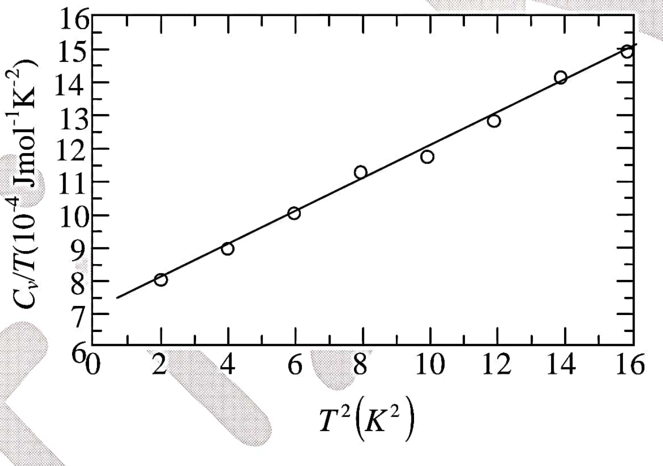
Q22. A flat plate is constantly being bombarded from one side by particles of mass m. If the number density of the particles is ρ and they strike the plate with speed v along the normal to the plate, the pressure exerted on the plate is

(a) $m\rho v^2$ (b) $2m\rho v^2$ (c) $m\rho v$ (d) $2m\rho v$

Q23. Helium atoms at low temperatures make a perfect closed pack structure of hexagonal lattice with parameters a = 0.36 nm and c = 0.59 nm. The density of the crystal is approximately

(a) 2000 kg/m^3 (b) 100 kg/m^3 (c) 123 kg/m^3 (d) 200 kg/m^3

Q24. The ratio of the specific heat capacity and temperature, C_v/T, of Cu is plotted as a function of T^2, the square of the absolute temperature, in the graph below:



The values of γ and β (the coefficients corresponding to the electronic and the vibrational components of the specific heat) are, approximately

(a) $\gamma = 7.0 \times 10^{-4}$ J mol^{-1} and $\beta = 5.0 \times 10^{-5}$ J mol^{-1} K^{-4}

(b) $\gamma = 5.0 \times 10^{-5}$ J mol^{-1} and $\beta = 7.0 \times 10^{-4}$ J mol^{-1} K^{-4}

(c) $\gamma = 1.4 \times 10^{-3}$ J mol^{-1} and $\beta = 7.0 \times 10^{-4}$ J mol^{-1} K^{-4}

(d) $\gamma = 5.0 \times 10^{-4}$ J mol^{-1} and $\beta = 7.0 \times 10^{-5}$ J mol^{-1} K^{-4}

Q25. A paramagnetic gas at room temperature is placed in an external magnetic field of 1.5 T (tesla). Each atom of the gas has a magnetic moment $\mu = 1.0\ \mu_B$, where $\mu_B = 9.3 \times 10^{-24}$ J/T is the Bohr magneton. The difference in energy when an atom is aligned along the magnetic field and opposite to it, is

(a) 2.8×10^{-23} J (b) 1.4×10^{-23} J

(c) 18.6×10^{-24} J (d) 9.3×10^{-24} J

Q26. The Fermi energy ε_F in metals depends on the number density n_e of mobile electrons, which may be thought of as a free Fermi gas. If n_e of one metal is larger by a factor of 1000 compared to another, then in comparison, its Fermi energy is

(a) 1000 times larger (b) smaller by a factor of 1/100

(c) 100 times larger (d) 10 times larger

Q27. The kinetic energy of a proton and an α-particle (not under the influence of any force) are given to be equal. If we denote the de Broglie wavelengths of the proton by λ_p and that of the α-particle by λ_α, then

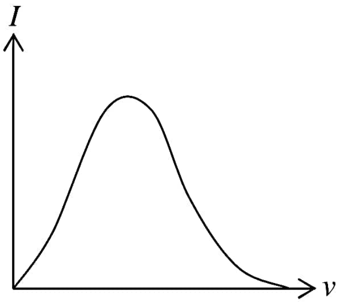
(a) $\lambda_p \approx \lambda_\alpha$ (b) $\lambda_p \approx 4\lambda_\alpha$ (c) $\lambda_p \approx ½\ \lambda_\alpha$ (d) $\lambda_p \approx 2\lambda_\alpha$

Q28. When a monochromatic point source of light is placed at a distance of 0.2 m from a photoelectric cell, the stopping potential V_s and the saturation current I_s are found to be 0.6 V and 18.0 mA, respectively. If the same source is now placed 0.6 m away from the photoelectric cell, one finds

(a) $V_s = 0.2$ V and $I_s = 6.0$ mA (b) $V_s = 0.6$ V and $I_s = 6.0$ mA

(c) $V_s = 0.6$ V and $I_s = 2.0$ mA (d) $V_s = 0.2$ V and $I_s = 18.0$ mA

Q29. The graph in the figure below shows the intensity I as a function of frequency ν of a perfect blackbody at a fixed temperature T:



The corresponding graph at temperature 2T can be obtained by which of the following operations?

For every point of the graph

(a) multiply the ν -coordinate by 1/2 and the I-coordinate by 8

(b) multiply the ν -coordinate by 2 and the I-coordinate by 8

(c) multiply the ν -coordinate by 1/2 and the I-coordinate by 16

(d) multiply the ν -coordinate by 2 and the I-coordinate by 16

Q30. What is the maximum theoretical accuracy ΔE to which an ideal experiment may determine the energy levels of the hydrogen atom?

[Hint: Use the fact that the age of the universe is estimated to be approximately 1.4 $\times 10^{10}$ years.]

(a) 4.7×10^{-26} eV (b) 9.4×10^{-33} eV

(c) 1.2×10^{-63} eV (d) 2.4×10^{-70} eV

Q31. A particle in one dimension is in the ground state (lowest energy quantum state) of the potential well given by

$$V(x) = \begin{cases} 0 & \text{for } |x| < \frac{L}{2} \\ \infty & \text{otherwise} \end{cases}$$

Let P_+ be the probability that the particle is found to move along the positive x-direction and p be the magnitude of the momentum for that state of motion. Then

(a) $P_+ = 0$ and $p = 0$

(b) $P_+ = \frac{1}{2}$ and $p = \frac{\pi}{2L}$

(c) $P_+ = \frac{1}{2}$ and $p = \frac{\pi}{L}$

(d) $P_+ = 1$ and $p = \frac{\pi}{L}$

Q32. A particle of mass m is moving in a three-dimensional potential

$$V(x, y, z) = \frac{1}{2} m\omega^2 \left(x^2 + 2y^2 + 4z^2\right)$$

The energy of the particle in the ground state (lowest energy quantum state) is

(a) $\frac{\sqrt{7}}{2}\hbar\omega$

(b) $\frac{3}{2}\hbar\omega$

(c) $\frac{7}{2}\hbar\omega$

(d) $\frac{\left(3+\sqrt{2}\right)}{2}\hbar\omega$

Q33. A nucleus may be modelled as a drop of liquid consisting of the nucleons (protons and neutrons). In this model, the dominant contribution to the nuclear binding energy is from the volume, which is proportional to A, the total number of nucleons. Then the two important subdominant contributions from the surface tension and the Coulomb repulsion of the protons are, proportional to

(a) $A^{2/3}$ and $Z/A^{1/3}$ respectively

(b) $A^{2/3}$ and $Z^2/A^{1/3}$ respectively

(c) $A^{1/3}$ and $Z^2/A^{2/3}$ respectively

(d) $A^{1/2}$ and $Z^2/A^{1/3}$ respectively