ONLINE CLASSES

SPECIAL THEORY OF RELATIVITY

Lecture 1

DR. DIPESH SATPATI,ISRO
ASSOCIATE SPACE SCIENTIST
INDIAN SPACE RESEARCH ORG
POST DOCTORAL RESEARCHER;ISCA

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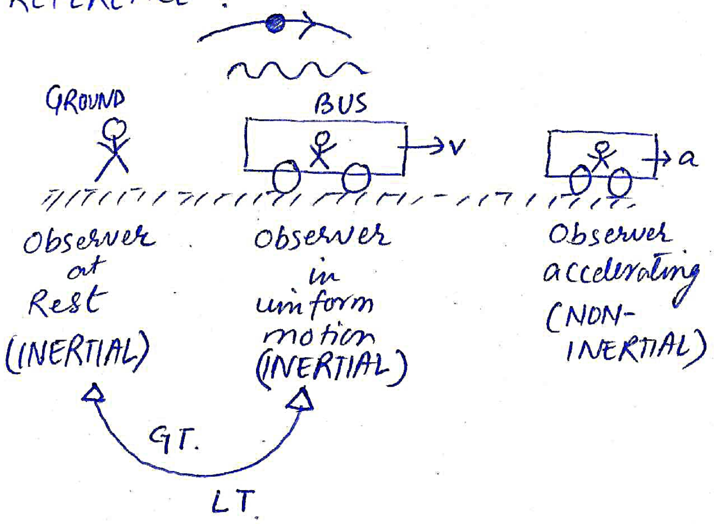
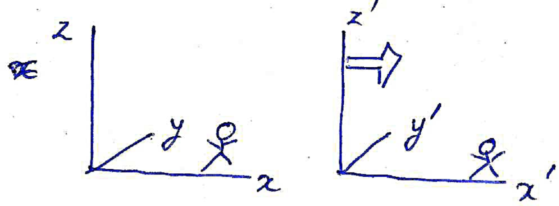
SPECIAL THEORY OF RELATIVITY

RELATIVISTIC MECHANICS

→ It deals with observations made wrt INERTIAL FRAMES OF REFERENCE.

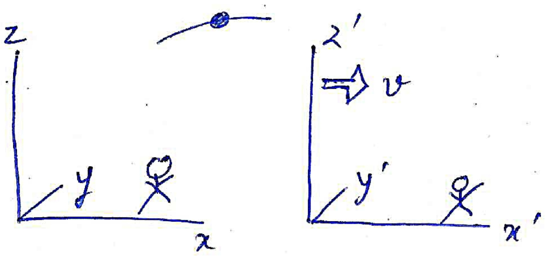


Galilean Transformations (GT)
(Newtonian Mech.)



$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\} \rightarrow \left\{\begin{aligned} u_x' &= u_x - v \\ u_y' &= u_y \\ u_z' &= u_z \end{aligned}\right.$$

(GT) Taking time-derivative (Velocity Transf.)



$$\boxed{F = ma} \xrightarrow[GT]{} \boxed{F' = ma'}$$

Mechanical laws are INVARIANT under a Galilean Transformatn.

$$\vec{\nabla}\cdot\vec{E} = 0 \;;\; \vec{\nabla}\cdot\vec{B} = 0 \qquad \vec{\nabla}\cdot\vec{E}' = 0 \;;\; \vec{\nabla}\cdot\vec{B}' = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \;;\; \vec{\nabla}\times\vec{B} = \mu_0\epsilon_0\frac{\partial \vec{E}}{\partial t} \;\not\xrightarrow{GT}\; \vec{\nabla}\times\vec{E}' = -\frac{\partial \vec{B}'}{\partial t} \;;\; \vec{\nabla}\times\vec{B}' = \mu_0\epsilon_0\frac{\partial \vec{E}'}{\partial t}$$

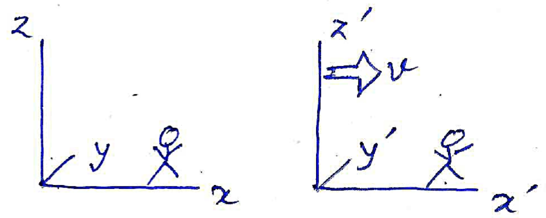
Maxwell's ~~Law~~ Eq^{n}s (Laws of Electromagnetism) are NOT INVARIANT under GT.

Postulates of Special Theory of Relativity :-

① Physical laws are same across all inertial frames of reference.

② Velocity of light in vaccum is same for all inertial observers

Lorentz Transformations (LT)



$$\left.\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right) \end{aligned}\right] \xrightarrow{v \ll c} \left[\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right.$$

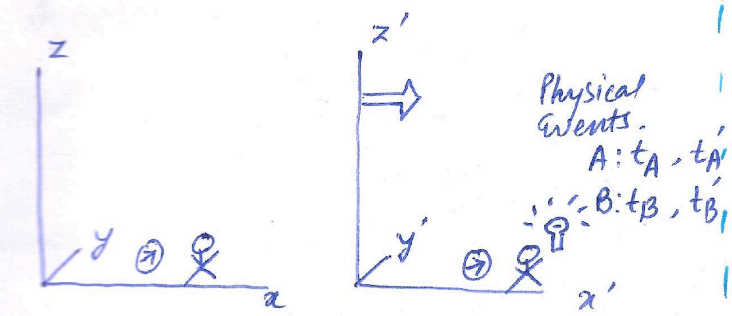
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Inverse Lorentz Transformation

$$x = \gamma(x' + vt')$$
$$y = y'$$
$$z = z'$$
$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

Consequences of Special Theory of Relativity

① Time Dilation :-



Acc to Galilean Transformatⁿ (GT) :-

$$(t_B - t_A) = (t'_B - t'_A)$$

$$\Rightarrow \Delta t_0 = \Delta t'$$

Acc to Lorentz Transformatⁿ (LT) :-

$$t_A = \gamma\left(t'_A + \frac{v x'_A}{c^2}\right),$$

$$t_B = \gamma\left(t'_B + \frac{v x'_B}{c^2}\right)$$

Then,

$$(t_B - t_A) = \gamma\left[t'_B + \frac{v x'_B}{c^2} - t'_A - \frac{v x'_A}{c^2}\right]$$

$$= \gamma\left[(t'_B - t'_A) + \frac{v}{c^2}\underbrace{(x'_B - x'_A)}_{0}\right]$$

$$= \gamma(t'_B - t'_A)$$

$$\Rightarrow \Delta t = \gamma \Delta t_0$$

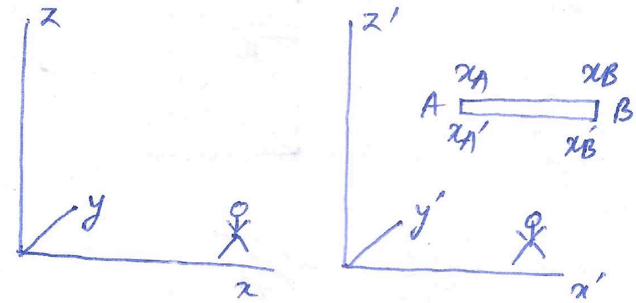
Δt : (Time periods b/w two events acc to a moving observer)

Δt_0 : Proper time, Δt_0 (Time period b/w two events happening in the rest frame of ref.)

or,

$$\boxed{\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}} \qquad \Delta t' \neq \Delta t_0$$

② Length contraction :-



Acc to GT :

$$(x_B - x_A) \Rightarrow (x'_B - x'_A)$$

$$\Delta l = \Delta l_0$$

Acc to LT :-

$$x'_B = \gamma(x_B - v t_B)$$

$$x'_A = \gamma(x_A - v t_A)$$

So,

$$x'_B - x'_A = \gamma\left[(x_B - x_A) - v\underbrace{(t_B - t_A)}_{0}\right]$$

$$= \gamma(x_B - x_A)$$

$$\Rightarrow \Delta l_0 = \gamma \Delta l$$

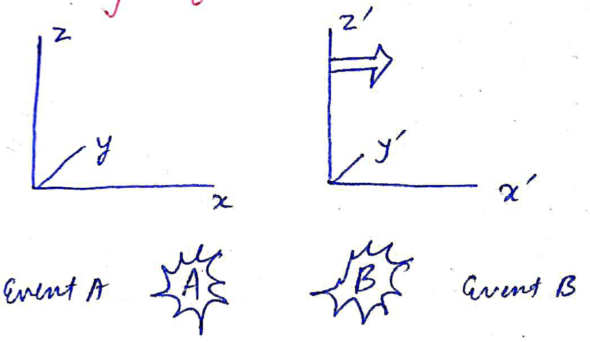
Δl_0 : Proper length (Length b/w two points in space wrt. an observer in rest frame wrt those two points)

Δl : Length b/w two points wrt an observer moving wrt. those two points

So,

$$\boxed{\Delta l = \sqrt{1 - \frac{v^2}{c^2}}\,\Delta l_0} \qquad \Delta l \neq \Delta l_0$$

③ Relativity of simultaneous events



x_A, t_A $\qquad$ x_B, t_B

x'_A, t'_A $\qquad$ x'_B, t'_B

$$t'_A = \gamma\left(t_A - \frac{v x_A}{c^2}\right)$$

$$t'_B = \gamma\left(t_B - \frac{v x_B}{c^2}\right)$$

Diff: $t'_B - t'_A = \gamma\left[(t_B - t_A) - \frac{v}{c^2}(x_B - x_A)\right]$

Lets suppose events are simultaneous wrt S- observer

$$\Rightarrow \; t_B = t_A$$

then $\quad t'_B - t'_A \neq 0$

$$\Rightarrow t'_B \neq t'_A$$

→ For one observer, both are simultaneous events

$$t_B = t_A$$

But, for the other observer, they are NOT

$$t'_B \neq t'_A$$

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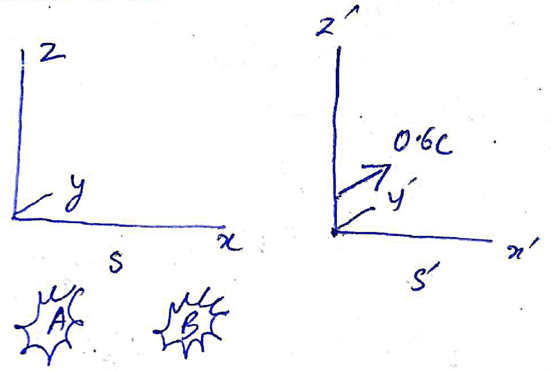

$A : (ct_A = 0,\ x_A = 0,\ y_A = 0,\ z_A = 0)$

$B : (ct_B = 0,\ x_B = 0,\ y_B = 2,\ z_B = 0)$

Relative motion along yy' axis:-

LT :

$$x' = x$$

$$y' = \gamma(y - vt)$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{vy}{c^2}\right)$$

So,

$$t'_A = \gamma\left(t_A - \frac{v}{c^2} y_A\right)$$

$$= \gamma\left(0 - \frac{v}{c^2} \times 0\right)$$

$$\Rightarrow ct'_A = 0$$

and

$$t'_B = \gamma\left(t_B - \frac{v}{c^2} y_B\right)$$

$$= \gamma\left(0 - \frac{v}{c^2} \times 2\right)$$

$$= \frac{1}{\sqrt{1 - \left(\frac{0.6c}{c}\right)^2}}\left(-\frac{0.6c}{c^2} \times 2\right)$$

$$= \frac{1}{\sqrt{1 - 0.36}}\left(-\frac{1.2}{c}\right)$$

$$= \frac{1}{\sqrt{0.64}}\left(-\frac{1.2}{c}\right) = \frac{1}{\cancel{0.8}_{\,2}}\left(-\frac{\cancel{1.2}^{\,3}}{c}\right)$$

$$= -\frac{3}{2c} \quad \Rightarrow ct'_B = -3/2 \rightarrow ⓐ$$