2019

AP® Physics C: Mechanics Free-Response Questions Set 1 ANSWERS

Que-1

$a = \frac{dv}{dt}$, $F = ma$

Que-2 $F_c = \frac{mv^2}{r}$

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$mgh = \frac{1}{2}mv^2$

Que-3 $\omega = \omega_0 + \alpha t$

$\alpha = \frac{d\omega}{dt}$, $\tau = I(\alpha)$

$\vec{\tau} = \vec{r} \times \vec{f}$, $KE = \frac{1}{2}I\omega^2$, $L = I\omega$

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit, $1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\ \text{MeV}/c^2$

Planck's constant, $h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s} = 4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}$

$hc = 1.99 \times 10^{-25}\ \text{J}\cdot\text{m} = 1.24 \times 10^{3}\ \text{eV}\cdot\text{nm}$

Vacuum permittivity, $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$

Coulomb's law constant, $k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9}\ (\text{N}\cdot\text{m}^2)/\text{C}^2$

Vacuum permeability, $\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$

Magnetic constant, $k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$

1 atmosphere pressure, $1\text{ atm} = 1.0 \times 10^{5}\ \text{N/m}^2 = 1.0 \times 10^{5}$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	$0°$	$30°$	$37°$	$45°$	$53°$	$60°$	$90°$
$\sin\theta$	0	1/2	3/5	$\sqrt{2}/2$	4/5	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	4/5	$\sqrt{2}/2$	3/5	1/2	0
$\tan\theta$	0	$\sqrt{3}/3$	3/4	1	4/3	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.

II. The direction of current is the direction in which positive charges would drift.

III. The electric potential is zero at an infinite distance from an isolated point charge.

IV. All batteries and meters are ideal unless otherwise stated.

V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^{\ 2} + 2a_x (x - x_0)$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{J} = \int \vec{F}\, dt = \Delta\vec{p}$$

$$\vec{p} = m\vec{v}$$

$$\left|\vec{F}_f\right| \le \mu \left|\vec{F}_N\right|$$

$$\Delta E = W = \int \vec{F} \cdot d\vec{r}$$

$$K = \frac{1}{2} m v^2$$

$$P = \frac{dE}{dt}$$

$$P = \vec{F} \cdot \vec{v}$$

$$\Delta U_g = mg\Delta h$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

$$I = \int r^2 dm = \sum m r^2$$

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

$$v = r\omega$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

$$K = \frac{1}{2} I \omega^2$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$$\vec{F}_s = -k\Delta\vec{x}$$

$$U_s = \frac{1}{2} k (\Delta x)^2$$

$$x = x_{max} \cos(\omega t + \phi)$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

$$T_p = 2\pi \sqrt{\frac{\ell}{g}}$$

$$\left|\vec{F}_G\right| = \frac{G m_1 m_2}{r^2}$$

$$U_G = -\frac{G m_1 m_2}{r}$$

ELECTRICITY AND MAGNETISM

$$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0} \left|\frac{q_1 q_2}{r^2}\right|$$

$$\vec{E} = \frac{\vec{F}_E}{q}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$E_x = -\frac{dV}{dx}$$

$$\Delta V = -\int \vec{E} \cdot d\vec{r}$$

$$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$$

$$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

$$\Delta V = \frac{Q}{C}$$

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$I = \frac{dQ}{dt}$$

$$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$$

$$R = \frac{\rho \ell}{A}$$

$$\vec{E} = \rho \vec{J}$$

$$I = N e v_d A$$

$$I = \frac{\Delta V}{R}$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$P = I\Delta V$$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$$\vec{F}_M = q\vec{v} \times \vec{B}$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$$

$$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$$

$$B_s = \mu_0 n I$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$U_L = \frac{1}{2} L I^2$$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

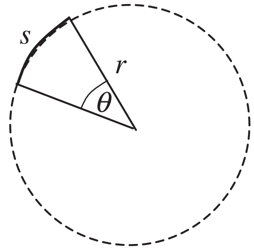
$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle



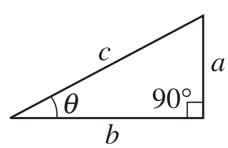
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



CALCULUS!

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1\,!$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS!

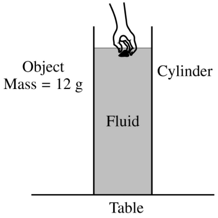
$$\vec{A}\bullet\vec{B} = AB\cos\theta$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

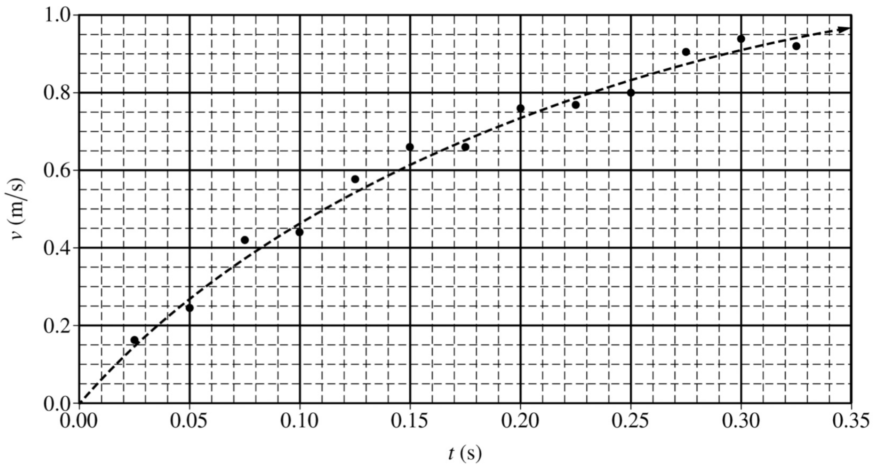
2019 AP® PHYSICS C: MECHANICS FREE-RESPONSE QUESTIONS

PHYSICS C: MECHANICS SECTION II Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



1. In an experiment, students used video analysis to track the motion of an object falling vertically through a fluid in a glass cylinder. The object of m = 12 g is released from rest at the top of the column of fluid, as shown above. The data for the speed v of the falling object as a function of time t are graphed on the grid below. The dashed curve represents the best fit chosen by the students for these data.



(a)

i. Does the speed of the object increase, decrease, or remain the same?

_✓_ Increase ____ Decrease ____ Remain the same

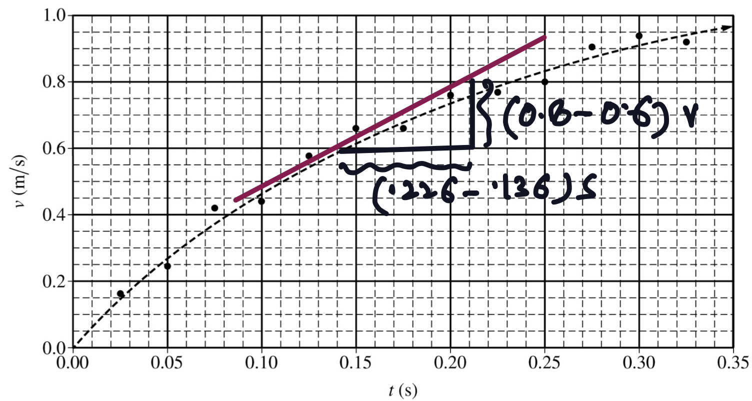
ii. In a brief statement, describe the direction of the object's acceleration and how the magnitude of this acceleration changed as the object fell.

Acceleration is down, magnitude of acceleration decreases as drag force increases.

iii. Using the graph, calculate an approximate value for the magnitude of the acceleration of the object at t = 0.20 s.

slope $a = \frac{\Delta v}{\Delta t} = \left(\frac{0.8 - 0.6}{0.226 - 0.136}\right)$

$= 2.22 \text{ m s}^{-2}$



The students use the equation $v = A(1 - e^{-Bt})$ to model the speed of the falling object and find the best fit coefficients to be $A = 1.18$ m/s and $B = 5s^{-1}$.

(b) Use the above equation to:

i. Derive an expression for the magnitude of the vertical displacement y(t) of the falling object as a function of time t .

$$v = A(1 - e^{-Bt}) \Rightarrow \frac{dy}{dt} = A(1 - e^{-Bt})$$

$$\int dy = \int_0^t A(1 - e^{-Bt})\,dt = A\int_0^t dt - A\int_0^t e^{-Bt}\,dt$$

$$y = A[t] + A\left[\frac{e^{-Bt}}{B}\right]_0^t = A(t) + \frac{A}{B}\left[e^{-Bt} - e^{0}\right]$$

$$= A(t) + \frac{A}{B}\left[e^{-Bt} - 1\right] = A\left\{t + \frac{1}{B}(e^{-Bt} - 1)\right\}$$

$$= (1.18)\left\{t + \frac{1}{5}(e^{-5t} - 1)\right\}$$

ii. Derive an expression for the magnitude of the net force F (t) exerted on the object as it falls through the fluid as a function of time t .

$$\text{acceleration} = \frac{dv}{dt} = \frac{d}{dt}\left\{A(1 - e^{-Bt})\right\}$$

$$= A\left[0 + e^{-Bt}\right] = Ae^{-Bt}$$

$$F = ma = mAe^{-Bt} = (.012)(1.18)(5)e^{-5t}$$

$$= 0.071\,e^{-5t}$$

The students repeat the experiment with a taller glass cylinder that is filled with the same fluid. The cylinder is tall enough so that the object reaches a constant speed.

(c)

i. Determine the constant speed of the object.
Justify your answer.

$v = A\left(1 - e^{-Bt}\right)$

constant speed will be attained at $t = \infty$

$v = A\left(1 - e^{-B\infty}\right) = A\left(1 - \frac{1}{e^{\infty}}\right) = A(1-0)$

$= A$

$= 1.18\ m s^{-1}$

ii. Determine the force exerted by the fluid on the object at this time.
Justify your answer.

Since it is moving with constant velocity then acceleration $= 0$

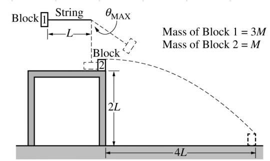
Net force $= 0$

Net force = weight of object − drag force

$0 = 12 \times 10^{-3} \times 10\ N$ − drag force

drag force $= 12 \times 10^{-3} \times 10$

$= 0.12\ N$



Note: Figure not drawn to scale.

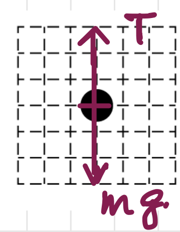
2. A pendulum of length L consists of block 1 of mass 3M attached to the end of a string. Block 1 is released from rest with the string horizontal, as shown above. At the bottom of its swing, block 1 collides with block 2 of mass M, which is initially at rest at the edge of a table of height 2L. Block 1 never touches the table. As a result of the collision, block 2 is launched horizontally from the table, landing on the floor a distance 4L from the base of the table. After the collision, block 1 continues forward and swings up. At its highest point, the string makes an angle θ_{max} to the vertical. Air resistance and friction are negligible. Express all algebraic answers in terms of M, L, and physical constants, as appropriate.

(a) Determine the speed of block 1 at the bottom of its swing just before it makes contact with block 2.

$$mgL = \frac{1}{2} m v^2$$

$$v = \sqrt{2gL} = \sqrt{2 \times 10 \times L} = \sqrt{20L}\ m s^{-1}$$

(b) On the dot below, which represents block 1, draw and label the forces (not components) that act on block 1 just before it makes contact with block 2. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot. Forces with greater magnitude should be represented by longer vectors.



(c) Derive an expression for the tension F_T in the string when the string is vertical just before block 1 makes contact with block 2. If you need to draw anything other than what you have shown in part (b) to assist in your solution, use the space below. Do NOT add anything to the figure in part (b).

$$F_T = 3mg + \frac{3mv^2}{L}$$

$$= 3mg + \frac{3m\left(\sqrt{2gL}\right)^2}{L}$$

$$= 3mg + 6mg = 9mg$$

For parts (d)–(g), the value for the length of the pendulum is L = 75 cm.

(d) Calculate the time between the instant block 2 leaves the table and the instant it first contacts the floor.

Consider vertical motion between starting point and final point

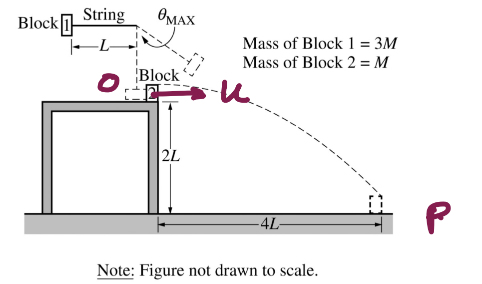
$$-2L = 0 - \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{4L}{g}} = \sqrt{\frac{4 \times 75}{100 \times 10}} = \sqrt{\frac{3}{10}} \text{ sec}$$

$$= 0.55 \text{ sec}$$

(e) Calculate the speed of block 2 as it leaves the table.

consider horizontal motion of point O and P.



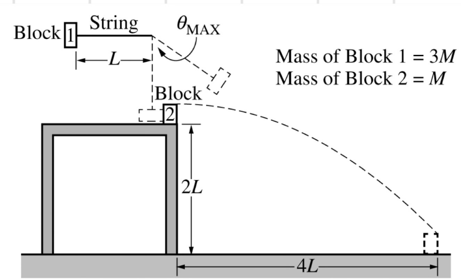
$$4L = u(t) \quad , \quad t = \sqrt{\frac{4L}{g}}$$

$$4L = u\sqrt{\frac{4L}{g}}$$

$$u = \frac{\sqrt{g}\ 4L}{\sqrt{4L}} = \sqrt{g(4L)} = \sqrt{\frac{10 \times 4 \times 75}{100}} = \sqrt{30}$$

$$= 5.47\ ms^{-1}$$

(f) Calculate the speed of block 1 just after it collides with block 2



Note: Figure not drawn to scale.

Apply conservation of linear momentum

$$3M\sqrt{2gL} + M(0) = 3MV' + M(5.47)$$

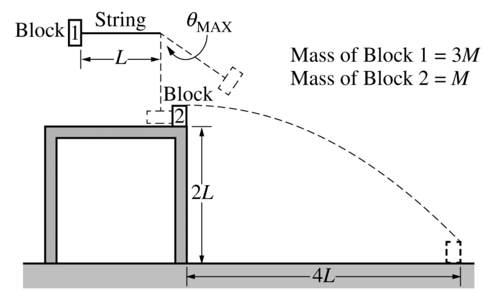
$$3\sqrt{2\times10\times75\times10^{-3}} = 3V' + 5.47$$

$$3V' = 3\sqrt{15} - 5.47$$

$$= 3(3.87) - 5.47$$

$$V' = 2.04\ ms^{-1}$$

(g) Calculate the angle θ_{max} that the string makes with the vertical, as shown in the original figure, when block 1 is at its highest point after the collision.



For block ①

$$\frac{1}{2}(3M)(2.04)^2 = 3Mgh$$

$$\frac{(2.04)^2}{2\times 10} = h = L - L\cos\theta$$

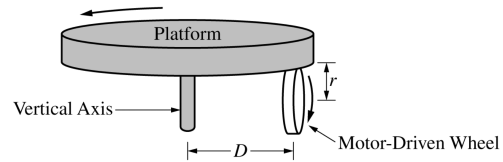
$$= L(1-\cos\theta)$$

$$(1-\cos\theta) = \frac{2.04\times 2.04\times 100}{2\times 10\times 75} = 0.2774$$

$$\cos\theta = 1 - 0.2774 = 0.722$$

$$\theta = \cos^{-1}(0.722)$$

$$= 43.897^0$$



3. A horizontal circular platform with rotational inertia I_p rotates freely without friction on a vertical axis. A small motor-driven wheel that is used to rotate the platform is mounted under the platform and touches it. The wheel has radius r and touches the platform a distance D from the vertical axis of the platform, as shown above. The platform starts at rest, and the wheel exerts a constant horizontal force of magnitude F tangent to the wheel until the platform reaches an angular speed ω_p after time Δt. During time Δt, the wheel stays in contact with the platform without slipping.

(a) Derive an expression for the angular speed ω_p of the platform. Express your answer in terms of ,
and physical constants, as appropriate.

$$\omega_p = \omega_0 + \alpha(\Delta t)$$

$$F D = I(\alpha) \Rightarrow \alpha = \frac{F D}{I_p}$$

$$\omega_0 = 0$$

$$\omega_p = \frac{F(D)}{I_p}(\Delta t) \Rightarrow \boxed{\omega_p = \frac{F D \Delta t}{I_p}}$$

(b) Determine an expression for the kinetic energy of the platform at the moment it reaches angular speed ω_p.
Express your answer in terms of I_p, r, D, F, Δt, and physical constants, as appropriate.

$$KE = \frac{1}{2} I_p \omega_p^2 = \frac{1}{2} I_p \left(\frac{F D \Delta t}{I_p}\right)^2$$

$$= \frac{1}{2} \frac{F^2 D^2 (\Delta t)^2}{I_p}$$

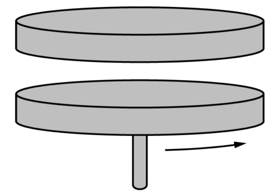
(c) Derive an expression for the angular speed of the wheel ω_w when the platform has reached angular speed ω_p. Express your answer in terms of D, r, ω_p, and physical constants, as appropriate.

Linear speed of platform = Linear speed of wheel

$$v_p = v_w$$

$$D \omega_p = r \omega_w$$

$$\boxed{\omega_w = \frac{D \omega_p}{r}}$$



When the platform is spinning at angular speed ω_p, the motor-driven wheel is removed. A student holds a disk directly above and concentric with the platform, as shown above. The disk has the same rotational inertia I_p as the platform. The student releases the disk from rest, and the disk falls onto the platform. After a short time, the disk and platform are observed to be rotating together at angular speed ω_f

(d) Derive an expression for ω_f. Express your answer in terms of ω_p, I_p, and physical constants, as appropriate.

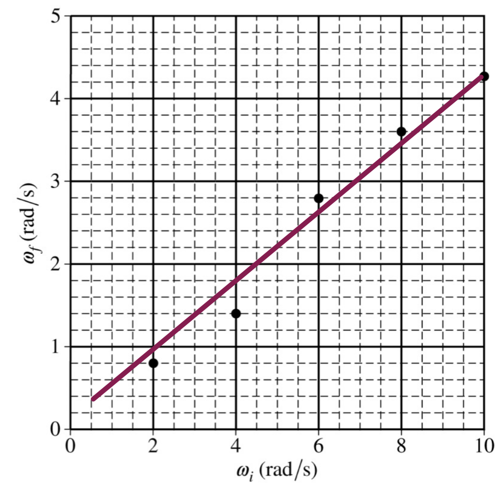
Apply conservation of angular momentum

$$I_p \omega_p + 0 = (I_p + I_p)\,\omega_f$$

$$\omega_f = \frac{I_p \omega_p}{2 I_p} = \frac{\omega_p}{2}$$

A student now uses the rotating platform (I_p = 3.1 kg m^2) to determine the rotational inertia I_U of an unknown object about a vertical axis that passes through the object's center of mass. The platform is rotating at an initial angular speed w_i when the unknown object is dropped with its center of mass directly above the center of the platform. The platform and object are observed to be rotating together at angular speed w_f. Trials are repeated for different values of w_i. A graph of w_f as a function of w_i is shown on the axes below.

(e)
i. On the graph on the previous page, draw a best-fit line for the data.
ii. Using the straight line, calculate the rotational inertia of the unknown object I_U about a vertical axis passing through its center of mass.



Apply conservation of angular momentum:

$$I_p w_i = (I_p + I_U) w_f$$

$$\frac{w_i}{w_f} = \frac{I_p + I_u}{I_p} = \frac{8}{3.6}$$

$$\frac{w_i}{w_f} = \frac{8}{3.6}$$

$$\frac{3.1 + I_u}{3.1} = \frac{8}{3.6}$$

$$(3.1)(8) = (3.1)(3.6) + 3.6\, I_u \Rightarrow 3.6\, I_u = (3.1)(8) - (3.1)(3.6)$$

$$I_U = 3.78 \text{ kg m}^2$$

(f) The kinetic energy of the spinning platform before the object is dropped on it is Ki . The total kinetic energy of the platform-object system when it reaches angular speed ω_f is K_f. Which of the following expressions is true?

_✓_ $K_f < K_i$ ___ $K_f = K_i$ ___ $K_f > K_i$

Justify your answer.

$K_f < K_i$ this is inelastic collision the KE will be lost.

(g) One of the students observes that the center of mass of the object is not actually aligned with the axis of the platform. Is the experimental value of I_U obtained in part (e) greater than, less than, or equal to the actual value of the rotational inertia of the unknown object about a vertical axis that passes through its center of mass? _✓_ Greater than Justify your answer.

____ Less than

____ Equal to

Because centre of mass of the object is off from the centre (axis of plat form)

And as per parallel axis theorem

$I' = I_{cm} + Mh^2$

From the calculation we get I' which is greater than I

STOP END OF EXAM