AP Physics C: Electricity and Magnetism Free-Response Questions Set 1 Answers

FORMULAE

Que-1 $\oint E \cdot ds = q/\epsilon_0$

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

Que-2

$$Q = Q_0 e^{-t/\tau}$$

$$C = \frac{k\epsilon_0 A}{d}$$

Que-3 $B = \frac{\mu_0}{4\pi}\frac{2I}{r}$, $e = -\frac{d\Phi}{dt}$

$$i = e/R$$

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1 \text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit, $1 \text{ u} = 1.66 \times 10^{-27} \text{ kg} = 931 \text{ MeV}/c^2$

Planck's constant, $h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$

$hc = 1.99 \times 10^{-25} \text{ J}\cdot\text{m} = 1.24 \times 10^{3} \text{ eV}\cdot\text{nm}$

Vacuum permittivity, $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$

Coulomb's law constant, $k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9}\ (\text{N}\cdot\text{m}^2)/\text{C}^2$

Vacuum permeability, $\mu_0 = 4\pi \times 10^{-7}$ (T•m)/A

Magnetic constant, $k' = \mu_0/(4\pi) = 1 \times 10^{-7}$ (T•m)/A

1 atmosphere pressure, $1 \text{ atm} = 1.0 \times 10^{5}\ \text{N/m}^2 = 1.0 \times 10^{5}$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.

II. The direction of current is the direction in which positive charges would drift.

III. The electric potential is zero at an infinite distance from an isolated point charge.

IV. All batteries and meters are ideal unless otherwise stated.

V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta \vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu |\vec{F}_N|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{G m_1 m_2}{r^2}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1 q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = N e v_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

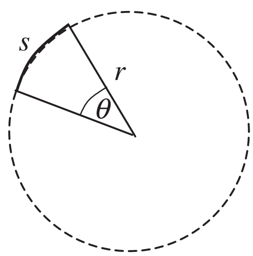
$a^2 + b^2 = c^2$

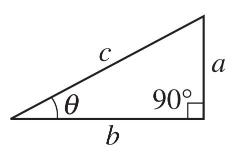
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS!

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1\,!$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS!

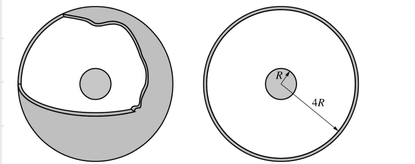
$\vec{A}\bullet\vec{B} = AB\cos\theta$

$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

PHYSICS C: ELECTRICITY AND MAGNETISM SECTION II

Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Cutout View   Cross-Section View

Note: Figures not drawn to scale.

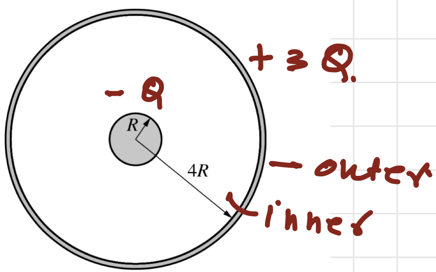
1. A nonconducting sphere of uniform volume charge density is surrounded by a thin concentric conducting spherical shell, as shown in the cutout view. The sphere has a charge of −Q and the shell has a charge of +3Q. The radii of the inner sphere and spherical shell are R and 4R, respectively, as shown in the cross-section view.

(a) Determine the charge on the outer surface of the shell.

$q_{net} = q_{outer} + q_{inner}$

$q_{outer} = q_{net} - q_{inner}$

$= 3Q - Q = 2Q$



Cross-Section View

(b) Using Gauss's law, derive an expression for the electric field a distance r from the center of the sphere for $r < R$. Express your answer in terms of Q, R, r, and physical constants, as appropriate.

Apply gauss theorem

$$\oint E\,ds = \frac{q}{\epsilon_0}$$

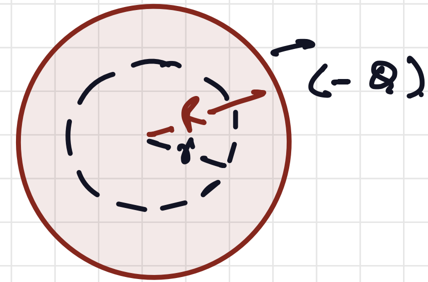
$$E.\,4\pi r^2 = \frac{q'}{\epsilon_0}$$

$$E.\,4\pi r^2 = \frac{-Q r^3}{R^3 \epsilon_0}$$

$$E = \frac{-Q}{4\pi R^3 \epsilon_0}$$

$$\frac{-Q}{\frac{4}{3}\pi R^3} = \frac{q'}{\frac{4}{3}\pi r^3}$$

$$q' = -\frac{Q r^3}{R^3}$$



$r < R$

ENERGY DENSITY WILL REMAIN - CONSTANT

(c) The magnitude of the electric field at r = R is 8N / C. Calculate the value of the electric field at r = 2R.

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \Rightarrow E \propto \frac{1}{r^2}$$

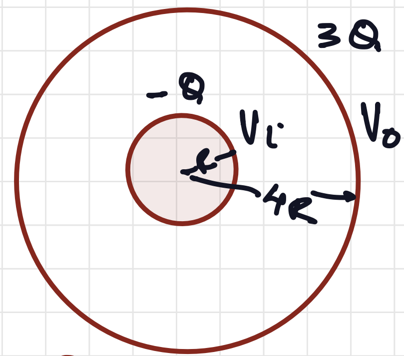
$$\frac{8}{E} = \frac{1}{R^2}\frac{(2R)^2}{(1)} = 4$$

$$E = \frac{8}{4} = 2\,N/C$$

(d) Derive an expression for the absolute value of the potential difference between the outer surface of the sphere and the inner surface of the shell. Express your answer in terms of Q, R, and physical constants, as appropriate.

$$V_o = \frac{1}{4\pi\epsilon_0}\frac{-Q}{4R} + \frac{1}{4\pi\epsilon_0}\frac{3Q}{4R}$$

$$V_i = \frac{1}{4\pi\epsilon_0}\frac{-Q}{R} + \frac{1}{4\pi\epsilon_0}\frac{3Q}{4R}$$

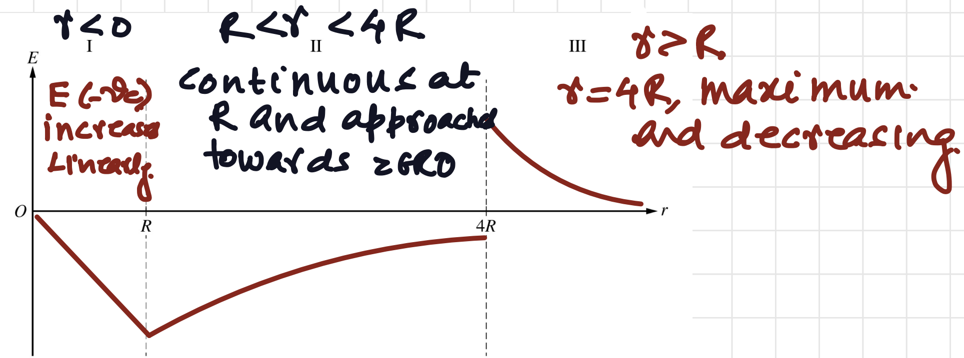


$$V_o - V_i = \frac{1}{4\pi\epsilon_0}\left[\frac{-Q}{4R} + \frac{3Q}{4R} + \frac{Q}{R} - \frac{3Q}{4R}\right]$$

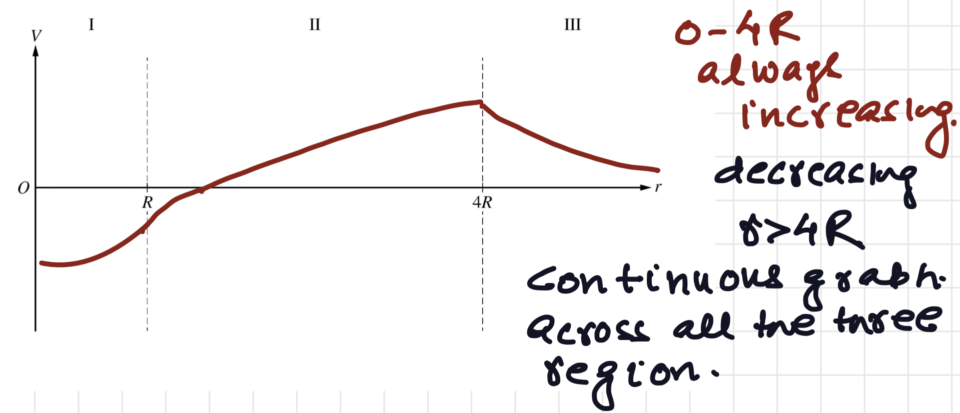
$$= \frac{Q}{4\pi\epsilon_0 (R)}\left(1 - \frac{1}{4}\right) = \frac{Q}{4\pi\epsilon_0 R}\left(\frac{3}{4}\right) = \frac{3Q}{16\pi\epsilon_0 R}$$

(e)

i. On the following axes that include regions I, II, and III, sketch a graph of the electric field E as a function of the distance r from the center of the sphere.

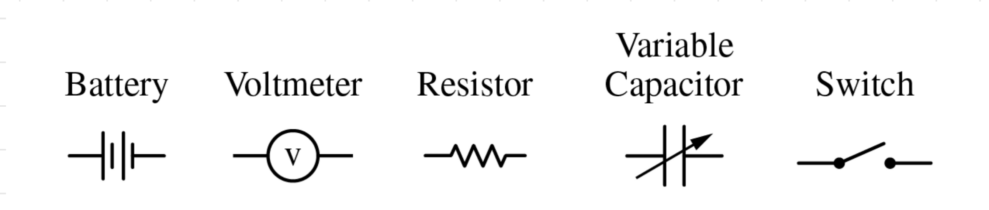


ii. On the following axes that include regions I, II, and III, sketch a graph of the electric potential V as a function of the distance r from the center of the sphere.

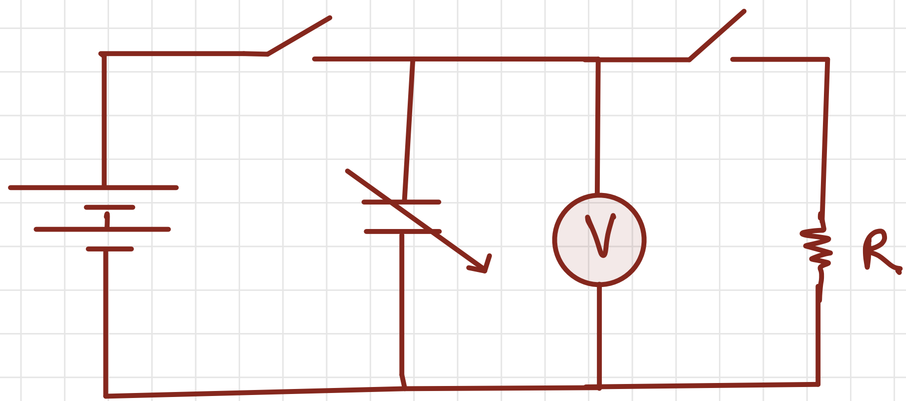


Begin your response to QUESTION 2 on this page.

2. The plates of a certain variable capacitor have an adjustable area. An experiment is performed to study the potential difference across the capacitor as it discharges through a resistor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0 a single voltmeter, a single resistor of resistance R, a single uncharged variable capacitor set to capacitance C, and one or more switches as needed.



(a) Using the symbols shown, draw a schematic diagram of a circuit that can charge the capacitor and may also be used to study the potential difference across the capacitor as it discharges through the resistor.



The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor. $-t$

(b) Show that the potential difference ΔV_c across the capacitor as a function of time t is $\Delta V_c(t) = \Delta V_0 e^{-t/RC}$ as the capacitor discharges.

$$V - IR = 0$$

$$\frac{Q}{C} = IR$$

$$\frac{Q}{C} = \frac{dQ}{dt}(R) \Rightarrow \int_{Q}^{Q_0} \frac{dQ}{Q} = \int_0^t \frac{dt}{RC} \Rightarrow \log_e \frac{Q}{Q_0} = -\frac{t}{RC}$$

$$\frac{Q}{Q_0} = e^{-t/RC}$$

$$\boxed{Q = Q_0 e^{-t/RC}}$$

$$Q = Q_0 e^{-t/RC}$$

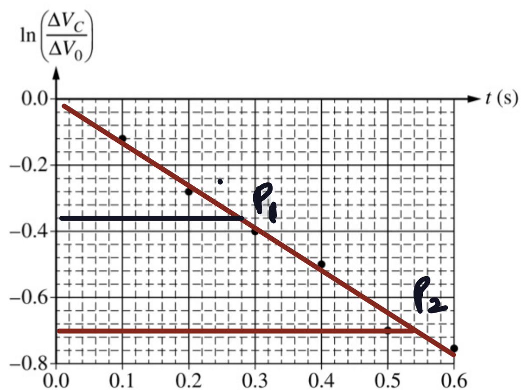
$$\frac{dQ}{dt} = Q_0 \left(-\frac{1}{RC}\right) e^{-t/RC}$$

$$\Delta V_c(t) = \left(-\frac{Q_0}{RC}\right) e^{-t/RC}$$

$$\boxed{\Delta V_c(t) = V_0 e^{-t/RC}}$$ proved

c) The experiment is performed using a resistor of R = 150 kΩ. Data for the potential difference ΔV_c across the

capacitor as a function of t are recorded and a plot of $\ln\left(\frac{\Delta V_C}{\Delta V_0}\right)$ as a function of t is created on the graph below.



i. Draw the best-fit line for the data.

ii. Using the best-fit line, calculate a value for the unknown capacitance C.

Slope of the graph

$$\frac{\ln\left(\frac{\Delta V_c}{\Delta V_0}\right)}{t} = m = \frac{\Delta y}{\Delta x} =$$

$P_1\,(+0.24, -0.32)$, $P_2\,(0.54, -0.72)$

$$\frac{\Delta y}{\Delta x} = \frac{-0.72 + 0.32}{0.54 - 0.24} = -1.33$$

$$\frac{-\log_e\left(\frac{\Delta V_c}{\Delta V_0}\right)}{t} = \frac{1}{RC}$$

$$\frac{1}{RC} = 1.33 \Rightarrow C = \frac{1}{R(1.33)} = \frac{1}{150\times10^{3}\times1.33}$$

$$= 5.01\ F$$

(d) The capacitor is adjusted so that the surface area of the plates is increased, and the experiment is repeated. Would the slope of the best-fit line in the second experiment be more steep, less steep, or unchanged compared to the slope of the best-fit line in part (c)?
___ More steep _✓_ Less steep ___ Unchanged Briefly justify your answer.

$$C = \frac{K \epsilon_0 A}{d}$$

when A is increased
the C will also increase

Hence slope will be less
steep

(e) The ideal battery is then replaced with a non-ideal battery with internal resistance r, and the experiment is repeated.
i. Would the slope of the graph in this final experiment change compared to the graph in part (c)? ___ Yes _✓_ No
Briefly justify your answer.

Capacitor is still discharge through
resistor R hence the slope will remain
same.

ii. Would the vertical intercept of the graph in this final experiment change compared to the graph in part (c)? ___ Yes ___ No ✓

Briefly justify your answer.

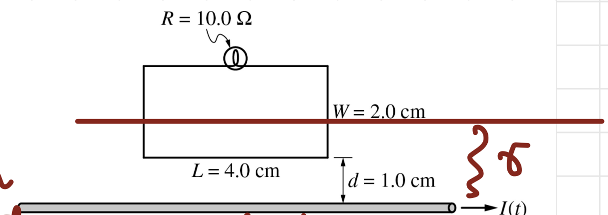
No → The best fit line does not change, because the internal resistance of the battery does not affect the potential difference across the charging capacitor.

3. A lightbulb of resistance R = 10.0 Ω is connected to a rectangular loop of wire of negligible resistance near a very long current-carrying wire. The rectangular loop has a length L = 4.0 cm and a width W = 2.0 cm and is positioned so one of the longer sides of the loop is a distance d = 1.0 cm above and parallel to the long wire, as shown. The current in the long wire is initially flowing to the right and is given by $I(t) = C - Dt$ where C = 10.0 A and D = 2.0 A/s. At time t = 5.0 s, the current in the long wire is instantaneously zero as the current changes direction.

(a) What is the direction, if any, of the magnetic field produced by the induced current in the rectangular loop as the current in the long wire changes direction?

___ Into the page ✓__ Out of the page ___ No direction, because the field is zero Justify your answer.

Direction of current reversed hence the magnetic flux inside the loop also decreases then as per lenz's law the induced current produced in such a way that It will produce magnetic field into the loop.



(b) Calculate the magnetic flux through the loop due to only the long wire at time t = 3.0 s.

$$I(t) = C - D(t) = 10 - 2(3) = 4\ A$$

$$\int d\phi = \int_d^{d+w} B(L)\,dr = \int_d^{(d+w)} \frac{\mu_0}{4\pi}\frac{2I}{r}(L)\,dr = \frac{\mu_0}{4\pi} 2I(L)\left[\log_e r\right]_d^{(d+w)}$$

$$= \frac{\mu_0 I L}{2\pi}\left[\log_e\left(\frac{d+w}{d}\right)\right]$$

$$= \frac{4\pi\times10^{-7}\times4\times.04}{2\pi}\log_e\left(\frac{.03}{.01}\right) = 3.52\times10^{-8}\ T\,m^2$$

(c) Calculate the current through the lightbulb at time t = 3.0 s.

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}\frac{\mu_0 I L}{2\pi}\left[\log_e\left(\frac{d+w}{d}\right)\right]$$

$$= -\frac{d}{dt}\frac{\mu_0 L}{2\pi}(C-Dt)\log_e\left(\frac{d+w}{d}\right)$$

$$= -\frac{\mu_0 L}{2\pi}\log_e\left(\frac{d+w}{d}\right)\frac{d}{dt}(C-Dt)$$

$$= \frac{\mu_0 D L}{2\pi}\log_e\left(\frac{d+w}{d}\right)$$

$$I = \frac{e}{R} = \frac{4\times10^{-7}\times 2\times .04}{2\pi\times10}\log_e\left(\frac{.03}{.01}\right)$$

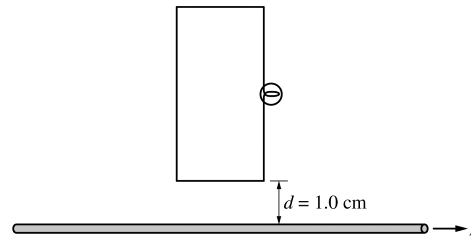
$$= 1.8\times10^{-9}\ A$$

(d) A group of students attempts to experimentally verify whether the current through the lightbulb is consistent with the current calculation from part (c). The current in the rectangular loop is measured to be greater than the current calculated in part (c). Which of the following could explain this discrepancy? Select one answer.

___ The students did not account for Earth's magnetic field.
___ The rectangular loop is tilted and is not in the same plane as the wire.
___ The resistance of the lightbulb is greater than the recorded value.
___ The long side of the rectangular loop is shorter than the recorded value.
✓ The current in the long wire changes at a faster rate than expected.

Briefly justify your answer.

If the current in the wire change at faster rate, there will be greater change of magnetic flux, so the induced current and induced emf will be larger.



(e) Later, the same rectangular loop with lightbulb is rotated such that a short side of the loop is 1.0 cm above and parallel to the long current-carrying wire, as shown. The current in the wire is again initially flowing from left to right and given by $I(t) = C - Dt$, where $C = 10.0$ A and $D = 2.0$ A/s. The current through the lightbulb in the loop's new orientation at time $t = 3.0$ s is I_2. Which of the following correctly relates the current I_2 to I_1 the current through the lightbulb in part (c)?

$I_2 < I_1$ $\quad I_2 = I_1$ $\quad$ ✓ $I_2 > I_1$

Justify your answer.

New orientation
some part of rectangle are further away from the straight wire, which means the magnetic field through the rectangle will be less, hence rate of change of flux decreases resulting smaller current

STOP

END OF EXAM