2022

AP Physics C: Mechanics Free-Response ANSWERS

Que-1

$T = mg$

$dW = \vec{F} \cdot d\vec{r}$

Que-2

$\text{Impulse} = F \cdot \Delta t$

$mgh = \frac{1}{2} m v^2$

Que-3

$\vec{\tau} = \vec{r} \times \vec{F}$

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass, $m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude, $e = 1.60 \times 10^{-19}$ C
Neutron mass, $m_n = 1.67 \times 10^{-27}$ kg	1 electron volt, $1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass, $m_e = 9.11 \times 10^{-31}$ kg	Speed of light, $c = 3.00 \times 10^{8}$ m/s
Avogadro's number, $N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$	Universal gravitational constant, $G = 6.67 \times 10^{-11}\ (\text{N}\cdot\text{m}^2)/\text{kg}^2$
Universal gas constant, $R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface, $g = 9.8\ \text{m/s}^2$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}$ J/K	

1 unified atomic mass unit, $1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\ \text{MeV}/c^2$

Planck's constant, $h = 6.63 \times 10^{-34}\ \text{J}\cdot\text{s} = 4.14 \times 10^{-15}\ \text{eV}\cdot\text{s}$

$hc = 1.99 \times 10^{-25}\ \text{J}\cdot\text{m} = 1.24 \times 10^{3}\ \text{eV}\cdot\text{nm}$

Vacuum permittivity, $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$

Coulomb's law constant, $k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9}\ (\text{N}\cdot\text{m}^2)/\text{C}^2$

Vacuum permeability, $\mu_0 = 4\pi \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$

Magnetic constant, $k' = \mu_0/(4\pi) = 1 \times 10^{-7}\ (\text{T}\cdot\text{m})/\text{A}$

1 atmosphere pressure, $1\text{ atm} = 1.0 \times 10^{5}\ \text{N}/\text{m}^2 = 1.0 \times 10^{5}$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.

II. The direction of current is the direction in which positive charges would drift.

III. The electric potential is zero at an infinite distance from an isolated point charge.

IV. All batteries and meters are ideal unless otherwise stated.

V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta \vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu |\vec{F}_N|$

$\Delta E = W = \int \vec{F} \cdot d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \cdot \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi \sqrt{\frac{m}{k}}$

$T_p = 2\pi \sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{G m_1 m_2}{r^2}$

$U_G = -\frac{G m_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0} \left| \frac{q_1 q_2}{r^2} \right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \cdot d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = N e v_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I \Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$\mathcal{E} = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L \frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

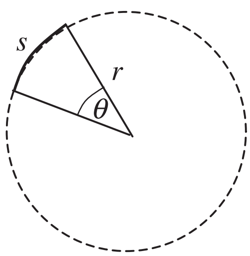
$a^2 + b^2 = c^2$

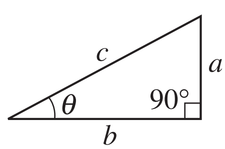
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS!

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1!$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS!

$\vec{A}\bullet\vec{B} = AB\cos\theta$

$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

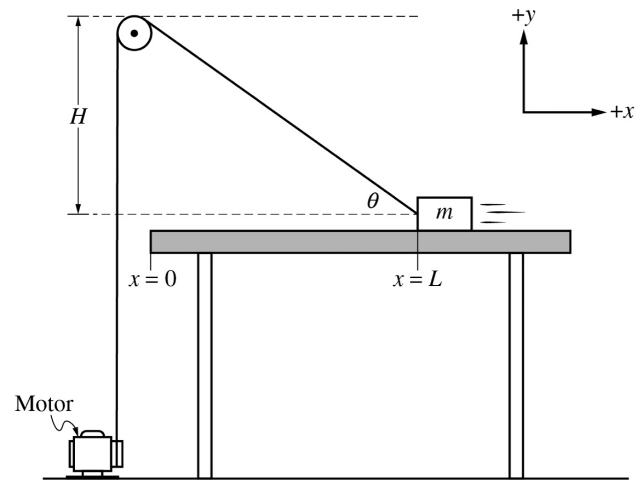
Begin your response to QUESTION 1 on this page.

PHYSICS C: MECHANICS SECTION II Time—45 minutes

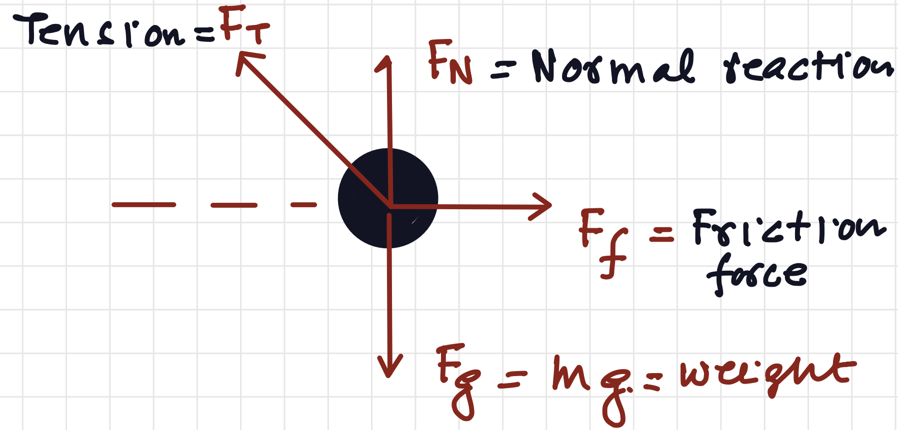
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

1. A block of mass m is pulled across a rough horizontal table by a string connected to a motor that is attached to the floor. The string passes over a pulley with negligible friction that is vertically aligned with the left edge of the table as shown. The string and pulley both have negligible mass. The pulley is at height H above the table. The motor exerts a constant force of tension FT on the string, and the block remains in contact with the table at all times as the block slides across the table from x = L to x = 0. The coefficient of kinetic friction between the table and the block is mk. Express all algebraic answers in terms of m, H, F_T, μ_K, L and physical constants as appropriate.



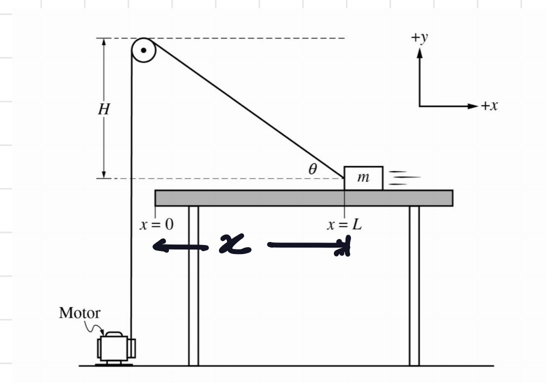
(a) On the dot below that represents the block, draw and label the forces (not components) that act on the block when the block is at $x = L$. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



(b) Derive an expression for the angle θ that the string makes with the horizontal as a function of x.

$$\tan\theta = \frac{H}{x}$$

$$\theta = \tan^{-1}\left(\frac{H}{x}\right)$$



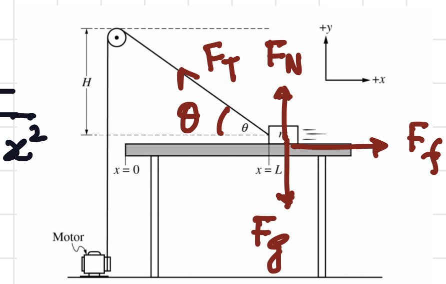
(c)

Continue your response to QUESTION 1 on this page.

i. Derive an expression for the normal force F_N exerted on the block by the table as a function of the block's position x.

$$\tan\theta = \frac{H}{x}$$

$$\sin\theta = \frac{H}{\sqrt{H^2+x^2}}$$



$$F_N + F_T \sin\theta - f_g = 0$$

$$F_N = F_g - F_T \sin\theta$$

$$= F_g - F_T\left(\frac{H}{\sqrt{H^2+x^2}}\right)$$

ii. Derive an expression for the magnitude of the net horizontal force F_{NET} exerted on the block as a function of the position x.

$$\cos\theta = \frac{x}{\sqrt{H^2+x^2}}$$

$$F_{NET} = F_T\cos\theta - F_f$$

$$= F_T\left(\frac{x}{\sqrt{H^2+x^2}}\right) - \mu_k F_N$$

$$= F_T\left(\frac{x}{\sqrt{(H^2+x^2)}}\right) - \mu_k\left(F_g - F_T\left(\frac{H}{\sqrt{H^2+x^2}}\right)\right)$$

(d) Write, but do not solve, an integral expression that could be used to solve for the work W done by the string on the block as the block moves from x = L to x = 0.

$$dW = \int_{x=L}^{x=0} F_{T,x}\, dx = \int -F_T\, dx \cos\theta$$

$$= \int_{x=L}^{x=0} -F_T \left(\frac{x}{\sqrt{x^2+H^2}}\right) dx$$

(e) Does the string do more, less, or the same amount of work on the block as the block moves from x = L to x = L/2 compared to when the block moves from x = L/2 to x = 0 ?

✓ More work when the block moves from x = L to x = L/2

______ Less work when the block moves from x = L to x = L/2

______ The same amount of work when the block moves from x = L to x = L/2

Justify your answer.

$x = L$ to $x = L/2$

$W_1 = F_T d \cos\theta_1$

$x = L/2$ to 0

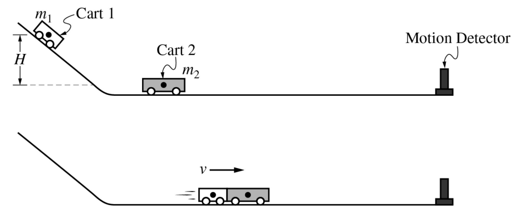
$W_2 = F_T d \cos\theta_2$

$\theta_2 > \theta_1$

$W_1 > W_2$

more work when block moves from $x = L$ to $x = L/2$

Begin your response to QUESTION 2 on this page.



2. Cart 1 of mass m_1 is held at rest above the bottom of an incline. Cart 2 has mass m_2 where $m_2 > m_1$ and is at rest at the bottom of the incline. At time t = 0, Cart 1 is released and then travels down the incline and smoothly transitions to the horizontal section. The center of mass of Cart 1 moves a vertical distance of H, as shown. At time t_c Cart 1 reaches the bottom of the incline and immediately collides with and sticks to Cart 2. After the collision, the two-cart system moves with constant speed v. Frictional and rotational effects are negligible.

(a) During the collision, is the impulse on Cart 1 from Cart 2 greater than, less than, or equal to the magnitude of the impulse on Cart 2 from Cart 1 ?

___ Greater than

___ Less than _✓_ Equal to

justify your answer

$F.\Delta T$ → same, exert same force on same time

Continue your response to QUESTION 2 on this page.

(b) On the following axes, draw graphs of the magnitude of the momentum of each cart as a function of time t, before and after t_c The collision occurs in a negligible amount of time. The grid lines on each graph are drawn to the same scale.

CART-1

$0 < t < t_c$

p increasing

at $t = t_c$ maximum momentum

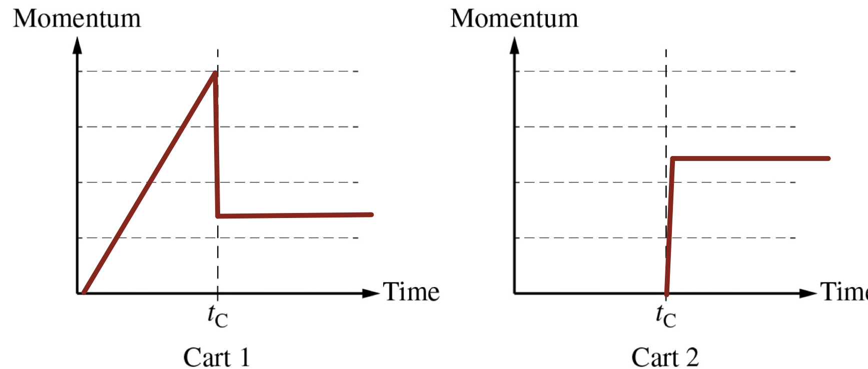
$t > t_c$ smaller momentum

CART-2

$0 < t < t_c$

$p = 0$

at $t > t_c$ momentum increases



(c) Show that the velocity v of the two-cart system after the collision is given by the equation .

$$v = \sqrt{2g}\left(\frac{m_1}{m_1 + m_2}\right)\sqrt{H}$$

Apply conservation of energy first

$$m_1 g H = \frac{1}{2} m_1 v_1^2 \Rightarrow v_1 = \sqrt{2gH}$$

Apply law of conservation of linear momentum.

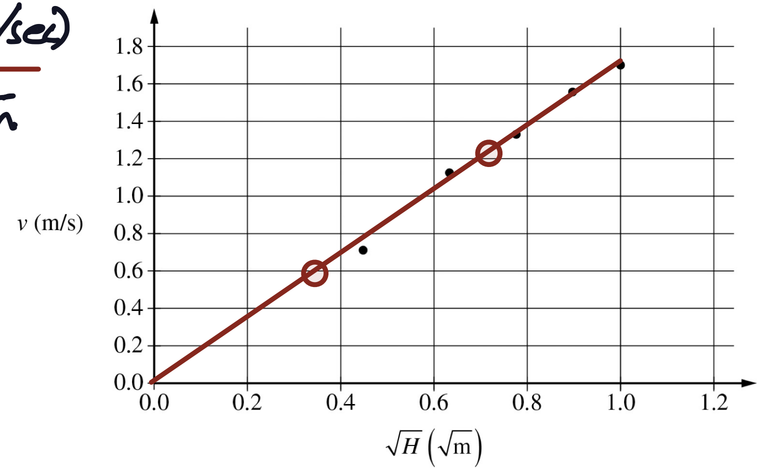
$$m_1 v_1 = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_1}{(m_1 + m_2)} = \frac{m_1 \sqrt{2gH}}{(m_1 + m_2)}$$

Proved

(d) A group of students use the setup to perform an experiment. They measure the mass of Cart 1 to be m_1 = 0.250 kg. The mass of Cart 2 is unknown. The students perform several trials and in each trial, Cart 1 is released from a different height H and the final velocity of the two-cart system is measured. The students graph v as a function of H, as shown below.

$$\text{slope of the graph} = \frac{v\ (m/sec)}{\sqrt{H}\ \sqrt{m}} = \frac{v}{\sqrt{H}} = \left(\frac{1.20 - 0.60}{0.70 - 0.35}\right) = 1.72\ \frac{\sqrt{m}}{s}$$



i Draw a line that represents the best fit to the data points shown.

ii. Use the best-fit line to calculate the mass of Cart 2.

$$\text{slope} = \left(\frac{m_1}{m_1 + m_2}\right)\sqrt{2g} \Rightarrow m_2 = \frac{m_1\sqrt{2g}}{\text{slope}} - m_1$$

$$= \frac{(0.25)\sqrt{2(9.8)}}{1.72} - 0.25 = 0.39\ \text{kg}$$

(e) After the experiment, the students use a balance to measure the mass of Cart 2 and find it to be less than what was determined in part (d). To explain this discrepancy, one of the students proposes that the mass of Cart 1 was incorrectly measured at the beginning of the experiment. The students measure the mass of Cart 1 again and record a new value, m_1'.

Should the students expect that m_1' will be greater than 0.250 kg, less than 0.250 kg, or equal to 0.250 kg ?

_____ $m_1' > 0.250$ kg Justify your answer.

✓____ $m_1' < 0.250$ kg _____ $m_1' = 0.250$ kg

A smaller m_2 indicates that initial energy and momentum was smaller, with identical slope of height H, this means that the mass m_1' must be smaller.

(a) Derive an expression for the mass m_B of the block.

$$m_B g = kx$$

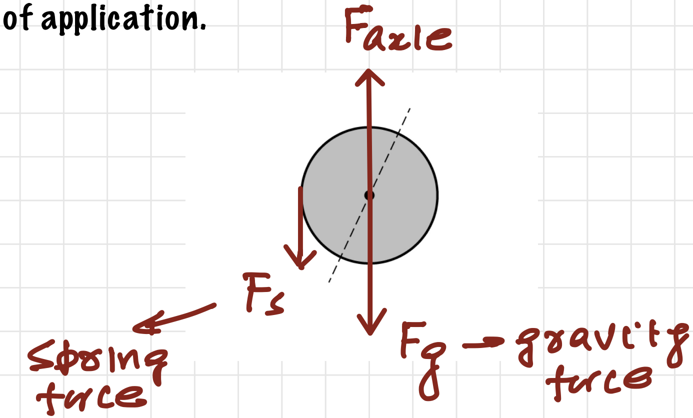
$$m_B g = kR(\theta)$$

$$\boxed{m_B = \frac{kR(\theta)}{g}}$$

$$\text{angle} = \frac{\text{arc}}{\text{Radius}}$$

$$\theta = \frac{\text{arc}}{R} \Rightarrow \text{arc} = R(\theta) = x$$

(b) At time t = 0, the string on the right side of the disk is cut and the block falls to the ground. On the circle below, which represents the disk, draw and label the forces (not components) that act on the disk immediately after the string is cut and the block is falling to the ground. Each force should be represented by an arrow that starts on and is directed away from the point of application.

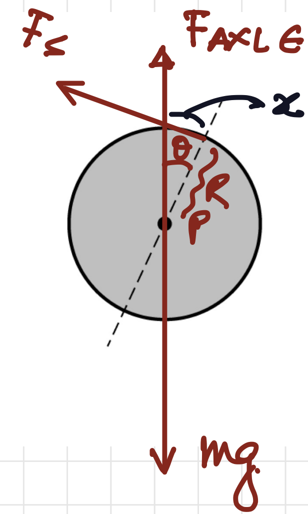


(c) Derive an expression for the angular acceleration a of the disk immediately after the string is cut.

Take Torque about point P.

$$F_s (R) = I (\alpha)$$

$$F_s = k(x) \qquad x = R(\theta)$$



$$-k(R)(\theta)(R) = \frac{1}{2} M_d R^2 (\alpha)$$

$$-\frac{2k(R)(\theta)(R)}{M_d R^2} = \alpha$$

$$\alpha = -\frac{2k\theta}{M_d}$$

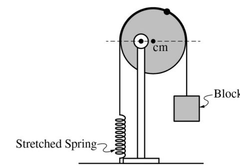


Figure 3

Note: Figure not drawn to scale.

(e) The disk is adjusted on the support so that the axle does not pass through the center of mass of the disk. The block is again hung on the right side of the disk and the spring-disk-block system comes to equilibrium, as shown in Figure 3. The axle does not exert a torque on the disk. For each force on the disk, indicate whether the magnitude of the torque about the axle caused by that force increases, decreases, or stays the same relative to part (b).

Torque due to spring force = Torque due to gravity force + Torque due to tension due to block

↓ Increase

Hence to make equilibrium Torque due to spring force also increases

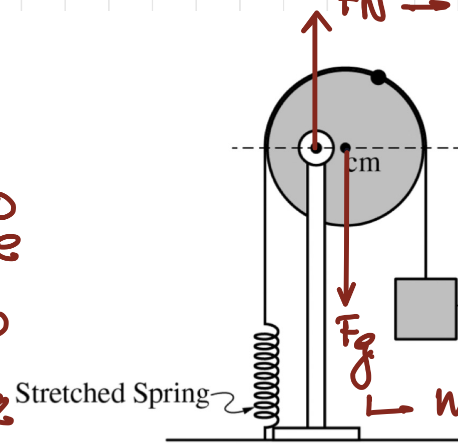


Figure 3

F_N → magnitude of torque remains same

F_g → magnitude of torque due to gravity increases

STOP

END OF EXAM

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