2021

AP® Physics C: Mechanics Free-Response Questions Set 1

Que-1 $F = m(a)$

Que-2 $F = \frac{mv^2}{r}$

$a_{NET} = \sqrt{a_c^2 + a_T^2}$

$mgh = \frac{1}{2}kx^2$

$KE + PE = \text{constant}$

Que-3 → $I = \int dm\, x^2$, $x_{cm} = \frac{\int x\,dm}{\int dm}$

ADVANCED PLACEMENT PHYSICS C TABLE OF INFORMATION

CONSTANTS AND CONVERSION FACTORS

Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg
Avogadro's number,	$N_0 = 6.02 \times 10^{23}$ mol^{-1}
Universal gas constant,	$R = 8.31$ J/(mol•K)
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K
Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
1 electron volt,	1 eV $= 1.60 \times 10^{-19}$ J
Speed of light,	$c = 3.00 \times 10^{8}$ m/s
Universal gravitational constant,	$G = 6.67 \times 10^{-11}\ (\text{N•m}^2)/\text{kg}^2$
Acceleration due to gravity at Earth's surface,	$g = 9.8\ \text{m/s}^2$
1 unified atomic mass unit,	1 u $= 1.66 \times 10^{-27}$ kg $= 931\ \text{MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}$ J•s $= 4.14 \times 10^{-15}$ eV•s
	$hc = 1.99 \times 10^{-25}$ J•m $= 1.24 \times 10^{3}$ eV•nm
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N•m}^2)$
Coulomb's law constant,	$k = 1/(4\pi\varepsilon_0) = 9.0 \times 10^{9}\ (\text{N•m}^2)/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ (T•m)/A
Magnetic constant,	$k' = \mu_0/(4\pi) = 1 \times 10^{-7}$ (T•m)/A
1 atmosphere pressure,	1 atm $= 1.0 \times 10^{5}\ \text{N/m}^2 = 1.0 \times 10^{5}$ Pa

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following assumptions are used in this exam.

I. The frame of reference of any problem is inertial unless otherwise stated.
II. The direction of current is the direction in which positive charges would drift.
III. The electric potential is zero at an infinite distance from an isolated point charge.
IV. All batteries and meters are ideal unless otherwise stated.
V. Edge effects for the electric field of a parallel plate capacitor are negligible unless otherwise stated.

ADVANCED PLACEMENT PHYSICS C EQUATIONS

MECHANICS

$v_x = v_{x0} + a_x t$

$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$

$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$

$\vec{F} = \frac{d\vec{p}}{dt}$

$\vec{J} = \int \vec{F}\, dt = \Delta \vec{p}$

$\vec{p} = m\vec{v}$

$|\vec{F}_f| \le \mu |\vec{F}_N|$

$\Delta E = W = \int \vec{F} \bullet d\vec{r}$

$K = \frac{1}{2} m v^2$

$P = \frac{dE}{dt}$

$P = \vec{F} \bullet \vec{v}$

$\Delta U_g = mg\Delta h$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$

$I = \int r^2 dm = \sum m r^2$

$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$

$v = r\omega$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

$K = \frac{1}{2} I \omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

a = acceleration
E = energy
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
P = power
p = momentum
r = radius or distance
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$\vec{F}_s = -k\Delta\vec{x}$

$U_s = \frac{1}{2} k (\Delta x)^2$

$x = x_{max} \cos(\omega t + \phi)$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$|\vec{F}_G| = \frac{Gm_1 m_2}{r^2}$

$U_G = -\frac{Gm_1 m_2}{r}$

ELECTRICITY AND MAGNETISM

$|\vec{F}_E| = \frac{1}{4\pi\varepsilon_0}\left|\frac{q_1 q_2}{r^2}\right|$

$\vec{E} = \frac{\vec{F}_E}{q}$

$\oint \vec{E} \bullet d\vec{A} = \frac{Q}{\varepsilon_0}$

$E_x = -\frac{dV}{dx}$

$\Delta V = -\int \vec{E} \bullet d\vec{r}$

$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$\Delta V = \frac{Q}{C}$

$C = \frac{\kappa \varepsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C (\Delta V)^2$

$R = \frac{\rho \ell}{A}$

$\vec{E} = \rho \vec{J}$

$I = N e v_d A$

$I = \frac{\Delta V}{R}$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = I\Delta V$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = radius or distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
Φ = flux
κ = dielectric constant

$\vec{F}_M = q\vec{v} \times \vec{B}$

$\oint \vec{B} \bullet d\vec{\ell} = \mu_0 I$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$

$\vec{F} = \int I\, d\vec{\ell} \times \vec{B}$

$B_s = \mu_0 n I$

$\Phi_B = \int \vec{B} \bullet d\vec{A}$

$\mathcal{E} = \oint \vec{E} \bullet d\vec{\ell} = -\frac{d\Phi_B}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2} L I^2$

ADVANCED PLACEMENT PHYSICS C EQUATIONS

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

$s = r\theta$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

Right Triangle

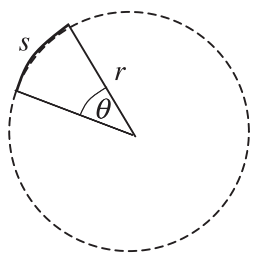
$a^2 + b^2 = c^2$

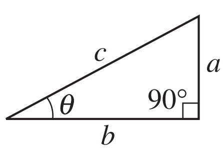
$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius
s = arc length
θ = angle





CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}(\ln ax) = \frac{1}{x}$$

$$\frac{d}{dx}[\sin(ax)] = a\cos(ax)$$

$$\frac{d}{dx}[\cos(ax)] = -a\sin(ax)$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$

$$\int \frac{dx}{x+a} = \ln|x+a|$$

$$\int \cos(ax)\,dx = \frac{1}{a}\sin(ax)$$

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax)$$

VECTOR PRODUCTS

$$\vec{A}\bullet\vec{B} = AB\cos\theta$$

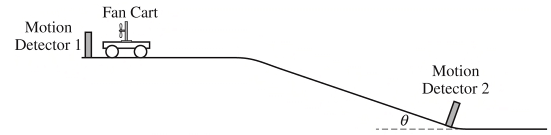
$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$$

Begin your response to QUESTION 1 on this page.

PHYSICS C: MECHANICS SECTION II Time—45 minutes

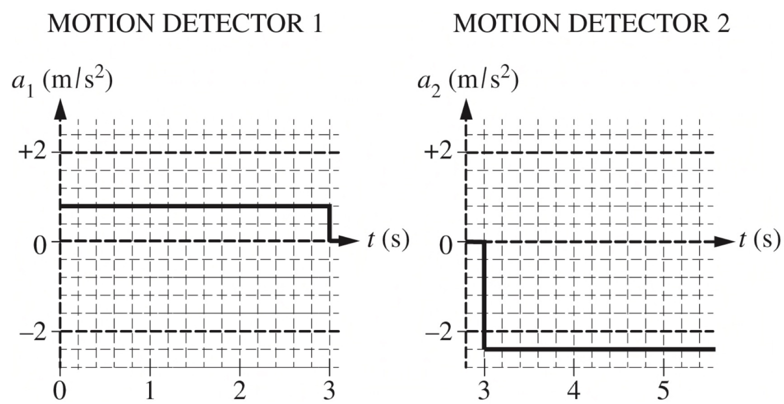
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



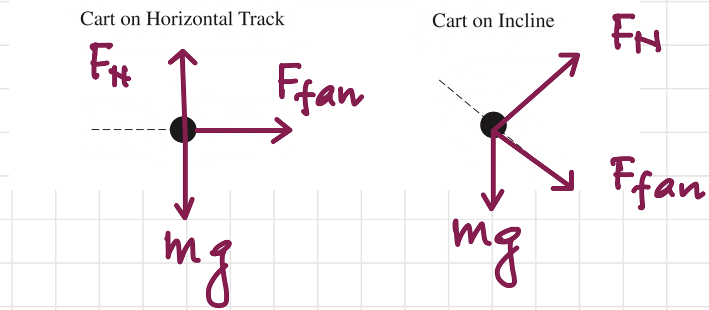
Note: Figure not drawn to scale.

1. A 0.50 kg fan cart is placed on a level, horizontal track of negligible friction, as shown. The fan is turned on, and the fan cart is released from rest and moves to the right. The cart travels along the horizontal track and then down an incline. Motion detector 1 measures the acceleration a of the cart from time t = 0 to t = 3 s. At t = 3 s, the cart makes a smooth transition to the incline, and motion detector 2 measures the acceleration of the cart after t = 3 s. The fan exerts the same magnitude of force on the cart during the entire motion. The graphs below show a as functions of t. For each motion detector, the positive direction is away from the detector.



Continue your response to QUESTION 1 on this page.

(a) On the dots below that represent the cart at two different locations, draw and label the forces (not components) that act on the cart at each location. Each force must be represented by a distinct arrow starting on, and pointing away from, the dot.



(b) Calculate the magnitude of the net force exerted on the fan cart when it is on the horizontal track.

$$F_{fan} = (M_{cart})(a_1)$$

$$= (0.50)(0.8) = 0.40\,N$$

(c) Calculate the angle θ of the incline.

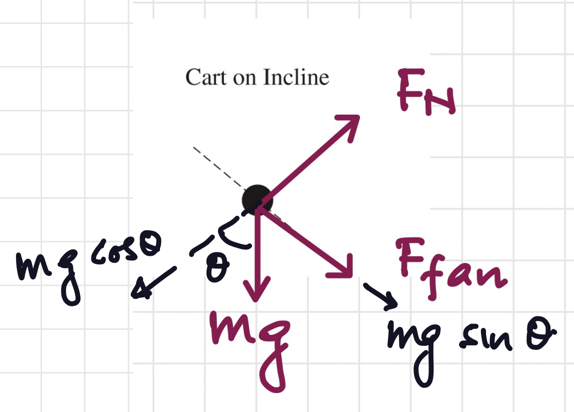
$$F_{fan} + mg\sin\theta = ma_2$$

$$mg\sin\theta = ma_2 - F_{fan}$$

$$\sin\theta = \frac{ma_2 - F_{fan}}{mg}$$

$$\sin\theta = \frac{(0.50)(2.4) - (0.50)(0.8)}{(0.50)(9.8)}$$

$$\theta = 9.4^\circ$$



(d) Suppose careful measurement determines the angle of the incline to be 3° larger than that calculated in part (c). Consider the following explanation.

"The scale used to measure the mass of the fan cart was not calibrated properly before the measurement, and this could account for the observed difference in the angle."

Does the explanation sufficiently account for the observed discrepancy? _____ Yes ___✓__ No
Justify your answer.

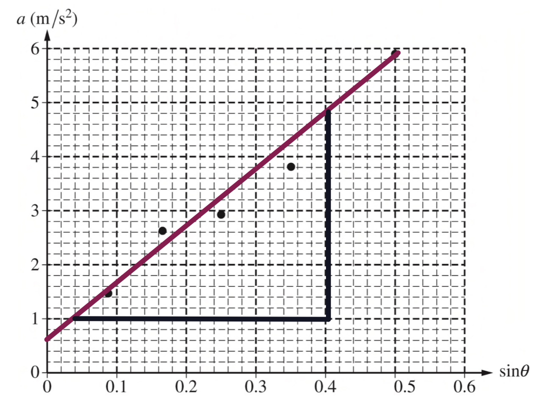
From the answer (c)

$$\sin\theta = \frac{ma_2 - f_{fan}}{mg} = \frac{\cancel{m}a_2 - ma_1}{\cancel{m}g}$$

↳ Hence mass of the cart cancels out the equation used to find the angle of the incline

Continue your response to QUESTION 1 on this page.

The experiment is repeated for several trials, each with a different angle for the incline. The acceleration of the cart down the incline is measured for each angle. The graph below shows the plot of the acceleration a of the cart as a function of the sine of the angle sin θ.



(e)
i. Draw a best-fit line for the data.
ii. Using the straight line, calculate an experimental value for the acceleration due to gravity g.

(f) If the cart were replaced with a second cart of mass 1.0 kg that has a fan that exerts the same magnitude of force as the original fan, explain how the graph given in part (e) would change.

$$\text{slope} = \frac{\Delta a}{\Delta \sin\theta} = \frac{5-1}{(0.42-.04)} = 10.52\ m\,s^{-2}$$

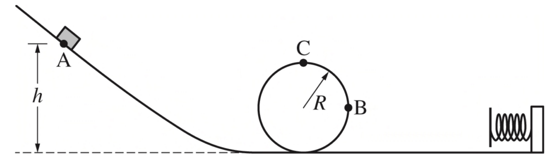
$$ma = F_{fan} + mg\sin\theta$$

$$a = g\sin\theta + \frac{F_{fan}}{m}$$

$$y = mx + b = (\text{slope})\sin\theta + y\ \text{intercept}$$

$$\text{slope} = g = 10.52\ m\,s^{-2}$$

Begin your response to **QUESTION 2** on this page.



Note: Figure not drawn to scale.

2. A block of mass m starts from rest at point A and travels with negligible friction through the loop onto a horizontal surface, where the block makes contact with a spring of spring constant $k = \frac{mg}{2R}$. All motion of the spring is in the horizontal direction. Point C is the highest point on the loop, and point B is the rightmost point on the loop. Express all algebraic answers in terms of m, h, R, and physical constants, as appropriate.

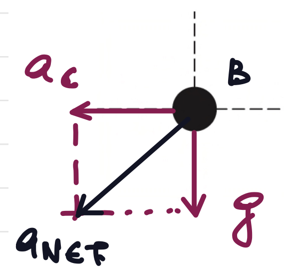
(a) On the dot below, which represents the block, draw an arrow that represents the direction of the acceleration of the block at point B in the figure above. The arrow must start on and point away from the dot.

Justify your answer

$a_c \rightarrow$ centripetal component towards the centre

$a_{downward} = g$

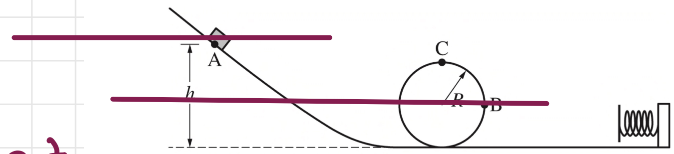
$a_{NET} = \sqrt{g^2 + a_c^2}$



(b)

i. Derive an expression for the speed v of the block at point B.

Apply conservation of energy at point A & point B.



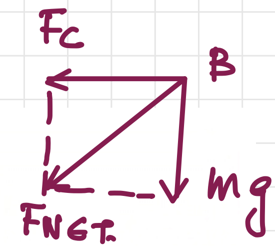
Note: Figure not drawn to scale.

$$(TE)_A = (TE)_B$$

$$mgh + \frac{1}{2}m(0)^2 = mgR + \frac{1}{2}m v_B^2$$

$$\cancel{m}g(h-R) = \frac{1}{2}\cancel{m}v_B^2 \Rightarrow v_B = \sqrt{2g(h-R)}$$

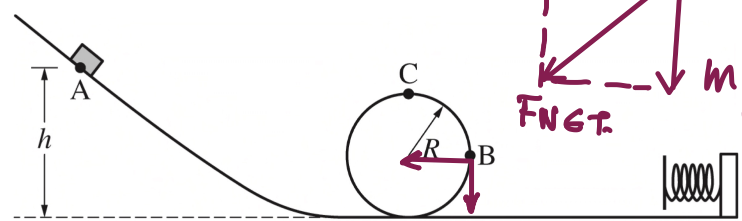
ii. Derive an expression for the magnitude of the net force F on the block at point B.



$$F_{NET} = \sqrt{(mg)^2 + (F_c)^2}$$

$$= \sqrt{m^2g^2 + \left(\frac{m v_B^2}{R}\right)^2}$$

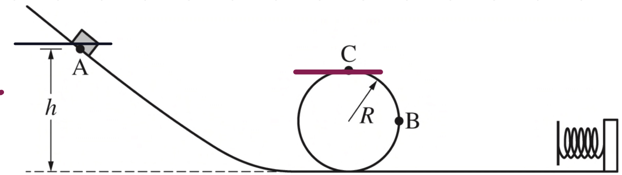
$$= \sqrt{m^2g^2 + \left(\frac{2mg}{R}(h-R)\right)^2}$$



Note: Figure not drawn to scale.

Continue your response to QUESTION 2 on this page.

(c) In terms of R, derive an expression for the minimum height h_{min} necessary for the block to maintain contact with the track through point C.



Note: Figure not drawn to scale.

$$(TE)_A = (TE)_C$$

$$mgh_A + \cancel{\frac{1}{2}mv_A^2}^{\,0} = mg(2R) + \frac{1}{2}mv_C^2$$

$$\cancel{m}gh = \cancel{m}g(2R) + \frac{1}{2}\cancel{m}v_C^2$$

$$gh = g(2R) + \frac{v_C^2}{2}$$

$$v_C = \sqrt{2g(h-2R)}$$

$$\sqrt{\cancel{g}R} = \sqrt{2\cancel{g}(h-2R)}$$

$$R = 2(h-2R)$$

$$R = 2h - 4R$$

$$2h = 5R \Rightarrow \boxed{h = \frac{5R}{2} = 2.5R}$$

For point C, minimum velocity to complete the circular motion, for which $N=0$

$$mg + N = \frac{mv_C^2}{R} \Rightarrow$$

put $N=0$

$$v_C = \sqrt{gR}$$

(d) It is determined that $h = 0.30$ m and $R = 0.10$ m. If the block is released from a height greater than that found in part (c), what would be the maximum compression x_{max} of the spring?

$$mgh = \frac{1}{2} k x_{max}^2$$

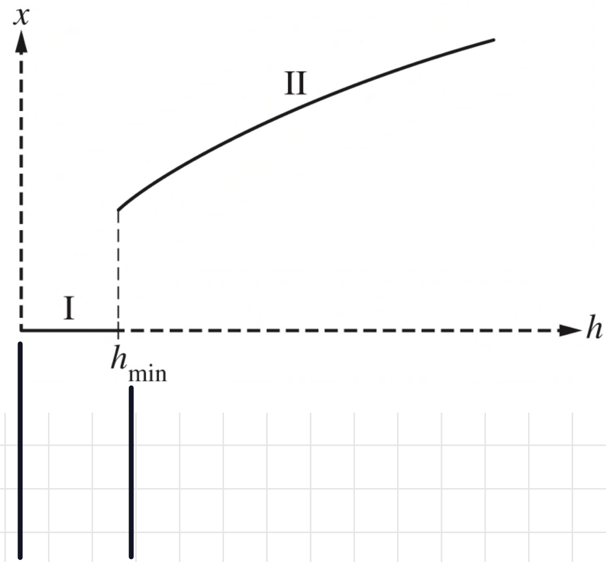
$$2mgh = k x_{max}^2 \Rightarrow x_{max} = \sqrt{\frac{2mgh}{k}}$$

$$= \sqrt{\frac{2mgh}{mg/2R}} = \sqrt{4hR}$$

$$= \sqrt{(4)(0.30)(0.10)}$$

$$= 0.35 \text{ m}$$

(e) A graph of the maximum compression of the spring as a function of height is shown below. The height hmin is the height calculated in part (c).



i. Explain why section I appears as a horizontal line segment on the horizontal axis.

Block does not make it through the loop at this height so it can not reach near to spring

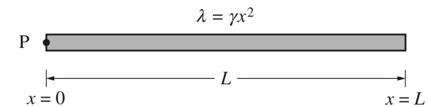
ii. Explain the reason for the shape of section II on the graph.

$$mgh = \frac{1}{2} kx^2$$

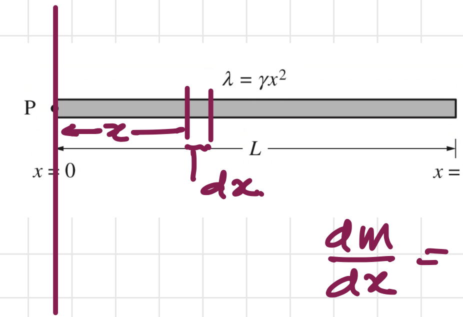
$$h \propto x^2$$

Hence parabolic

Begin your response to QUESTION 3 on this page.



3. A triangular rod of length L and mass M has a nonuniform linear mass density given by the equation $\lambda = \gamma x^2$ where $\gamma = \frac{3M}{L^3}$, and x is the distance from point P at the left end of the rod.
L3
(a) Using integral calculus, show that the rotational inertia I of the rod about an axis perpendicular to the page and through point P is $\frac{3}{5}ML^2$.



$$dI = dm\, x^2$$

$$= \lambda\, dx\, x^2$$

$$\int dI = \gamma x^2 dx\, x^2 = \int_0^L \gamma x^4 dx$$

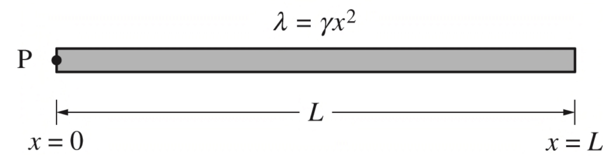
$$I = \gamma \left[\frac{x^5}{5}\right]_0^L = \frac{\gamma}{5}\left[L^5 - 0^5\right]$$

$$= \frac{\gamma}{5} L^5 = \frac{3M}{L^3(5)} L^5 = \frac{3}{5} M L^2$$

$$\frac{dm}{dx} = \lambda$$

$$dm = \lambda\, dx$$

(b) Determine the horizontal location of the center of mass of the rod relative to point P. Express your answer in terms of L.



$$x_{cm} = \frac{\int dm\, x}{\int dm}$$

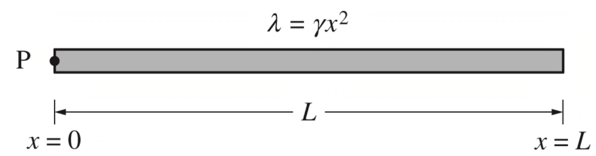
$$\frac{dm}{dx} = \lambda \Rightarrow dm = \lambda\, dx$$

$$= \frac{\int^{L} \lambda x\, dx}{\int_0 \lambda\, dx} = \frac{\int_0^L \gamma x^2 x \cdot dx}{\int_0^L \gamma x^2 dx} = \frac{\int_0^L \gamma x^3 dx}{\int_0^L \gamma x^2 dx}$$

$$= \frac{\gamma \left[\frac{x^4}{4}\right]_0^L}{\gamma \left[\frac{x^3}{3}\right]_0^L} = \frac{\cancel{\gamma}\left[\frac{L^4}{4} - 0\right]}{\cancel{\gamma}\left[\frac{L^3}{3} - 0\right]} = \frac{L^4 (3)}{4 \cdot L^3} = \frac{3L}{4}$$

(c) For an axis perpendicular to the page, is the value of the rotational inertia of the rod around point P greater than, less than, or equal to the value of the rotational inertia of the rod around the rod's center of mass?

✓_____ Greater than _____ Less than _____ Equal to Justify your answer.



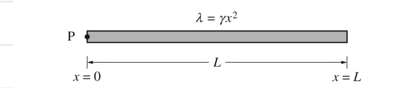
$I_P = I_c + M\,(x_{cm})^2$ — As per perpendicular axis theorem

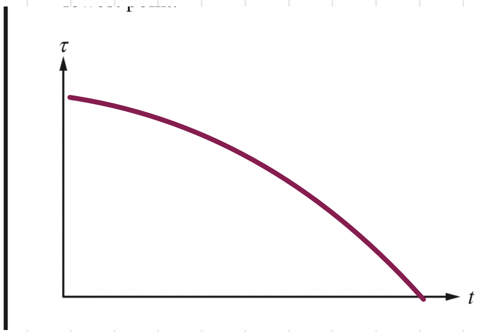
$I_P > I_c$

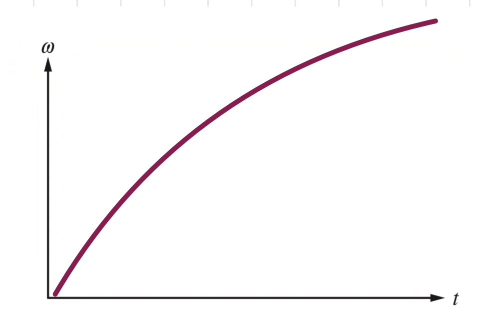
Continue your response to QUESTION 3 on this page.

The rod is released from rest in the position shown, and the rod begins to rotate about a horizontal axis perpendicular to the page and through point P.

(d) On the axes below, sketch graphs of the magnitude of the net torque t on the rod and the angular speed w of the rod as functions of time t from the time the rod is released until the time its center of mass reaches its lowest point.







$\tau = \vec{r} \times \vec{F}$

$= \text{force} \times \perp \text{distance}$

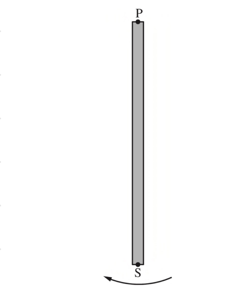
$w = w_0 + \alpha t$

(e) As the rod rotates from the horizontal position down through vertical, is the magnitude of the angular acceleration on the rod increasing, decreasing, or not changing?
_____ Increasing _____✓ Decreasing _____ Not changing Justify your answer.

$$\tau = \vec{r} \times \vec{f} = I(\alpha)$$

$$r f \sin\theta = I(\alpha)$$

As θ approaches to ZERO hence Torque is also approaching to ZERO, hence angular acceleration tends to zero.



(f) The mass of the rod is 3.0 kg, and the length of the rod is 1.0 m . Calculate the linear speed v of point S as the rod swings through the vertical position shown.

$$Mgh = \frac{1}{2} I \omega^2$$

$$\cancel{M} g \left(\frac{3}{4} L\right) = \frac{1}{2}\left(\frac{3}{5} \cancel{M} L^2\right)\left(\frac{V}{L}\right)^2$$

$$\frac{\cancel{3} g L}{{}_2\cancel{4}} = \frac{1}{\cancel{2}}\left(\frac{\cancel{3}}{5}\right)\frac{V^2}{\cancel{L^2}}$$

$$V = \sqrt{\frac{5gL}{2}} = \sqrt{\frac{5 \times 9.8 \times 1}{2}} = 4.9\,m s^{-1}$$