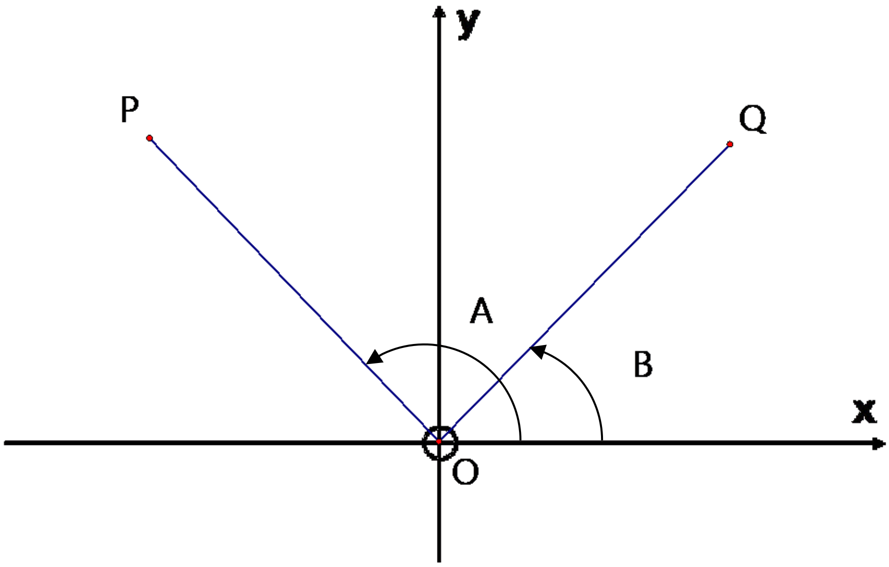
TRIGONOMETRY

COMPOUND ANGLES

Out of Interest:

1. Prove that $\cos(A-B)=\cos A\cos B+\sin A\sin B$



OP = OQ = r = 1 unit

2. Hence, prove that $\cos(A+B)=\cos A\cos B-\sin A\sin B$

3. Hence, prove that $\sin(A-B)=\sin A\cos B-\sin B\cos A$

4. Hence, prove that $\sin(A+B)=\sin A\cos B+\sin B\cos A$

COMPOUND ANGLE FORMULA

$$\cos(A-B)=\cos A\cos B+\sin A\sin B \qquad \cos(A+B)=\cos A\cos B-\sin A\sin B$$
$$\sin(A-B)=\sin A\cos B-\sin B\cos A \qquad \sin(A+B)=\sin A\cos B+\sin B\cos A$$

Examples

Expand and simplify, without using a calculator:

a) $\cos(45^\circ-30^\circ)$

b) $\sin(60^\circ+45^\circ)$

<u>Exercise A</u>

Expand each of the following using the <u>compound angle formula:</u>

a) $\cos(x-40°)=$

b) $\sin(x-50°)=$

c) $\cos(2x+y)=$

d) $\sin(10°+B)=$

e) $\cos(60°-A)=$

f) $\sin(45°-x)=$

<u>Examples</u>

1. Simplify each of the following, <u>without using a calculator</u>, to one term only:

a) $\sin 2\theta.\cos\theta-\cos 2\theta.\sin\theta$

b) $\cos 70°.\cos x-\sin 70°.\sin x$

c) $\sin x.\sin y-\cos x.\cos y$

d) $\cos x.\sin y-\sin x.\cos y$

e) $\cos 2x.\sin 3x-\cos 3x.\sin 2x$

f) $\sin 4\theta.\sin 5\theta-\cos 5\theta.\cos 4\theta$

2. Calculate each of the following, <u>without using a calculator</u>:

a) $\cos 320°.\cos 20°-\sin 140°.\sin 200°$

b) $\cos 10°.\sin 160°-\sin 10°.\sin 110°$

c) $\cos 48°+\sin 48°.\tan 24°$

Exercise B

1. Calculate each of the following, without using a calculator:

a) $\sin 280^\circ.\cos 160^\circ - \cos 100^\circ.\sin 200^\circ$

b) $\cos 95^\circ.\sin 355^\circ - \sin 85^\circ.\cos 175^\circ$

c) $\cos 65^\circ.\cos 295^\circ - \sin 115^\circ.\cos 205^\circ$

d) $\cos 50^\circ.\sin 260^\circ + \cos 10^\circ.\sin 140^\circ$

Examples

1. a) Show that $\cos(60^\circ + A) + \cos(60^\circ - A) = \cos A$

   b) Hence, evaluate $\cos 105^\circ + \cos 15^\circ$, without using a calculator.

$$\left[\frac{\sqrt{2}}{2}\right]$$

2. Calculate $\sin 75^\circ$, without using a calculator:

$$\left[\frac{\sqrt{6}+\sqrt{2}}{4}\right]$$

Exercise C

1. Simplify:

a) $\cos(A+B)+\cos(A-B)$

b) $\cos(A+B)-\cos(A-B)$

c) $\sin(A+B)-\sin(A-B)$

2. a) Simplify $\sin(A+B)+\sin(A-B)$

b) Hence, simplify $\sin 5A+\sin A$

c) Hence, simplify $\sin 5A+\sin 3A$

N.B.

3. Without using a calculator, prove that $\cos 80^\circ+\cos 40^\circ=\cos 20^\circ$

4. Example: Evaluate $\sin 15^\circ$, without using a calculator:

Now evaluate, without using a calculator:

a) $\cos 105^\circ$

b) $\sin 75^\circ$

5. Example: If $\sin\hat{P} = \frac{-3}{5}$ *and* $\hat{P} \in [0^\circ\ ;\ 270^\circ]$ *and* $\cos\hat{Q} = \frac{4}{5}$ *and* $\hat{Q} \in [180^\circ\ ;\ 360^\circ]$, determine, without using a calculator, $\cos(Q - P)$

$\left[-\frac{7}{25}\right]$

a) If $\cos A = -\frac{3}{5}$ *where* $A \in [180^\circ\ ;\ 360^\circ]$ *and* $\cos B = \frac{12}{13}$ *where* $B \in [180^\circ\ ;\ 360^\circ]$, determine, without using a calculator, $\sin(A + B)$

$\left[-\frac{33}{65}\right]$

b) If $\sin A = \frac{3}{5}$ *and* $\cos A < 0$ *and* $\sin B = -\frac{12}{13}$ *and* $\tan B > 0$, determine, <u>without using a calculator</u>, $\cos(A+B)$

6. <u>Example</u>: If $\sin 20° = m$, express the following in terms of m.

i) $\cos 110°$

ii) $\sin 80°$

a) If $\cos 10° = k$, express the following in terms of k.

i) $\sin 280°$

ii) $\cos 55°$

b) If $\sin 40° = m\cos 40°$, express the following in terms of m.

i) $\tan 50°$

ii) $\sin 70°$

c) If $\sin 25° = k \sin 65°$, express the following in terms of k.

i) $\tan 25°$ ii) $\tan 65°$ iii) $\cos 70°$

TRIG EQUATIONS USING COMPOUND ANGLES

Examples

1. Find the general solution in each of the following:

a) $\sin 40° \cos\theta - \sin\theta \cos 40° = \cos\theta \cos 40° - \sin\theta \sin 40°$

b) $\sin\theta \cos 40° + \cos\theta \sin 40° = \sin\theta \cos\theta + \cos\theta \sin\theta$

c) $\sin 3A \cos A - \sin A \cos 3A = -\frac{1}{2}$

d) $\sin 43° \cos\theta + \cos 43° . \sin\theta = \sin 86°$

N.B.

2. If $\cos(x + 30°) = 2\cos x$, prove, without using a calculator, that $\tan x = \sqrt{3} - 4$

N.B.

3. If $\sin(x + 30°) = \frac{1}{2}\sin x$, show, without using a calculator, that $\tan x = \frac{1}{1 - \sqrt{3}}$

CHALLENGING QUESTIONS (PROBLEM SOLVING)

1. In Δ ABC, $\hat{C}$ is an obtuse angle. Calculate the value of $\sin A$, if $\sin B = \frac{3}{5}$ and $\sin C = \frac{12}{13}$

2. In acute-angled Δ ABC prove that: $\sin\frac{A}{2} = \cos\frac{B+C}{2}$

DOUBLE ANGLES

Complete:

$\sin 2\theta =$

$\cos 2\theta =$

$=$

$=$

Remember:

$\sin^2\theta + \cos^2\theta = 1$

$1 - \cos^2\theta = \sin^2\theta$

$1 - \sin^2\theta = \cos^2\theta$

Exercise D

1. Express each of the following as 1 term in simplest form: (Remember – Factorise)

a) $1 + \cos x + \cos 2x$

b) $1 + \sin 2x + \cos 2x$

c) $1-\sin 2x$

d) $1+\sin 2x$

e) $\cos 2x+\cos x$

f) $\cos 2x+\sin x$

g) $1+\cos 2x$

h) $1-\cos 2x$

i) $\sin^2 x-\cos^2 x$

j) $-\sin^2 x-\cos^2 x$

k) $\dfrac{1-\cos 2x}{\sin 2x}$

l) $\cos^4 x-\sin^4 x$

m) $\dfrac{\sin 2x+\sin x}{\cos 2x+1+\cos x}$

n) $\dfrac{\sin 9x}{\sin 3x}-\dfrac{\cos 9x}{\cos 3x}$

o) $\dfrac{\sin 3x}{\sin x}-\dfrac{\cos 3x}{\cos x}$

Exercise E

1. Show that $\sin 8\theta = 8\sin\theta.\cos\theta.\cos 2\theta.\cos 4\theta$

2. Prove that $\cos 3A = 4\cos^3 A - 3\cos A$

3. Prove that $\cos 4A = 8\cos^4 A - 8\cos^2 A + 1$

4. Prove that $\sin 3A = 3\sin A - 4\sin^3 A$

5. Prove that $\dfrac{\cos 2A + \cos A}{\sin 2A - \sin A} = \dfrac{\cos A + 1}{\sin A}$

6. Prove that $\dfrac{1 + \cos 2A}{\cos 2A} = \dfrac{\tan 2A}{\tan A}$

7. Prove that $(\sin x + \cos x)^2 = 1 + \sin 2x$

8. Prove that $\dfrac{\cos 2x}{\cos x - \sin x} = \cos x + \sin x$

9. Prove that $\sin(45° - A)\sin(45° + A) = \dfrac{\cos 2A}{2}$

10. Prove that $\left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\cos 2\theta = 1-\tan^2\theta$

11. Prove that $\frac{\tan 2x}{\tan x} = \frac{\cos 2x + 1}{\cos 2x}$

12. Prove that $\frac{1-\sin 2A}{\sin A - \cos A} = \sin A - \cos A$

13. Prove that $\tan x + \dfrac{\cos x}{\sin x} = \dfrac{2}{\sin 2x}$

14. Prove that $\dfrac{\left(\cos^2 x - \sin^2 x\right)^2}{\cos^4 x - \sin^4 x} = \cos 2x$

15. Prove that $\dfrac{\sin 2x - \sin x}{\cos 2x + \cos x} = \dfrac{\sin x}{1 + \cos x}$

16. Prove that $\dfrac{\sin 2A}{\cos 2A} + \dfrac{1}{\cos 2A} = \dfrac{\sin A + \cos A}{\cos A - \sin A}$

17. Prove that $\dfrac{1}{1-\cos(180^\circ - x)} + \dfrac{1}{1-\sin(90^\circ - x)} = \dfrac{2}{\sin^2 x}$

18. Prove that $\dfrac{\cos A + 2\cos 2A - 3}{5\sin A + 2\sin 2A} = \dfrac{\cos A - 1}{\sin A}$

19. Prove that $\cos 6A + \cos 2A = 2\cos 2A.\cos 4A$

Exercise F

1. Use the double-angle formulae of sine to expand the following once:

a) $\sin 4A$ b) $\sin A$ c) $\sin 50°$

2. Use the double-angle formulae of cosine to expand the following only once:

a) $\cos 6A$ b) $\cos A$ c) $\cos 80°$

3. Write as a single trigonometric function:

a)	$2\sin x.\cos x$ $=$	b)	$2\sin 3A.\cos 3A$ $=$	c)	$2\sin 20°.\cos 20°$ $=$
d)	$\sin\theta.\cos\theta$ $=$	e)	$\cos^2 3\theta - \sin^2 3\theta$ $=$	f)	$1 - 2\sin^2\frac{\theta}{2}$ $=$
g)	$\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ $=$	h)	$2\cos^2\frac{\theta}{2} - 1$ $=$	i)	$\sin(45° - A)\sin(45° + A)$ $=$
j)	$2\cos^2 2\theta - 1$ $=$	k)	$1 - \sin^2\frac{\alpha}{2}$ $=$	l)	$\sin^2 42° - \cos^2 42°$ $=$

m)	$1-2\cos^2 22{,}5°$ $=$	n)	$(\cos 15° - \sin 15°)^2$ $=$ $=$ $=$	o)	$1-\sin^2 30°$ $=$ $=$ $=$
p)	$1-2\sin^2 30°$ $=$ $=$ $=$	q)	$2\cos 15°.\sin 15°$ $=$ $=$ $=$	r)	$\cos^2 45° - 1$ $=$ $=$ $=$
s)	$2\cos^2 45° - 1$ $=$ $=$ $=$				

<u>Examples</u>

Evaluate, <u>without using a calculator</u>:

a) $\dfrac{\sin 35°.\cos 35°}{\sin 150°.\cos 200°}$ **[-1]**

b) $\dfrac{\sin 36°}{\sin 12°} - \dfrac{\cos 36°}{\cos 12°}$ **[2]**

c) $\dfrac{\sin(-63°).\sin 27°}{\sin 1206°}$ **[-½]**

d) $\dfrac{\sin 118°.\tan 152°.\cos 332°}{\sin(-124°)}$ $\left[-\dfrac{1}{2}\right]$

<u>Exercise G</u>

Determine the value of each of the following, <u>without using a calculator</u>:

1. $\dfrac{\cos^2(-325°)-\sin^2 145°}{\cos 340°.\tan 200°}$ **[1]**

$$\frac{\left(\cos(325°)\right)^2 - \left(\sin(180°-35°)\right)^2}{\cos(360° -}$$

N.B.

2. $\cos 40° + 2\sin^2 200°$ **[1]**

3. $\tan 15° + \dfrac{\cos 15°}{\sin 15°}$ **[4]**

4. $\cos^2 15° + \sin 22{,}5°.\cos 22{,}5° - \sin^2 15°$ $\left[\dfrac{2\sqrt{3}+\sqrt{2}}{4}\right]$

5. $\sin 110° + \cos 70°.\tan 190°$

[1]

<u>"IF" TYPE OR PYTHAGORAS (DIAGRAM) TYPE QUESTIONS INVOLVING COMPOUND AND DOUBLE ANGLES</u>

<u>Examples</u>

1. If $\cos 25° = k$, determine the following in terms of k:

a) $\cos 155°$ b) $\cos 50°$ c) $\sin 25°$ d) $\sin 50°$

2. If $\sin 18^\circ = t$, determine the following in terms of t:

a) $\cos 18^\circ$ b) $\sin 78^\circ$ c) $\sin 36^\circ$ d) $\tan 198^\circ$

Exercise H

N.B.

1. If $\cos 9^\circ = k$, determine the value of $\sin 18^\circ$ in terms of k.

2. If $\cos 55^\circ = p$, determine the value of $\cos 5^\circ$ in terms of p.

Examples

1. In each of the following, find the general solution:

a) $\sin 2x = \cos 2x$

b) $\sin 2x - 1 = 2\sin x - \cos x$

N.B.

c) $\sin 2x + 2\cos 2x = 1$

d) $4\cos^2 x + \sin 2x - 1 = 0$

Exercise I

Find the general solution for each of the following:

1. $-3\sin 2\theta = 1 + \cos 2\theta$

2. $8\sin^2\theta = \sin\theta\cos\theta + 2$

3. $\cos 2\theta - \sin\theta = 0$

4. $2\sin^2\theta - \cos\theta - \sin 2\theta + \sin\theta = 0$

Pattern B: $\boxed{\cos\, angle = \sin(diff\ angle)}$

Example: $\cos(\theta + 20°) = \sin(2\theta - 80°)$ OR $\sin 2\theta = \sin(\theta + 20°)$

Pattern C: $\boxed{a\ \cos\,(angle) = b\ \sin(angle)}$

Example: $2\cos 3\theta = \sin 3\theta$

Pattern D: $\boxed{a\ \cos\,(angle) = b\ \sin(diff\ \ angle)}$ use compound angles

Example:

a) $3\sin x = 2\sin(x - 60°)$

b) $3\sin(x + 30°) = 2\cos x$

c) $\sin 3x - 2\sin x\cos x = 0$