Factorising Quadratic Equations

We know that a quadratic equation has the form of:

$y = ax^2 + bx + c$

Some quadratic equations can be solved using factorisation technique, which uses algebra to reduce the quadratic equation into two brackets. For example, $(x + 3)(x + 2) = 0$→ this will provide us with two solutions:

$(x + 3) = 0$ → Move the 3 over the other side to make x the subject → x = -3

$(x + 2) = 0$ → Move the 2 over the other side to make x the subject → x = -2

Therefore, the solutions are x = -3 and x = -2

Method 1 Factorising Quadratic Equations

The steps to factorise a quadratic equation:

Step 1 → find the value of a, b and c of the quadratic equation

Step 2 → find the product, Product = a x c

Step 3 → find the sum, Sum = b

Step 4 → find two numbers (let's call the two numbers "s" and "t"), that if we multiply them, we get the same number as the Product (Step 2) and if we add the same two numbers, we get the Sum (Step 3)

Step 5 → place the two numbers inside the quadratic equation by replacing "bx" with sx and tx, such as:

$$ax^2 + bx + c = 0$$

$ax^2 + sx + tx + c = 0$→ because sx + tx = bx

Step 6 → cut the equation in half:

$$ax^2 + sx \,||\, + tx + c = 0$$

Step 7 → find the common features in the first half of the equation and place it in a bracket

Step 8 → find the common features in the second half of the equation and place it in a bracket

Step 9 → put both bracket in step 7 and 8 together → what is in both brackets must be the same

Step 10 → eliminate one bracket and put those values outside of the bracket inside a bracket

Example

Factorise $x^2 + 3x - 10$

Step 1:

$ax^2 + bx + c = 0$

$x^2 + 3x - 10$ → a=1 b=3 c=-10

Step 2:

Product = a x c = 1 x -10 = **-10**

Step3:

Sum = b = **3**

Step 4: find two numbers, that if we multiply them, we get the same number as the Product (Step 2) and if we add the same two numbers, we get the Sum (Step 3)

What are the multiples to get 10?
2x5 or 1x10

How can we get b = 3 from the above multiples?
1x10 or 1 and 10 cannot be used to find 3 if added together

2 and 5 can be used to find 3 if added together → But 2 + 5 =7 [NOT WHAT WE WANT]

5 -2 =3 [WHAT WE WANT]

Therefore, the two numbers are **5 and -2**, as 5 x -2 = -10 (product) and 5 + (-2) =3 (sum)

Step 5:

$$x^2 + 3x - 10$$

$x^2 - 2x + 5x - 10$ → replaced 3x by -2x and 5x which equals to 3x (-2x + 5x = 3x or 5x - 2x = 3x)

Step 6:

$$x^2 - 2x \mid\mid + 5x - 10$$

Step 7:

$x^2 - 2x$ = $x\ (x - 2)$ → the common feature in both is x, so we take it outside the bracket and we adjust inside the bracket to make it look like the original equation $x^2 - 2x$
So, if we open the bracket we get $x^2 - 2x$
→ $x\ X\ x\ = x^2\ and\ x\ X\ 2\ = 2x$

Step 8:

$+5x - 10\ =\ +5\ (x\ - 2)$ → the common feature (something that is common between the two numbers) is 5

Step 9:

$x\ (x - 2)\ + 5\ (x\ - 2)$

Step 10: eliminate one bracket and put those values outside of the bracket inside a bracket

We take out (Remove) the first bracket, and what is outside the brackets is x and +5, so we place them in a bracket (x+5)

$x\ (x - 2)\ + 5\ (x\ - 2)$ → x (x -2) + 5 (x - 2) → $(x\ + 5)\ (x\ - 2)$

Final Answer: $(x\ + 5)\ (x\ - 2)$

Do we need to do all these steps when answering a question?

You don't need to do the steps one by one; you can jump steps or do them mentally

So, if you would need to solve this in an exam, you will do it in this way:

Factorise $x^2 + 3x - 10$

Product = a x c = 1 x -10 = -10
Sum = b = 3

The two numbers = -2 and 5

$$x^2 - 2x + 5x - 10$$

$x^2 - 2x$ = $x\ (x - 2)$ and the second half $+5x - 10\ =\ +5\ (x\ - 2)$

$x\ (x - 2)\ + 5\ (x\ - 2)$

$(x\ + 5)\ (x\ - 2)$

Method 2 Factorising Quadratic Equations [But this is only used when a=1]

Factorise $x^2 + 3x - 10$

Find two numbers that if multiplied gives -10 (a x c = 1 x -10 = -10) and if added gives +3 (b = 3)

→ Step 4

The two numbers 5 and -2 as

-5 x 2 = -10 → -5 +2 = -3 → Not Correct

-5 x -2 = +10 → -5 + -2 = -7 → Not Correct

5 x -2 = -10 → 5 + -2 = 3 → Correct

So, we know that $(x + \ldots)\ (x + \ldots)$

And we have the two numbers of 5 and -2, so we place them instead of the dotted line/ missing values:

$(x\ + 5)\ (x\ - 2)$

HOWEVER, METHOD 2 CANNOT BE USED TO FACTORISE EQUATIONS LIKE THE ONE BELOW:

Factorise the following $3x^2 + 33x + 30 = 0$

Using Method 1 approach to factorise and comparing the equation to $y = ax^2 + bx + c$

a= 3 b= 33 c= 30

Product = a x c = 3 x 30 =90

Sum = b = 33

The two numbers are two numbers that if we multiply, we get 90 and if we add them, we get 33?

The two numbers are 3 and 30

$$3x^2 + 3x \,||\, + 30x + 30 = 0$$

What is common in the LHS $3x^2 + 3x = 3x(x + 1)$

What is common in the RHS $30x + 30 = 30(x + 1)$

Placing them together:

$$3x(x + 1) + 30(x + 1)$$

Scrap one of the brackets and put the two values outside of the brackets (3x and +30), inside a bracket

$(x + 1)(3x + 30)$ → Answer

Solving a quadratic equation using Factorisation

This technique is the same for both methods of factorisation

To solve a factorisation question → if the question asks us to find the solution through factorizing, then we do:

$(x + 5)(x - 2) = 0$ → referring to the previous example in page 2 and 4

$(x + 5) = 0$ and $(x - 2) = 0$

In each case we solve for x by making x the subject. → move the numbers over the other side

x = -5 and x = 2

Question 1

Solve $x^2 + 5x - 14 = 0$ [Hint: you can use both methods]

Using **Method 2** we get:

Two numbers that if multiplied will provide -14 and added will give us 5 is 7 and -2

$(x + \ldots)(x + \ldots) = 0$ → replacing the dotted line or missing values with 7 and -2

$(x + 7)(x - 2) = 0$

This implies that $(x + 7) = 0$ and $(x - 2) = 0$

Solving $(x + 7) = 0$ → need to make x the subject by moving 7 over the other side

$x = -7$

Solving $(x - 2) = 0$ → need to make x the subject by moving -2 over the other side

$x = 2$

So, the solution of the quadratic equation is $x = -7$ and $x = 2$

Question 2

Solve $2x^2 + 11x + 12 = 0$ [Hint cannot use method 2]

The two numbers are 3, 8 → as 3x8 =24 (Product) and 3+8 =11 (sum)

Option 1: $2x^2 + 3x + 8x + 12$
Option 2: $2x^2 + 8x + 3x + 12$ → best to use this one as it's easier to factorise due to 2 and 8 are common, also 3 and 12 are common

$$2x^2 + 8x|| + 3x + 12 = 0$$

What is common in the LHS $2x^2 + 8x = 2x(x + 4)$

What is common in the RHS $+3x + 12 = +3(x + 4)$

Joined together:

$$2x(x + 4) + 3(x + 4) = 0$$

Scrap one of the **brackets** and put the two values outside of the brackets inside a bracket:

$$(2x + 3)(x + 4) = 0$$

This implies that $(2x + 3) = 0$ and $(x + 4) = 0$

Solving $(2x + 3) = 0$ → need to make x the subject by moving 3 over the other side and dividing by 2

$x = -\frac{3}{2}$

Solving $(x + 4) = 0$ → need to make x the subject by moving 4 over the other side

$x = -4$

So, the solution of the quadratic equation is $x = -\frac{3}{2}$ and $x = -4$