Surds

Simplification of Surds

Surds are used when numbers are square rooted such as $\sqrt{12}$ and breaking it down to simplify the square root. Surds are also used because square rooting a non-squared number will not give us a whole number such as:

$\sqrt{12} = 3.464101615$ → not easy to write

Whereas, the square root of a squared number such as 16 is:

$$\sqrt{16} = 4$$

Therefore, to simplify using surds the $\sqrt{12}$, we firstly need to find the multiples of 12 that can be consist of square numbers?

4 x 3 =12 → 4 is a square number

$\sqrt{12} = \sqrt{4}$ x $\sqrt{3}$= 2x $\sqrt{3}$ = $2\sqrt{3}$ → **simplest form**

NOTE: we write surds in the form of number and then square root → $a\sqrt{b}$ where "a"is a whole number and $\sqrt{b}$ is the surd.

Example 1

Simplify $\sqrt{72}$

Find the multiples that will give 72:
- 2 x 36 =72
- 3 x 24
- 4 x 18
- 6 x 12
- 8 x 9

Choose the multiple that has a square number
- Square numbers are 4,9,16,25,36,49,64,81,100,121,144
- This gives us 2 x 36 and 8 x 9
- Choose the one that gives us the smallest numbers 2 x 36 as 8 x 9 give $3\sqrt{8}$

$\sqrt{72} = \sqrt{2}$x $\sqrt{36}$= $\sqrt{2}$x 6 = 6 $\sqrt{2}$ → by choosing 2 x 36 it gives a smaller root Number, as $\sqrt{2}$is smaller than $\sqrt{8}$

Example 2

Simplify $3\sqrt{12}$

We know that 12 = 3 x 4
3 ($\sqrt{3}$x$\sqrt{4}$)

3 ($\sqrt{3}$x 2) → both 2 and 3 are like terms

3 x 2 x $\sqrt{3}$

6 x $\sqrt{3}$

$6\sqrt{3}$

Multiplication of Surds

For each value we need to simplify and then multiply, this can be done in two ways:

- Simplify each surd and then multiply
- Bring like terms together and then multiply together

Which method to use depends upon how much you can simplify the expression

Example 1

$2\sqrt{3}$ x $3\sqrt{5}$

We cannot simplify each surd as $\sqrt{3}$ and $\sqrt{5}$ are in the lowest surd form, so we use the second strategy by bringing like terms together:

2 x 3 x $\sqrt{3}$ x $\sqrt{5}$ → the 2 and 3 are whole numbers, which means they are like terms and $\sqrt{3}$ and $\sqrt{5}$are both surds

6 x $\sqrt{15}$

$6\sqrt{15}$

Example 2 (first and second method together)

$\sqrt{8}$ x $\sqrt{27}$

As each surd is able to be simplified, we use the first way of attempting to multiply the surds. Firstly, we find multiples of of each surd that has root numbers

$\sqrt{8}$= $\sqrt{2}$x$\sqrt{4}$= $\sqrt{2}$x 2 = $2\sqrt{2}$

$\sqrt{27}$= $\sqrt{3}$x$\sqrt{9}$= $\sqrt{3}$x 3 = $3\sqrt{3}$

Now we put the two together and then put like terms together (use second method):

$2\sqrt{2}$x $3\sqrt{3}$

2 x 3 x $\sqrt{2}$x $\sqrt{3}$

$6\sqrt{6}$

Division of Surds

For each value or part of the expression we need to simplify and then divide, this can be done in two ways:

- Simplify each surd and then divide
- Bring like terms together and then divide them together

Which method to use depends upon how much you can simplify the expression

Example 1

$$\frac{\sqrt{24}}{\sqrt{6}}$$

We should use method 1 as we can simplify and then divide the like terms

We know that 24 = 6 x 4

$$\frac{(\sqrt{6} \times \sqrt{4})}{\sqrt{6}}$$ → the $\sqrt{6}$ cancel out each other

$\sqrt{4}$

= 2

Example 2

$$\frac{\sqrt{121}}{\sqrt{12}}$$

121 = 11 x 11, so $\sqrt{121}$ = 11

12 = 3 x 4

So therefore,

$$\frac{11}{\sqrt{3} \times \sqrt{4}}$$

$\frac{11}{\sqrt{3} \times 2}$

$\frac{11}{2\sqrt{3}}$

Adding Surds

Adding surds will need to be done if the surds are the same, because you can only add same terms, such as:

$2x + 3x = 5x$ → both numbers have a common factor of x, so we can add them

This strategy is the same under surds:

$2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$ → I treat the $\sqrt{3}$as the common factor, which is the same as how i treated x

Note: this is only possible if both additions have the same common factor (for example both have $\sqrt{3}$). Therefore, we first simplify the expression so that we can have the same common factors and then add them together.

BOTH VALUES MUST HAVE THE SAME ROOT!

Example 1

$\sqrt{27}+\sqrt{75}$

Firstly, we simplify each value in the expression and find a common surd in both, so we know that:

27 = 3 x 9

75 = 3 x 25 → So the common factor is 3

$\sqrt{3} \times \sqrt{9} + \sqrt{3} \times \sqrt{25}$ → simplifying each expression

$\sqrt{3} \times 3 + \sqrt{3} \times 5$

$3\sqrt{3} + 5\sqrt{3}$ → we can add them together as they both have a common factor of $\sqrt{3}$

$(3+5)\sqrt{3}$

$8\sqrt{3}$

Example 2

$\sqrt{54}+\sqrt{45}$

54 = 9 x 6

45 = 9 x 5 → common factor is 9

$\sqrt{9}x\sqrt{6}+ \sqrt{9}x\sqrt{5}$

$3x\sqrt{6}+ 3\ x\sqrt{5}$

$3\sqrt{6}+ 3\sqrt{5}$ → we **CANNOT** add the two together as both values have a different surd as $\sqrt{6}$ and $\sqrt{5}$ are not the same

Subtracting Surds

This is exactly the same as adding but we minus the two expressions

Subtracting surds will need to be done if the surds are the same, because you can only subtract same terms, such as:

$7x - 3x \ = 4x$ → both numbers have a common factor of x, so we can add them

This strategy is the same under surds:

$7\sqrt{3} \ - \ 3\sqrt{3}= 4\sqrt{3}$ → treat the $\sqrt{3}$ as the common factor, which is the same as how I treated x

Note: this is only possible if both numbers have the same common factor (for example both have $\sqrt{3}$). Therefore, we first simplify the expression so that we can have the same common factors and then subtract them together.

Example 1

$\sqrt{27}$-$\sqrt{12}$

27 = 3 x 9

12 = 3 x 4

$\sqrt{3}$x$\sqrt{9}$ -$\sqrt{3}$x$\sqrt{4}$

$\sqrt{3}$x3 -$\sqrt{3}$x2

$3\sqrt{3}$ - $2\sqrt{3}$

(3-2) $\sqrt{3}$

1 $\sqrt{3}$

$\sqrt{3}$

Example 2

$\sqrt{54}$-$\sqrt{45}$

54 = 9 x 6

45 = 9 x 5 → common factor is 9

$\sqrt{9}$x$\sqrt{6}$- $\sqrt{9}$x$\sqrt{5}$

3x$\sqrt{6}$ - 3 x$\sqrt{5}$

$3\sqrt{6}$ - $3\sqrt{5}$ → we **CANNOT** subtract the two together as both values have a different surd as $\sqrt{6}$ and$\sqrt{5}$are not the same

Surds and Brackets

We treat the surds as if it is an x or y and we open out the bracket by multiplying them out, then we simplify by merging common factors and then simplify using the surd techniques we did before.

Example 1

$$(\sqrt{3}+\sqrt{6})^{2}$$

$(\sqrt{3}+\sqrt{6})(\sqrt{3}+\sqrt{6})$ → duplicate the brackets, as the square of the whole brackets means that there is two of the same brackets multiplied

$\sqrt{3}\text{x}\sqrt{3}+\sqrt{3}\text{x}\sqrt{6}+\sqrt{6}\text{x}\sqrt{3}+\sqrt{6}\text{x}\sqrt{6}$ → multiply each of the values to open the bracket

We know that $\sqrt{6}$can be written as $\sqrt{3}\text{x}\sqrt{2}$

$\sqrt{3}\blacksquare^{2}+\sqrt{3}\text{x}(\sqrt{3}\text{x}\sqrt{2})+(\sqrt{3}\text{x}\sqrt{2})\text{x}\ \sqrt{3}+\sqrt{6}\blacksquare^{2}$

$3+\sqrt{3}\ {}^{2}\text{x}\sqrt{2}+\sqrt{3}\blacksquare^{2}\text{x}\sqrt{2}+6$ → put the like terms together and simplify any terms

$3+6+3\sqrt{2}+3\sqrt{2}$ → we know that both $3\sqrt{2}$have the same surd, which means we can add them together $3\sqrt{2}+3\sqrt{2}=6\sqrt{2}$→ see the addition section of surds on page 4.

$9+6\sqrt{2}$

Example 2

$$(\sqrt{8}+\sqrt{2})(\sqrt{6}-\sqrt{4})$$

$\sqrt{8}\text{x}\sqrt{6}-\sqrt{8}\text{x}\sqrt{4}+\sqrt{2}\text{x}\sqrt{6}-\sqrt{2}\text{x}\sqrt{4}$

8 = 4 x 2
6 = 3 x 2

$(\sqrt{4}\text{x}\sqrt{2})\text{ x }(\sqrt{3}\text{x}\sqrt{2})+(\sqrt{4}\text{x}\sqrt{2})\text{ x }2+\sqrt{2}\text{x}(\sqrt{3}\text{x}\sqrt{2})-\sqrt{2}\text{x }2$

$(2x\sqrt{2})$ x $(\sqrt{3}x\sqrt{2})$ + $(2x\sqrt{2})$ x 2 + $\sqrt{2}x(\sqrt{3}x\sqrt{2})$ - $\sqrt{2}$x 2
$2x\sqrt{2}x\sqrt{2}x\sqrt{3} + 2x2x\sqrt{2} + \sqrt{2}x\sqrt{2}x\sqrt{3} - 2\sqrt{2}$

$2x\sqrt{2}^{2}x\sqrt{3} + 2^{2}x\sqrt{2} + \sqrt{2}^{2}x\sqrt{3} - 2\sqrt{2}$

$2x2x\sqrt{3} + 4x\sqrt{2} + 2x\sqrt{3} - 2\sqrt{2}$

$4\sqrt{3} + 4\sqrt{2} + 2\sqrt{3} - 2\sqrt{2}$

$4\sqrt{2} - 2\sqrt{2} + 2\sqrt{3} + 4\sqrt{3}$

$2\sqrt{2} + 6\sqrt{3}$

Example 3

Show that $(\sqrt{2} + 3\sqrt{8})^{2} = 98$

We take the Left hand side and simplify it as much as possible in order to see if it equals to 98

$(\sqrt{2} + 3\sqrt{8})^{2} = (\sqrt{2} + 3\sqrt{8})(\sqrt{2} + 3\sqrt{8})$

$\sqrt{2}x\sqrt{2} + \sqrt{2}x3\sqrt{8} + 3\sqrt{8}x\sqrt{2} + 3\sqrt{8}x3\sqrt{8}$

We know that 8 = 4 x 2

$\sqrt{2}^{2} + \sqrt{2}x3(\sqrt{4}x\sqrt{2}) + \sqrt{2}x3(\sqrt{4}x\sqrt{2}) + 3x3x\sqrt{8}x\sqrt{8}$

$2 + \sqrt{2}x3(2x\sqrt{2}) + \sqrt{2}x3(2x\sqrt{2}) + 3^{2}x\sqrt{8}^{2}$

$2 + 3x2x\sqrt{2}x\sqrt{2} + 3x2x\sqrt{2}x\sqrt{2} + 9x8$

$2 + 6x\sqrt{2}^{2} + 6x\sqrt{2}^{2} + 72$

2 + 72+ 6x2 + 6x2

2 + 72+ 12 + 12

98