## Portfolio and CAPM Questions

## Question 1

The returns of the common stock of " S " are perfectly positively correlated with those of the market portfolio. In addition, the equilibrium rate of return for $S$ stock is 25 percent and the market's expected rate of return is 15 percent. If the risk-free rate is 10 percent and the standard deviation in the market return is 12 percent, what is the standard deviation in the returns of S's stock? What is the beta coefficient for S's stock?

## Answer:

Beta from the CAPM equation is
$\beta=\frac{E\left(r_{i}\right)-r_{f}}{E\left(r_{m}\right)-r_{f}}=\frac{25-10}{15-10}=3$
Also
$\beta=\frac{\sigma_{i m}}{\sigma_{m}^{2}} \Rightarrow \sigma_{i m}=\beta \sigma_{m}^{2}=3 \times 0.12^{2}=0.0432$

We also know that correlation coefficient is
$\rho_{\mathrm{im}}=\frac{\sigma_{\mathrm{im}}}{\sigma_{\mathrm{i}} \sigma_{\mathrm{m}}}$
Then,
$\sigma_{\mathrm{im}}=\rho_{\mathrm{im}} \sigma_{\mathrm{i}} \sigma_{\mathrm{m}}$
And
$\sigma_{\mathrm{i}}=\frac{\sigma_{\mathrm{im}}}{\rho_{\text {im }} \sigma_{\mathrm{m}}}=\frac{0.0432}{1 \times 0.12}=0.36=36 \% /$ T T T D D S

## Question 2

The market portfolio has an expected rate of return of 19 percent and a standard deviation of 8 percent, and the risk-free rate is 10 percent. The TIQ Investment Company is considering the acquisition of two stocks. Company A's stock has a beta of 1.2; company B's beta is 0.80 .
a. What rate of return should TIQ expect on each stock investment?
b. If the price of both A's and B's stock for the next year is expected to be \$100 and neither pays an interim cash dividend, what is the value of each stock today?

## Answer:

a)

$$
\begin{aligned}
& \mathrm{E}\left(\mathrm{R}_{\mathrm{A}}\right)=0.1+1.2(0.19-0.1)=0.208 \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{B}}\right)=0.1+0.8(0.19-0.1)=0.172
\end{aligned}
$$

b)

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{A} 0}=\frac{\mathrm{P}_{\mathrm{A} 1}}{1+\mathrm{E}\left(\mathrm{R}_{\mathrm{A}}\right)}=\frac{100}{1+0.208}=82.78 \\
& \mathrm{P}_{\mathrm{B} 0}=\frac{\mathrm{P}_{\mathrm{B} 1}}{1+\mathrm{E}\left(\mathrm{R}_{\mathrm{B}}\right)}=\frac{100}{1+0.172}=85.32
\end{aligned}
$$

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## Question 3

Ella is a young talented painter but not very good with numbers. Ella has inherited $£ 200,000,000$ from her mother that she wants to invest in a stock portfolio promising a return of 35 percent and a standard deviation of 40 percent. John Kyriakopoulos, a banker and friend who thinks very highly of her paintings and is confident of her future earnings potential, has offered to lend her $£ 300,000,000$ at a rate of 10 percent. Athena plans to invest the total of $£ 500,000,000$ in the stock portfolio. What would be the expected rate of return and standard deviation of Ella's levered portfolio?

## Answer:

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\(E\left(r_{p}\right)=w_{1} E\left(r_{1}\right)+w_{f} E\left(r_{f}\right)\)
\(\mathrm{w}_{1}+\mathrm{w}_{\mathrm{f}}=500000\)
borrow :
\(\frac{-3}{2}=-1.5\)
invest:
\(1-(-1.5)=2.5\)
\(\mathrm{E}\left(\mathrm{r}_{\mathrm{p}}\right)=2.5 \times 0.35+(-1.5) \times 0.10=72.5 \%\)
Since
\(\sigma_{\mathrm{f}}=0\)
Then,
\(\sigma_{\mathrm{p}}^{2}=\mathrm{w}_{1}^{2} \sigma_{1}^{2}=2.5 \times 0.4^{2}=1\)
\(\sigma_{\mathrm{p}}=1=100 \%\)
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## Question 4

Assume that the return on the market is 14 percent and the risk-free rate is 8 percent.
(a) Use the CAPM to determine whether the following stocks are overpriced or under-priced

| stock | Expected return \% | Beta |
| :---: | :---: | :---: |
| A | 17 | 1.4 |
| B | 14 | 0.2 |
| C | 15 | 1.2 |
| D | 16 | 1.3 |

(b) Determine which stocks would be overpriced if the market return were 16 percent and the risk-free rate 9 percent.

## Answer:

Using the CAPM formula, we find that:

$$
\begin{aligned}
& \mathrm{E}\left(\mathrm{R}_{\mathrm{A}}\right)=0.08+1.4(0.14-0.08)=16.4 \% \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{B}}\right)=0.08+0.2(0.14-0.08)=9.2 \% \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{C}}\right)=0.08+1.2(0.14-0.08)=15.2 \% \\
& \mathrm{E}\left(\mathrm{R}_{\mathrm{D}}\right)=0.08+1.3(0.14-0.08)=15.8 \%
\end{aligned}
$$

If expected return from CAPM is greater than the expected return given, i.e. when stock lies below the SML, stock is overpriced and vice versa. Hence:

Stock A: 16.4\%<17\% - under-priced
Stock B: $9.2 \%<14 \%$ - under-priced
Stock C: 15.2\%>15\% - overpriced
Stock D: 15.8\%<16\% - under-priced

