

1. UNITS AND DIMENSIONS

Differentiation	Integration
1. $\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{(n+1)} + c,$ provided $n \neq -1$ Here c is constant of integration
2. $\frac{d}{dx}(x) = 1$	$\int dx = x + c$
3. $\frac{d}{dx}(\log_e x) = \frac{1}{x}$	$\int \frac{dx}{x} = \log_e x + c$
4. $\frac{d}{dx}(\sin x) = \cos x$	$\int \cos x \cdot dx = \sin x + c$
5. $\frac{d}{dx}(\cos x) = -\sin x$	$\int \sin x \cdot dx = -\cos x + c$
6. $\frac{d}{dx}(\tan x) = \sec^2 x$	$\int \sec^2 x \cdot dx = \tan x + c$
7. $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x \cdot dx = -\cot x + c$
8. $\frac{d}{dx}(\sec x) = \sec x \cdot \tan x$	$\int \sec x \cdot \tan x \cdot dx = \sec x + c$

1. Simple Conversion of Units

- $1 \text{ kg m}^{-3} = 10^{-3} \text{ g cm}^{-3}$
 $1 \text{ g cm}^{-3} = 10^3 \text{ kg m}^{-3}$
 $1 \text{ N} = 10^5 \text{ dyn}$
 $1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$
 $1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$
 $1 \text{ ly} = 9.46 \times 10^{15} \text{ m}$
 $1 \text{ persec} = 3.08 \times 10^{16} \text{ m}$
 $1 \text{ \AA} = 10^{-10} \text{ m} = 0.1 \text{ nm}$

2. Order of Magnitude

To determine the order of magnitude of a number N , we express it as

$$N = n \times 10^x$$

If $0.5 < n \leq 5$, then x will be the order of magnitude of N .

3. Indirect Methods for Long Distances

1. Reflection or echo method

$$S = \frac{c \times t}{2} \quad \text{or} \quad \frac{v \times t}{2}$$

2. Triangulation method

(i) Height of an accessible object

$$h = x \tan \theta$$

where x is the distance of observation point from the foot of the object.

(ii) Height of an inaccessible object

$$h = \frac{d}{\cot \theta_2 - \cot \theta_1}$$

where d is the distance between the two observation points.

3. Parallax method. The distance of an astronomical object

$$S = \frac{\text{Basic}}{\text{Parallax angle}} = \frac{b}{\theta}$$

4. Size of the an astronomical object,

Linear diameter = Distance \times angular diameter

$$\text{or} \quad D = S \times \theta$$

4. Indirect Methods of Small Distances: -

1. Molar volume = Volume of 1 mole of a gas at S.T.P

$$= 22.4 \text{ L}$$

2. Volume of a sphere = $\frac{4}{3} \pi r^3$

3. Thickness of an oil film =

$$\frac{\text{Volume of oil drop}}{\text{Area of the film}}$$

5. Magnification of Sizes

1. Linear magnification =

$$\frac{\text{Final size}}{\text{Initial size}} = \frac{\text{Size of image}}{\text{Size of object}}$$

2. Linear magnification = $\sqrt{\text{Areal magnification}}$

6. Measurement of Time

Fractional error in time =

$$\frac{\text{Difference in time}}{\text{Time interval}} = \frac{\Delta t}{t}$$

7. Significant Figures

1. Rules for rounding off a measurement.

(i) If the digit to be dropped is smaller than 5, then the preceding digit is left unchanged.

(ii) If the digit to be dropped is greater than 5, then the



preceding digit is increased by 1.

(iii) If the digit to be dropped is 5 followed by non-zero digits, then the preceding digit is increased by 1.

(iv) If the digit to be dropped is 5, then the preceding digit is left unchanged if it is even.

(v) If the digit to be dropped is 5, then the preceding digit is increased by 1 if it is odd.

2. Rules for determining the number of significant figures.

(i) All non-zero digits are significant. So 13.75 has four significant figures.

(ii) All zero between two non-zero digits are significant. Thus 100.05 km has five significant figures.

(iii) All zeros to the right of a non-zero digit but to the left of an understood decimal point are not significant.

For example, 86400 has three significant figures. But such zeros are significant if they came from a measurement. For example, 86400 s has five significant figures.

(iv) All zeros to the right of a non-zero digit but to the left of a decimal point are significant. For example 648700. Has six significant figure.

(v) All zeros to the right of a decimal point are significant. So 161 cm, 1610 cm and 161.00 cm have three, four and five significant figures respectively.

(vi) All zeros to the right of a decimal point but to the left of a non-zero digit are not significant. So 0.161 cm and 0.161 cm, both have three significant figures.

Moreover, zero conventionally placed to the left to the decimal point is not significant.

(vii) *The number of significant figures does not depend on the system of units.* So 16.4 cm, 0.164m and 0.000164 km, all have three significance figures.

8. Errors in Measurements

1. True value. If $a_1, a_2, a_3, \dots, a_n$ are the readings of an experiment, then true value of the quantity is given by the arithmetic mean,

$$\bar{a} = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n} = \frac{1}{n} \sum_{i=1}^n a_i$$

2. Absolute error = True value – Measured value

or $\Delta a_1 = \bar{a} - a_i$

3. Final absolute error

= Arithmetic mean of absolute errors

$$\Delta \bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \dots + |\Delta a_n|}{n}$$

$$= \frac{1}{n} \sum_{i=1}^n |\Delta a_i|$$

4. Relative error or fractional error

$$= \frac{\text{Final_absolute_error}}{\text{True_value}} \text{ or } \delta a = \frac{\Delta \bar{a}}{\bar{a}}$$

5. Percentage error = $\frac{\Delta \bar{a}}{\bar{a}} \times 100\%$

9. Combination of Errors

1. If $Z = A + B$, then the maximum possible error in Z,

$$\Delta Z = \Delta A + \Delta B$$

2. If $Z = A - B$, then the maximum possible error in Z,

$$\Delta Z = \Delta A + \Delta B$$

3. If $Z = AB$, then the maximum fractional error in Z

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

4. If $Z = A/B$, then the maximum fractional error in Z,

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

5. If $Z = A^n$, then the maximum fractional error in Z,

$$\frac{\Delta Z}{Z} = n \cdot \frac{\Delta A}{A}$$

6. If $Z = \frac{A^p B^q}{C^r}$, then the maximum fractional error in Z,

$$\frac{\Delta Z}{Z} = p \frac{\Delta A}{A} + q \frac{\Delta B}{B} + r \frac{\Delta C}{C}$$

The percentage error in Z

$$\frac{\Delta Z}{Z} \times 100 = p \frac{\Delta A}{A} \times 100 + q \frac{\Delta B}{B} \times 100 + r \frac{\Delta C}{C} \times 100$$

MOTION IN A STRAIGHT LINE-3

1. Calculation of Distance Covered Displacement,

Average Speed and Average Velocity

1. Distance covered = Length of actual path traversed by the body

2. Displacement = Vector drawn from initial to final position of the body

$$3. \text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

2. Instantaneous Velocity and Instantaneous

Acceleration

$$1. v_{av} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta s}{\Delta t}$$

$$2. v = \lim_{\Delta t \rightarrow 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$$

$$3. a_{av} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

$$4. a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

3. Motion with Uniform Acceleration

1. Equations of motion in conventional form,

$$(i) v = u + at$$

$$(ii) s = ut + \frac{1}{2} at^2$$

$$(iii) v^2 - u^2 = 2as \text{ or } v^2 = u^2 + 2as$$

$$(iv) s_{nth} = u + \frac{a}{2} (2n - 1)$$

2. Equations of motion in Cartesian form,

$$(i) v(t) = v(0) + at$$

$$(ii) v(t') = v(t) + a(t' - t)$$

$$(iii) x(t) = x(0) + v(0)t + \frac{1}{2} at^2$$

$$(iv) x(t') = x(t) + v(t)(t' - t) + \frac{1}{2} a(t' - t)^2$$

$$(v) [v(t')]^2 - [v(t)]^2 = 2a[x(t') - x(t)]$$

4. Motion under Gravity

1. For a freely falling body, the equations of motion are

$$(i) v = u + gt \quad (ii) s = ut + \frac{1}{2} gt^2$$

$$(iii) v^2 - u^2 = 2gs$$

2. For a body falling freely under the action of gravity, g is taken *positive*.

3. For a body thrown vertically upward, g is taken *negative*

4. When a body is just dropped, $u = 0$

5. For a body thrown vertically up with initial velocity u .

$$(i) \text{Maximum height reached, } h = \frac{u^2}{2g}$$

$$(ii) \text{Time of ascent} = \text{Time of descent} = \frac{u}{g}$$

$$(iii) \text{Total time of flight} = \frac{2u}{g}$$

(iv) Velocity of fall at the point of projection = u

(v) Velocity attained by a body dropped from height h ,

$$v = \sqrt{2gh}$$

5. Position -Time and Velocity -Time Graphs

1. Slope of position-time ($x-t$) graph gives velocity.

$$v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

2. Slope of velocity-time ($v-t$) graph gives acceleration

$$a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$$

3. Distance travelled = Area between the ($v-t$) graph and time-axis

4. Change in velocity = Area between the ($a-t$) graph and time-axis

6. Relative Velocity

1. Relative velocity of object A w.r.t. object B,

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

2. Relative velocity of object B w.r.t. object A,

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

where \vec{v}_A and \vec{v}_B are the velocities w.r.t. the ground

3. When the objects A and B move in the same direction,

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

4. When the object B moves in the opposite direction of A,

$$v_{AB} = v_A + v_B$$



4-MOTION IN A PLANE

1. Composition of Vectors

1. By triangle law or parallelogram law of vector addition, the magnitude of resultant \vec{R} of two vectors \vec{P} and \vec{Q} inclined to each other at angle θ , is given by

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

2. If resultant \vec{R} makes an angle β with \vec{P} , then

$$\tan \beta = \frac{Q \sin \theta}{P + Q \cos \theta}$$

2. Expressing the Vectors in terms of Base

Vectors and Rectangular Components of

Vectors

1. If A_x, A_y, A_z are the rectangular components of \vec{A} and $\hat{i}, \hat{j}, \hat{k}$ are the unit vectors along X-, Y- and Z-axis respectively, then $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$$2. |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$3. \hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{A_x \hat{i} + A_y \hat{j} + A_z \hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

4. If vector \vec{A} makes angle θ with the horizontal, then horizontal component of $\vec{A} = A_x = A \sin \theta$
vertical component of $\vec{A} = A_y = A \sin \theta$

and
$$A = \sqrt{A_x^2 + A_y^2}$$

3. Scalar or DOT Product of two Vectors

$$1. \vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta = AB \cos \theta$$

$$2. \text{ If } \vec{A} \perp \vec{B}, \theta = 90^\circ \text{ and } \vec{A} \cdot \vec{B} = 0$$

3. Angle θ between \vec{A} and \vec{B} is given by

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

4. In terms of rectangular components

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$5. \text{ Work done, } W = \vec{F} \cdot \vec{S}$$

$$6. \text{ Power } P = \vec{F} \cdot \vec{v}$$

4. Vector or Cross of two Vectors

$$1. \vec{A} \times \vec{B} = AB \sin \theta \hat{n}$$

2. Unit vector \hat{n} perpendicular to the plane of vectors \vec{A} and \vec{B} is given by $\hat{n} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

3. Angle θ between vectors \vec{A} and \vec{B} is given by

$$\sin \theta = \frac{|\vec{A} \times \vec{B}|}{|\vec{A}| |\vec{B}|}$$

4. In terms of rectangular components, we have

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

or

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

5. For parallel vectors, $\vec{A} \times \vec{B} = \vec{0}$

6. Moment of a force or torque, $\vec{\tau} = \vec{r} \times \vec{F}$

5. MOTION IN A PLANE

1. Distance is the length of actual path traversed by a moving body between its initial and final positions.

2. Displacement is the shortest distance between the initial and final positions of a body.

$$3. \text{ Average speed} = \frac{\text{Displacement covered}}{\text{Time taken}}$$

$$5. \text{ Instantaneous velocity, } \vec{v} = \frac{d\vec{r}}{dt}$$

$$6. \text{ Instantaneous acceleration, } \vec{a} = \frac{d\vec{v}}{dt}$$

6. Relative Velocity of two inclined Motions

1. The relative velocity of A w.r.t B, $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

2. The relative velocity of B w.r.t A, $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$



3. For two objects moving with velocities v_A and v_B at an angle θ , the relative velocity of an object A w.r.t. B is given by

$$v_{AB} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos \theta}$$

4. If velocity v_{AB} makes angle β with v_A , then

$$\tan \beta = \frac{v_B \sin \theta}{v_A - v_B \cos \theta}$$

7. Projectile Fired Horizontally

1. Positive of the projectile after time t

$$x = ut, \quad y = \frac{1}{2}gt^2$$

2. Equations of trajectory: $y = \frac{g}{2u^2} \cdot x^2$

3. Velocity after time t : $v = \sqrt{u^2 + g^2 t^2}$

$$\beta = \tan^{-1} \frac{gt}{u}$$

4. Time of flight: $T = \sqrt{\frac{2h}{g}}$

5. Horizontal range: $R = u \times T = u \sqrt{\frac{2h}{g}}$

8. Projectile Fired at an Angle with the Horizontal

1. For a projectile fired with velocity u at an angle θ with the horizontal : $u_x = u \cos \theta, \quad u_y = u \sin \theta$

, $a_x = 0, \quad a_y = -g$ 2.

Position after time t : $x = (u \cos \theta)t,$

$$y = (u \sin \theta)t - \frac{1}{2}gt^2$$

3. Equation of trajectory :

$$y = x \tan \theta - \frac{g}{2u^2 \cos^2 \theta} \cdot x^2$$

4. Maximum height: $H = \frac{u^2 \sin^2 \theta}{g}$

5. Time of flight, $T = \frac{2u \sin \theta}{g}$

6. Horizontal range, $R = \frac{u^2 \sin 2\theta}{g}$

7. Maximum horizontal range is attained at

$\theta = 45^\circ$ and its value is $R_{\max} = \frac{u^2}{g}$

8. Velocity after time t , $v_x = u \cos \theta, v_y = u \sin \theta - gt$

$$v = \sqrt{v_x^2 + v_y^2} \quad \text{and} \quad \tan \beta = \frac{v_y}{v_x}$$

9. Uniform Circular Motion

1. Angular displacement, $\theta = \frac{s}{r}$

2. Angular velocity, $\omega = \frac{\theta}{t}$

3. Also, $\omega = \frac{2\pi}{T} = 2\pi v$

4. Linear velocity $v = r\omega$

5. Centripetal acceleration, $a = \frac{v^2}{r} = r\omega^2$

6. Linear acceleration, $a = r\alpha$

5-LAWS OF MOTION

1. Linear Momentum and Newton's Second Law of Motion

1. Linear momentum, $p = mv$
2. According to Newton's second law,
Applied force = Rate of change of linear momentum

$$\text{or } F = \frac{dp}{dt} = ma = m \left(\frac{v-u}{t} \right)$$

2. Impulse of a Force

1. Impulse = Force x time = Change in momentum

$$\text{or } J = F \times t = m(v-u)$$

2. $\vec{J} = \int_{t_1}^{t_2} \vec{F}.dt$ = Area under force – time (F – t) graph.

3. Newton's Third Law and Motion in a Lift

1. Reaction = - Action
2. The apparent weight of a man in a lift:
(i) When the lift moves upwards with acceleration a ,

$$R = m(g + a)$$

- (ii) When the lift moves downwards with acceleration a , $R = m(g - a)$

- (iii) When the lift falls freely, $a = g$, so

$$R = m(g - a) = m(g - g) = 0$$

- (iv) When the lift is at rest or moves with uniform velocity, $a = 0$, so

$$R = m(g - 0) = mg$$

4. Conservation of Linear Momentum

1. In the absence of any external force, vector sum of the linear momenta of a system of particles remains constant.

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + \dots + m_n \vec{v}_n = \text{constant}$$

2. For a two body system,

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$3. \text{ Recoil velocity of a gun, } V = -\frac{mv}{M}$$

where M is the mass of the gun, m the mass of bullet and v is the velocity of the bullet.

5. Rocket Propulsion

1. Resultant force on the rocket
 $F = \text{upthrust on the rocket} - \text{weight of the rocket}$

$$= u \frac{dm}{dt} - mg$$

2. Acceleration of the rocket after time t

$$a = \left[\frac{u}{m_0 - t \frac{dm}{dt}} \right] \frac{dm}{dt} - g$$

3. Velocity of the rocket after time t ,

$$v = v_0 + u \log_e \frac{m_0}{m} - gt$$

If the effect of gravity is neglected, then

$$F = u \frac{dm}{dt}$$

$$a = \left[\frac{u}{m_0 - t \frac{dm}{dt}} \right] \frac{dm}{dt}; v = v_0 + u \log_e \frac{m_0}{m}$$

4. Burn-out speed of the rocket,

$$v_b = v_0 + u \log_e \frac{m}{m_r}$$

Here :

u = Velocity of exhaust gases

v_0 = Initial velocity of the rocket

v = Velocity of the rocket at any instant t

m_0 = Initial mass of the rocket

m = Final mass of the rocket

m_r = Mass of the empty rocket

dm/dt = Rate of ejection of fuel

6. Equilibrium of Concurrent Forces

1. A number of forces acting at the same point are called concurrent forces.
2. A number of concurrent forces are said to be in



equilibrium if there resultant is zero.

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots \vec{F}_n = \vec{0}$$

3. If \vec{F}_1, \vec{F}_2 and \vec{F}_3 are three concurrent forces in equilibrium.

$$(i) \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$$

$$(ii) \frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma} \quad (\text{Lami's theorem})$$

7. Motion of Connected Bodies

1. When a number of bodies are connected together by strings, rods, etc., it is convenient to draw a free body diagram for each body separately by showing all the forces acting on it.
2. Equation of motion for each body is written by equating the net force acting on the body to its mass times the acceleration produced.

8. Coefficient of friction and Angle of Friction

1. Coefficient of limiting friction =

$$\frac{\text{Limiting friction}}{\text{Normal reaction}}$$

$$\text{or } \mu_s = \frac{f_s^{\max}}{R} \quad \text{or} \quad f_s^{\max} = \mu_s R$$

2. Coefficient of kinetic friction =

$$\frac{\text{Kinetic friction}}{\text{Normal reaction}}$$

$$\text{or } \mu_k = \frac{f_k}{R} \quad \text{or} \quad f_k = \mu_k R$$

3. For a body placed on horizontal surface,

$$R = mg$$

$$f_s^{\max} = \mu_s mg \quad \text{and} \quad f_k = \mu_k mg$$

4. Static friction, $f_s \leq f_s^{\max}$ or $f_s \leq \mu_s R$

5. Kinetic friction, $f_k < f_s^{\max}$

6. If θ is the angle of friction, then $\mu_s = \tan \theta$

7. If ϕ is the angle of repose, then $\mu_s = \tan \phi$

8. Angle of repose = Angle of friction i.e. $\theta = \phi$

9. For a body moving on a rough horizontal

surface with retardation a , $\mu = \frac{f}{R} = \frac{ma}{mg} = \frac{a}{g}$

$$10. f_r = \mu_r \cdot \frac{R}{r} \quad \text{and} \quad \mu_r < \mu_k < \mu_s$$

Where μ_r is the coefficient of rolling friction, f_r is the rolling friction and r is the radius of the rolling body.

9. Motion along Rough Inclined Plane

1. For a body placed on an inclined plane of inclination θ ,

$$\text{Normal reaction, } R = mg \cos \theta,$$

$$\text{Friction, } f = \mu R = \mu mg \cos \theta,$$

2. When a body moves down an inclined plane without any acceleration, net downward force needed is

$$F = mg \sin \theta - f = mg (\sin \theta - \mu \cos \theta)$$

$$\text{Work done, } W = Fs = mg (\sin \theta - \mu \cos \theta) s$$

3. When a body moves up an inclined plane without acceleration, net upward force needed is

$$F = mg \sin \theta + f = mg (\sin \theta + \mu \cos \theta)$$

$$W = mg (\sin \theta + \mu \cos \theta) s$$

4. When a body moves up an inclined plane, with acceleration a , net upward force needed is

$$F = ma + mg \sin \theta + f$$

$$= m(a + g \sin \theta + \mu g \cos \theta)$$

$$W = m(a + g \sin \theta + \mu g \cos \theta) s$$

10. Centripetal Force

1. For a body moving, along a horizontal circular path, centripetal force is

$$F = \frac{mv^2}{r} = mr\omega^2 = mr(2\pi v)^2 = mr \left(\frac{2\pi}{T} \right)^2$$

2. Centrifugal force is equal to centripetal force in magnitude but acts away from the centre

11. Banking of Roads and Bending of a Cyclist

1. A vehicle taking a circular turn on a level road. If μ is the coefficient of friction between tyres and

road, then the maximum velocity with which the vehicle can safely take a circular turn of radius r is given by

$$v_{\max} = \sqrt{\mu rg}$$

2. *Banking of tracks (roads).* The maximum velocity with which a vehicle (in the absence of friction) can negotiate a circular road of radius r and banked at an angle θ is given by

$$v = \sqrt{\mu rg}$$

When the frictional forces are also taken into account, the maximum safe velocity is given by

$$v_{\max} = \sqrt{rg \left(\frac{\mu + \tan \theta}{1 - \mu \tan \theta} \right)}$$

3. *Bending of a cyclist.* In order to take a circular turn of radius r with speed v , the cyclist should bend himself through an angle θ from the vertical such that.

$$\tan \theta = \frac{v^2}{rg}$$

12. Motion in a Vertical Circle

1. Velocity of the body at any point at a height h from the lowest point.

$$v = \sqrt{u^2 - 2gh}$$

2. Tension in the string at any point

$$T = \frac{m}{r} (u^2 - 3gh + gr)$$

3. Tension at the lowest point,

$$T_L = \frac{m}{r} (u^2 + gr)$$

4. Tension at the highest point,

$$T_H = \frac{m}{r} (u^2 - 5gr)$$

5. Difference in tensions at the highest and lowest points,

$$T_L - T_H = 6mg$$

6. Minimum velocity at the lowest point for looping the vertical loop,

$$v_L = \sqrt{5gr}$$

7. Velocity at the highest point for looping the loop,

$$v_H = \sqrt{gr}$$

6-WORK, ENERGY AND POWER

1. Work Done by a Constant Force

1. $W = \vec{F} \cdot \vec{s} = F s \cos \theta$

2. If a body of mass m is raised through height h , then

$$W = mgh$$

3. If a body moves up a plane inclined at an angle θ with a constant speed, then $W = mg \sin \theta \times s$

2. Work done by a Variable Force

1. $W = \sum_i \vec{F}_i \cdot \vec{s}_i$

2. $W = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s}$

3. W = Area under the force-displacement curve between the initial and final positions of the body.

3. K.E. and W.E. Theorem

1. Kinetic energy, $K = \frac{1}{2}mv^2$

2. According to work-energy theorem,

$$W = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

4. P.E. and Conservation of Energy

1. Gravitational P.E., $U = mgh$

2. For a conservative force, $F = -\frac{dU}{dx}$

3. $\Delta U = U_f - U_i = -W = -\int_{x_i}^{x_f} F dx$

4. When work is done only by conservative forces only, mechanical energy is conserved.

$$K + U = \text{constant}$$

5. Potential Energy of a Spring

1. According to Hooke's law, $F = -kx$

2. Force constant, $k = \frac{F}{x}$

3. Work done on a spring or P.E of a spring stretched through distance x , $W = U = \frac{1}{2}kx^2$

6. Mass-Energy Equivalence

According to Einstein, energy equivalent of mass m is $E = mc^2$ where c = speed of light in free space = $3 \times 10^8 \text{ ms}^{-1}$.

7. Power

1. Power = $\frac{\text{Work}}{\text{Time}}$ or $P = \frac{W}{t}$

2. Also $P = \vec{F} \cdot \vec{v}$ when $\theta = 0^\circ$, $P = Fv$

8. Collisions

1. Linear momentum is conserved both in elastic and inelastic collisions.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

2. Kinetic energy is conserved in elastic collision.

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

3. In one-dimensional elastic collision, velocities after the collision are given by

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2$$

$$v_2 = \frac{2m_1}{m_1 + m_2}u_1 + \frac{m_2 - m_1}{m_1 + m_2}u_2$$

4. Coefficient of restitution for a collision is given by

$$e = -\frac{v_1 - v_2}{u_1 - u_2} = \frac{|v_1 - v_2|}{|u_1 - u_2|}$$

5. For a ball rebounding from a floor, $e = \frac{v}{u}$

6. For an elastic collision (involving no loss of K.E.),

$$e = 1$$

7. For an inelastic collision (involving loss of K.E.),

$$e < 1$$

7-SYSTEMS OF PARTICLES & ROTATIONAL MOTION

1.Centre of Mass

1. For a system of N particles, the position vector of centre mass is

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \dots + m_N\vec{r}_N}{m_1 + m_2 + \dots + m_N} = \frac{\sum_{i=1}^N m_i \cdot \vec{r}_i}{M}$$

2. The position vector of the centre mass of a two particle system is

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

3. The Cartesian co-ordinates of the centre of mass are given by

$$x = \frac{m_1x_1 + m_2x_2 + \dots + m_Nx_N}{m_1 + m_2 + \dots + m_N} = \frac{\sum_{i=1}^N m_i x_i}{M}$$

$$y = \frac{m_1y_1 + m_2y_2 + \dots + m_Ny_N}{m_1 + m_2 + \dots + m_N} = \frac{\sum_{i=1}^N m_i y_i}{M}$$

$$z = \frac{m_1z_1 + m_2z_2 + \dots + m_Nz_N}{m_1 + m_2 + \dots + m_N} = \frac{\sum_{i=1}^N m_i z_i}{M}$$

4. For a continuous mass distribution

$$\vec{R}_{CM} = \frac{1}{M} \int \vec{r} dm$$

where dm is the mass of small element located at position \vec{r}

$$\text{Also } x_{CM} = \frac{1}{M} \int x dm, \quad y_{CM} = \frac{1}{M} \int y dm,$$

$$z_{CM} = \frac{1}{M} \int z dm$$

5. The algebraic sum of the moments of masses of various particles of a system about its centre of mass is zero.

6. Velocity of CM of a two particle system is

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

2. Equations of Rotational Motion

For a body in rotational motion under constant angular acceleration, the equations of motion can be written as

$$1. \quad \omega = \omega_0 + \alpha t$$

$$2. \quad \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$3. \quad \omega^2 - \omega_0^2 = 2\alpha t$$

3. Torque, Power of a Torque, Work done by a Torque and Angular Momentum

1. Torque = Force x its perpendicular distance from the axis of rotation or $\tau = Fd$

$$2. \text{ Torque, } \tau = rF \sin \theta \quad \text{or} \quad \vec{\tau} = \vec{r} \times \vec{F}$$

3. Power of a torque = Torque x Angular velocity
or $P = \tau \omega$

4. Work done by a torque
= Torque x Angular displacement
or $W = \tau \theta$

5. Angular momentum = Linear momentum x its perpendicular distance from the axis of rotation
or $L = pd$

$$6. \text{ Angular momentum, } L = rp \sin \theta \quad \text{or} \quad \vec{L} = \vec{r} \times \vec{p}$$

7. For a particle of mass m moving with uniform speed v along a circle of radius r , $L = mvr$

8. Torque = Rate of change of angular momentum
or $\tau = \frac{dL}{dt}$

4. Moment of Inertia, Radius of Gyration and Rotational K.E

1. Moment of inertia of a body about the given axis of rotation

$$I = m_1r_1^2 + m_2r_2^2 + \dots + m_nr_n^2 = \sum_{i=1}^n m_i r_i^2$$

2. Radius of gyration K is given by

$$I = MK^2 \quad \text{or} \quad K = \sqrt{\frac{I}{M}}$$

When all the particles are of same mass,

$$K = \sqrt{\frac{r_1^2 + r_2^2 + \dots + r_n^2}{n}}$$

3. Theorem of perpendicular axes: $I_z = I_x + I_y$

4. Theorem of parallel axes, $I = I_{CM} + Md^2$

5. M.I of a circular ring about an axes through its centre



and perpendicular to its plane, $I = MR^2$

6. M.I of a thin ring about any diameter, $I = \frac{1}{2}MR^2$

7. M.I of thin ring about any tangent in its plane,

$$I = \frac{3}{2}MR^2$$

8. M.I. of a circular disc about an axis through its centre

and perpendicular to its plane, $I = \frac{1}{2}MR^2$

9. M.I. of a circular disc about any diameter

$$I = \frac{1}{4}MR^2$$

10. M.I. of a circular disc about a tangent in its plane,

$$I = \frac{5}{4}MR^2$$

11. M.I. of a thin rod about an axis through its middle

point ad perpendicular to rod, $I = \frac{1}{12}ML^2$

12. M.I. of a thin rod about an axis through its one end

and perpendicular to rod, $I = \frac{1}{3}ML^2$

13. M.I. of a rectangular lamina of sides l and b about an axis through its centre and perpendicular to its plane

$$I = M \left(\frac{l^2 + b^2}{12} \right)$$

14. M.I. of a right circular solid cylinder about its symmetry axis.

$$I = \frac{1}{2}MR^2$$

15. M.I. of a right circular hollow cylinder about its axis

$$I = MR^2$$

16. M.I. of a solid sphere about an axis through its centre,

$$I = \frac{2}{5}MR^2$$

17. M.I. of a solid sphere about any tangent, $I = \frac{7}{5}MR^2$

18. M.I. of a hollow sphere about an axis through its centre,

$$I = \frac{2}{3}MR^2$$

19. M.I. of a hollow sphere about any tangent

$$I = \frac{5}{3}MR^2$$

20. Rotational K.E. = $\frac{1}{2}I\omega^2$

21. Total K.E. = Rotational K.E. + Translational K.E.

$$= \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$$

5. Relations between Torque, Angular momentum and Moment of Inertia

1. Torque = M.I. x angular acceleration

$$\text{or } \tau = I\alpha$$

2. Work done by a torque, $W = \tau\theta$

3. Angular momentum = M.I. x angular velocity

$$\text{or } L = I\omega$$

6. Law of Conservation of Angular Momentum

In the absence of any external torque,

$$L = I\omega = \text{a constant}$$

$$\text{or } I_1\omega_1 = I_2\omega_2 \quad \text{or} \quad I_1 \cdot \frac{2\pi}{T_1} = I_2 \cdot \frac{2\pi}{T_2}$$

7. Motion of a Cylinder Rolling without Slipping on an Inclined Plane

For a cylinder of mass M and radius R rolling without slipping down plane inclined at angle θ with the horizontal,

1. Force of friction between the plane and cylinder

$$f = \frac{1}{3}Mg \sin \theta$$

2. Linear acceleration, $a = \frac{2}{3}g \sin \theta$

3. Condition for rolling without slipping is

$$\mu_s > \frac{1}{3} \tan \theta$$



8-GRAVITATION

1. Newton's Law of Gravitation and Principle of Superposition

1. Newton's law of gravitation, $F = \frac{Gm_1m_2}{r^2}$
2. Mass of planet or satellite, $M = \frac{4\pi^2 r^3}{GT^2}$
3. Principle of superposition of gravitational forces

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n$$

2. Mass and Density of Earth

1. Acceleration due to gravity on the earth's surface,

$$g = \frac{GM}{R^2}$$

2. Mass of the earth, $M = \frac{gR^2}{G}$
3. Mean density of earth, $\rho = \frac{3g}{4\pi GR}$
4. From Kepler's law of periods, $M = \frac{4\pi^2 r^3}{GT^2}$
5. Weight of a body, $W = mg$

3. Variation of 'g' with Altitude

1. $g_h = g \frac{R^2}{(R+h)^2}$, when h is comparable to R
2. $g_h = g \left(1 - \frac{2h}{R}\right)$, When $h \ll R$

4. Variation of 'g' with Depth

1. At a depth d , $g_d = g \left(1 - \frac{d}{R}\right)$
2. When $d = 2h$, $g_h = g_d$

5. Variation of 'g' with Rotation of the Earth

1. At latitude λ , $g_\lambda = g - R\omega^2 \cos^2 \lambda$
2. At equator, $\lambda = 0^\circ$, so $g_e = g - R\omega^2$
3. At poles, $\lambda = 90^\circ$, so $g_p = g$
4. $g_p - g_e = R\omega^2$

6. Gravitational Intensity, Potential and Potential Energy

1. Intensity of gravitational field, $E = \frac{F}{m} = \frac{GM}{r^2}$
2. Gravitational potential,
 $V = \frac{\text{Work done}}{\text{Mass}} = -\frac{GM}{r}$
3. Gravitational potential energy
 $U = \text{Gravitational potential} \times \text{mass} = -\frac{GMm}{r}$
4. $E = -\frac{dV}{dr}$
5. Total energy of a body in a gravitational field,
 $= \text{K.E.} + \text{P.E.} = \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right)$

7. Escape Velocity of a satellite

$$1. v_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} = \sqrt{\frac{8}{3}\pi G\rho R^2}$$

8. Orbital Velocity of Satellite

1. Orbital velocity at a height h
 $v_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}} = R\sqrt{\frac{g}{R+h}}$
2. When a satellite revolves close to earth's surface
 $v_0 = \sqrt{gR}$
 $v_e = \sqrt{2}v_0$
3. Time period of a satellite

$$T = \frac{2\pi(R+h)}{v_0} = 2\pi\sqrt{\frac{(R+h)}{GM}} = \frac{2\pi}{R}\sqrt{\frac{(R+h)^3}{g}}$$

$$= \sqrt{\frac{3\pi(R+h)}{G\rho R^3}}$$

4. Height of a satellite, $h = \left[\frac{T^2 R^2 g}{4\pi^2} \right]^{1/3} - R$

9. Total energy and Binding Energy of Satellite

1. Potential energy, $U = -\frac{GMm}{r}$

2. Kinetic energy, $K =$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{GM}{r}\right) = \frac{1}{2}\frac{GMm}{r}$$

3. Total energy, $E = K + U =$

$$\frac{1}{2}mv_0^2 - \frac{GMm}{r} = -\frac{GMm}{2r}$$

4. As $E = \frac{U}{2} = -K \quad \therefore \Delta K = -\Delta E$ and

$$\Delta U = 2\Delta E$$

5. Binding energy = $\frac{GMm}{2r}$

10. Kepler's law of Planetary Motion

1. Angular momentum, $L = mvr = \text{constant}$

2. Law of areas, $\frac{\Delta A}{\Delta t} = \text{constant}$

3. Law of periods, $T^2 \propto r^3$ or $T^2 = kr^3$. For a

satellite of earth, $k = \frac{4\pi^2}{GM_E} = 10^{-13} s^2 m^{-3}$

4. $\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3}$



9-MECHANICAL PROPERTIES OF SOLIDS

1.Young's Modulus

1. Stress = $\frac{\text{Force}}{\text{Area}} = \frac{F}{A}$

2. Longitudinal strain = $\frac{\text{Change in length}}{\text{Original length}} = \frac{\Delta l}{l}$

3. Young's modulus
= $\frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$ or $Y = \frac{F/A}{\Delta l/l} = \frac{F}{A} \cdot \frac{l}{\Delta l}$

4. Percentage increase in length

$$\frac{\Delta l}{l} \times 100 = \frac{F}{AY} \times 100$$

2. Bulk Modulus

1. Volumetric stress = $\frac{F}{A} = p$, the applied pressure

2. Volumetric strain = $\frac{\Delta V}{V}$

3. Bulk modulus = $\frac{\text{Volumetric stress}}{\text{Volumetric strain}}$

or $\kappa = -\frac{F/A}{\Delta V/V} = -\frac{p}{\Delta V/V} = -V \frac{p}{\Delta V}$

Negative sign indicated the decrease in volume with the increase in stress

4. Compressibility = $\frac{1}{\kappa} = -\frac{\Delta V}{pV}$

3. Modulus of Rigidity

1. Shearing stress = $\frac{\text{Tangential Force}}{\text{Area}} = \frac{F}{A}$

2. Shearing strain = $\theta = \frac{\Delta l}{l}$

3. Modulus of rigidity = $\frac{\text{Shearing stress}}{\text{Shearing strain}}$

or $\eta = \frac{F/A}{\theta} = \frac{F/A}{\Delta l/l}$

4. Elastic Potential Energy

1. Total P.E. stored in a stretched wire,

$$U = \frac{1}{2} \text{Stretching force} \times \text{extension} = \frac{1}{2} F \Delta l$$

or $U = \frac{1}{2} \text{Stress} \times \text{strain} \times \text{volume of wire}$

2. P.E. stored per unit volume of a stretched wire.

$$u = \frac{1}{2} \text{Stress} \times \text{strain}$$

or $u = \frac{1}{2} \text{Young's modulus} \times \text{strain}^2$

5. Poisson's Ratio

Poisson's ratio = $\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

or $\sigma = \frac{\Delta D/D}{\Delta l/l}$

10-MECHANICAL PROPERTIES OF FLUIDS

1. Thrust and Pressure

1. Thrust = Total force exerted by a liquid on the surface in contact

$$2. \text{ Pressure} = \frac{\text{Thrust}}{\text{Area}} \quad \text{or} \quad P = \frac{F}{A}$$

2. Pascal's Law and Hydraulic Lift

1. According to Pascal's law, pressure applied at any point of an enclosed mass of fluid is transmitted equally in all directions.

$$2. \text{ For a hydraulic lift, } P = \frac{f}{a} = \frac{F}{A}$$

3. Pressure Exerted by a Liquid Column and Gauge Pressure

1. Pressure exerted by a liquid column of height h and density ρ is $P = h \rho g$

2. Absolute pressure
= Atmospheric pressure + Gauge pressure
 $P = P_a + P_g$

4. Archimedes' Principle and Law of Floatation

1. According to Archimedes' principle, Loss in weight of a body in a liquid = Weight of liquid displaced = Volume \times Density of liquid $\times g$

2. Apparent weight of solid in a liquid
= True weight – Weight of liquid displaced,
 $= mg - V' \rho' g = mg - \frac{m}{\rho} \rho' g = mg \left(1 - \frac{\rho'}{\rho} \right)$,

where ρ' is the density of the liquid and ρ that of solid.

3. What a body just floats. Weight of the body
= Weight of liquid displaced

$$\text{or } V \rho g = V' \rho' g \quad \text{or} \quad \frac{V'}{V} = \frac{\rho}{\rho'}$$

$$\frac{\text{volume of immersed part}}{\text{Total volume of the solid}} = \frac{\text{Density of solid}}{\text{Density of liquid}}$$

4. Relative density =

$$\frac{\text{Density of substance}}{\text{Density of water at } 4^\circ \text{C}}$$

5. Relative density of a solid

$$= \frac{\text{Weight of solid in air}}{\text{Loss in weight in water}}$$

6. Relative density of a liquid

$$= \frac{\text{loss in weight in liquid}}{\text{Loss in weight in water}}$$

5. Coefficient of Viscosity

$$1. \text{ Velocity gradient} = \frac{dv}{dx}$$

2. Newton's formula for viscous force between two parallel layers is $F = -\eta A \frac{dv}{dx}$

6. Poiseuille's Formula

Poiseuille's formula for the volume of a liquid flowing out per second through a narrow pipe is

$$Q = \frac{V}{t} = \frac{\pi r^4}{8 \eta l}$$

7. Stokes' law and Terminal Velocity

1. According to Stokes' law, force of viscosity acting on a spherical body of radius r moving with velocity v through a fluid of viscosity η is

$$F = 6\pi \eta r v$$

2. Terminal velocity of a spherical body of density ρ and radius r moving through a liquid of density ρ' is

$$v = \frac{2}{9} \frac{r^2}{\eta} (\rho - \rho') g$$

8. Reynold's Number

1. For a liquid of viscosity η , density ρ and flowing through a pipe of diameter D , Reynold's

number is given by

$$R_e = \frac{\rho v D}{\eta}$$

2. Flow is laminar for R_e between 0 and 2000.

The fluid velocity corresponding to $R_e = 2000$ is called critical velocity.

$$v_c = \frac{2000 \times \eta}{\rho D}$$

3. Flow is turbulent for R_e above 3000.

4. Flow is unstable for R_e between 2000 and 3000.

9. Equation of Continuity and Bernoulli's Theorem

1. Volume of a liquid flowing per second through

a pipe of cross-section a with velocity v , $Q = av$

2. Equation of continuity, $av = \text{constant}$

or $a_1 v_1 = a_2 v_2$

3. First form of Bernoulli's theorem,

$$\frac{p}{\rho} + gh + \frac{1}{2} v^2 = \text{constant}$$

or Pressure energy per unit mass + P.E. per unit mass + K.E. per unit mass = constant

4. Second form of Bernoulli's theorem

$$\frac{p}{\rho g} + h + \frac{v^2}{2g} = \text{constant}$$

or Pressure head + Gravitational head + Velocity head = constant

5. Volume of a liquid flowing out per second through a venturimeter,

$$Q = a_1 a_2 \sqrt{\frac{2gh}{a_1^2 - a_2^2}}$$

where a_1 and a_2 are the areas of cross-section of bigger and smaller tubes respectively.

6. Torricelli's theorem, velocity of efflux of a liquid through an orifice at depth h from the liquid surface $v = \sqrt{2gh}$

10. Surface Tension and Surface Energy

1. Surface tension = $\frac{\text{Force}}{\text{Length}}$ or $\sigma = \frac{F}{l}$

2. Increase in surface energy or work done, $W = \text{surface tension} \times \text{increase in area of the liquid surface}$

11. Excess Pressure in Drops & Bubbles

1. Excess pressure inside a liquid drop,

$$p = \frac{2\sigma}{R} \text{ (with one free surface).}$$

2. Excess pressure inside a soap bubble,

$$p = \frac{4\sigma}{R} \text{ (with two free surface)}$$

3. Excess pressure in an air bubble,

$$p = \frac{2\sigma}{R} \text{ (with one free surface)}$$

12. Capillarity: Ascent Formula

1. When a capillary tube of radius r is dipped in a liquid of density ρ and surface tension σ , the liquid rises or falls through a distance,

$$h = \frac{2\sigma \cos \theta}{r\rho g}$$

where θ is the angle of contact.

11-THERMAL PROPERTIES OF MATTER

1. Measurement of Temperature

1. If T_C , T_F , T_R and T are the temperatures of a body on Celsius, Fahrenheit and Kelvin scales respectively, then

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32} = \frac{T_R - 0}{80 - 0} = \frac{T - 273.15}{100}$$

$$\text{or } \frac{T_C}{5} = \frac{T_F - 32}{9} = \frac{T_R}{4} = \frac{T - 273.15}{5}$$

$$2. T_C - \frac{5}{9}(T_F - 32), T_F = \frac{9}{5}T_C + 32$$

$$3. T = T_C + 273.15, T_C = T - 273.15$$

$$4. T_F = \frac{9}{5}(T - 273.15) + 32 = \frac{9}{5}T - 459.67$$

$$\text{or } T = \frac{5}{9}T_F + 255.37$$

5. For a constant volume air thermometer

$$T = T_0 \times \frac{P}{P_0}$$

In terms of triple point of water, $T = T_{tr} \times \frac{P}{P_{tr}}$

6. For a platinum resistance thermometer, resistance of platinum at $t^\circ\text{C}$, $R = R_0(1 + \alpha)$

$$\text{Temperature coefficient of resistance, } \alpha = \frac{R - R_0}{R_0 \times t}$$

2. Thermal Expansion

1. Change in length, $l' - l = l \alpha (T' - T)$ or $\Delta l = l \alpha \Delta T$

2. Coefficient of linear expansion, $\alpha = \frac{\Delta l}{l \Delta T}$

3. Final length, $l' = l(1 + \alpha \Delta T)$

4. Change in surface area, $S' - S = S \beta (t' - t)$

$$\text{or } \Delta S = S \beta \Delta T$$

5. Coefficient of superficial expansion, $\beta = \frac{\Delta S}{S \Delta T}$

6. Final surface area, $S' = S(1 + \beta \Delta T)$

7. Change in volume, $V' - V = V \gamma (t' - t)$

$$\text{or } \Delta V = V \gamma \Delta T$$

8. Coefficient of cubical expansion, $\gamma = \frac{\Delta V}{V \Delta T}$

9. Final volume $V' = V(1 + \gamma \Delta T)$

10. Relation between α , β and γ

$$\frac{\alpha}{1} = \frac{\beta}{2} = \frac{\gamma}{3} \quad \beta = 2\alpha \text{ and } \gamma = 3\alpha$$

11. Final density, $\rho' = \rho(1 - \gamma \Delta T)$

3. Specific Heat and Latent Heat

1. Heat gained or lost, $Q = mc \Delta T$

2. According to the principle of calorimetry,

$$\text{Heat gained} = \text{Heat lost}$$

3. Water equivalent, $w = mc$ (gram)

4. Heat capacity = mc ($\text{cal } ^\circ\text{C}^{-1}$)

5. Latent heat of vaporization or fusion, $Q = ml$

4. Thermal Conductivity

1. The amount of heat that flows in time t across the opposite faces of a slab of thickness x and cross-section A ,

$$Q = \frac{KA(T_1 - T_2)t}{x}$$

where T_1 and T_2 are the temperatures of hot and cold faces and K is the coefficient of thermal conductivity of the material of the slab.

2. Rate of flow of heat,

$$\frac{dQ}{dt} = -KA \frac{dT}{dx}$$

Here dt/dx is the rate of fall of temperature with distance and is called temperature gradient.

5. Newton's Law of Cooling

Newton's law of cooling. If the temperature difference between body and surroundings is small, then Rate of loss of heat \propto Temperature difference from the body.

Rate of loss of heat from the body is

$$mc \frac{(T_1 - T_2)}{t} = k(T - T_0) = k \left(\frac{T_1 + T_2}{2} - T_0 \right)$$

Here temperature of the body falls from T_1 to T_2 in time-interval t .

6. Stefan's Law

1. *Stefan's law*. Energy emitted per second per unit area by a black body at a absolute temperature T ,

$$E = \sigma T^4, \text{ where } \sigma = \text{Stefan's constant.}$$

2. *Stefan-Boltzmann law*. When a black body at temperature T is placed in an enclosure at temperature T_0 the net heat energy radiated per unit area,

$$E = \sigma(T^4 - T_0^4)$$

3. Energy radiated by a surface of emissivity ε , area A in time t ,

(i) $E = \varepsilon \sigma T^4 \times A \times t$ (Stefan's law)

(ii) $E = \varepsilon \sigma (T^4 - T_0^4) \times A \times t$ (Stefan-Boltzmann law)

7. Wien's Displacement Law

1. *Wien's displacement*: The wavelength λ_m corresponding to maximum energy emission by a black body at absolute temperature T is given by

$$\lambda_m = \frac{b}{T}$$

Where b = Wien's constant = 0.002898 mK

12-THERMODYNAMICS

1. Work Done during a Cyclic Process

1. Work done during the expansion or compression of a gas is equal to the area enclosed between the P-V curve and the volume axis.
2. Work done per cycle
= Area of the loop representing the cycle
3. If the loop is traced *clockwise*, the work done is *positive* and work is done by the system.
4. If the loop is traced *anticlockwise*, the work done is *negative* and work is done on the system.

2. First Law of Thermodynamics

1. According to first law of thermodynamics
 $dQ = dU + dW = dU + PdV$
2. For change of state, $dQ = mL$
3. For rise in temperature, $dQ = mC \Delta T$
4. Change in internal energy, $dU = U_f - U_i$

3. Relation between Two Specific Heats of a Gas

1. For one mole of a gas,
(i) $C_p - C_v = R$ (When C_p, C_v are in units of work)
(ii) $C_p - C_v = \frac{R}{J}$ (when C_p, C_v are in units of heat)
2. For 1 g of a gas
(i) $c_p - c_v = r$ (when c_p, c_v are in units of work)
(ii) $c_p - c_v = \frac{r}{J}$ (when c_p, c_v are in units of heat)

where $r = \frac{R}{M}$ = gas constant for 1 g of a gas

3. Heat lost or gained by a gas.
(i) $Q = nC_p \Delta T$ (At constant pressure)
(ii) $Q = nC_v \Delta T$ (At constant volume)

where n = Number of moles of gas
$$= \frac{\text{Mass of gas}}{\text{Molecular mass}}$$

4. Isothermal and Adiabatic Processes

1. Equation for isothermal process
 $PV = \text{constant}$ or $P_1V_1 = P_2V_2$
2. Equations for adiabatic processes,

$$(i) P_1V_1^\gamma = P_2V_2^\gamma \quad (ii) T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$
$$(iii) \frac{P_1^{\gamma-1}}{T_1^\gamma} = \frac{P_2^{\gamma-1}}{T_2^\gamma}, \text{ Where } \gamma = C_p / C_v$$

3. Work done when 1 mole of a gas expands isothermally,

$$(i) W_{iso} = 2.303RT \log \frac{V_2}{V_1}$$

$$(ii) W_{iso} = 2.303RT \log \frac{P_1}{P_2}$$

4. Work done when 1 mole of a gas expands adiabatically and its temperature falls from T_1 to T_2 ,

$$(i) W_{adi} = \frac{R}{\gamma-1} [T_1 - T_2]$$

$$(ii) W_{adi} = \frac{1}{\gamma-1} [P_1V_1 - P_2V_2]$$

5. Carnot Engine

1. Efficiency of a heat engine,
$$\eta = \frac{\text{Work output}}{\text{Heat input}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$
2. Efficiency of a Carnot's engine (an ideal heat engine),

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

where Q_1 = Heat extracted from the source
 Q_2 = heat rejected to the sink
 T_1 = temperature of the source
 T_2 = temperature of the sink

6. Refrigerator

Coefficient of performance of a refrigerator,

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

Where Q_2 = heat drawn per cycle from sink
 W = work done per cycle on refrigerator



13-KINETIC THEORY OF GASES

1. Gas laws and ideal Gas Equation

1. *Boyle's law* = At constant temperature,

$$PV = \text{Constant or } P_1V_1 = P_2V_2$$

2. *Charles' law* = At constant pressure,

$$V \propto T \quad \text{or} \quad \frac{V_2}{V_1} = \frac{T_2}{T_1}$$

3. *Gay Lussac's law* = At constant volume,

$$P \propto T \quad \text{or} \quad \frac{P_2}{P_1} = \frac{T_2}{T_1}$$

4. Perfect gas equation is $PV = nRT$

$$\text{or} \quad \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

5. Boltzmann's constant $k_B = \frac{R}{N}$

2. Kinetic Theory of Gases & Kinetic Interpretation of Temperature

1. Pressure exerted by a gas,

$$P = \frac{1}{3} \frac{M}{V} v_{rms}^2 = \frac{1}{3} \rho v_{rms}^2$$

$$2. v_{rms} = \sqrt{\frac{3P}{\rho}}$$

3. Mean K.E. per molecule of a gas,

$$\bar{E} = \frac{1}{2} m v^2 = \frac{3}{2} k_B T$$

4. Mean K.E. per mole of a gas,

$$E = \frac{1}{2} M v_{rms}^2 = \frac{3}{2} RT = \frac{3}{2} k_B NT$$

5. K.E. of 1 g of a gas = $\frac{1}{2} v_{rms}^2 = \frac{3}{2} \frac{R}{M} T$

6. Avogadro's number = $\frac{\text{Molecular mass}}{\text{Mass of 1 molecule}}$

$$\text{or } N = \frac{M}{m}$$

7. No. of moles, $n = \frac{\text{Mass of gas}}{\text{Molecular mass}}$

3. Average, R.M.S. & Most Probable Speeds

1. Average speed, $\bar{v} = \frac{v_1 + v_2 + v_3 + \dots + v_n}{n}$

$$2. \bar{v} = \sqrt{\frac{8k_B T}{\pi m}} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8PV}{\pi M}}$$

3. R.M.S. speed, $v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + \dots + v_n^2}{n}}$

$$4. v_{rms} = \sqrt{\frac{3k_B T}{m}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3PV}{M}}$$

5. Most probable speed,

$$v_{mp} = \sqrt{\frac{2k_B T}{m}} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2PV}{M}}$$

4. Degrees of Freedom, Specific Heats of Monoatomic Diatomic and Polyatomic Gases

1. Energy associated with each degree of freedom

$$\text{per molecular} = \frac{1}{2} k_B T$$

2. For a gas of *polyatomic molecules* having f degrees of freedom,

Energy associated with 1 mole of gas, $U = \frac{f}{2} RT$

$$C_V = \frac{f}{2} R, \quad C_P = \left(1 + \frac{f}{2}\right) R, \quad \gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$$

3. For *monoatomic gas* $f = 3$, so

$$U = \frac{3}{2} RT, \quad C_V = \frac{3}{2} R, \quad C_P = \frac{5}{2} R, \quad \gamma = 1.66$$

4. For a *diatomic gas*, $f = 5$

$$U = \frac{5}{2} RT, \quad C_V = \frac{5}{2} R, \quad C_P = \frac{7}{2} R, \quad \gamma = 1.4$$

5. For a *triatomic gas of non-linear molecules* $f = 6$, so

$$U = 3RT, \quad C_V = 3R, \quad C_P = 4R, \quad \gamma = 1.33$$

6. For a *triatomic gas of linear molecules* $f = 7$, so

$$U = \frac{7}{2} RT, \quad C_V = \frac{7}{2} R, \quad C_P = \frac{9}{2} R, \quad \gamma = 1.28$$

14-OSCILLATIONS

1. Periodic and Harmonic Functions

1. A function which can be represented by a single sine or cosine function is a harmonic function otherwise non-harmonic.
2. A periodic function can be expressed as the sum of sine and cosine functions of different time periods with suitable coefficients.

2. Displacement, Velocity, Acceleration and Time Period of SHM

1. Displacement, $x = A \cos(\omega t + \phi_0)$

where A = amplitude, ω = angular frequency and

ϕ_0 = initial phase of particle in SHM

2. Velocity, $v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi_0)$
 $= -\omega \sqrt{A^2 - x^2}$

Maximum velocity, $v_{\max} = \omega A$

3. Acceleration,

$$a = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi_0) = -\omega^2 x$$

Maximum acceleration, $a_{\max} = \omega^2 A$

4. Restoring force, $F = -kx = -m\omega^2 x$

where k = force constant and $\omega^2 = k/m$.

5. Angular frequency, $\omega = 2\pi\nu = 2\pi/T$.

6. Time period, $T = 2\pi \sqrt{\frac{\text{Displacement}}{\text{Acceleration}}} = 2\pi \sqrt{\frac{x}{a}}$

7. Time period, $T = 2\pi \sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}} = 2\pi \sqrt{\frac{m}{k}}$.

3. Energy of S.H.M

1. P.E. at displacement y from the mean position

$$E_p = \frac{1}{2}ky^2 = \frac{1}{2}m\omega^2 y^2 = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$$

2. K.E. at displacement y from the mean position

$$E_k = \frac{1}{2}k(A^2 - y^2) = \frac{1}{2}m\omega^2 (A^2 - y^2)$$

$$= \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$$

3. Total energy at any point.

$$E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2 A^2 = 2\pi^2 mA^2 \nu^2$$



15-WAVES

1.Relation Between Frequency, Wavelength and Wave Velocity

1. Wave velocity = Frequency x Wavelength
or $v = v\lambda$

2. Wave velocity = $\frac{\text{Wavelength}}{\text{Timeperiod}}$ or $v = \frac{\lambda}{T}$

3. Wavelength = $\frac{\text{Wave_velocity}}{\text{Frequency}}$ or $\lambda = \frac{v}{\nu}$

2. Velocity of Transverse Waves in Solids and Strings

1. Velocity of transverse waves in a solid of modulus of rigidity η and density ρ ,

$$v = \sqrt{\frac{\eta}{\rho}}$$

2. Velocity of transverse waves in a string of mass per unit length m and stretched under tension T ,

$$v = \sqrt{\frac{T}{m}}$$

3. Velocity of Longitudinal Waves

1. Velocity of longitudinal waves in a solid of bulk modulus κ , modulus of rigidity η and density ρ is given by

$$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}}$$

2. Velocity of longitudinal waves in a long rod of Young's modulus Y and density ρ is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

3. Velocity of longitudinal waves in liquid of bulk modulus κ and density ρ is given by

$$v = \sqrt{\frac{\kappa}{\rho}}$$

4. *Newton's formula* for the velocity of sound in a gas is

$$v = \sqrt{\frac{\kappa_{iso}}{\rho}} = \sqrt{\frac{P}{\rho}}$$

where P = pressure of a gas

5. *Laplace formula* for the velocity of sound in a gas is

$$v = \sqrt{\frac{\kappa_{adia}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}, \text{ where } \gamma = \frac{C_p}{C_v}$$

4. Factors affecting Velocity of Sound through Gases

1. *Effect the pressure.* There is no effect of pressure on velocity of sound.

2. *Effect of density* $v \propto \frac{1}{\sqrt{\rho}}$

or $\frac{v_1}{v_2} = \sqrt{\frac{\rho_2}{\rho_1}}$

3. *Effect of temperature* $v \propto \sqrt{T}$

or $\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$

Also $v = \sqrt{\frac{\gamma RT}{M}}$

where M = molecular mass of the gas.

4. *Temperature coefficient of sound.* It is give by

$$\alpha = \frac{v_t - v_0}{t}$$

for air, $\alpha = 0.61 \text{ ms}^{-1} \text{ } ^\circ\text{C}^{-1}$.

5. Progressive Waves

1. A plane progressive harmonic wave travelling along positive direction of X- axis can be represented by any of the following expressions:

(i) $y = A \sin(\omega t - kx)$, $k = 2\pi / \lambda$

(ii) $y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$

(iii) $y = A \sin \frac{2\pi}{\lambda} (vt - x)$

where λ is the wavelength, v is the velocity, A the amplitude and x is the distance of observation

point from the origin.

2. For a progressive wave travelling along – ve X-axis.

$$y = A \sin(\omega t + kx)$$

$$\text{or } y = A \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right) = A \sin \frac{2\pi}{\lambda} (vt + x)$$

$$3. \text{ Phase, } \phi = 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) + \phi_0$$

where ϕ_0 is the initial phase.

$$4. \text{ Phase change with time, } \Delta\phi = \frac{2\pi}{T} \Delta t$$

$$5. \text{ Phase change with position, } \Delta\phi = -\frac{2\pi}{\lambda} \Delta x$$

6. Instantaneous particle velocity

$$u = \frac{dy}{dt} = \frac{2\pi A}{T} \cos 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$$

$$\text{Velocity amplitude, } u_0 = \frac{2\pi A}{T} = \omega A$$

7. Instantaneous particle acceleration

$$f = \frac{du}{dt} = -\frac{4\pi^2}{T^2} A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) = -\omega^2 y$$

$$\text{Acceleration amplitude, } f_0 = \frac{4\pi^2}{T^2} A = \omega^2 A$$

6. Equation of Stationary Waves

$$1. \text{ Let } y_1 = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) \quad (\text{incident wave})$$

$$y_2 = \pm a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda} \right) \quad (\text{refracted wave})$$

Then stationary wave formed by the superposition is given by

$$y = y_1 + y_2 = \pm 2a \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi t}{T}$$

It involves the product of separate harmonic functions of time t and position x .

2. For (+) sign in the above equation, antinodes are formed at the positions $x = 0, x/2, x, 3x/2, \dots$

And nodes are formed at $x = x/4, 3x/4, 5x/4, \dots$

3. For (-) sign, antinodes are formed at the

positions

$x = \lambda/4, 3\lambda/4, 5\lambda/4, \dots$ and nodes at

$$x = 0, \lambda/2, \lambda, 3\lambda/2,$$

4. The distance between two successive nodes or antinodes is $\lambda/2$ and that between a node and nearest antinodes is $\lambda/4$.

7. Modes of Vibrations of Strings

$$1. \text{ Fundamental frequency, } v = \frac{v}{2L} = \frac{1}{2L} \sqrt{\frac{T}{m}}$$

2. When the stretched string, vibrates in p loops,

$$v_p = \frac{p}{2L} \sqrt{\frac{T}{m}} = pv$$

3. For a string of diameter D and density ρ ,

$$v = \frac{1}{LD} \sqrt{\frac{T}{\pi\rho}}$$

4. Law of length, $v \propto 1/L$ or $vL = \text{constant}$

$$\text{or } v_1 L_1 = v_2 L_2$$

8. Organ Pipes and Rods clamped in the Middle

1. In an organ closed at one end, only odd harmonics are present.

$$\text{Fundamental mode, } v_1 = \frac{v}{4L} = v \text{ (First harmonic)}$$

$$\text{Second mode, } v_2 = 3v \text{ (third harmonic or first overtone)}$$

$$\text{Third mode, } v_3 = 5v \text{ (fifth harmonic or second overtone)}$$

$$\text{nth mode, } v_n = (2n-1)v \text{ (2n-1)th harmonic or (n-1)th overtone}$$

2. In an organ pipe open at both ends. Both odd and even harmonics are present.

$$\text{Fundamental mode, } v'_1 = \frac{v}{2L} = v' \text{ (First harmonic)}$$

$$\text{Second mode, } v'_2 = 2v' \text{ (second harmonic or first overtone)}$$

$$\text{Third mode } v'_3 = 3v'$$



(Third harmonic or second overtone)

n th mode $v'_n = nv$

(n th harmonic or $(n-1)$ th overtone)

Clearly, $v'_1 = 2v_1$

3. *Resonance tube*. If L_1 and L_2 are the first and second resonance lengths with a tuning fork of frequency v , then the speed of sound,

$$v = 4v(L_1 + 0.3D)$$

D = internal diameter of resonance tube

or $v = 2v(L_2 - L_1)$

$$\text{End correction} = 0.3 D = \frac{L_2 - 3L_1}{2}$$

9. Beats Formation

1. Beat frequency = Number of beats per second.

= Difference frequencies of two sources

or $b(v_1 - v_2)$ or $(v_2 - v_1)$

2. $v_2 = v_1 \pm b$

3. If the prong of tuning fork is *filed*, its frequency increases. If the prong of a tuning fork is *loaded* with a little wax, its frequency decreases. These facts can be used to decide about + or – sign in the above equation.

10. Doppler Effect in Sound

1. If v , v_0 , v_s and v_m are the velocities of sound, observer, source and medium respectively, then

the apparent frequency $v' = \frac{v + v_m - v_0}{v + v_m - v_s} \times v$

2. If the medium is at rest ($v_m = 0$), then $\frac{v - v_0}{v - v_s} \times v$

3. All the velocities are taken *positive* in the *source to observer* ($S \rightarrow O$) direction and *negative* in the opposite ($O \rightarrow S$) direction.



1-ELECTROSTATICS

1. $q = ne$
2. Mass transferred during charging = $m_e \times n$

2. Coulomb's Law

1. $F_{vac} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$
2. $F_{med} = \frac{1}{4\pi\epsilon_0 K} \cdot \frac{q_1 q_2}{r^2}$

3. Principle of Superposition of Electric Forces

$$1. \vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \dots + \vec{F}_{1N}$$

$$F = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos \theta}$$

4. Relation between Electric Field Strength and Force

$$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{or} \quad \vec{F} = q_0 \vec{E}$$

5. Electric Fields of Point Charges

1. $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$
2. By the principle of superposition, electric field due to a number of point charges,
$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$$

6. Continuous Charge Distributions

1. Volume charge density, $\rho = \frac{dq}{dV}$
2. Surface charge density, $\sigma = \frac{dq}{dS}$
3. Linear charge density, $\lambda = \frac{dq}{dL}$
4. Force exerted on a charge q_0 due to a continuous charge distribution,

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

5. Electric field due to a continuous charge

distribution,

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r^2} \hat{r}$$

7. Dipole Moment, Dipole Field and Torque on a Dipole

1. Dipole moment, $p = q \times 2a$; where $2a$ is the distance between the two charges.
2. Dipole field at an axial point at distance r from the centre of the dipole is

$$E_{axial} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2}$$

When $r \gg a$,

$$E_{axial} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2p}{r^3}$$

3. Dipole field at an equatorial point at distance r from the centre of the dipole is

$$E_{equa} = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{(r^2 + a^2)^{3/2}}$$

When $r \gg a$,

$$E_{equa} = \frac{1}{4\pi\epsilon_0} \cdot \frac{p}{r^3}$$

4. Torque, $\tau = pE \sin \theta$ where θ is the angle between \vec{p} and \vec{E}

8. Electric Flux and Gauss's Theorem

1. Electric flux through a plane surface area S held in a uniform electric field \vec{E} is

$$\phi_E = \vec{E} \cdot \vec{S} = ES \cos \theta$$

where θ is the angle which the normal to the outward drawn normal to surface area \vec{S} makes with the field \vec{E} .

2. According to Gauss's theorem the total electric flux through a closed surface S enclosing charge q is

$$\phi_E = \oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

$$3. \text{ Flux density} = \frac{\text{Total flux}}{\text{Area}} = \frac{\phi_E}{S}$$

9. Applications of Gauss's Theorem

1. Electric field of a long straight wire of uniform linear charge density λ ,

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

where r is the perpendicular distance of the observation point from the wire.

2. Electric field of an infinite plane sheet of uniform surface charge density σ ,

$$E = \frac{\sigma}{2\epsilon_0}$$

3. Electric field of two positively charged parallel plates with charge densities σ_1 and σ_2 such that $\sigma_1 > \sigma_2 > 0$,

$$E = \pm \frac{1}{2\epsilon_0} (\sigma_1 + \sigma_2) \quad (\text{Outside the plates})$$

$$E = \frac{1}{2\epsilon_0} (\sigma_1 - \sigma_2) \quad (\text{Inside the plates})$$

4. Electric field of two equally and oppositely charged parallel plates.

$$E = 0 \quad (\text{For outside points})$$

$$E = \frac{\sigma}{\epsilon_0} \quad (\text{For inside points})$$

5. Electric field of a thin spherical shell of charge density σ and radius R ,

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \quad \text{For } r > R \text{ (Outside points)}$$

$$E = 0 \quad \text{For } r < R \text{ (inside points)}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2} \quad \text{For } r = R \text{ (At the surface)}$$

$$\text{Here } q = 4\pi R^2 \sigma$$

6. Electric field of a solid sphere of uniform charge density ρ and radius R :

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2} \quad \text{For } r > R \text{ (Outside points)}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{qr}{R^3} \quad \text{For } r < R \text{ (Inside points)}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2} \quad \text{For } r = R \text{ (At the surface)}$$

$$\text{Here } q = \frac{4}{3}\pi R^3 \rho$$

2-ELECTRIC POTENTIAL AND CAPACITANCE

1. Electric Potential

1. Potential difference = $\frac{\text{Workdone}}{\text{Charge}}$ or $V = \frac{W}{q}$

2. Electric potential due to a point charge q at distance r from it

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$

3. Electric potential at a point due to N point charges

$$V = \frac{1}{4\pi\epsilon_0} \cdot \sum_{i=1}^N \frac{q_i}{r_i}$$

4. Electric potential at a point due to a dipole,

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\vec{p} \cdot \vec{r}}{r^3}$$

2. Relation between Electric Field and Potential

1. Electric field in a region can be determined from the electric potential by using relation.

$$E = -\frac{dV}{dr}$$

or $E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$

2. Electric field between two parallel conductors

$$E = \frac{V}{d}$$

3. Electric potential in a region can be determined from the electric field by using the relation

$$V = -\int_{\infty}^r \vec{E} \cdot d\vec{r}$$

3. Electric Potential Energy

1. Electric potential energy of a system of N point charges

$$U = \frac{1}{4\pi\epsilon_0} \sum_{\text{all pairs}} \frac{q_j q_k}{r_{jk}}$$

2. Electric potential of a system of two point charges

$$U = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

3. Potential energy of an electric dipole in a uniform electric field,

$$U = -pE(\cos \theta_2 - \cos \theta_1)$$

If initially the dipole is perpendicular to the field E , $\theta_1 = 90^\circ$ and $\theta_2 = \theta$ (say), then

$$U = -pE \cos \theta = -\vec{p} \cdot \vec{E}$$

If initially the dipole is parallel to the field E , $\theta_1 = 0^\circ$ and $\theta_2 = \theta$ (say), then

$$U = -pE(\cos \theta - 1) = pE(1 - \cos \theta)$$

4. Capacitance of Spherical Conductors

1. Capacitance of a spherical conductor of radius R ,

$$C = 4\pi\epsilon_0 r$$

2. Capacitance = $\frac{\text{Charge}}{\text{Potential}}$ or $C = \frac{q}{V}$

5. Capacitance of Air-Filled Capacitors

1. Capacitance, $C = \frac{q}{V}$

2. Capacitance of a parallel plate capacitor,

$$C = \frac{\epsilon_0 A}{d}$$

3. P.D. between the two plates of a capacitor having charges q_1 and q_2 ,

$$V = \frac{q_1 - q_2}{2C}$$

4. Capacitance of a spherical capacitor,

$$C = 4\pi\epsilon_0 \frac{ab}{b-a}$$

Here a and b are the radii of inner and outer shells of the spherical capacitor.

5. Capacitance of a cylindrical capacitor

$$C = 2\pi\epsilon_0 \frac{L}{\log_e \frac{b}{a}} = 2\pi\epsilon_0 \frac{L}{2.303 \log_{10} \frac{b}{a}}$$

Here a and b are the radii of inner and outer coaxial cylinders and L is the length of the capacitor.

6. Grouping of Capacitors

1. In series combination, $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$
2. In parallel combination,
 $C_p = C_1 + C_2 + C_3 + \dots$
3. In series combination charge on each capacitor is same (equal to the charge supplied by battery) but potential difference across the capacitors may be different.
4. In parallel combination potential difference on each capacitor is same but the charges on the capacitors may be different.

7. Energy Stored in Capacitors

1. Energy stored in capacitor

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \cdot \frac{q^2}{C} = \frac{1}{2} qV$$

2. Energy stored per unit volume or the energy density of the electric field of a capacitor

$$u = \frac{1}{2} \epsilon_0 E^2$$

3. Electric field between capacitor plates, $E = \frac{\sigma}{\epsilon_0}$

8. Capacitors Filled with Dielectrics and Conductors

1. Capacitance of a parallel plate capacitor filled with a dielectric constant κ ,

$$C = \kappa C_0 = \frac{\epsilon_0 \kappa A}{d}$$

2. Capacitance of a parallel plate capacitor with a dielectric slab of thickness t ($< d$) in between its plates,

$$C = \frac{\epsilon_0 A}{d - t \left(1 - \frac{1}{\kappa} \right)}$$

3. Capacitance of a parallel plate capacitor with a conducting slab of thickness t ($< d$) in between its plates.

$$C = \frac{\epsilon_0 A}{d - t}$$

4. Capacitance of a spherical capacitor filled with a dielectric,

$$C = 4\pi \epsilon_0 \kappa \cdot \frac{ab}{b - a}$$

5. Capacitance of a cylindrical capacitor filled with a dielectric,

$$C = \frac{2\pi \epsilon_0 \kappa l}{2303 \log_{10} \frac{b}{a}}$$

6. Effect of dielectric with battery disconnected from the capacitor

$$Q = Q_0, V = \frac{V_0}{\kappa}, E = \frac{E_0}{\kappa}, C = \kappa C_0, U = \frac{U_0}{\kappa}$$

7. Effect of dielectric with battery connected across the capacitor

$$Q = \kappa Q_0, V = V_0, E = E_0, C = \kappa C_0, U = \kappa U_0$$

3-CURRENT ELECTRICITY

1. Definitions of Electric Current

1. Electric current = $\frac{\text{Charge}}{\text{Time}}$ or $I = \frac{q}{t}$
2. As $q = ne$, so $I = \frac{ne}{t}$
3. In case of an electron revolving in a circle of radius r with speed v , period of revolution of the electron is

$$T = \frac{2\pi r}{v}$$

Frequency of revolution, $\nu = \frac{1}{T} = \frac{v}{2\pi r}$

Current at any point of the orbit is

$I =$ charge flowing in 1 revolution

\times No. of revolutions per second

or $I = ev = \frac{ev}{2\pi r}$

2. Ohm's law, Resistance, Resistivity, Conductance, Conductivity, Current Density and Colour Code of Carbon Resistors

1. Ohm's law, $R = \frac{V}{I}$ or $V = IR$
2. Resistance of a uniform conductor, $R = \rho \frac{l}{A}$
3. Resistivity or specific resistance, $\rho = \frac{RA}{l}$
4. Conductance = $\frac{1}{R}$
5. Conductivity = $\frac{1}{\text{Resistivity}}$ or $\sigma = \frac{1}{\rho} = \frac{1}{RA}$
6. Current density = $\frac{\text{Current}}{\text{Area}}$ or $j = \frac{1}{A}$
7. Colour code of carbon resistor.

3. Drift Velocity

1. Current in terms of drift velocity (v_d) is $I = enAv_d$
2. Current density, $j = env_d$

3. No. of atoms in one gram atomic mass of an element, $N = \text{Avogadro's number} = 6.023 \times 10^{23}$
4. In terms of relaxation time τ

$$R = \frac{ml}{ne^2 \tau A} \quad \text{and} \quad \rho = \frac{m}{ne^2 \tau}$$

5. Relation between current density and electric field,

$$j = \sigma E \quad \text{or} \quad E = \rho j$$

4. Mobility of Charge Carriers

1. Mobility, $\mu = \frac{v_d}{E} = \frac{q\tau}{m}$
2. Electric current, $I = enAv_d = enA\mu E$
3. Conductivity of metallic conductor, $\sigma = ne\mu_e$
4. Conductivity of semiconductor
 $\sigma = ne\mu_e + pe\mu_h$

5. Temperature Variation of Resistance

Temperature coefficient of resistance

$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$$

if $t_1 = 0^\circ\text{C}$ and $t_2 = t^\circ\text{C}$, then

$$\alpha = \frac{R_t - R_o}{R_o \times t} \quad \text{or} \quad R_t = R_o(1 + \alpha t)$$

6. Combination of Resistances in Series and Parallel

1. The equivalent resistance R_s of a number of resistances connected in series is given by

$$R_s = R_1 + R_2 + R_3 + \dots$$

2. The equivalent resistance R_p of a number of resistances connected in parallel is given by

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

3. For two resistances in parallel

Currents through the two resistors will be

$$I_1 = \frac{R_2 I}{R_1 + R_2} \quad \text{and} \quad I_2 = \frac{R_1 I}{R_1 + R_2}$$



7. EMF, Internal Resistance, Terminal Potential Difference and Grouping of Cells

1. EMF of a cell, $E = \frac{W}{q}$
2. For a cell of internal resistance r , the emf is
 $E = V + Ir = I(R + r)$
3. Terminal p.d. when a current is being drawn from the cell.
 $V = E - Ir$
4. Terminal p.d. of a cell, $V = IR = \frac{ER}{R + r}$
5. Terminal p.d.. when the cell is being charged,
 $V = E + Ir$
6. Internal resistance of a cell, $r = R \left[\frac{E - V}{V} \right]$

8. Grouping of Cells

1. For n cells in series, $I = \frac{nE}{R + nr}$
2. For n cells in parallel, $I = \frac{nE}{nR + r}$
3. For mixed grouping, $I = \frac{mnE}{mR + nr}$
where n = no of cells in series in one row.
 m = no. of rows of cells in parallel.
4. For maximum current, the external resistance must be equal to the total internal resistance.

i.e. $\frac{nr}{m} = R$

or $nr = mR$

9. Heating Effect of Current, Electric Power and Electric Energy

1. Heat produced by electric current,

$$H = I^2 R t \text{ joule} = \frac{I^2 R t}{4.18} \text{ cal}$$

or $H = V I t \text{ joule} = \frac{V I t}{4.18} \text{ cal}$

2. Electric power, $P = \frac{W}{t} = VI = I^2 R = \frac{V^2}{R}$

3. Electric power, $W = Pt = V I t = I^2 R t$

10. Kirchhoff's Laws

1. $\sum I = 0$ (Junction rule)
or Total incoming current = Total outgoing current
2. $\sum E = \sum IR$ (Loop rule)

11.(i) Comparison of E.M.F.s of two Cells (ii) Measurement of internal Resistance of a cell by a Potentiometer

1. For comparing e.m.f.s of two cells, $\frac{E_2}{E_1} = \frac{l_2}{l_1}$
2. For measuring internal resistance of a cell,
 $r = \frac{l_1 - l_2}{l_2} \times R$
3. Potential gradient of the potentiometer wire,
 $k = \frac{V}{l}$
4. Unknown emf balanced against length l , $E = kl$

12. (i) Wheatstone Bridge (ii) Slide Wire Bridge

1. For a balanced Wheatstone bridge, $\frac{P}{Q} = \frac{R}{S}$

If X is the unknown resistance

$$\frac{P}{Q} = \frac{R}{X} \quad \text{or} \quad X = \frac{RQ}{P}$$

2. In a slide wire bridge, if balance point is obtained at l cm from zero end, then

$$\frac{P}{Q} = \frac{R}{X} = \frac{l}{100 - l} \quad \text{or} \quad X = \left(\frac{100 - l}{l} \right) R$$

4-MAGNETIC EFFECT OF CURRENT

1. Biot-Savart Law

Biot-Savart law, $dB = \frac{\mu_0}{4\pi} \frac{Idl \sin \theta}{r^2}$

2. Magnetic field due to straight Current Carrying Conductor

1. Magnetic field due to a straight conductor of finite length,

$$B = \frac{\mu_0 I}{4\pi a} (\sin \phi_1 + \sin \phi_2)$$

2. Magnetic field due to an infinitely long straight conductor

$$B = \frac{\mu_0 I}{2\pi a}$$

3. Magnetic Field due to a Circular Coil

1. Magnetic field at the centre of a circular loop

$$B = \frac{\mu_0 I}{2r}$$

2. Magnetic field at an axial of a circular loop,

$$B = \frac{\mu_0 I a^2}{2(r^2 + a^2)^{3/2}}$$

4. Ampere's Circuital law and Magnetic Field due to (i) Straight Solenoid (ii) Toroidal Solenoid

1. Ampere's circuital law, $\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

When B is directed along tangent to every point on closed curve L, $BL = \mu_0 I$

2. Magnetic field due to straight solenoid

(i) At a point well inside the solenoid, $B = \mu_0 nI$

(ii) At either end of the solenoid $B_{end} = \frac{1}{2} \mu_0 nI$

Here n is the number of turns per unit length.

3. Magnetic field inside a toroidal solenoid,

$$B = \mu_0 nI$$

Magnetic field is zero outside the toroid.

5. Force on Moving Charges in a Magnetic Field

Force on a charge q moving with velocity v in a magnetic field at an angle θ with it is

$$F = qvB \sin \theta$$

The direction of the force is given by Fleming's left hand rule.

6. Motion of Charges in Electric and Magnetic Fields

1. Electric force on a charge, $F_e = qE$

2. Magnetic force on a charge, $F_m = qvB \sin \theta$

3. In a perpendicular magnetic field, the charge follows a circular path.

$$qvB = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{qB}$$

$$T = \frac{2\pi m}{qB} \quad \text{and} \quad f = \frac{qB}{2\pi m}$$

4. When \vec{v} makes angle θ with \vec{B} , the charge follows helical path.

$$r = \frac{mv_{\perp}}{qB} = \frac{mv \sin \theta}{qB}; T = \frac{2\pi r}{v_{\perp}} = \frac{2\pi m}{qB}$$

$$\text{Pitch of helix, } h = v_{\parallel} T = \cos \theta \cdot T = \frac{2\pi m v \cos \theta}{qB}$$

5. K.E. gained by an electron when accelerated through a potential difference V,

$$\frac{1}{2} mv^2 = eV \quad v = \sqrt{\frac{2eV}{m}}$$

7. Cyclotron

For the accelerated charged particle.

1. Velocity $v = \frac{qBr}{m}$

2. Period of revolution, $T = \frac{2\pi m}{qB}$

3. Cyclotron frequency, $f_c = \frac{qB}{2\pi m}$

4. Maximum kinetic energy, $k_{\max} = \frac{q^2 B^2 R^2}{2m}$

Where R is the radius of the dees.

8. Force on a Current Carrying Conductor in a Magnetic Field

1. $\vec{F} = I(\vec{l} \times \vec{B})$ 2. $F = IlB \sin \theta$

3. $F_{\max} = IlB$

9. Forces between Parallel Current-Carrying Wires

1. Force per unit length $f = \frac{\mu_0 I_1 I_2}{2\pi r}$

2. Force on length l of one of the wires

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

10. Torque on Current Loops

Torque on a current loop in a magnetic field,

$$\tau = NIBA \sin \theta = mB \sin \theta$$

where $m = NIA$ = magnetic dipole moment of the current loop.

In vector form, $\vec{\tau} = \vec{m} \times \vec{B}$

11. Moving Coil Galvanometer and its Sensitivity

1. In a moving coil galvanometer

Current, $I = \frac{k}{NBA} \alpha$

Deflection produced, $\alpha = \frac{NBA}{k} I$

2. Fig. of merit, $G = \frac{I}{\alpha} = \frac{k}{NBA}$

3. Current sensitivity, $I_s = \frac{\alpha}{I} = \frac{NBA}{k}$

4. Voltage sensitivity, $V_s = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NBA}{kR}$

12. Conversion of Galvanometer into (i) Ammeter and (ii) Voltmeter, and Measurement of Current and Voltage

1. For conversion of a galvanometer into ammeter the shunt resistance,

$$R_s = \frac{I_g}{I - I_g} \times R_g; \text{ Here } I_g = \frac{R_s}{R_g + R_s} \times I$$

2. Resistance of an ammeter, $R_A = \frac{R_g R_s}{R_g + R_s}$

3. For conversion of a galvanometer into a voltmeter, the value of high series resistance,

$$R = \frac{V}{I_g} - R_g; \text{ Here } I_g = \frac{V}{R_g + R}$$

4. Resistance of a voltmeter, $R_V = R_g + R$

5. For a galvanometer, $I_g = nk$

where n = no of divisions on the galvanometer scale.

k = current required to produce deflection of one scale division or figure of merit of the galvanometer.

5-MAGNETISM

1. Coulomb's Law and Dipole Moment of a Magnet

1. Magnetic dipole moment, $m = q_m \times 2l$

2. Coulomb's law, $F = \frac{\mu_0}{4\pi} \cdot \frac{q_{m1} q_{m2}}{r^2}$

2. Magnetic Field of a Bar Magnet

Magnetic field of a bar magnet of length $2l$ and dipole moment m at a distance r from its centre,

1. $B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2mr}{(r^2 - l^2)^2}$ (on the axial line)

2. $B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 - l^2)^{3/2}}$ (on the equatorial line)

For a short magnet, $l < r$, so

3. $B_{axial} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$ (on the axial line)

4. $B_{equa} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$

3. Torque and Potential Energy of a Dipole, and Magnetic Moment of a Current Loop

1. Torque, $\tau = mB \sin \theta$ or $\vec{\tau} = \vec{m} \times \vec{B}$

2. Work done in turning the dipole or P.E. of a dipole

$$W = U = -mB(\cos \theta_2 - \cos \theta_1)$$

3. If initially the dipole is perpendicular to the field,

$$U = -mB \sin \cos \theta$$

(i) When \vec{m} is parallel to \vec{B} , $\theta = 0^\circ$, $U = -mB$

Potential energy of the dipole is minimum. It is in a state of stable equilibrium.

(ii) When \vec{m} is perpendicular to \vec{B} , $\theta = 90^\circ$, $U = 0$.

(iii) When \vec{m} is antiparallel

$$\vec{B}, \theta = 180^\circ, U = +mB.$$

Potential energy of the dipole is maximum. It is in a state of unstable equilibrium.

4. Magnetic moment of a current loop, $m = NIA$.

5. Orbital magnetic moment of an electron in n th orbit.

$$\mu_l = \frac{evr}{2} = \frac{e}{2m_e} l = n \left(\frac{eh}{4\pi m_e} \right)$$

6. Bohr magneton is the magnetic moment of an electron in first ($n = 1$) orbit.

$$\mu_B = (\mu_l)_{\min} = \frac{eh}{4\pi m_e}$$

4. Earth's Magnetism and Neutral Points

1. Declination (α) = Angle between geographic meridian and magnetic meridian.

2. Relations between elements of earth's magnetic field are

$$B_H = B \cos \delta$$

and $B_V = B \sin \delta$

$$\frac{B_V}{B_H} = \tan \delta$$

and $B = \sqrt{B_H^2 + B_V^2}$

3. For a magnet placed with its N-pole pointing north, neutral points lie at its equatorial line.

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{m}{(r^2 + l^2)^{3/2}} = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^3}$$

(for a short magnet)

4. For a magnet placed with its N-pole pointing south, neutral points lie on its axial line.

$$B_H = \frac{\mu_0}{4\pi} \cdot \frac{2m}{(r^2 - l^2)^{3/2}} = \frac{\mu_0}{4\pi} \cdot \frac{2m}{r^3}$$

(for a short magnet)

5. Magnetic Properties of Materials

1. Intensity of magnetization, $M = \frac{m}{V}$

$$2. \mu = \frac{B}{B_H}$$

$$3. \mu_r = \frac{\mu}{\mu_0}$$

$$4. \chi_m = \frac{M}{H}$$

$$5. \chi_m = \frac{C}{T} \quad (\text{Curie's law})$$

$$6. B = \mu_0(H + M)$$

$$7. \mu_r = 1 + \chi_m$$

6-ELECTROMAGNETIC INDUCTION

(i) Magnetic Flux (ii) Laws of Electromagnetic Induction

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \mu_0 n_1 n_2 Al$$

$$\text{where } n_1 = \frac{N_1}{l}, n_2 = \frac{N_2}{l}$$

1. Magnetic flux, $\phi = BA \cos \theta = \vec{B} \cdot \vec{A}$
2. Induced emf, $E = -N \frac{d\phi}{dt}$
3. Average induced emf, $E = -N \frac{\phi_2 - \phi_1}{t}$
4. Induced current, $I = \frac{|E|}{R}$.

2. Motional EMF

1. The emf induced in a conductor of length l moving with velocity v perpendicular to field B ,
 $E = Blv$.
2. Induced emf developed between the two ends of rod rotating at its one end in perpendicular magnetic field, $E = \frac{1}{2} Bl^2 \omega$

3. Induced EMF in a Rotating Coil

1. $E = E_0 \sin \omega t$
2. $E_0 = NBA\omega$ Where $\omega = 2\pi f$
3. Maximum induced current, $I_0 = \frac{E_0}{R}$

4. Self-induction and Mutual Induction

1. For self-induction, $\phi = LI$
2. Self induced emf, $E = -L \frac{dI}{dt}$
3. For mutual induction, $\phi = MI$
4. Mutual induced emf, $E = -M \frac{dI}{dt}$
5. Self-induced of long solenoid,

$$L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 Al, \text{ where } n = \frac{N}{l}$$

6. Mutual inductance of two closely wound solenoids,



7-ALTERNATING CURRENT AND ELECTRICAL MACHINES

1. (i) Mean (ii) Effective (iii) Instantaneous Values of Alternating Currents and Voltages

1. Instantaneous value of a.c. $I = I_0 \sin \omega t$, where I_0 is the peak or maximum value of a.c.
2. Average or mean value of a.c. over half cycle,

$$I_{av} = \frac{2}{\pi} I_0 = 0.637 I_0$$

3. Effective or rms or virtual value of a.c.

$$I_{eff} \text{ or } I_{rms} \text{ or } I_v = \frac{1}{\sqrt{2}} I_0 = 0.707 I_0$$

4. For alternating voltages, we have

$$E = E_0 \sin \omega t, E_{av} = 0.637 E_0, E_{rms} = \frac{1}{\sqrt{2}} E_0$$

2. (i) Inductive reactance (ii) Capacitive reactance

1. For an a.c. circuit containing inductor only,

- (i) Inductive reactance, $X_L = \omega L = 2\pi f L$

- (ii) Current amplitude, $I_0 = \frac{E_0}{X_L} = \frac{E_0}{\omega L}$

- (iii) Effective current, $I_{rms} = \frac{E_{rms}}{X_L} = \frac{E_{rms}}{\omega L} = \frac{E_0}{\sqrt{2} \cdot \omega L}$

2. For an a.c. circuit containing capacitor only,

- (i) Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

- (ii) Current amplitude, $I_0 = \frac{E_0}{X_C} = \frac{E_0}{1/\omega C}$

- (iii) Effective current,

$$I_{rms} = \frac{E_{rms}}{X_C} = \frac{E_{rms}}{1/\omega C} = \frac{E_0}{\sqrt{2} \cdot 1/\omega C}$$

3. Series LR-Circuit

1. Impedance,

$$Z = \frac{E_{rms}}{I_{rms}} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

$$2. \text{ Current, } I_{rms} = \frac{E_{rms}}{Z}$$

3. Phase angle ϕ is given by

$$\tan \phi = \frac{X_L}{R} = \frac{\omega L}{R} \quad \text{or} \quad \cos \phi = \frac{R}{Z}$$

4. Instantaneous current, $I = I_0 \sin(\omega t - \phi)$

4. Series CR-Circuit

1. Impedance,

$$Z = \frac{E_{rms}}{I_{rms}} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$2. \text{ Current, } I_{rms} = \frac{E_{rms}}{Z}$$

3. Phase angle ϕ is given by

$$\tan \phi = \frac{X_C}{R} = \frac{1/\omega C}{R} \quad \text{or} \quad \cos \phi = \frac{R}{Z}$$

4. Instantaneous current, $I = I_0 \sin(\omega t + \phi)$

5. Series LCR- Circuit, its Resonance and Q-factor

1. Impedance of a series LCR-circuit.

$$Z = \frac{E_{rms}}{I_{rms}} = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

2. Phase angle ϕ between current and voltage is given by

$$\tan \phi = \frac{X_L - X_C}{R} \quad \text{or} \quad \cos \phi = \frac{R}{Z}$$

3. Resonant frequency of LCR-series circuit (when $X_L = X_C$)

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$4. \text{ Q-Factor} = \frac{\omega_r}{\omega_2 - \omega_1} = \frac{\omega_r L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

where ω_1 and ω_2 are the frequencies at which current falls to $1/\sqrt{2}$ times its resonant value.

6. Energy and Power associated with A.C. Circuits



1. Average power consumed per cycle in any a.c.

$$\text{circuit, } P_{av} = E_{rms} I_{rms} \cos \phi$$

$E_{rms} I_{rms}$ is the apparent power

$$2. \text{ Power factor, } \cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}}$$

3. Average power consumed per cycle in a pure resistive circuit,

$$P_{av} = \frac{E_0^2}{2R} = E_{rms} I_{rms} = \frac{E_{rms}^2}{R}$$

$$3. \text{ Energy stored in an inductor, } U = \frac{1}{2} LI^2$$

5. Average power consumed per cycle in pure inductive circuit = 0

6. Energy stored in a capacitor,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \frac{Q^2}{C}$$

7. Average power consumed per cycle in a pure capacitive circuit = 0.

8. For an LCR in resonance

$$X_L = X_C \text{ and } f_r = \frac{1}{2\pi\sqrt{LC}}$$

7. LC-Oscillations

1. Angular frequency of free oscillations of an LC-circuit.

$$\omega = \frac{1}{\sqrt{LC}}$$

2. Frequency of free oscillations of an LC-circuit,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

3. Instantaneous charge on the capacitor,

$$q = q_0 \cos \omega t$$

4. Instantaneous current in the LC-circuit

$$I = -\frac{dq}{dt} = I_0 \sin \omega t, \text{ where } I_0 = \omega q_0$$

5. Electrical energy stored in the capacitor at any instant,

$$U_E = \frac{1}{2} \cdot \frac{q^2}{C}$$

$$U_E^{\max} = \frac{1}{2} \cdot \frac{q_0^2}{C}$$

6. Magnetic energy stored in the inductor at any instant,

$$U_B = \frac{1}{2} LI^2$$

$$U_B^{\max} = \frac{1}{2} LI_0^2$$

7. Total energy stored in the LC-circuit,

$$U = U_E + U_B = \frac{1}{2} \cdot \frac{q_0^2}{C} = \frac{1}{2} LI_0^2$$

8. Transformers and Long Distance Power

Transmission

1. The voltages and currents in a transformer are related as

$$\frac{E_2}{E_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = k \text{ u}$$

where suffix 1 refers 1 to primary coil, 2 to secondary coil and k is the transformation or turns ratio.

2. $E_1 I_1$ (Power in primary coil)

$$= E_2 I_2 \text{ (Power in secondary coil)}$$

3. Efficiency of a transformer

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100\%$$

4. Power is transmitted from power stations to sub-stations at very high voltages to reduce cost reduce losses.

9. Generators

For an a.c. generator,

1. Flux linked, $\phi = NBA \cos \omega t$

2. Instantaneous induced emf, $E = E_0 \sin \omega t$

3. Maximum induced emf, $E_0 = NBA \omega$

4. Instantaneous current, $I = I_0 \sin \omega t$

5. Maximum current, $I_0 = \frac{E_0}{R} = \frac{NBA \omega}{R}$

8-ELECTROMAGNETIC WAVES

1. Displacement, Current and Modified

Ampere's Circuital law

1. Displacement current, $I_D = \epsilon_0 \frac{d\phi_E}{dt}$

$$\begin{aligned} \text{Also } I_D &= \epsilon_0 \frac{d}{dt}(EA) = \epsilon_0 A \frac{dE}{dt} \\ &= \epsilon_0 \frac{d}{dt}\left(\frac{V}{d}\right) = \frac{\epsilon_0 A}{d} \frac{dV}{dt} = C \frac{dV}{dt} \end{aligned}$$

2. Modified Ampere's circuital law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_C + I_D)$$

2. Equation, Speed, Amplitude and Average Density of an Electromagnetic Wave

1. Wave velocity, $c = v\lambda$

2. Energy of photon = $E = h\nu = \frac{hc}{\lambda}$

3. Speed of e.m. wave in vacuum, $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

4. Speed of e.m. wave in a material medium,

$$c = \frac{1}{\sqrt{\mu\epsilon}}$$

5. For a wave of frequency ν , wavelength λ , propagating along x -direction, the equations for electric and magnetic fields are

$$E_y = E_0 \sin(kx - \omega t) = E_0 \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

$$B_z = B_0 \sin(kx - \omega t) = B_0 \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

6. Amplitude ratio of electric and magnetic fields,

$$\frac{E_0}{B_0} = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

7. Propagating constant, $k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$

8. Average energy density of B-field,

$$u_B = \frac{1}{4\mu_0} B_0^2 = \frac{1}{2\mu_0} B_{rms}^2$$

9. Average energy density of E-field,

$$u_E = \frac{1}{4}\epsilon_0 E_0^2 = \frac{1}{2}\epsilon_0 E_{rms}^2$$

10. Average energy density of e.m. wave,

$$u_{av} = \frac{1}{2}\epsilon_0 E_{rms}^2 + \frac{1}{2\mu_0} B_{rms}^2 = \epsilon_0 E_{rms}^2 = \frac{B_{rms}^2}{\mu_0}$$

$$\text{or } u_{av} = \frac{1}{4}\epsilon_0 E_0^2 + \frac{1}{4\mu_0} B_0^2 = \frac{1}{2}\epsilon_0 E_0^2 = \frac{1}{2\mu_0} B_0^2$$

11. Momentum delivered by an e.m. wave.

$$p = \frac{U}{c}$$

12. Intensity of a wave = $\frac{\text{Energy / time}}{\text{Area}} = \frac{\text{Power}}{\text{Area}}$

$$\text{or } I = u_{av} c = \epsilon_0 E_{rms}^2 c.$$

9-RAY OPTICS AND OPTICAL INSTRUMENT

1.FORMATION OF IMAGE BY SPHERICAL MIRRORS

1. For any spherical mirror, $f = R/2$
2. Mirror formula, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{R}$
3. Magnification, $m = \frac{h_2}{h_1} = -\frac{v}{u} = \frac{f}{f-u} = \frac{f-u}{f}$
4. Magnification m is -ve for real images and +ve for virtual images.
5. f and R are -ve for a concave mirror and +ve for a convex mirror.
6. For a real object u is -ve, v is -ve for real image and +ve for virtual image.
7. Do not give any sign to unknown quantity. The sign will automatically appear in the final result.

2. Refraction of Light (ii) Lateral shift and (iii) Real and Apparent Depths

1. Refractive index = $\frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$

or $\mu = \frac{c}{v}$

2. $\mu = \frac{\text{Wavelength in vacuum}}{\text{Wavelength in medium}} = \frac{\lambda}{\lambda'}$

3. Snell's law, ${}^1\mu_2 = \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$

or $\mu_1 \sin i = \mu_2 \sin r$

4. ${}^1\mu_2 = \frac{1}{{}^2\mu_1}$

5. ${}^1\mu_2 \times {}^2\mu_3 \times {}^3\mu_1 = 1$ or ${}^2\mu_3 = \frac{{}^1\mu_3}{{}^1\mu_2}$

6. Lateral shift of a ray through a rectangular slab,

$$x = \frac{t}{\cos r} \sin(i - r)$$
$$= t \sin i \left[1 - \frac{\cos i}{(\mu^2 - \sin^2 i)^{1/2}} \right]$$

7. $\mu = \frac{\text{Real Depth}}{\text{Apparent depth}} = \frac{1}{\text{Apparent depth}}$

Apparent depth = $\frac{t}{\mu}$

8. Apparent shift = $t \left(1 - \frac{1}{\mu} \right)$

9. Total apparent shift for compound media

$$= t_1 \left(1 - \frac{1}{\mu_1} \right) + t_2 \left(1 - \frac{1}{\mu_2} \right)$$

3. Total Internal Reflection

1. Critical angle, i_c = Angle of incidence in denser medium for which angle of refraction is 90° in rarer medium.
2. Refractive index of denser medium, $\mu = \frac{1}{\sin i_c}$
3. Total internal refraction occurs when $i > i_c$

4. Refraction through Spherical Surfaces

1. For refraction from rarer to denser medium,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

2. For refraction from denser to rarer medium,

$$\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$$

3. Power of a surface, $P = \frac{\mu_2 - \mu_1}{R} = \frac{\mu - 1}{R}$ (for air)

4. First principal focal length $f_1 = -\frac{\mu_1 R}{\mu_2 - \mu_1}$

5. Second principal focal length $f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1}$

$$\frac{f_2}{f_1} = -\frac{\mu_2}{\mu_1}$$

5. Len's Maker's formula

1. For the lens of material of refractive index μ_2 placed in a medium of refractive index μ_1

$$\frac{1}{f} = \left(\frac{\mu_2 - \mu_1}{\mu_1} \right) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

2. When the lens is placed in air,

$$\mu_1 = 1 \text{ and } \mu_2 = \mu$$

$$\frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

3. f and R are positive for convex surfaces and negative for concave surfaces.

6. Thin Lens Formula and Linear

Magnification

1. Focal length of any lens is given by the thin lens formula

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

2. Magnification, $m = \frac{h_2}{h_1} = \frac{v}{u} = \frac{f}{f+u} = \frac{f-v}{f}$

3. In Cartesian sign convention, u is taken negative

4. In case of convex lens, v is positive for real image and negative for virtual image and f is positive.

5. In case of concave lens u , v and f are all negative.

6. Magnification m is positive for virtual image and negative for real image.

7. (i) Power of Lenses (ii) Combination of Lenses

1. Power of a lens, $P = \frac{1}{f(m)} = \frac{100}{f(cm)}$

$$2. P = \frac{1}{f} = (\mu - 1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right]$$

3. For a combination of lenses

$$m = m_1 \times m_2 \times m_3 \times \dots$$

4. For two lenses in contact, equivalent focal length F is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \text{ or Power } P = P_1 + P_2$$

For n lenses in contact, $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \dots + \frac{1}{f_n}$

5. The equivalent focal length F two lenses separated by distance d is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

or Power, $P = P_1 + P_2 - d.P_1.P_2$

8. Image formation by a combination of a lens and a mirror

We first find the position of the image formed by the lens by using thin lens formula. Taking this image as real (or virtual) object for the mirror, we use mirror formula to locate the position of the final image formed by the combination.

9. Refraction and Dispersion of Light through a prism

1. For refraction through a prism.

$$A + \delta = i + i' \text{ and } r + r' = A$$

2. In the condition of minimum deviation,

$$i = i', r = r', \delta = \delta_m; \mu = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}}$$

3. Deviation produced by a prism of small angle.

$$\delta = (\mu - 1)A$$

4. Angular dispersion $= \delta_V - \delta_R = (\mu_V - \mu_R)A$

5. Dispersive power, $\omega = \frac{\delta_V - \delta_R}{\delta} = \frac{\mu_V - \mu_R}{\mu - 1}$

6. Mean deviation, $\delta = \frac{\delta_V + \delta_R}{2}$

7. Mean refractive index, $\mu = \frac{\mu_V + \mu_R}{2}$

10. Defects of Vision

1. Correction of myopia or short sightedness. A concave lens of focal length f equal to the distance x of the far point from the defective eye is used.

$$f = -x \text{ and } P = -\frac{1}{x}$$

2. Correction of hypermetropia or long sightedness. A convex lens of focal length f is

used, where

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Here

x = distance of the near point from the defective eye and

D = the least distance of distinct vision.

11. Simple Microscope

1. When the final image is formed at the least distance of distinct vision, the magnifying power

is $m = 1 + \frac{D}{f}$

2. When the final image is formed at infinity, the magnifying power is $m = \frac{D}{f}$

12. Compound Microscope

1. Magnifying power, $m = m_o \times m_e$

2. When the final image is formed at the least distance of distinct vision,

$$m = \frac{v_o}{u_o} \left(1 + \frac{D}{f_e} \right) = -\frac{L}{f_o} \left(1 + \frac{D}{f_o} \right)$$

3. When the final image is formed at infinity

$$m = \frac{v_o}{u_o} \cdot \frac{D}{f_e} = -\frac{L}{f_o} \cdot \frac{D}{f_e}$$

13. Telescopes

1. *Astronomical telescope* (i) In normal adjustment

$$m = \frac{f_o}{f_e}$$

Distance between objective and eyepiece = $f_o + f_e$

(ii) When the final image is formed at the least

distance of distinct vision, $m = \frac{f_o}{f_e} \left(1 + \frac{f_e}{D} \right)$

Distance between objective and eyepiece

$$= f_o + u_e = f_o = \frac{f_e D}{f_e + D}$$

2. *Terrestrial telescope*. (i) In normal adjustment,

$$m = \frac{f_o}{f_e}$$

Distance between objective and eyepiece

$$f_o + 4f + f_e$$

where f = focal length of the erecting lens.

3. *Galileo's telescope*. In normal adjustment,

$$m = \frac{f_o}{f_e}$$

Distance between objective and eyepiece $f_o - f_e$

4. *Reflecting telescope*. $m = \frac{f_o}{f_e} = \frac{R/2}{f_e}$

Where f_o = focal length of concave mirror.

f_e = focal length of eyepiece.

10-WAVE OPTICS

1. Reflection and Refraction of Light Waves

1. Snell's law, ${}^1\mu_2 = \frac{\sin i}{\sin r}$
2. $\mu = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{Speed of light in medium}}$
3. Speed of light in vacuum, $c = v\lambda$
4. $\mu = \frac{\lambda}{\lambda'} = \frac{\text{wavelength in vacuum}}{\text{Wavelength in medium}}$
5. Wavelength in medium, $\lambda' = \frac{\lambda}{\mu}$
6. Optical path (in vacuum) = $\mu \times$ Path in medium
7. Frequency of light remains unchanged during its reflection or refraction

2. Amplitude and Intensity at any point in an Interference Pattern

1. Resultant amplitude, $a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos \phi}$
2. Resultant intensity, $I = I_1 + I_2 + 2\sqrt{I_1I_2} \cos \phi$
3. When
 $I_1 = I_2 = I_0, I = 2I_0(1 + \cos \phi) = 4I_0 \cos^2 \frac{\phi}{2}$

3. Young's Double Slit Experiment

1. For the bright fringe, path difference, $p = n\lambda$
2. For a dark fringe, $p = (2n-1)\frac{\lambda}{2}, n = 1, 2, 3$
3. Distance of n th bright fringe from the centre of the screen,

$$x_n = n \frac{D\lambda}{d}, \quad n = 1, 2, 3, \dots$$

4. Distance of n th dark fringe from the centre of the screen,

$$x'_n = (2n-1) \frac{D\lambda}{2d}$$

5. Fringe width, $\beta = \frac{D\lambda}{d}$

6. Wavelength of light used, $\lambda = \frac{\beta d}{D}$

7. Angular position of n th bright fringe,

$$\theta'_n = \frac{x_n}{D} = \frac{n\lambda}{d}$$

8. Angular position of n th dark fringe

$$\theta'_n = \frac{x'_n}{D} = (2n-1) \frac{\lambda}{2d}$$

4. Intensity Ratio at Maxima and Minima of an Interference Pattern

1. Intensity of light \propto Width of slit
2. Ratio of slit widths, $\frac{w_1}{w_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$
3. Intensity at maxima, $I_{\max} \propto (a_1 + a_2)^2$
4. Intensity at minima, $I_{\min} \propto (a_1 - a_2)^2$
5. Intensity ratio at maxima and minima

$$\frac{I_{\max}}{I_{\min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \left(\frac{r+1}{r-1} \right)^2$$

where $r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}}$ = amplitude ratio of two waves.

5. Interference in Thin Film

1. For reflected system of light,
 - (i) Maxima: $2\mu t \cos r = (2n+1) \frac{\lambda}{2}$
 - (ii) Minima: $2\mu t \cos r = n\lambda$
2. For transmitted system of light,
 - (i) Maxima: $2\mu t \cos r = n\lambda$
 - (ii) Minima: $2\mu t \cos r = (2n+1) \frac{\lambda}{2}$

Where $n = 0, 1, 2, 3, \dots$

6. Displacement of Interference Fringes

1. When a thin transparent sheet of thickness t and refractive index μ is inserted in one of the interfering beams, path difference introduced,
 $p = (\mu - 1)t$

2. Displacement of the central bright fringe,

$$\Delta x = \frac{\beta}{\lambda}(\mu - 1)t = \frac{D}{d}(\mu - 1)t.$$

7. Diffraction of Light and Fresnel's distance

1. For diffraction at a single slit of width d .

(i) Condition for n th minimum is

$$d \sin \theta = n\lambda \quad \text{where } n = 1, 2, 3, \dots$$

(ii) Condition of n th secondary maximum is

$$d \sin \theta = (2n + 1) \frac{\lambda}{2}, \quad \text{where } n = 1, 2, 3, \dots$$

(iii) Angular position or direction of n th minimum

$$\theta_n = \frac{n\lambda}{d}$$

(iv) Distance of n th minimum from the centre of the screen.

$$x_n = \frac{nD\lambda}{d}$$

(v) Angular position of n th secondary maximum,

$$\theta'_n = (2n + 1) \frac{\lambda}{2d}$$

(vi) Distance of n th secondary maximum from the centre of the screen.

$$x'_n = (2n + 1) \frac{D\lambda}{2d}$$

(vii) Width of central maximum, $\beta_0 = 2\beta = \frac{2D\lambda}{d}$

(viii) Angular spread of central maximum on

either side, $\theta = \pm \frac{\lambda}{d}$

(ix) Total angular spread of central maximum,

$$2\theta = \frac{2\lambda}{d}$$

2. For diffraction at a circular aperture of diameter d ,

(i) Angular spread of central maximum,

$$\theta = \frac{1.22\lambda}{d}$$

(ii) Linear spread, $x = D\theta$

(iii) Areal spread, $x^2 = (D\theta)^2$

where D is the distance at which the effect is

considered.

3. Fresnel distance, $D_F = \frac{d^2}{\lambda}$

4. Size of Fresnel zone, $d_f = \sqrt{\lambda D}$

8. Resolving Power of (i) Telescope (ii) Microscope

1. Limit of resolution of a telescope, $d\theta = \frac{1.22\lambda}{D}$

2. Resolving power of a telescope = $\frac{1}{d\theta} = \frac{D}{1.22\lambda}$

where D = diameter of the objective lens.

3. Limit of resolution of a microscope,

$$d = \frac{\lambda}{2\mu \sin \theta}$$

4. Resolving power of a microscope =

$$\frac{1}{d} = \frac{2\mu \sin \theta}{\lambda}$$

where θ = half angle of cone of light from the point object. The factor $\mu \sin \theta$ is called numerical aperture (N.A.).

9. The law of Malus

Law of Malus, $I = I \cos^2 \theta$

10. Brewster Law

1. Brewster law, $\mu = \tan i_p$

2. $i_p + r_p = 90^\circ$

3. ${}^2\mu_3 = \frac{{}^1\mu_2}{{}^1\mu_3}$

11. Doppler Effect of Light

1. $\frac{\Delta v}{v} = \frac{v' - v}{v} = \pm \frac{v}{c}$

2. $\frac{\Delta \lambda}{\lambda} = \frac{\lambda' - \lambda}{\lambda} = \mp \frac{v}{c}$

11-DUAL NATURE OF RADIATION AND MATTER

1. Photons and photoelectric effect

1. Energy of a photon, $E = hv = \frac{hc}{\lambda}$

2. Number of photons emitted per second $N = \frac{P}{E}$

3. Momentum of photon, $p = mc = \frac{hv}{c} = \frac{h}{\lambda}$

4. Equivalent mass of a photon, $m = \frac{hv}{c^2}$

5. Work function, $W_0 = hv_0 = \frac{hc}{\lambda_0}$

6. Kinetic energy of photoelectrons is given by Einstein's photoelectron equation,

$$K_{\max} = \frac{1}{2}mv_{\max}^2 = hv - W_0 = h(v - v_0) = h\left[\frac{c}{\lambda} - \frac{c}{\lambda_0}\right]$$

7. If V_0 is the stopping potential, the maximum kinetic energy of the ejected photo electrons,

$$K = \frac{1}{2}mv_{\max}^2 = eV_0$$

8. Intensity of radiation = $\frac{\text{Energy}}{\text{Area} \times \text{time}} = \frac{\text{Power}}{\text{Area}}$

Incident power = Incident intensity x area

2. de-Broglie Waves

1. Kinetic energy, $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

Momentum, $p = \sqrt{2mK}$

2. De-Broglie wavelength of an electron beam accelerated through a potential difference of V volts is

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{1.23}{\sqrt{V}} \text{ nm}$$

4. Bragg's equation for crystal diffraction is $2d \sin \theta = n\lambda$, n is order of the spectrum.



12-ATOMS

1. Distance of Closest Approach and Impact

Parameter

1. K.E. of α -particles, $K = \frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{r_0}$

2. Distance of closest approach

$$r_0 = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{K} = \frac{1}{4\pi\epsilon_0} \cdot \frac{4Ze^2}{mv^2}$$

3. Impact parameter

$$b = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2 \cot \frac{\theta}{2}}{K} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\cot \frac{\theta}{2}}{\frac{1}{2}mv^2}$$

2. Bohr's Theory of Hydrogen Atom

1. $\frac{kZe^2}{r^2} = \frac{mv^2}{r}$

2. $L = mvr = \frac{nh}{2\pi}$

3. $h\nu = E_{n_2} - E_{n_1}$

4. $r_n = \frac{n^2 h^2}{4\pi^2 m k Z e^2}$

5. $v_n = \frac{2\pi k e^2}{nh} = \alpha \cdot \frac{C}{n}$ where $\alpha = \frac{2\pi k e^2}{ch}$ is fine

structure constant

6. $K.E. = \frac{kZe^2}{2r}$

7. $P.E. = \frac{-kZe^2}{r}$

8. Total energy,

$$E_n = -\frac{2\pi^2 m k^2 Z^2 e^4}{n^2 h^2} = -\frac{Rhc}{n^2} = \frac{-13.6}{n^2} eV$$

9. $E_n = -K.E.$ or $K.E. = -E_n$; $P.E. = -2 K.E. =$

$2 E_n$

10. Frequency, $\nu = \frac{2\pi^2 m k^2 e^4}{h^3} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$

11. Wave number, $\bar{\nu} = \frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$

Where $R = \frac{2\pi^2 m k^2 e^4}{ch^3}$, is Rydberg's constant

12. Ionization potential = $-\frac{13.6Z^2}{n^2}$ volt

13. $T_n = \frac{2\pi r_n}{v_n} = \frac{n^3 h^3}{4\pi^2 m k^2 Z e^4} = T_1 n^3$



11-NUCLEI

1. Equivalent Energy, Atomic Mass, Nuclear Size and Nuclear Density

1. Einstein's mass-energy equivalence, $E = mc^2$

2. $1 \text{ amu} = \frac{1}{12} \times \text{Mass of C-12 atom}$

3. Nuclear radius, $R = R_0 A^{1/3}$

where $R_0 = 1.2 \times 10^{-15} \text{ m}$

4. $\rho_{nu} = \frac{\text{Nuclear mass}}{\text{Nuclear volume}} = \frac{m_{nu}}{\frac{4}{3}\pi R^3}$

5. Average atomic mass of an element
= Weighted average of the masses of all isotopes

$$4. N = N_0 \left(\frac{1}{2} \right)^n, \text{ where } n = \frac{t}{T_{1/2}}$$

$$5. \text{Mean life: } \tau = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44 T_{1/2}$$

$$\text{or } T_{1/2} = 0.693 \tau$$

6. Decay rate or activity of a substance:

$$R = \left| \frac{dN}{dt} \right| = \lambda N = \lambda N_0 e^{-\lambda t}$$

7. Time required to reduce the radioactive substance

$$t = \frac{2.303}{\lambda} \log \frac{N_0}{N}$$

$$8. \text{Decay constant: } \lambda = \frac{2.303}{t} \log \frac{N_0}{N}$$

2. Binding Energy of a Nucleus

1. Mass defect, $\Delta m = [Zm_p + (A - Z)m_n - m_N]$

2. B.E. = $(\Delta m)c^2$

3. B.E./nucleon = $\frac{B.E.}{A}$

4. Packing fraction = $\frac{\Delta m}{A}$.

3. Radioactivity

1. Displacement laws for radioactive transformation are as follows:

(i) α - decay: ${}_Z^A X \rightarrow {}_{Z-2}^{A-4} Y + {}_2^4 \text{He}$

(ii) β - decay: ${}_Z^A X \rightarrow {}_{Z+1}^A Y + {}_{-1}^0 e + \bar{\nu}$

(iii) γ - decay: ${}_Z^A X \rightarrow {}_Z^A X + \gamma$

(Excited state) (Ground state)

2. Radioactive decay law

$$(i) -\frac{dN}{dt} = \lambda N \quad (ii) N = N_0 e^{-\lambda t}$$

where λ = decay constant or disintegration constant

$$3. \text{Half life } T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$$



14-SEMICONDUCTOR DEVICES AND DIGITAL CIRCUITS

1. Intrinsic and Extrinsic Semiconductor

1. In an intrinsic semiconductor, $n_e = n_h = n_i$
2. At equilibrium in any semiconductor,
 $n_e n_h = n_i^2$
3. In an n -type semiconductor, $N_D = n_e \gg n_h$
4. In a p -type semiconductor, $N_A = n_h \gg n_e$
5. Minimum energy required to create a hole-electron pair,

$$E_g = h\nu_{\min} = \frac{hc}{\lambda_{\max}}$$

2. Conductivity of Semiconductors

1. Mobility of a charge carrier, $\mu = \frac{v}{E}$
2. Electric current, $I = eA(n_e v_e + n_h v_h)$.
3. Electrical conductivity, $\sigma = \frac{1}{\rho} = e(n_e \mu_e + n_h \mu_h)$
4. Variation of conductivity with temperature.

$$\sigma = \sigma_0 \exp\left(-\frac{E_g}{2k_B T}\right)$$

3. p-n Junction

1. The d.c. resistance of a junction diode, $r_{dc} = \frac{V}{I}$
2. The dynamic or a.c. resistance of a junction diode,

$$r_d \quad \text{or} \quad r_{ac} = \frac{\Delta V}{\Delta I}$$

3. Voltage equation for a diode circuit.

Applied voltage, $V = V_d + IR$

4. For a zener diode, $V_Z = V_i - RI$

Series resistor, $R = \frac{V_i - V_Z}{I}$

5. Average value of d.c. obtained from a half-

wave rectifier, $I_{dc} = \frac{I_0}{\pi}$

6. Average value of d.c. obtained from a full wave rectifier $I_{dc} = \frac{2I_0}{\pi}$

15-COMMUNICATION SYSTEMS

1. Amplitude Modulation

Modulation factor, $\mu = \frac{A_m}{A_c}$ or $\mu = \frac{A_m}{A_c} \times 100\%$

2. If A_{\max} and A_{\min} are the maximum and minimum amplitudes of the carrier wave, then

$$\mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} \times 100\%$$

3. Modulating voltage, $m(t) = A_m \sin \omega_m t$

4. Carrier voltage, $c(t) = A_c \sin \omega_c t$

5. Instantaneous voltage of A.M. wave is

$$\begin{aligned} c_m(t) &= A_c(1 + \mu \sin \omega_m t) \sin \omega_c t \\ &= A_c \sin \omega_c t - \frac{\mu A_c}{2} \cos(\omega_c + \omega_m)t \\ &\quad + \frac{\mu A_c}{2} \cos(\omega_c - \omega_m)t \end{aligned}$$

6. Component of frequencies of A.M. wave are:

(a) Carrier frequency = f_c

(b) $USB = f_c + f_m$ (c) $LSB = f_c - f_m$

7. Bandwidth = $(f_c + f_m) - (f_c - f_m) = 2f_m$

2. Length of Antenna and Demodulation Process

Length of a dipole antenna, $l = \frac{\lambda}{2} = \frac{c}{2v}$

2. Condition for satisfactory detection by a diode

$$\frac{1}{f_e} < RC < \frac{1}{f_m}$$

3. Bandwidth of Communication channels

Number of channels =

$$\frac{\text{Total_bandwidth_of_channel}}{\text{Bandwidth_needed_per_channel}}$$

4. Propagation of Radiowaves and Range of TV Transmission

1. Critical frequency for sky wave propagation,

$$f_c = 9(N_{\max})^{1/2}$$

Where N_{\max} is the maximum electron density in the ionosphere. Frequencies below f_c are reflected back to earth.

2. Ground wave propagation is limited to 1500 kHz.

3. Frequencies above 20 MHz are carried through satellite communication.

4. Wave of UHF/VHF regions can propagate through space wave or tropospheric wave.

5. The range of TV transmission.

$$d = \sqrt{2hR}$$

Where h = height of antenna and

R = radius of earth.

6. Population covered = Population density x area

7. Maximum distance covered in LOS communication,

$$d_M = d_T + d_R = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$