ELECTROSTATICS

1. ELECTRIC CHARGE

1.1 Definition

Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects.

1.2 Type

There exists two types of charges in nature

(i) Positive charge

(ii) Negative charge

Charges with the same electrical sign repel each other, and charges with opposite electrical sign attract each other.

1.3 Unit and dimensional formula

S.I. unit of charge is *coulomb* (*C*),

$\left(1\text{mC}=10^{-3}\text{C},\ 1\mu\text{C}=10^{-6}\text{C},\ 1\text{nC}=10^{-9}\text{C}\right)$.

C.G.S. unit of charge is *e.s.u.* $1\text{C}=3\times10^{9}$ esu

Dimensional formula [Q] = [AT].

1.4 Point Charge

Whose spatial size is negligible as compared to other distances.

1.5 Properties of charge

(i) Charge is a Scalar Quantity : Charges can be added or subtracted algebrically.

(ii) Charge is transferable : If a charged body is put in contact with an uncharged body, uncharged body becomes charged due to transfer of electrons from one body to the other.

(iii) Charge is always associated with mass, *i.e.*, charge can not exist without mass though mass can exist without charge.

(iv) Charge is conserved : Charge can neither be created nor be destroyed.

(v) Invariance of charge : The numerical value of an elementary charge is independent of velocity.

(vi) Charge produces electric field and magnetic field : A charged particle at rest produces only electric field in the space surrounding it. However, if the charged particle is in unaccelerated motion it produces both electric and magnetic fields. And if the motion of charged particle is accelerated it not only produces electric and magnetic fields but also radiates energy in the space surrounding the charge in the form of electromagnetic waves.

(vii) Charge resides on the surface of conductor : Charge resides on the outer surface of a conductor because like charges repel and try to get as far away as possible from one another and stay at the farthest distance from each other which is outer surface of the conductor. This is why a solid and hollow conducting sphere of same outer radius will hold maximum equal charge and a **soap bubble expands on charging.**

(viii) Quantization of charge : When a physical quantity can have only discrete values rather than any value, the quantity is said to be quantised. The smallest charge that can exist in nature is the charge of an electron. If the charge of an electron $\left(-1.6\times10^{-19}\text{C}\right)$ is taken as elementary unit *i.e.* quanta of charge the charge on any body will be some integral multiple of *e* *i.e.*, $Q=\pm ne$ with $n=0, 1, 2, 3......$

Charge on a body can never be 0.5 e, ±17.2 e or $\pm10^{-5}$ e *etc.*

1.6 Comparison of Charge and Mass

We are familiar with role of mass in gravitation, and we have just studied some features of electric charge. We can compare the two as shown below :

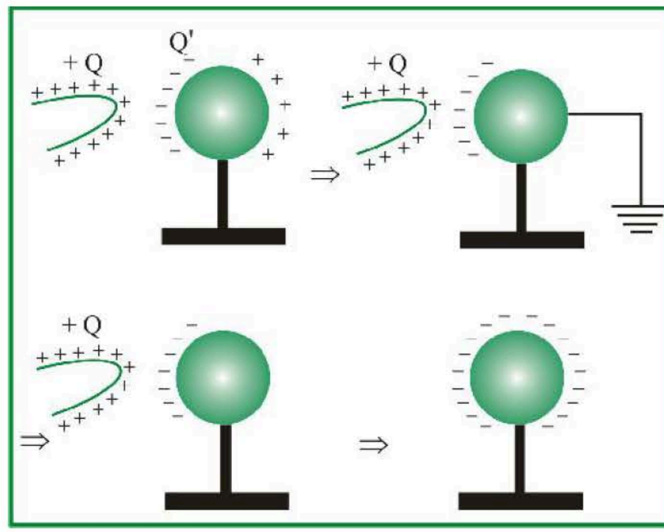
Charge	Mass
1. Electric charge can be positive, negative or zero.	1. Mass of a body is a positive quantity.
2. Charge carried by a body does not depend upon velocity of the body.	2. Mass of a body increases with its velocity as $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$ where c is velocity of light in vaccum, m is the mass of the velocity v and m_0 is rest mass of the body.
3. Charge is quantized.	3. The quantization of mass is yet to be established.
4. Electric charge is always conserved.	4. Mass is not conserved as it can be changed into energy and vice–versa.
5. Force between charges can be attractive or repulsive, according as charges are unlike or like charges.	5. The gravitational force between two masses is always attractive.

1.7 Methods of Charging

A body can be charged by following methods :

(i) **By friction :** In friction when two bodies are rubbed together, electrons are transferred from one body to the other. As a result of this one body becomes positively charged while the other negatively charged, e.g., when a glass rod is rubbed with silk, the rod becomes positively charged while the silk becomes negatively charged. However, ebonite on rubbing with wool becomes negatively charged making the wool positively charged. Clouds also become charged by friction. In charging by friction in accordance with conservation of charge, **both positive and negative charges in equal amounts appear simultaneously due to transfer of electrons from one body to the other.**

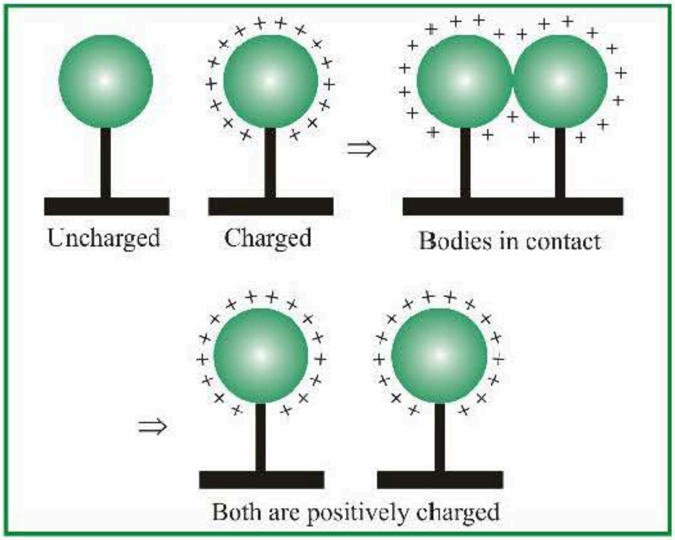
(ii) **By electrostatic induction :** If a charged body is brought near an uncharged body, the charged body will attract opposite charge and repel similar charge present in the uncharged body. As a result of this one side of neutral body (closer to charged body) becomes oppositely charged while the other is similarly charged. This process is called electrostatic induction.



Note..

Inducting body neither gains nor loses charge.

(iii) **Charging by conduction :** Take two conductors, one charged and other uncharged. Bring the conductors in contact with each other. The charge (whether –ve or +ve) under its own repulsion will spread over both the conductors. Thus the conductors will be charged with the same sign. This is called as charging by conduction (through contact).



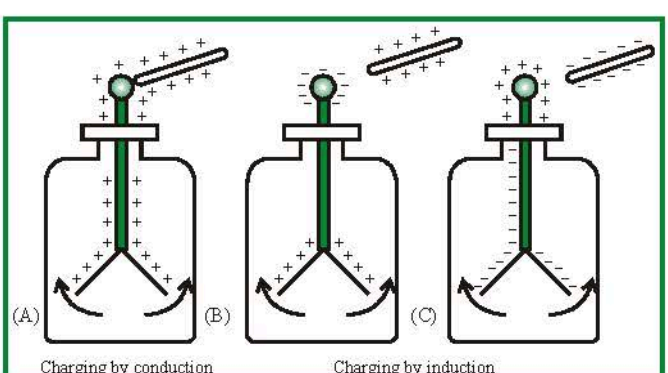
Note..

A truck carrying explosives has a metal chain touching the ground, to conduct away the charge produced by friction.

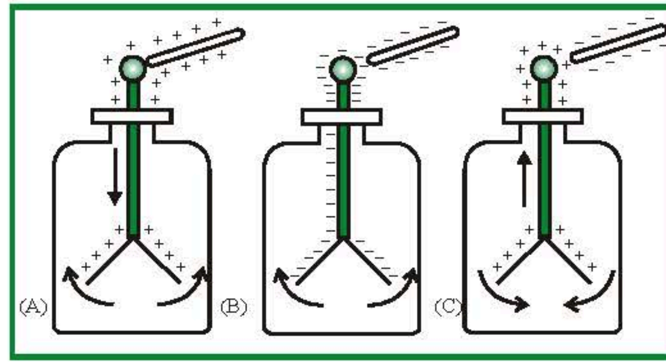
1.8 Electroscope

It is a simple apparatus with which the presence of electric charge on a body is detected (see figure). When metal knob is touched with a charged body, some charge is transferred to the gold leaves, which then diverges due to repulsion. The separation gives a rough idea of the amount of charge on the body. If a charged body brought near a charged electroscope the leaves will also diverge. If the charge on body is similar to that on electroscope and will usually converge if opposite. If the induction effect is strong enough leaves after converging may again diverge.

(1) Uncharged electroscope



(2) Charged electroscope



2. COULOMB'S LAW

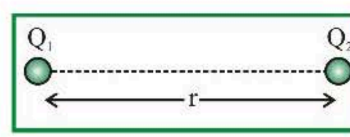
If two stationary and point charges Q_1 and Q_2 are kept at a distance r, then it is found that force of attraction or repulsion between them is Mathematically, Coulomb's law can be written as

$$F = k\frac{q_1 q_2}{r^2}$$

where k is a proportionality constant.

In SI units k has the value, $k = 8.988 \times 10^9\ \text{N m}^2/\text{C}^2$

$\simeq 9.0 \times 10^9\ \text{N m}^2/\text{C}^2$



(a) The direction of force is always along the line joining the two charges.

(b) The force is repulsive if the charges have the same sign and attractive if their signs are opposite.

(c) This force is conservative in nature.

(d) This is also called inverse square law.

2.1 Variation of *k*

Constant *k* depends upon system of units and medium between the two charges.

2.1.1 Effect of units

(a) In C.G.S. for air $k = 1$, $F = \frac{Q_1 Q_2}{r^2}$ *Dyne*

(b) In S.I. for air $k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 \frac{N-m^2}{C^2}$,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1 Q_2}{r^2} \text{ Newton } (1 \text{ Newton} = 10^5 \text{ Dyne})$$

Note..

- ε_0 = Absolute permittivity of air or free space

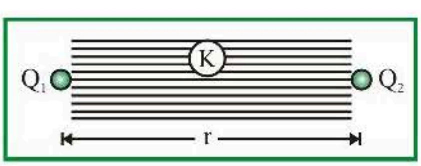
$$= 8.85\times10^{-12} \frac{C^2}{N-m^2}\left(= \frac{\text{Farad}}{m}\right).$$

Dimension is $\left[M^{-1}L^{-3}T^4A^2\right]$

- ε_0 Relates with absolute magnetic permeability (μ_0) and velocity of light (c) according to the following relation $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

2.1.2 Effect of medium

(a) When a dielectric medium is completely filled in between charges rearrangement of the charges inside the dielectric medium takes place and the force between the same two charges decreases by a factor of K known as **dielectric constant**, K is also called relative permittivity ε_r of the medium (relative means with respect to free space).



Hence in the presence of medium

$$F_m = \frac{F_{air}}{K} = \frac{1}{4\pi\varepsilon_0 K} \cdot \frac{Q_1 Q_2}{r^2}$$

Here $\varepsilon_0 K = \varepsilon_0 \varepsilon_r = \varepsilon$ (permittivity of medium)

Medium	K
Vacuum / air	1
Water	80
Mica	6
Glass	5–10
Metal	∞

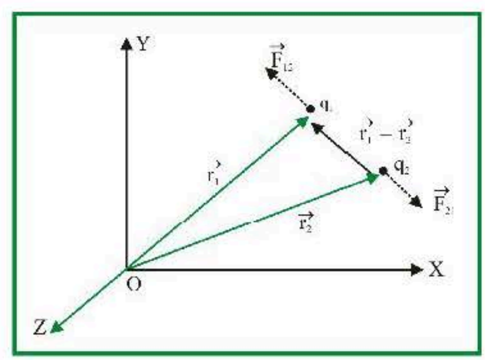
2.2 Vector form of coulomb's law

It is helpful to adopt a convention for subscript notation.

F_{12} = force on 1 due to 2 $\quad$ F_{21} = force on 2 due to 1

Suppose the position vectors of two charges q_1 and q_2 are $\vec{r}_1$ and $\vec{r}_2$, then, electric force on charge q_1 due to charge q_2 is,

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

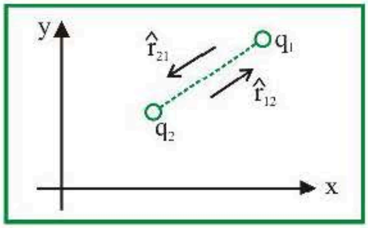


Similarly, electric force on q_2 due to charge q_1 is

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1)$$

Force is a vector, so in vector form the Coulomb's law is written as

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{12}$$



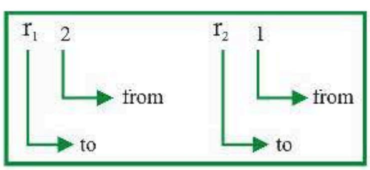
where $\hat{r}_{12}$ is a unit vector directed toward q_1 from q_2.

Note..

$$\hat{r}_{12} = -\hat{r}_{21}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_1}{r^2}(-\hat{r}_{21})$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{q_2 q_1}{r^2}\hat{r}_{21} = -\vec{F}_{21}$$



Remember convention for $\hat{r}$.

Here q_1 and q_2 are to be substituted with sign. Position vector of charges q_1 and q_2 are $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ respectively. Where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the co-ordinates of charges q_1 and q_2.

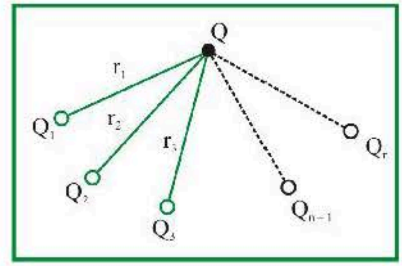
2.3 Principle of superposition

According to the principle of super position, total force acting on a given charge due to number of charges is the vector sum of the individual forces acting on that charge due to all the charges.

Consider number of charge Q_1, Q_2, Q_3...are applying force on a charge *Q*

Net force on *Q* will be

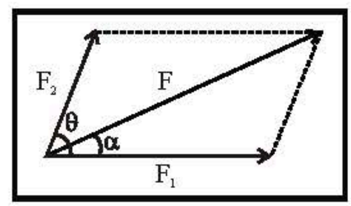
$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + ... + \vec{F}_{n-1} + \vec{F}_n$$



The magnitude of the resultant of two electric force is given by

$F = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$ and the force direction is given by

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$



3. ELECTRIC FIELD

A positive charge or a negative charge is said to create its field around itself. Thus space around a charge in which another charged particle experiences a force is said to have electrical field in it.

3.1 Electric field intensity $(\vec{E})$

The electric field intensity at any point is defined as the force experienced by a unit positive charge placed at that point. $\vec{E} = \dfrac{\vec{F}}{q_0}$

Where $q_0 \to 0$ so that presence of this charge may not affect the source charge Q and its electric field is not changed, therefore expression for electric field intensity can be better written as $\vec{E} = \lim_{q_0 \to 0} \dfrac{\vec{F}}{q_0}$

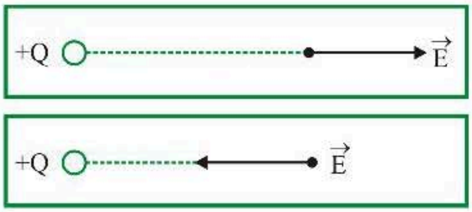
(a) **Unit and Dimensional formula :** It's S.I. unit –

$$\frac{\text{Newton}}{\text{coulomb}} = \frac{\text{volt}}{\text{meter}} = \frac{\text{Joule}}{\text{coulomb} \times \text{meter}} \quad \text{and}$$

C.G.S. unit = *Dyne/stat coulomb.*

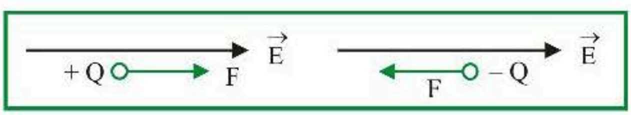
Dimension : $[E] = [MLT^{-3}A^{-1}]$

(b) **Direction of electric field :** Electric field (intensity) $\vec{E}$ is a vector quantity. Electric field due to a positive charge is always away from the charge and that due to a negative charge is always towards the charge



3.2 Relation between electric force and electric field

In an electric field $\vec{E}$ a charge (*Q*) experiences a force F = QE. If charge is positive then force is directed in the direction of field while if charge is negative force acts on it in the opposite direction of field

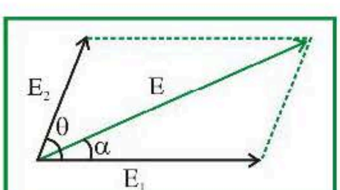


3.3 Super position of electric field

The resultant electric field at any point is equal to the vector sum of electric fields at that point due to various charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$$

The magnitude of the resultant of two electric fields are given by



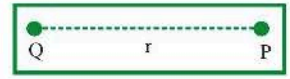
$E = \sqrt{E_1^2 + E_2^2 + 2E_1E_2 \cos\theta}$ and the direction is given by

$$\tan\alpha = \frac{E_2 \sin\theta}{E_1 + E_2 \cos\theta}$$

3.4 Point Charge

Point charge produces its electric field at a point P which is distance r from it given by

$$E_P = \frac{Q}{4\pi\varepsilon_0 r^2} \quad \text{(Magnitude)}$$



- For + ve point charge, E is directed away from it.
- For – ve point charge, E is directed towards it.

3.5 Continuous charge distributions

There are infinite number of ways in which we can spread a continuous charge distribution over a region of space. Mainly three types of charge distributions will be used. We define three different charge densities.

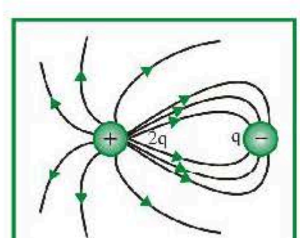
Symbol	Definition	SI units
(lambda) λ =	Charge per unit length	C/m
(sigma) σ =	Charge per unit area	[illegible]^2
(rho) ρ =	Charge per unit volume	C/m^3

If a total charge q is distributed along a line of length ℓ, over a surface area A or throughout a volume V, we can calculate charge densities from.

$$\lambda = \frac{q}{\ell}, \quad \sigma = \frac{q}{A}, \quad \rho = \frac{q}{V}$$

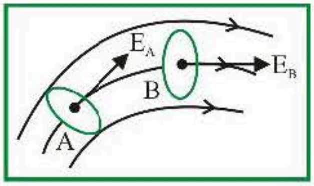
3.6 Properties of Electric Field Lines

1. Electric field lines originate from a **positive charge** & terminate on a **negative charge**.
2. The number of field lines originating/terminating on a charge is **proportional** to the magnitude of the charge.



3. The number of Field Lines passing through perpendicular unit area will be **proportional** to the **magnitude** of Electric Field there.

4. Tangent to a Field line at any point gives the **direction** of Electric Field at that point. This will be the **instantaneous** path charge will take if kept there.

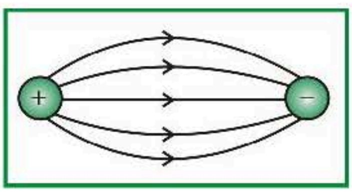


5. Two or more field lines can **never intersect** each other.

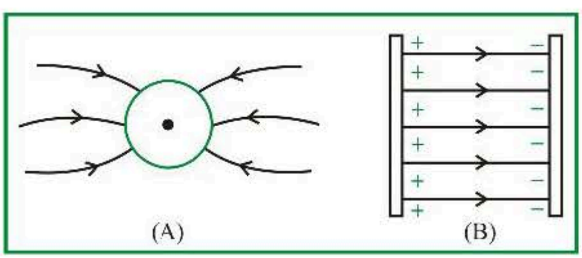
   [they cannot have multiple directions]

6. **Uniform field lines** are straight, parallel & uniformly placed.

7. Field lines cannot form a **loop**.



8. Electric field lines originate & terminate **perpendicular** to the surface of the conductor. Electric field lines do not exist inside a conductor.



9. Field lines always flow from higher **potential** to lower potential.

10. If in a region electric field is absent, there will be **no field lines**.

3.7 Motion of Charged Particle in an Electric Field

(a) When charged particle initially at rest is placed in the uniform field :

Let a charge particle of mass *m* and charge *Q* be initially at rest in an electric field of strength *E*

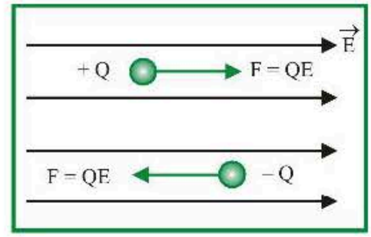


Fig. (A)

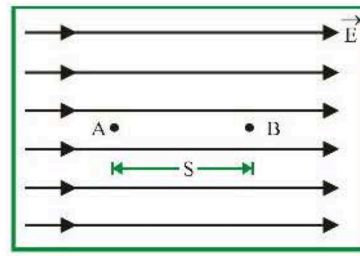


Fig. (B)

(i) Force and acceleration : The force experienced by the charged particle is F = QE. Positive charge experiences force in the direction of electric field while negative charge experiences force in the direction opposite to the field. [Fig. (A)]

Acceleration produced by this force is $a = \frac{F}{m} = \frac{QE}{m}$

Since the field E in constant the acceleration is constant, thus motion of the particle is uniformly accelerated.

(ii) Velocity : Suppose at point *A* particle is at rest and in time *t*, it reaches the point *B* [Fig. (B)]

V = Potential difference between *A* and *B*;

S = Separation between *A* and *B*

(a) By using

$$v = u + at, \quad v = 0 + Q\frac{E}{m}t,$$

$$\Rightarrow v = \frac{QEt}{m}$$

(b) By using

$$v^2 = u^2 + 2as, \; v^2 = 0 + 2\times\frac{QE}{m}\times s = \sqrt{\frac{2QEs}{m}}$$

(iii) Momentum : Momentum $p = mv$,

$$p = m \times \frac{QEt}{m} = QEt$$

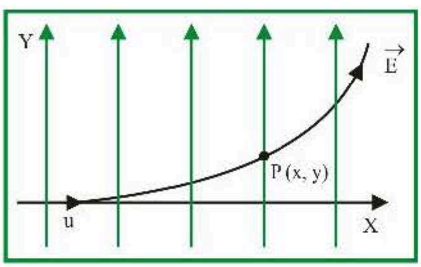
(iv) Kinetic energy : Kinetic energy gained by the particle in time *t* is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{(QEt)^2}{m} = \frac{Q^2E^2t^2}{2m}$$

(b) When a charged particle enters with an initial velocity at right angle to the uniform field

When charged particle enters perpendicularly in an electric field, it describe a parabolic path as shown

(i) Equation of trajectory : Throughout the motion particle has uniform velocity along *x*-axis and horizontal displacement (*x*) is given by the equation $x = ut$



Since the motion of the particle is accelerated along y-axis, we will use equation of motion for uniform acceleration to determine displacement y. From $S = ut + \frac{1}{2}at^2$

We have u = 0 (along y-axis) so $y = \frac{1}{2}at^2$

i.e., displacement along y-axis will increase rapidly with time (since $y \propto t^2$)

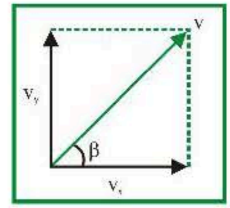
From displacement along x-axis t = x/u

So $y = \frac{1}{2}\left(\frac{QE}{m}\right)\left(\frac{x}{u}\right)^2$; this is the equation of parabola which shows $y \propto x^2$

(ii) Velocity at any instant : At any instant *t*, $v_x = u$ and

$$v_y = \frac{QEt}{m}$$

So
$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \frac{Q^2E^2t^2}{m^2}}$$



If β is the angle made by *v* with *x*-axis than

$$\tan\beta = \frac{v_y}{v_x} = \frac{QEt}{mu}.$$

4. ELECTRIC POTENTIAL ENERGY

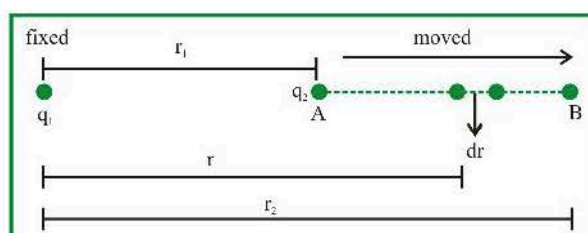
4.1 Potential energy of 2 charges system

It is always change in potential energy that is designed as

$\Delta U = -W_{\text{conservative force}} = -W_{\text{Coulomb force}}$

Potential energy is defined of a system of charges in a particular configuration.

Consider a system of two charges q_1 and q_2. Suppose, the charge q_1 is fixed and the charge q_2 is taken from a point A to B.



The electric force on the charge q_2 is $\vec{F} = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}\hat{r}$

The total work done as the charge q_2 moves from B to C is

$$W = \int_{r_1}^{r_2} \frac{q_1q_2}{4\pi\varepsilon_0 r^2}dr = \frac{q_1q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

The change in potential energy $U(r_2) - U(r_1)$ is, therefore,

$$U(r_2) - U(r_1) = -W = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

The potential energy of the two-charge system is assumed to be zero when they have infinite separation.

The potential energy when the separation is r is

$$U(r) = U(r) - U(\infty) = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$$

The potential energy depends essentially on the separation between the charges and is independent of the spatial location of the charged particles.

Equation gives the electric potential energy of a pair of charges.

Note..

* Electric potential energy is a scalar quantity so in the above formula take sign of Q_1 and Q_2.

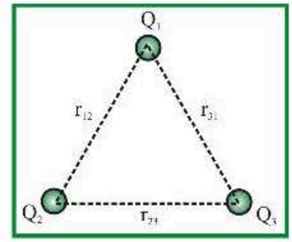
4.2 Electron volt (*eV*)

It is the smallest practical unit of energy used in atomic and nuclear physics. As electron volt is defined as "the energy acquired by a particle having one quantum of charge 1*e* when accelerated by 1*volt*" *i.e.*

$$1\,eV = 1.6\times10^{-19}\,C \times \frac{1J}{C} = 1.6\times10^{-19}\,J = 1.6\times10^{-12}\ \text{erg}$$

4.3 Potential energy of a system of *n* charges

In a system of *n* charges electric potential energy is calculated for each pair and then all energies so obtained are added algebraically. *i.e.* $U = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1Q_2}{r_{12}} + \frac{Q_2Q_3}{r_{23}} + \dots\dots\dots\right]$ and in case of continuous distribution of charge. As $dU = dQ.V \Rightarrow U = \int V\,dQ$

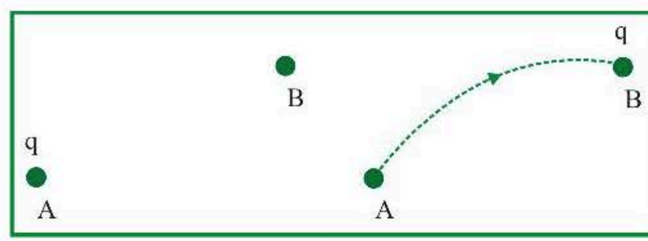


e.g. Electric potential energy for a system of three charges

$$\text{Potential energy} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1Q_2}{r_{12}} + \frac{Q_2Q_3}{r_{23}} + \frac{Q_3Q_1}{r_{31}}\right]$$

5. ELECTRIC POTENTIAL

Suppose, a test charge q is moved in an electric field from a point A to a point B while all the other charges in question remain fixed. If the electric potential energy changes by $U_B - U_A$ due to this displacement, we define the potential difference between the point A and the point B as



$$\Delta V = \frac{\Delta U}{q} \quad \text{i.e.} \quad V_B - V_A = \frac{U_B - U_A}{q} = \frac{W_{ext}}{q} \quad [\Delta KE = 0]$$

Conversely, if a charge q is taken through a potential difference $V_B - V_A$, the electric potential energy is increased by $U_B - U_A = q(V_B - V_A)$.

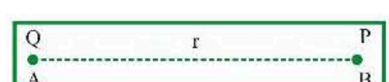
Also $W_{ext} = q(V_B - V_A)$ $[\Delta KE = 0]$

Potential difference between two points give us an idea about work which has to be done in moving a charge between those points.

5.1 Electric Potential due to a point charge

Consider a point charge Q placed at a point A.

The potential at P is,



$$V_P - V_\infty = \frac{U_P - U_\infty}{q} = \frac{Q}{4\pi\varepsilon_0 r} \Rightarrow V_P = \frac{Q}{4\pi\varepsilon_0 r}$$

($\because V_\infty$ is taken as 0)

The electric potential due to a system of charges may be obtained by finding potentials due to the individual charges using equation and then adding them. Thus,

$$V = \frac{1}{4\pi\varepsilon_0}\sum \frac{Q_i}{r_i}$$

Electric potential is a scalar quantity, hence sign of charges is to taken in expression it is denoted by *V*

5.2 Unit and dimensional formula

$$\text{S. I. unit} - \frac{\text{Joule}}{\text{Coulomb}} = \text{volt}$$

$[V] = [ML^2T^{-3}A^{-1}]$

5.3 Types of electric potential

According to the nature of charge potential is of two types

(i) Positive potential : Due to positive charge.

(ii) Negative potential : Due to negative charge.

Note..

- At the centre of two equal and opposite charge $V = 0$ but $E \neq 0$.
- At the centre of the line joining two equal and similar charge $V \neq 0, E = 0$.
- **If left free to move,**

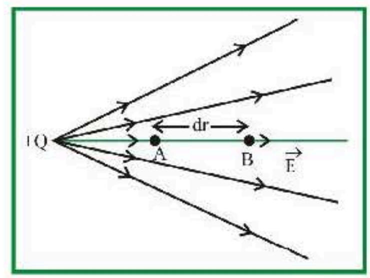
  Positive charge will always move from higher to lower potential points.

  Negative charge will always move from lower to higher potential points.

  (Because this motion will decrease potential energy of a system)

6. RELATION BETWEEN ELECTRIC FIELD & POTENTIAL

In an electric field rate of change of potential with distance is known as potential gradient. It is a vector quantity and it's direction is opposite to that of electric field. Potential gradient relates with electric field according to the following relation $E = -\frac{dV}{dr}$; This relation gives another unit of electric field is $\frac{\text{volt}}{\text{meter}}$. In the above rleation negative sign indicates that in the direction of electric field potential decreases.



In space around a charge distribution we can also write $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

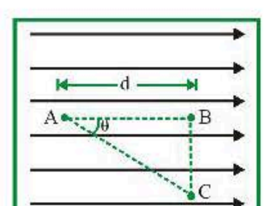
where $E_x = -\frac{dV}{dx}$, $E_y = -\frac{dV}{dy}$ and $E_z = -\frac{dV}{dz}$

Suppose *A*, *B* and *C* are three points in an uniform electric field as shown in figure.

(i) Potential difference between point *A* and *B* is

$$V_B - V_A = -\int_A^B \vec{E} \cdot \vec{dr}$$

Since displacement is in the direction of electric field, hence $\theta = 0°$



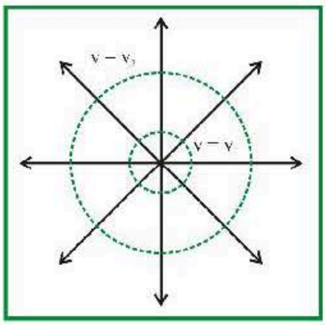
$$\text{So, } V_B - V_A = -\int_A^B E \cdot dr \cos 0 = -\int_A^B E \cdot dr = -Ed$$

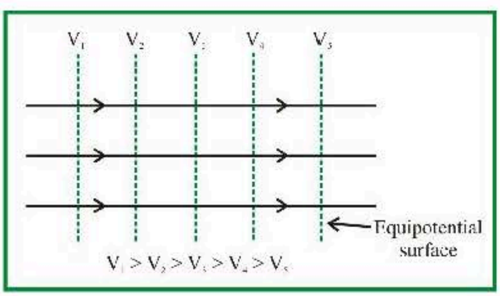
Equipotential Surface or Lines

(1) If every point of a surface is at same potential, then it is said to be an equipotential surface dark for a given charge distribution, locus of all points having same potential is called "equipotential surface" regarding equipotential surface following points should keep in mind :

(2) The direction of electric field is perpendicular to the equipotential surfaces or lines.

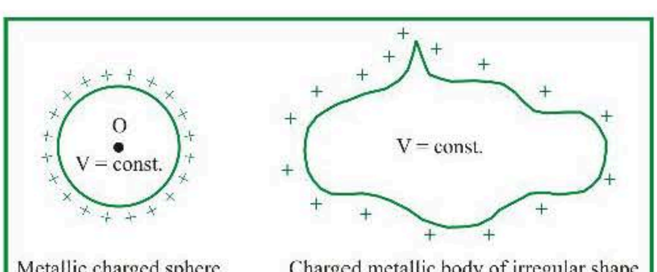
(3) The equipotential surfaces produced by a point charge or a spherically charge distribution are a family of concentric spheres.





(4) For a uniform electric field, the equipotential surfaces are a family of plane perpendicular to the field lines.

(5) A metallic surface of any shape is an equipotential surface *e.g.* When a charge is given to a metallic surface, it distributes itself in a manner such that its every point comes at same potential even if the object is of irregular shape and has sharp points on it.



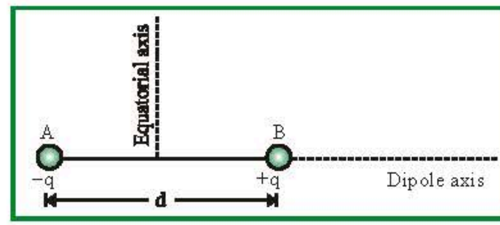
(6) Equipotential surfaces can never cross each other.

It is a common misconception that the path traced by a positive test charge is a field line but actually the path traced by a unit positive test charge represents a field full line only when it moves along a straight line.

7. ELECTRIC DIPOLE

7.1 General information

System of two equal and opposite charges separated by a small fixed distance is called a dipole.



(i) **Dipole axis :** Line joining negative charge to positive charge of a dipole is called its axis. It may also be termed as its longitudinal axis.

(ii) **Equatorial axis :** Perpendicular bisector of the dipole is called its equatorial or transverse axis as it is perpendicular to length.

(iii) **Dipole length :** The distance between two charges is known as dipole length (d)

(iv) **Dipole moment :** It is a quantity which gives information about the strength of dipole. It is a vector quantity and is directed from negative charge to positive charge along the axis. It is denoted as $\vec{p}$ and is defined as the product of the magnitude of either of the charge and the dipole length.

i.e. $\vec{p} = q(\vec{d})$

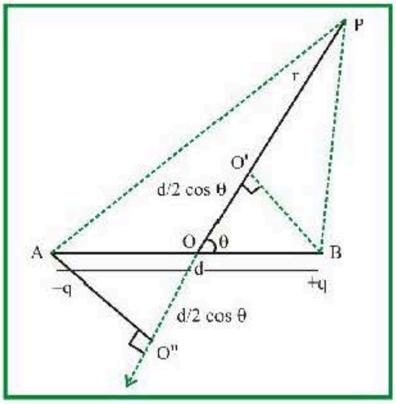
Its S.I. unit is ***coulomb-metre*** or ***Debye*** (1 *Debye* = 3.3×10^{-30} $C \times m$) and its dimensions are $M^0L^1T^1A^1$.

Note..

- A region surrounding a stationary electric dipole has electric field only.
- When a dielectric is placed in an electric field, its atoms or molecules are considered as tiny dipoles.

7.2 Electric field and potential due to an electric dipole

(a) Electric Potential due to a dipole



$$V_P = \frac{k(-q)}{AP} + \frac{k(+q)}{BP}$$

r >> d (distance 'r' is large as compared to d)

$$AP \approx O''P\,; \qquad BP \approx O'P$$

$$O''P = r + d/2\ \text{Cos}\theta, \quad O'P = r - d/2\ \text{Cos}\theta$$

$$V_p = \frac{k(-q)}{(r + d/2\cos\theta)} + \frac{k(+q)}{(r - d/2\cos\theta)}$$

$$= k(+q)\left[\frac{1}{r - d/2\cos\theta} - \frac{1}{r + d/2\cos\theta}\right].$$

$$= kq\left[\frac{r + d/2\cos\theta - r + d/2\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta}\right] = \frac{kq\,d\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta}$$

$$V_p = \frac{k(qd)\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta} = \frac{kp\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta}\ [\because p = qd].$$

since r >> d

$$V_p = \frac{kp\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}.$$

θ is angle with the axis of dipole ; r is distance from centre of dipole.

(b) Electric Field due to dipole

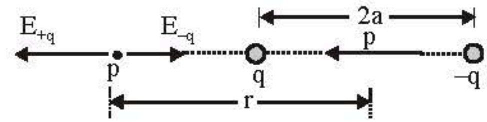
(i) For points on the axis

Let the point P be at distance r from the centre of the dipole on the side of the charge q, as shown in fig (a). Then

$$E_{-q} = -\frac{q}{4\pi\varepsilon_0(r+a)^2}\hat{p}$$

where $\hat{p}$ is the unit vector along the dipole axis (from –q to q). Also

$$E_{+q} = \frac{q}{4\pi\varepsilon_0(r-a)^2}\hat{p}$$



(a) a point on the axis.

The total field at P is

$$E = E_{+q} + E_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{4ar}{(r^2 - a^2)^2}\hat{p}$$

For r >> a

$$E = \frac{4qa}{4\pi\varepsilon_0 r^3}\hat{p} \qquad (r >> a) \qquad ...(i)$$

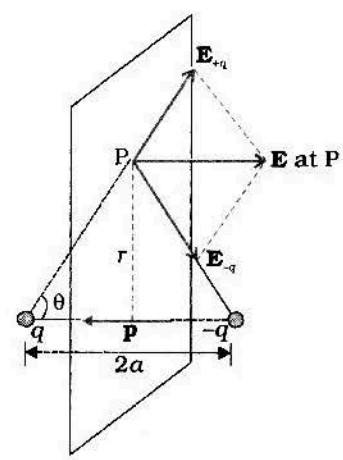
(ii) For points on the equatorial plane

The magnitudes of the electric fields due to the two charges + q and –q are given by

$$E_{+q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2 + a^2}$$

$$E_{-q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2 + a^2}$$

and are equal.



(b) a point on the equatorial plane of the dipole. p is the dipole moment vector of magnitude p = q × 2a and directed from –q to q.

The directions of E_{+q} and E_{-q} are as shown in fig. (b). Clearly, the components normal to the dipole axis cancel away. The components along the dipole axis add up. The total electric field is opposite to $\hat{p}$. We have

$$E = -(E_{+q} + E_{-q})\cos\theta\ \hat{p}$$

$$= -\frac{2qa}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}\hat{p}$$

At large distances (r >> a), this reduces to

$$E = -\frac{2qa}{4\pi\varepsilon_0 r^3}\hat{p}\ (r >> a) \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), it is clear that the dipole field at large distances does not involve q and a separately ; it depends on the product qa. This suggests the definition of dipole is defined by

$$P = q \times 2a\ \hat{p}$$

that is, it is a vector whose magnitude is charge q times the separation 2a (between the pair of charges q, –q) and the direction is along the line from –q to q. In terms of p, the electric field of a dipole at large distances takes simple forms :

At a point on the dipole axis

$$\boxed{E = \frac{2p}{4\pi\varepsilon_0 r^3} = \frac{2kp}{r^3}\ (r >> a)}$$

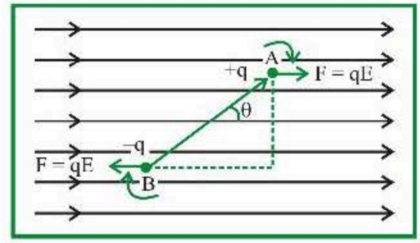
At a point on the equatorial plane

$$\boxed{E = \frac{p}{4\pi\varepsilon_0 r^3} = \frac{-kp}{r^3}\ (r >> a)}$$

7.3 Electric Dipole in uniform electric field

(i) Force and Torque : If a dipole is placed in a uniform field such that dipole (*i.e.* $\vec{p}$) makes an angle θ with direction of field then two equal and opposite force acting on dipole constitute a couple whose tendency is to rotate the dipole hence a torque is developed in it and dipole tries to align it self in the direction of field.

Consider an electric dipole in placed in a uniform electric field such that dipole (i.e. $\vec{p}$) makes an angle θ with the direction of electric field as shown

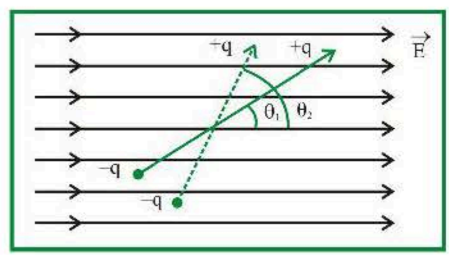


(a) Net force on electric dipole $F_{net} = 0$

(b) . . . . . . $\tau = pE\sin\theta\ \left(\vec{\tau} = \vec{p}\times\vec{E}\right)$

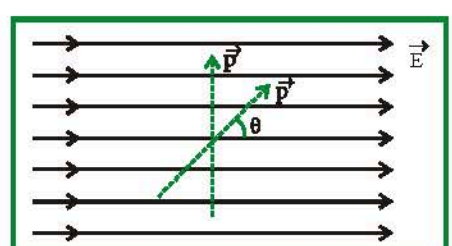
(ii) Work : From the above discussion it is clear that in an uniform electric field dipole tries to align itself in the direction of electric field (i.e. equilibrium position). To change it's angular position some work has to be done.

Suppose an electric dipole is kept in an uniform electric field by making an angle θ_1 with the field, if it is again turn so that it makes an angle θ_2 with the field, work done in this process is given by the formula



$$W = pE\left(\cos\theta_1 - \cos\theta_2\right)$$

(iii) Potential energy : In case of a dipole (in a uniform field), potential energy of dipole is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction i.e. if $\theta_1 = 90°$ and $\theta_2 = \theta$ then



$$W = \Delta U = U_\theta - U_{90°} = -pE\cos\theta$$

$$\Rightarrow U_\theta = -pE\cos\theta \quad \because [U(90°) = 0] \text{ or } \boxed{U = -\vec{p}.\vec{E}}$$

8. ELECTRIC FIELD

8.1 Continuous charge distributions

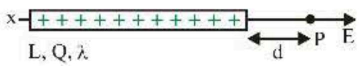
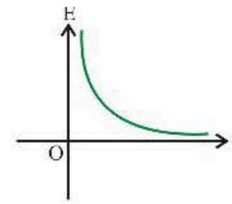
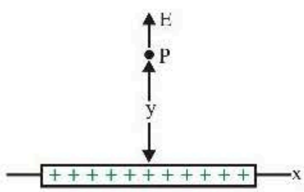
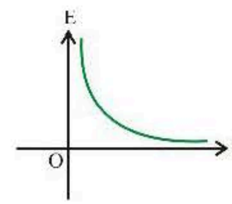
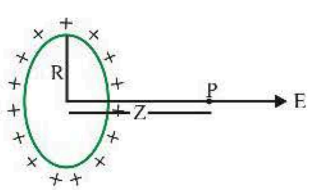
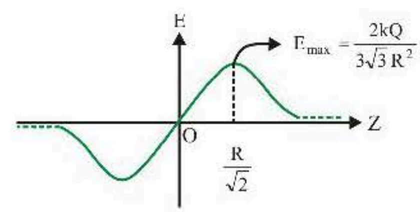
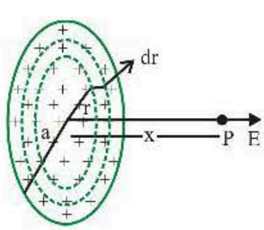
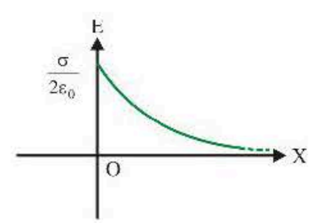
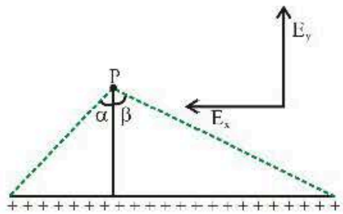
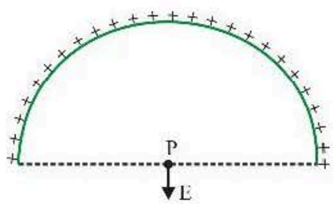
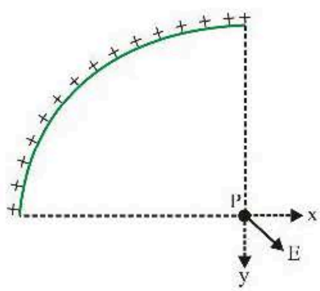
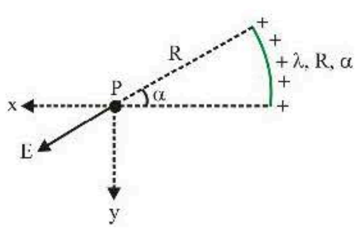
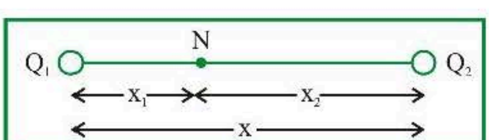
Diagram	Data	Graph
	$E_P = \dfrac{k\lambda L}{d(L+d)} = \dfrac{kQ}{d(L+d)}$	
	$E = \dfrac{k\lambda L}{y\sqrt{y^2 + \left(\dfrac{L}{2}\right)^2}}$	
	$E_z = \dfrac{kQz}{\left(z^2 + R^2\right)^{3/2}}$	
	$E_x = \dfrac{\sigma}{2\varepsilon_0}\left[1 - \dfrac{1}{\sqrt{\dfrac{a^2}{x^2} + 1}}\right]$	

Diagram	Data
	$E_y = \frac{k\lambda}{y}(\sin\alpha + \sin\beta)$ $E_x = \frac{k\lambda}{y}(\cos\alpha - \cos\beta)$
	$E = \frac{2k\lambda}{R}$
	$E_y = \frac{k\lambda}{R}$ $E_x = \frac{k\lambda}{R}$ $E = \frac{\sqrt{2}k\lambda}{R}$
	$E_x = \frac{k\lambda}{R}\sin(\alpha)$ $E_y = \frac{k\lambda}{R}[1 - \cos(\alpha)]$ $E_{net} = \frac{2k\lambda}{R}\sin\left(\frac{\alpha}{2}\right) = \frac{kQ}{R^2}\frac{\sin(\alpha/2)}{(\alpha/2)}$

8.2 Neutral Point

A neutral point is a point where resultant electrical field is zero. Thus neutral points can be obtained only at those points where the resultant field is subtractive.

(a) At an internal point along the line joining two like charges (Due to a system of two like point charge) : Suppose two like charges. Q_1 and Q_2 are separated by a distance x from each other along a line as shown in following figure.



If N is the neutral point at a distance x_1 from Q_1 and at a distance x_2 $(= x - x_1)$ from Q_2 then for natural pt. at N,

$|$E.F. due to $Q_1| = |$E.F. due to $Q_2|$ i.e.,

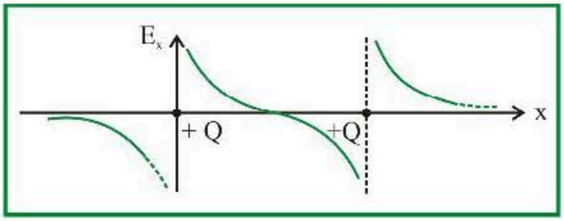
$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{|Q_1|}{x_1^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{|Q_2|}{x_2^2} \Rightarrow \frac{|Q_1|}{|Q_2|} = \left(\frac{x_1}{x_2}\right)^2$$

Short trick :

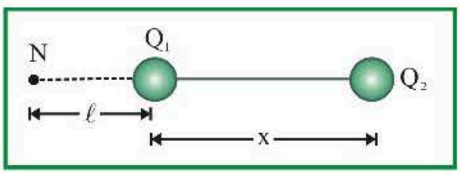
$$x_1 = \frac{x}{1 + \sqrt{|Q_2|/|Q_1|}} \text{ and } x_2 = \frac{x}{1 + \sqrt{|Q_1|/|Q_2|}}$$

Note..

In the above formula if $Q_1 = Q_2$, neutral point lies at the centre so remember that resultant field at the midpoint of two equal and like charges is zero.



(b) At an external point along the line joining two unlike charges (Due to a system of two unlike point charge) : Suppose two unlike charge Q_1 and Q_2 separated by a distance *x* from each other.



Here neutral point lies outside the line joining two unlike charges and also **it lies nearer to charge which is smaller in magnitude.**

If $|Q_1| < |Q_2|$ then neutral point will be obtained on the side of Q_1, suppose it is at a distance *l* from Q_1

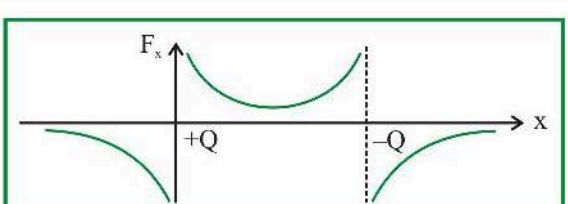
Hence at neutral point ;

$$\frac{k|Q_1|}{\ell^2} = \frac{k|Q_2|}{(x+\ell)^2} \Rightarrow \frac{|Q_1|}{|Q_2|} = \left(\frac{\ell}{x+\ell}\right)^2$$

Short trick : $\ell = \dfrac{x}{\left(\sqrt{|Q_2|/|Q_1|} - 1\right)}$

Note..

In the above discussion if $|Q_1| = |Q_2|$ neutral point will be at infinity.



8.3 Equilibrium of Charge

(a) Definition : A charge is said to be in equilibrium, if net force acting on it is zero. A system of charges is said to be in equilibrium if each charge is in equilibrium.

(b) Type of equilibrium : Equilibrium can be divided in following type:

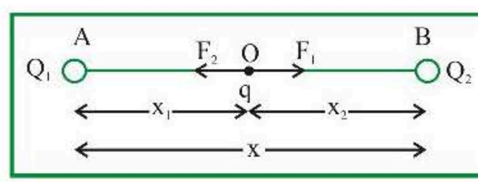
(i) Stable equilibrium : After displacing a charged particle from it's equilibrium position, if it returns back then it is said to be in stable equilibrium. If U is the potential energy then in case of stable equilibrium U **is minimum.**

(ii) Unstable equilibrium : After displacing a charged particle from it's equilibrium position, if it never returns back then it is said to be in unstable equilibrium and in unstable equilibrium, ***U* is maximum.**

(iii) Neutral equilibrium : After displacing a charged particle from it's equilibrium position if it neither comes back, nor moves away but remains in the position in which it was kept it is said to be in neutral equilibrium and in neutral equilibrium, **U is constant.**

(c) Different cases of equilibrium of charge

Suppose three similar charge Q_1, q and Q_2 are placed along a straight line as shown below



Case –1 :

Charge *q* will be in equilibrium if $|F_1| = |F_2|$ *i.e.,* $\dfrac{Q_1}{Q_2} = \left(\dfrac{x_1}{x_2}\right)^2$;

This is the condition of equilibrium of charge *q*. After following the guidelines we can say that charge *q* is in stable equilibrium and this system is not in equilibrium.

Note..

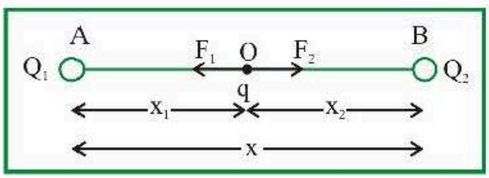
$$x_1 = \frac{x}{1+\sqrt{Q_2/Q_1}} \text{ and } x_2 = \frac{x}{1+\sqrt{Q_1/Q_2}}$$

e.g. if two charges +4 μC and +16 μC are separated by a distance of 30 *cm* from each other then for equilibrium a third charge should be placed between them at a distance

$$x_1 = \frac{30}{1+\sqrt{16/4}} = 10 \text{ cm or } x_2 = 20 \text{ cm}$$

Case –2 :

Two similar charge Q_1 and Q_2 are placed along a straight line at a distance *x* from each other and a third dissimilar charge *q* is placed in between them as shown below



Charge *q* will be in equilibrium if $|F_1| = |F_2|$

i.e., $\frac{Q_1}{Q_2} = \left(\frac{x_1}{x_2}\right)^2$.

Note..

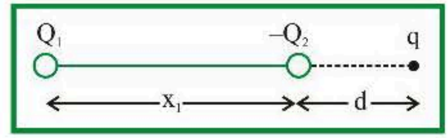
Same short trick can be used here to find the position of charge *q* as we discussed in Case–1 *i.e.,*

$$x_1 = \frac{x}{1+\sqrt{Q_2/Q_1}} \text{ and } x_2 = \frac{x}{1+\sqrt{Q_1/Q_2}}$$

It is very important to know that magnitude of charge *q* can be determined if one of the extreme charge (either Q_1 or Q_2) is in equilibrium *i.e.* if Q_2 is in equilibrium then $|q| = Q_1(x_2/x)^2$ and if Q_1 is in equilibrium then $|q| = Q_2(x_1/x)^2$ (It should be remember that sign of *q* is opposite to that of Q_1 (or Q_2))

Case –3 :

Two dissimilar charge Q_1 and Q_2 are placed along a straight line at a distance x from each other, a third charge q should be placed out side the line joining Q_1 and Q_2 for it to experience zero net force.



(Let $|Q_2| < |Q_1|$)

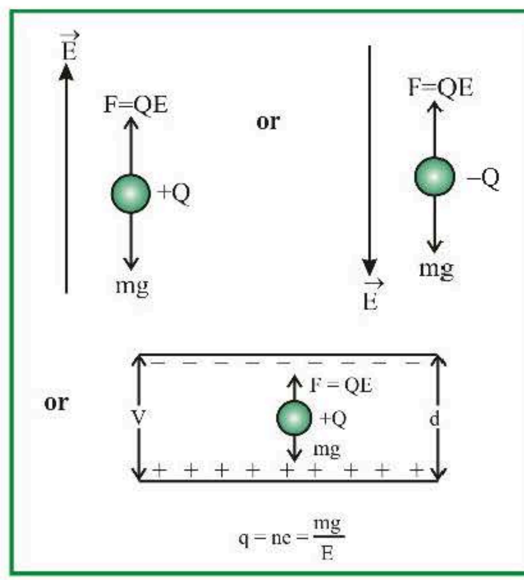
Short Trick :

For it's equilibrium. Charge *q* lies on the side of charge which is smallest in magnitude and

$$d = \frac{x}{\sqrt{Q_1/Q_2} - 1}$$

(d) Equilibrium of suspended charge in an electric field

(i) **Freely suspended charged particle :** To suspend a charged a particle freely in air under the influence of electric field it's downward weight should be balanced by upward electric force for example if a positive charge is suspended freely in an electric field as shown then



In equilibrium $QE = mg \Rightarrow E = \frac{mg}{Q}$

Note..

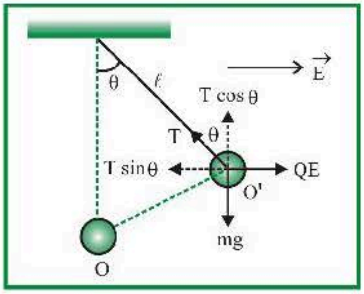
In the above case if direction of electric field is suddenly reversed in any figure then acceleration of charge particle at that instant will be $a = 2g$.

(ii) Charged particle suspended by a massless insulated string (like simple pendulum) : Consider a charged particle (like Bob) of mass m, having charge Q is suspended in an electric field as shown under the influence of electric field. It turned through an angle (say θ) and comes in equilibrium.

So, in the position of equilibrium (O' position)

$T\sin\theta = QE$ ...(i)

$T\cos\theta = mg$ ...(ii)



By squaring and adding equation (i) and (ii)

$$T = \sqrt{(QE)^2 + (mg)^2}$$

Dividing equation (i) by (ii) $\tan\theta = \dfrac{QE}{mg}$

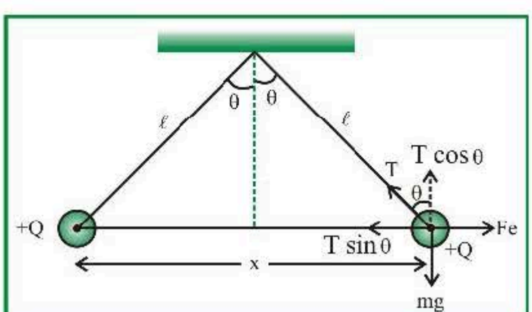
$$\Rightarrow \quad \theta = \tan^{-1}\frac{QE}{mg}$$

(iii) Equilibrium of suspended point charge system : Suppose two small balls having charge $+Q$ on each are suspended by two strings of equal length l. Then for equilibrium position as shown in figure.

$T\sin\theta = F_e$ ...(i)

$T\cos\theta = mg$ ...(ii)

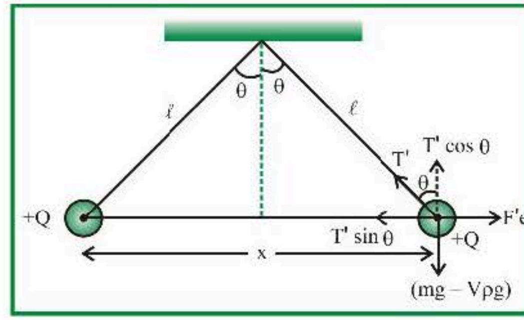
$T^2 = (F_e)^2 + (mg)^2$



and $\tan\theta = \dfrac{F_e}{mg}$;

here $F_e = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q^2}{x^2}$ and $\dfrac{x}{2} = \ell\sin\theta$

(iv) Equilibrium of suspended point charge system in a liquid : In the previous discussion if point charge system is taken into a liquid of density ρ such that θ remain same then



In equilibrium

$$Fe' = T'\sin\theta \text{ and } (mg - V\rho g) = T'\cos\theta$$

$$\therefore \quad \tan\theta = \frac{Fe'}{(mg - V\rho g)} = \frac{Q^2}{4\pi\varepsilon_0 K(mg - V\rho g)x^2}$$

When this system was in air

$$\tan\theta = \frac{Fe}{mg} = \frac{Q^2}{4\pi\varepsilon_0\, mgx^2}$$

So equating these two gives us

$$\frac{1}{m} = \frac{1}{k(m - V\rho)} \Rightarrow K = \frac{m}{m - V\rho} = \frac{1}{\left(1 - \frac{V}{m}\rho\right)}$$

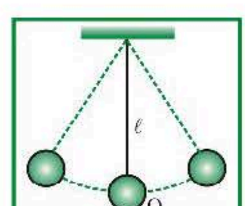
If σ is the density of material of ball then

$$K = \frac{1}{\left(1 - \frac{\rho}{\sigma}\right)} = \frac{\sigma}{\sigma - \delta}$$

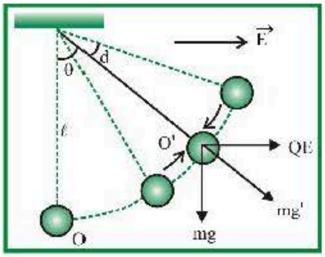
8.4 Time Period of Oscillation of a Charged Body

(a) **Simple pendulum based :** If a simple pendulum having length l and mass of bob m oscillates about it's mean position than it's time period of oscillation

$$T = 2\pi\sqrt{\ell / g}$$



Case –1 : If some charge say $+Q$ is given to bob and an electric field E is applied in the direction as shown in figure then equilibrium position of charged bob (point charge) changes from O to O′.



On displacing the bob from it's equilibrium position O′. It will oscillate under the effective acceleration g', where

$$mg' = \sqrt{(mg)^2 + (QE)^2} \Rightarrow g' = \sqrt{g^2 + (QE/m)^2}$$

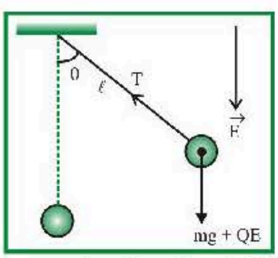
Hence the new time period is $T_1 = 2\pi\sqrt{\frac{\ell}{g'}}$

$$\Rightarrow \quad T_1 = 2\pi\sqrt{\frac{\ell}{\left(g^2 + (QE/m)^2\right)^{\frac{1}{2}}}}$$

Since $g' > g$, hence $T_1 < T$

i.e. time period of pendulum will decrease.

Case –2 : If electric field is applied in the downward direction then.

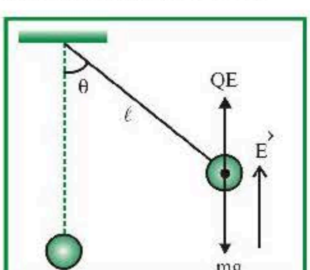


Effective acceleration $g' = g + QE/m$

So new time period $T_2 = 2\pi\sqrt{\frac{\ell}{g + (QE/m)}}$

$T_2 < T$

Case –3 : In case 2 if electric field is applied in upward direction then, effective acceleration.



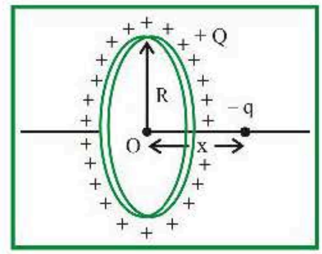
$g' = g - QE/m$

So new time period

$$T_3 = 2\pi\sqrt{\frac{\ell}{g - (QE/m)}}$$

$T_3 > T$

(b) **<u>Charged circular ring</u> :** A thin stationary ring of radius R has a positive charge $+Q$ unit. If a negative charge $-q$ (mass m) is placed at a small distance x from the centre. Then motion of the particle will be simple harmonic motion.



Electric field at the location of $-q$ charge

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qx}{\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

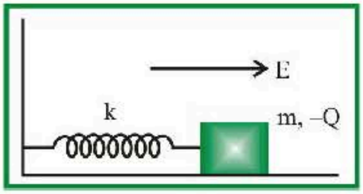
Since $x << R$, So x^2 neglected hence $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qx}{R^3}$

Force experienced by charge $-q$ is $F = -q\frac{1}{4\pi\varepsilon_0} \cdot \frac{Qx}{R^3}$

$\Rightarrow$ $F \propto -x$ hence motion is simple harmonic

Having time period $T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR^3}{Qq}}$

(c) **<u>Spring mass system</u> :** A block of mass m containing a negative charge $-Q$ is placed on a frictionless horizontal table and is connected to a wall through an unstretched spring of spring constant k as shown. If electric field E applied as shown in figure the block experiences an electric force, hence spring compress and block comes in new position. This is called the equilibrium position of block under the influence of electric field. If block compressed further or stretched, it execute oscillation having time period $T = 2\pi\sqrt{m/k}$. . Maximum compression in the spring due to electric field = QE/k



9. ELECTRIC POTENTIAL ENERGY

For the expression of total potential energy of a system of n charges consider $\frac{n(n-1)}{2}$ number of pair of charges.

Using Work energy theorem

$$\boxed{W_{ext} = \Delta KE + \Delta U}$$

If only conservative forces are there (e.g. gravity / spring / coulomb force), then $W_{ext} = 0$

$\Delta KE + \Delta U = 0$ or, $KE_f + U_f = KE_i + U_i$

$\text{Work} = \Delta KE + \Delta U$

$\boxed{\text{If } \Delta KE = 0}$ $\Delta U = W_{ext} = -W_{\text{Coulomb force}}$

If charges are assembled from infinity : $\Delta U = U(r) - U(\infty) = U(r)\,[U(\infty) = 0]$

We know, $\Delta U = W_{ext}$ [when $\Delta K.E. = 0$]

If $U(\infty) = 0 \Rightarrow U(r) = W_{ext}$

$\boxed{\text{If } W_{ext} = 0}$ $\Delta KE + \Delta U = 0$ or $KE_i + U_i = KE_f + U_f$

10. ELECTRIC POTENTIAL

10.1 Potential due to charge distribution

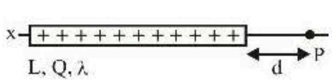
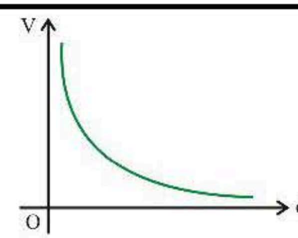
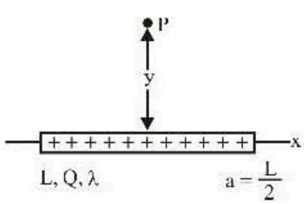
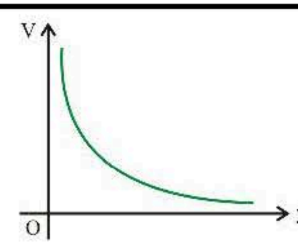
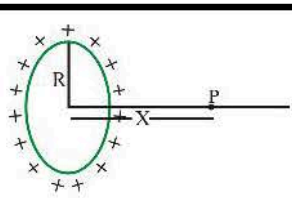
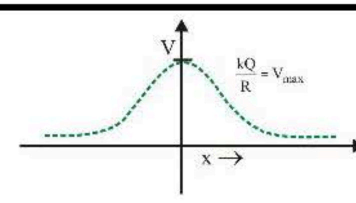
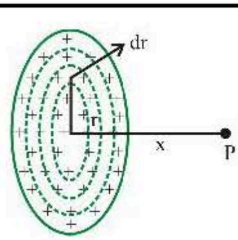
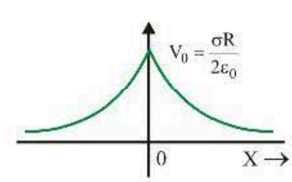
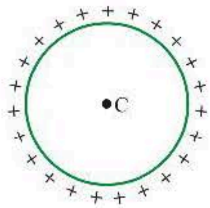
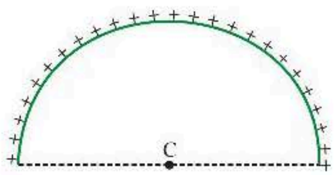
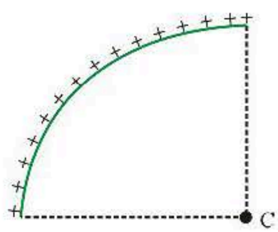
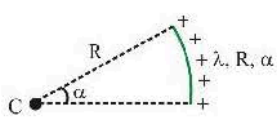
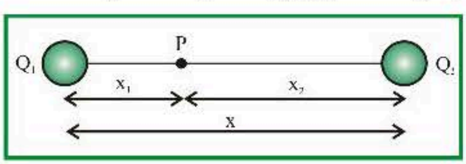
Diagram	Data	Graph
	$V_P = k\lambda\,\ell n\left(\frac{L+d}{d}\right)$	
	$V_P = 2k\lambda\,\ell n\left(\frac{a+\sqrt{a^2+y^2}}{y}\right)$	
	$V_P = V(x) = \frac{kQ}{\sqrt{x^2+R^2}}$	
	$V_P = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+x^2} - x\right]$	

Diagram	Data
	$V_C = \frac{kQ}{R} = k\lambda 2\pi$
	$V_C = \frac{kQ}{R} = k\lambda\pi$
	$V_C = \frac{kQ}{R} = k\lambda\frac{\pi}{2}$
	$V_C = \frac{kQ}{R} = k\lambda\alpha$

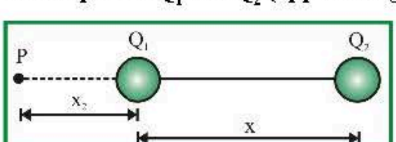
10.2 Zero Potential due to a System of Two Point Charge

I. For internal point : Q_1 and Q_2 (opposite signs)



$$\frac{Q_1}{x_1} = \frac{Q_2}{(x - x_1)} \Rightarrow x_1 = \frac{x}{(|Q_2|/|Q_1| + 1)}$$

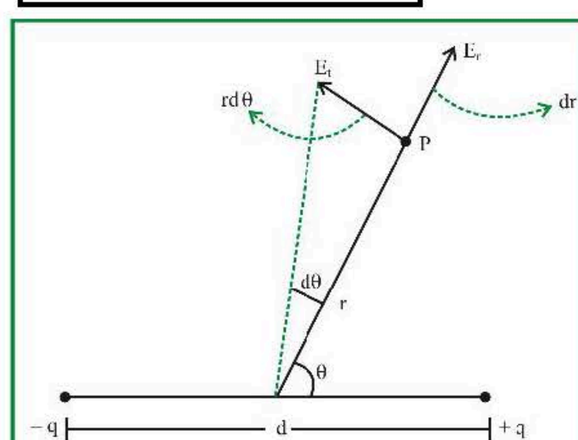
II. For External point : Q_1 and Q_2 (opposite signs) $|Q_1| < |Q_2|$



$$\frac{|Q_1|}{x_2} = \frac{|Q_2|}{(x + x_2)} \Rightarrow x_2 = \frac{x}{(|Q_2|/|Q_1| - 1)}$$

11. ELECTRIC DIPOLE

11.1 Electric field due to a dipole



Using the concept that if we know potential electric field can be calculated we have already calculated

$$V_p = \frac{kp\cos\theta}{r^2}.$$

To Calculate net electric field at P we need E_r (Radial Component) & E_t (tangential component) of electric field at P.

$$E_r = \frac{-dV}{dr}.\ \text{[When we travel in the radial direction].}$$

$$E_t = -\frac{dV}{rd\theta}\ \text{[When we travel in the tangential direction].}$$

$$V_p = \frac{kP\cos\theta}{r^2}$$

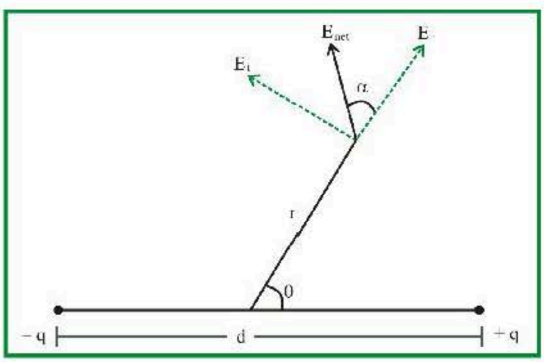
$$E_r = \frac{-d}{dr}\left(\frac{kP\cos\theta}{r^2}\right) = \frac{2kP\cos\theta}{r^3}.$$

$$E_t = \frac{-d}{rd\theta}\left(\frac{kP\cos\theta}{r^2}\right) = \frac{-kP}{r^3}\frac{d}{d\theta}\cos\theta = \frac{kP\sin\theta}{r^3}.$$

$$E_{net} = \sqrt{E_r^2 + E_t^2} = \sqrt{\left(\frac{kP}{r^3}\right)^2\left[4\cos^2\theta + \sin^2\theta\right]}$$

$$= \sqrt{\left(\frac{kP}{r^3}\right)^2\left[1+3\cos^2\theta\right]}$$

$$E_{net} = \frac{kP}{r^3}\sqrt{1+3\cos^2\theta}$$



$$\tan\alpha = \frac{E_t}{E_r} = \frac{\frac{kP}{r^3}\sin\theta}{2\frac{kP\cos\theta}{r^3}} = \frac{\tan\theta}{2} \qquad \alpha = \tan^{-1}\left[\frac{\tan\theta}{2}\right]$$

[**Note :** α is the angle with the radial direction]

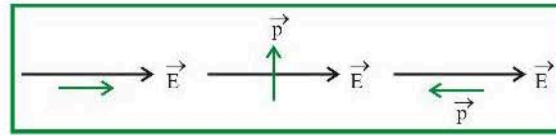
11.2 Equilibrium of dipole

We know that, for any equilibrium net torque and net force on a particle (or system) should be zero.

We already discussed when a dipole is placed in an uniform electric field net force on dipole is always zero. But net torque will be zero only when $\theta = 0°$ or $180°$.

When $\theta = 0°$ *i.e.* dipole is placed along the electric field it is said to be in stable equilibrium, because after turning it through a small angle, dipole tries to align itself again in the direction of electric field.

When $\theta = 180°$ *i.e.* dipole is placed opposite to electric field, it is said to be in unstable equilibrium.



$\theta = 0°$	$\theta = 90°$	$\theta = 180°$
Stable equilibrium		Unstable equilibrium
$\tau = 0$	$\tau_{max} = pE$	$\tau = 0$
$W = 0$	$W = pE$	$W_{max} = 2pE$
$U_{min} = -pE$	$U = 0$	$U_{max} = pE$

11.3 Angular SHM

In a uniform electric field (intensity E) if a dipole (electric) is slightly displaced from it's stable equilibrium position it executes angular SHM having period of oscillation. If I = moment of inertia of dipole about the axis passing through it's centre and perpendicular to it's length.

For electric dipole : $T = 2\pi\sqrt{I/pE}$

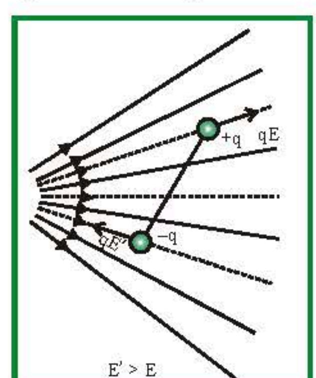
11.4 Dipole-point charge interaction

If a point charge is placed in dipole field at a distance r from the mid point of dipole then force experienced by point charge varies according to the relation $F \propto \frac{1}{r^3}$

11.5 Electric dipole in non-uniform electric field

When an electric dipole is placed in a non-uniform field, the two charges of dipole experiences unequal forces, therefore the net force on the dipole is not equal to zero.

Due to two unequal forces, a torque is produced which rotate the dipole so as to align it in the direction of field.



So in non-uniform electric field

(i) Motion of the dipole is translatory and rotatory

(ii) Torque on it may be zero.

GAUSS'S LAW

1. ELECTRIC FLUX

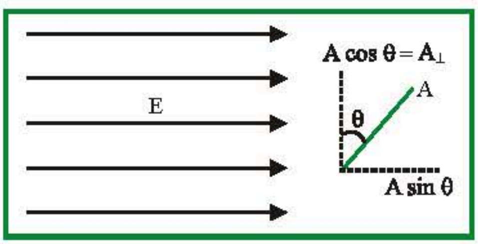
1.1 Definition

Electric flux is defined as proportional to number of field lines crossing or cutting any area of cross section in space.

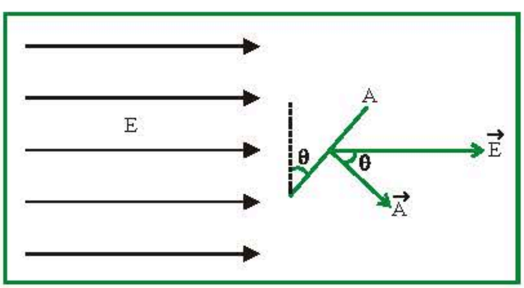
"The number of field lines passing through perpendicular unit area will be proportional to the magnitude of Electric Field there" (Theory of Field Lines)

$$\frac{N}{A_\perp} \propto E \quad \Rightarrow \quad N \propto E A_\perp$$

$\therefore$ Electric Flux, $\Phi_{A_\perp} = EA_\perp$



As θ increases, flux through area A decreases. If we draw a vector of magnitude A along the positive normal, it is called the **area vector,** $\vec{A}$ corresponding to the area A.



$\therefore$ Electric Flux, $\Phi_A = EA\cos\theta = \vec{E}\cdot\vec{A}$

(Assuming Electric Field is uniform over whole area)

Note..

If Electric field is not constant over the area of cross section, then

$$\Phi = \int_A \vec{E}\cdot d\vec{A}$$

1.2 Unit and Dimension

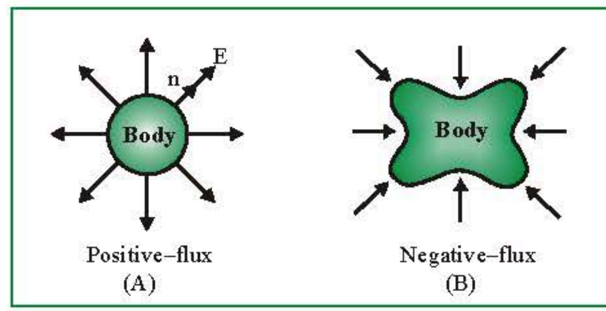
Flux is a scalar quantity.

S.I. unit : (volt × m) or $\frac{N.m^2}{C}$

It's Dimensional formula : $(ML^3T^{-3}A^{-1})$

1.3 Types of flux

For a closed body outward flux is taken to be **positive**, while inward flux is taken to be **negative.**



2. GAUSS'S LAW

2.1 Definition

According to Gauss's law, total electric flux through a closed surface enclosing a charge is $\frac{1}{\varepsilon_0}$ times the magnitude of the charge enclosed.

i.e. $\phi_{net} = \frac{1}{\varepsilon_0}(Q_{enc.})$

i.e. $\oint \vec{E}.d\vec{A} = \frac{Q_{en}}{\varepsilon_0}$.

Note..

Gauss's law is only applicable for a closed surface.

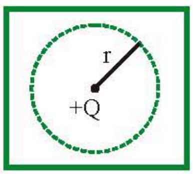
2.2 Gaussian Surface

The closed surface on which Gauss law is applicable is defined as a Gaussian surface.

Note..

- Gaussian surface can be of any shape & size, only condition is that it should be closed.
- Gaussian surface is hypothetical in nature. It does not have a physical existence.

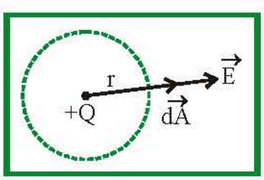
2.3 Deriving Gauss's law from Coulomb's law



Lets take a spherical gaussian surface with charge '+Q' kept at the centre.

We know field lines for a +ve charge are always radially outward.

Angle between $d\vec{A}$ & $\vec{E}$ is zero.



$$E = \frac{kQ}{r^2} = \frac{Q}{4\pi \in_0 r^2}$$

$$\text{Net flux} = \oint \vec{E}.\overrightarrow{dA} = \oint \frac{Q}{4\pi \in_0 r^2} dA$$

$$= \frac{Q}{4\pi \in_0 r^2} \oint dA = \frac{Q}{4\pi \in_0 r^2} . 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

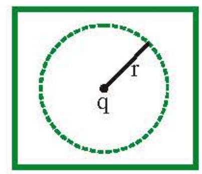
Hence **Net flux = Q/ε_0**.

Note..

Although we derived gauss law for a spherical surface it is valid for any shape of gaussian surface and for any charge kept anywhere inside the surface.

2.4 Coulomb's law from Gauss's law

We choose an imaginary sphere (Gaussian surface) of radius r centred on the charge +q. Due to symmetry, E must have the same magnitude at any point on the surface, and $\vec{E}$ points radially outward, parallel to $\overrightarrow{dA}$. Hence we write the integral in Gauss's law as



$$\phi_{net} = \oint \vec{E}.\overrightarrow{dA} = \oint E\, dA = E\oint dA = E\left(4\pi r^2\right)$$

$$Q_{enclosed} = q$$

Thus, $E\left(4\pi r^2\right) = \frac{q}{\varepsilon_0}$ or $E = \frac{q}{4\pi\varepsilon_0 r^2}$

From the definition of the electric field, the force on a point charge q_0 located at a distance r from the charge q is $F = q_0E$. Therefore,

$$\mathbf{F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}}$$

which is Coulomb's law.

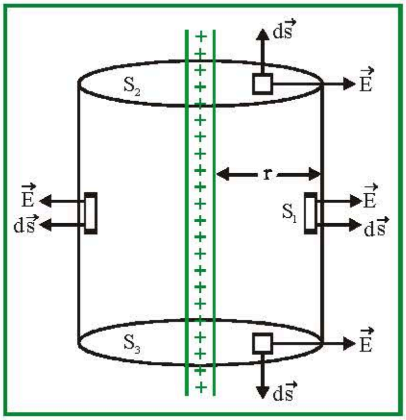
3. APPLICATIONS OF GAUSS'S LAW

Using Gauss's law to derive 'E' due to various charge distributions.

3.1 Electric Field due to a Line Charge

Consider an infinite line which has a linear charge density **λ**. Using Gauss's law, let us find the electric field at a distance 'r' from the line charge.

The **cylindrical symmetry** tells us that the field strength will be the same at all points at a fixed distance r from the line. Thus, if the charges are positive. The field lines are directed radially outwards, perpendicular to the line charge.



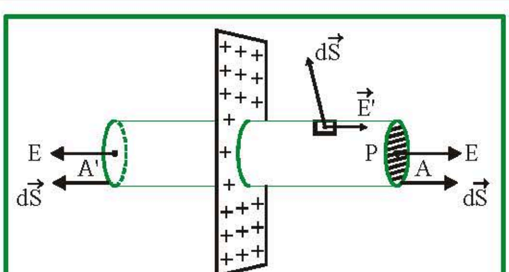
The appropriate choice of Gaussian surface is a cylinder of **radius r** and **length L**. On the flat end faces, S_2 and S_3, $\vec{E}$ is perpendicular $d\vec{S}$, which means flux is zero on them. On the curved surface S_1, $\vec{E}$ is parallel to $d\vec{S}$, so that $\vec{E}.\overrightarrow{dS} = EdS$. The charge enclosed by the cylinder is Q = λL. Applying Gauss's law to the curved surface, we have

$$E\oint dS = E(2\pi rL) = \frac{\lambda L}{\varepsilon_0} \quad \text{or} \quad \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}} = \frac{2k\lambda}{r}$$

Note..

This is the field at a distance r from the line. It is directed away from the line if the charge is positive and towards the line if the charge is negative.

3.2 Electric Field due to a Plane Sheet of Charge



Consider a large plane sheet of charge with surface charge density (charge per unit area) **σ**. We have to find the electric field E at a point P in front of the sheet.

Note..

If the charge is positive, the field is away from the plane.

To calculate the field E at P. Choose a cylinder of area of cross-section A through the point P as the Gaussian surface. The flux due to the electric field of the plane sheet of charge passes only through the two circular caps of the cylinder.

According to Gauss law $\oint \vec{E}.d\vec{S} = q_{in}/\varepsilon_0$

$$\int_{\text{I circular surface}} \vec{E}.\overrightarrow{dS} + \int_{\text{II circular surface}} \vec{E}.\overrightarrow{dS} + \int_{\text{cylindrical surface}} \vec{E}.\overrightarrow{dS} = \frac{\sigma A}{\varepsilon_0}$$

or $$EA + EA + 0 = \frac{\sigma A}{\varepsilon_0}$$

or $$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

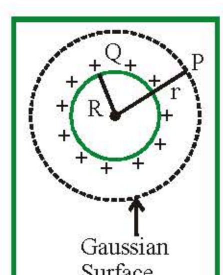
Note..

We see that the field is uniform and does not depend on the distance from the charge sheet. This is true as long as the sheet is large as compared to its distance from P.

3.3 Uniform Spherical Charge Distribution

3.3.1 Outside the Sphere

P is a point outside the sphere at a distance r from the centre.



According to Gauss law, $\oint \vec{E}.d\vec{s} = \dfrac{Q}{\varepsilon_0}$ or $E\left(4\pi r^2\right) = \dfrac{Q}{\varepsilon_0}$

Electric field at P (Outside sphere)

$$E_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2} = \frac{\sigma R^2}{\varepsilon_0 r^2} \quad \text{and}$$

$$V_{out} = -\int_{\infty}^{r} \vec{E}.d\vec{r} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r} = \frac{\sigma R^2}{\varepsilon_0 r}$$

Note..

$$\left\{\begin{aligned} Q &= \sigma \times A \\ &= \sigma \times 4\pi R^2 \end{aligned}\right\}$$

3.3.2 At the surface of sphere

At surface r = R

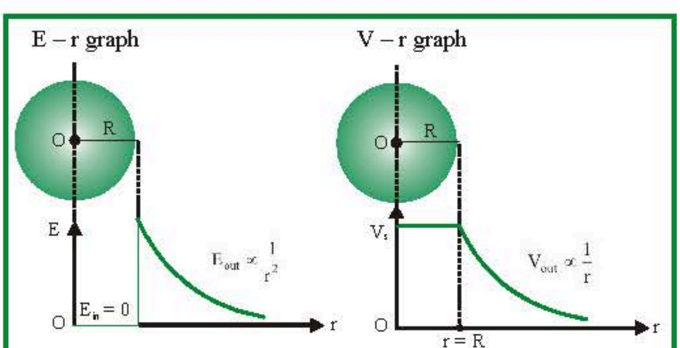
$$\text{So, } E_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2} = \frac{\sigma}{\varepsilon_0} \quad \text{and} \quad V_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{\sigma R}{\varepsilon_0}$$

3.3.3 Inside the sphere

Inside the conducting charged sphere electric field is zero and potential remains constant every where and equals to the potential at the surface.

$$E_{in} = 0 \text{ and } V_{in} = \text{constant} = V_s = \frac{Q}{4\pi\epsilon_0 R}$$

Graphical variation of electric field and potential with distance

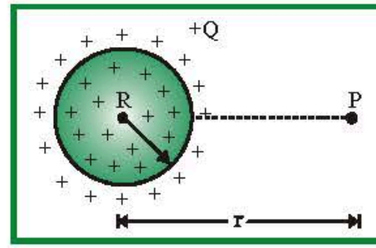


3.4 Uniform Spherical Volume Charge Distribution

We consider a spherical uniformly charge distribution of radius R in which total charge Q is uniformly distributed throughout the volume.

The charge density

$$\rho = \frac{\text{total charge}}{\text{total volume}} = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{3Q}{4\pi R^3}$$



3.4.1 Outside the sphere at P (r ≥ R)

According to Gauss law $\oint \vec{E}.\vec{ds} = \dfrac{Q}{\varepsilon_0}$ or $E\left(4\pi r^2\right) = \dfrac{Q}{\varepsilon_0}$

$$E_{out} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r^2} \text{ and } V_{out} = -\int_{\infty}^{r} \vec{E}.d\vec{r} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{r}$$

$$\text{using } \rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$E_{out} = \frac{\rho R^3}{3\varepsilon_0 r^2} \text{ and } V_{out} = \frac{\rho R^3}{3\varepsilon_0 r} \quad (V(\infty) = 0)$$

3.4.2 At the surface of sphere

At surface r = R

$$E_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R^2} = \frac{\rho R}{3\varepsilon_0} \text{ and } V_s = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = \frac{\rho R^2}{3\varepsilon_0}$$

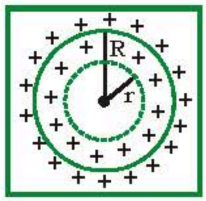
3.4.3 Inside the sphere

At a distance r from the centre. (r ≤ R)

$$\oint \vec{E}_{in}.\vec{ds} = \frac{q_{in}}{\varepsilon_0} = \frac{Qr^3}{\varepsilon_0 R^3} \text{ or } E_{in}\left(4\pi r^2\right) = \frac{Qr^3}{\varepsilon_0 R^3}$$

$$E_{in} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qr}{R^3} = \frac{\rho r}{3\varepsilon_0} \{E_{in} \propto r\} \text{ and}$$

$$V_{in} = -\int_R^r \vec{E}.d\vec{r} = \frac{1}{4\pi\varepsilon_0} \frac{Q\left[3R^2 - r^2\right]}{2R^3} = \frac{\rho\left(3R^2 - r^2\right)}{6\varepsilon_0}$$

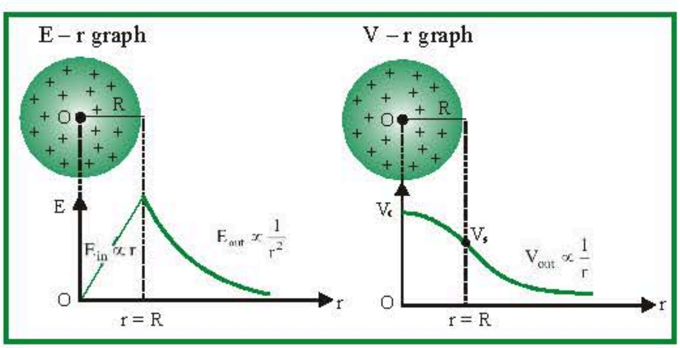


Note..

At centre (r = 0). $V_{centre} = \frac{3}{2} \times \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R} = \frac{3}{2} V_s$

i.e., $V_{centre} > V_{surface} > V_{out}$

Graphical variation of electric field and potential with distance



4. PROPERTIES OF CONDUCTORS

1. Inside a conductor, electrostatic field is zero

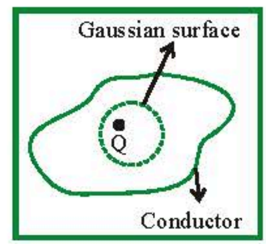
Consider a conductor, neutral or charged. There may also be an external electrostatic field. In the static situation the electric field is zero everywhere inside the conductor. As long as electric field is not zero, the free charge carriers would experience force and drift. In the static situation, the free charges have so distributed themselves that the electric field is zero everywhere inside. Electrostatic field is zero inside a conductor.

2. At the surface of a charged conductor, electrostatic field must be normal to the surface at every point

If E were not normal to the surface, it would have some non-zero component along the surface. Free charges on the surface of the conductor would then experience force and move. In the static situation, therefore, E should have no tangential component. Thus electrostatic field at the surface of a charged conductor must be normal to the surface at every point. (For a conductor without any surface charge density, field is zero even at the surface).

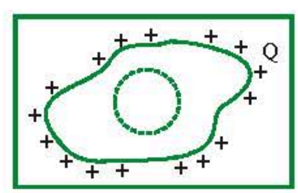
3. The charge kept in the material of a conductor will come to its outermost surface.

We know electric field at all points inside the material of a conductor is zero. This means 'E' at all points on the Gaussian surface is zero.



$$\int \vec{E}.\overrightarrow{dA} = \frac{Q_{en}}{\varepsilon_0} \Rightarrow E = 0 \Rightarrow \boxed{Q_{en} = 0}$$

Charge cannot remain inside so it comes outside dotted surface.



4. Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface

This follows from results 1 and 2 above. Since E = 0 inside the conductor and has no tangential component on the surface, no work is done in moving a small test charge within the conductor and on its surface. That is, there is no potential difference between any two points inside or on the surface of the conductor. Hence, the result. If the conductor is charged, electric field normal to the surface exists; this means potential will be different for the surface and a point just outside the surface.

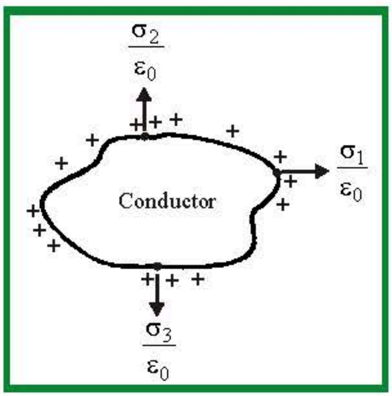
In a system of conductors of arbitrary size, shape and charge configuration, each conductor is characterised by a constant value of potential, but this constant may differ from one conductor to the other.

5. Electric field at the surface of a charged conductor

$$E = \frac{\sigma}{\varepsilon_0}\hat{n}$$

where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction.

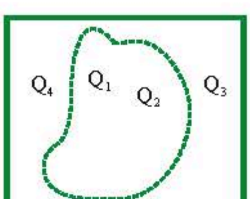
For $\sigma > 0$, electric field is normal to the surface outward ; for $\sigma > 0$, electric field is normal to the surface inward.



5. GAUSS LAW

5.1 Some important points about Gauss Law

1. $$\underset{\text{(Net flux)}}{\oint \vec{E}.\overrightarrow{dA}} = \frac{Q_{en}}{\epsilon_0}$$



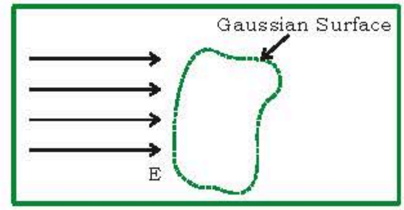
In the above expression, charge enclosed is (Q_1 & Q_2).

Net flux will only depend on Q_1 & Q_2.

But 'E' in the Gauss Law will be due to all the charges Q_1, Q_2, Q_3 & Q_4. Hence electric field calculated through Gauss law is not just due to enclosed charges but due to all the charges.

2. If Q_{en} by a gaussian surface is given to be zero then it does not necessarily imply that $E = 0$. It may or may not be zero.

For example :



$$Q_{en} = 0$$

but electric field on the Gaussian surface is present.

3. If E at all points on the gaussian surface is zero then it mean Q_{en} has to be zero.

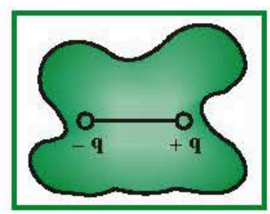
Because $\oint \vec{E}.\overrightarrow{dA} = \frac{Q_{en}}{\varepsilon_0}$.

5.2 Zero flux

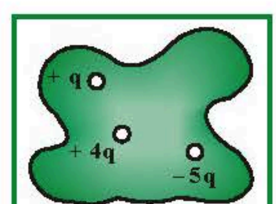
... **Net charge equals to zero**

If a dipole is enclosed by a surface

$\phi = 0$; $Q_{enc} = 0$



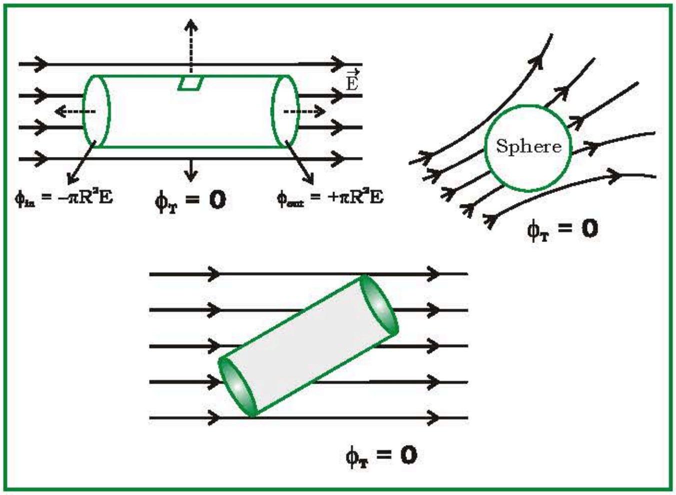
If the magnitude of positive and negative charges are equal inside a closed surface.



$Q_{enc} = 0$, so, $\phi = 0$

(ii) Charges are absent

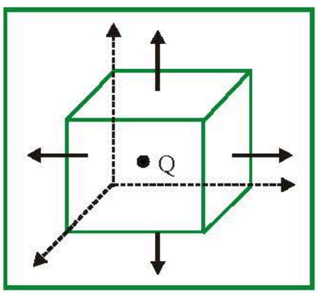
If a closed body (not enclosing any charge) is placed in an electric field (either uniform or non-uniform) total flux linked with it will be zero



5.3 Observe Flux through Common Geometrical Figures

5.3.1 Cube

(i) Charge at the centre of cube.

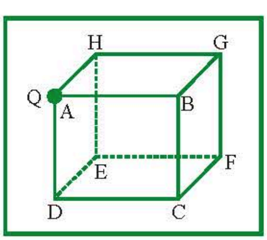


Note..

$$\phi_{total} = \frac{1}{\varepsilon_0}.(Q)$$

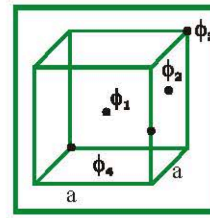
Flux through each face, $\phi_{face} = \frac{Q}{6\varepsilon_0}$

(ii) Charge situated at the corner of a cube.



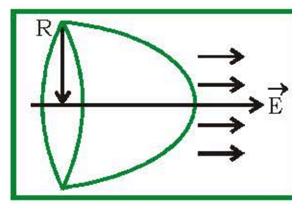
For the charge at the corner, we require eight cube to symmetrically enclose it in a Gaussian surface. The total flux $\phi_T = \frac{Q}{\varepsilon_0}$. Therefore the flux through one cube will be $\phi_{cube} = \frac{Q}{8\varepsilon_0}$. The cube has six faces and flux linked with three faces (through A) is zero (ABCD, AHED, ABGH), so flux linked with remaining three faces will $\frac{Q}{8\varepsilon_0}$. Now as the remaining three are identical so flux linked with each of the three faces will be $= \frac{1}{3} \times \left[\frac{1}{8}\left(\frac{Q}{\varepsilon_0}\right)\right] = \mathbf{\frac{1}{24}\frac{Q}{\varepsilon_0}}$.

Charge Position	ϕ_T
Cube centre	q/ε_0
Face centre	$q/2\varepsilon_0$
At corner	$q/8\varepsilon_0$
At centre of edge	$q/4\varepsilon_0$

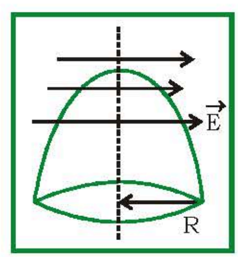


5.3.2 Hemisphere

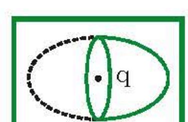
$$\phi_{out} = \phi_{in} = \pi R^2 E$$



$\phi_T = 0$



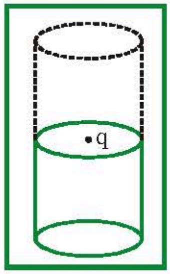
$$\phi_T = \frac{q}{\varepsilon_0}, \quad \phi_{hemisphere} = \frac{q}{2\varepsilon_0}$$



(dotted part shows imaginary part to enclose the charge completely)

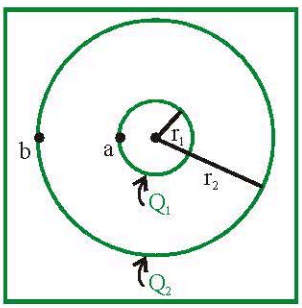
5.3.3 Cylinder

$$\phi_T = \frac{q}{\varepsilon_0}, \ \phi_{cyl.} = \frac{q}{2\varepsilon_0}$$

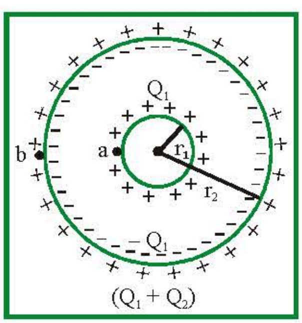


6. CASES OF CONCENTRIC SHELLS

Initially



Final distribution

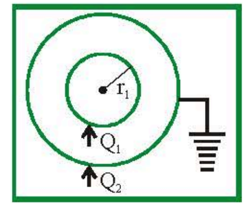


$$V_a = \frac{kQ_1}{r_1} + \frac{k(-Q_1)}{r_2} + \frac{k(Q_1+Q_2)}{r_2}$$

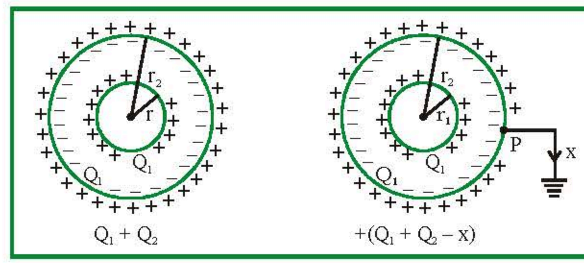
$$V_b = \frac{kQ_1}{r_2} + \frac{k(-Q_1)}{r_2} + \frac{k(Q_1+Q_2)}{r_2}$$

(Refer Section 3.3)

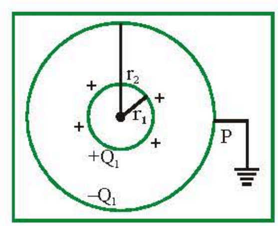
7. CASES OF EARTHING A CONDUCTOR



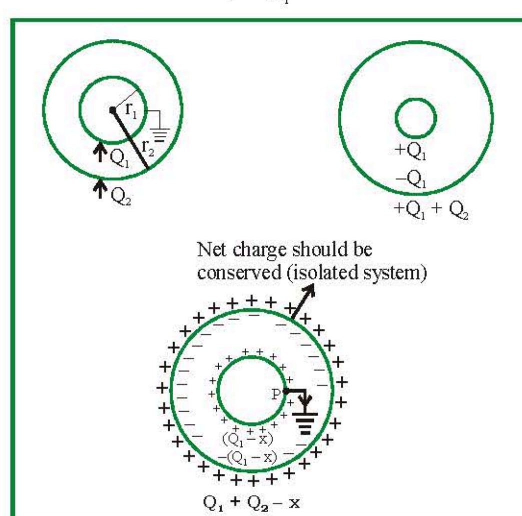
1. Firstly do charge distribution before earthing.
2. After charge distribution, assume some 'x' charge flown to ground (after earthing).
3. Do re-distribution of charge.
4. Take a point on the conductor (which is earthed) & do net potential of it equals 0. Calculate x.



$$V_P = 0 = \frac{kQ_1}{r_2} + \frac{k(-Q_1)}{r_2} + \frac{k(Q_1+Q_2-x)}{r_2} \Rightarrow x = Q_1 + Q_2$$



Charge is flown from outer surface because as long as Q_1 remains on inner shell, '$-Q_1$' will be induced on inner shell.



$$V_P = 0 = \frac{k(Q_1 - x)}{r_1} + \frac{k(-(Q_1 - x))}{r_2} + \frac{k(Q_1 + Q_2 - x)}{r_2}$$

$$\frac{Q_1 - x}{r_1} - \frac{(Q_1 - x)}{r_2} + \frac{Q_1 + Q_2 - x}{r_2} = 0$$

$$\Rightarrow \quad \frac{Q_1}{r_1} + \frac{Q_2}{r_2} = \frac{x}{r_1} = \boxed{x = Q_1 + Q_2\frac{r_1}{r_2}}$$

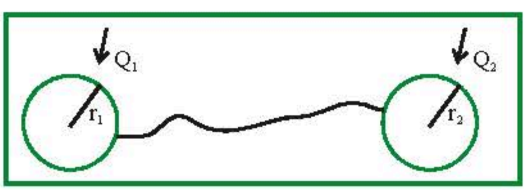
Note..

As it can be seen not all charge on the surface flows to ground. When the outermost conductor is earthed then the charge residing on the outermost surface of outer conductor will flow to ground.

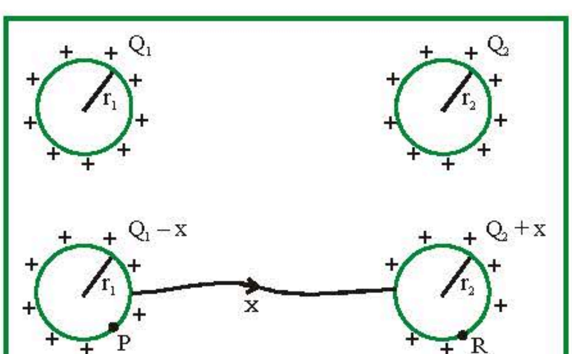
8. CONNECTION OF CHARGED CONDUCTORS

Steps

1. Do charge distribution before connection.
2. Assume 'x' charge flows from one conductor to another.
3. Do redistribution of charges.
4. Equate net potential of conductor (1) equal to net potential of conductor (2).

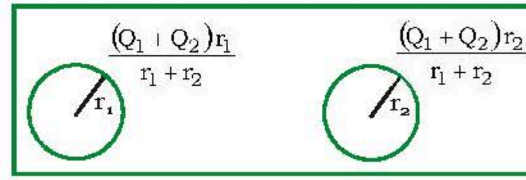


Assumption : Distance between them is very large.

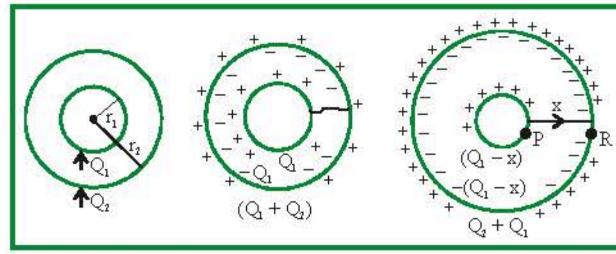


$$V_P = V_R \quad \Rightarrow \quad \frac{k(Q_1 - x)}{r_1} = \frac{k(Q_2 + x)}{r_2} = x = \frac{Q_1 r_2 - Q_2 r_1}{r_1 + r_2}$$

Final charges



$$\text{Final common potential} = k\left(\frac{Q_1 + Q_2}{r_1 + r_2}\right).$$



$V_P = V_R$

$$V_P = \frac{k(Q_1 - x)}{r_1} + \frac{k - (Q_2 - x)}{r_2} + \frac{k(Q_1 + Q_2)}{r_2}$$

$$\Rightarrow \quad V_R = \frac{k(Q_1 - x)}{r_2} + \frac{k - (Q_1 - x)}{r_2} + \frac{k(Q_1 + Q_2)}{r_2}$$

$V_P = V_R$

$$\Rightarrow \quad \frac{k(Q_1 - x)}{r_1} = \frac{k(Q_1 - x)}{r_2} \Rightarrow (Q_1 - x)\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 0$$

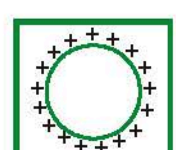
$\Rightarrow \quad x = Q_1$

This indicates that all the charge on shell (1) will flow to shell (2).

9. SELF ENERGY OF CHARGED SPHERE

Consider a uniformly charged sphere of radius R having a total charge Q. The electric potential energy of this sphere is equal to the work done in bringing the charges from infinity to assemble the sphere.

$$U = \frac{3Q^2}{20\pi\varepsilon_0 R} \qquad U = \frac{Q^2}{8\pi\varepsilon_0 R}$$



10. ENERGY DENSITY

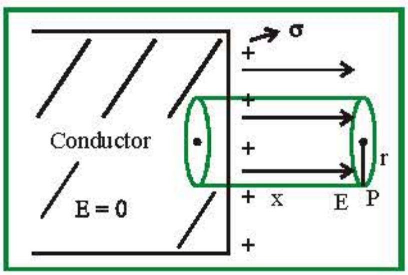
The energy stored per unit volume around a point in an electric field is given by $u_e = \dfrac{U}{\text{Volume}} = \dfrac{1}{2}\varepsilon_0 E^2$. If in place of vacuum some medium of dielectric constant K is present then $u_e = \dfrac{1}{2}K\varepsilon_0 E^2$.

11. PLATE THEORY

11.1 Charged Conducting Plate

$$\text{Net Flux} = \frac{Q_{en}}{\varepsilon_0} = \frac{\sigma\pi r^2}{\varepsilon_0} \text{ (cylindrical Gaussian surface)}$$

$$E\pi r^2 = \frac{\sigma\pi r^2}{\varepsilon_0}$$

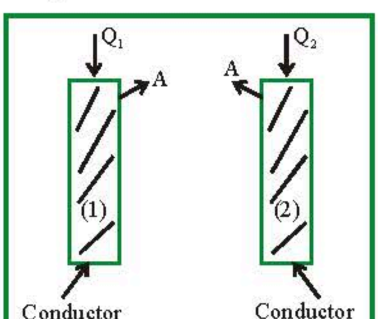


$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

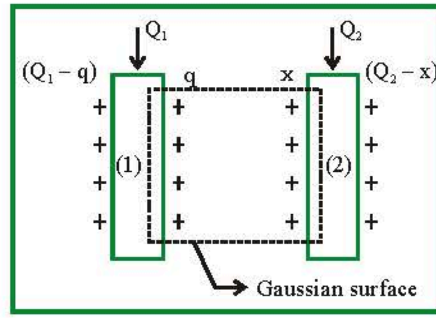
Net electric field at point P, near a conducting surface, ... σ' is given by [σ/ε_0].

11.2 Parallel Plate Theory

To find charge distribution on each surface of plates

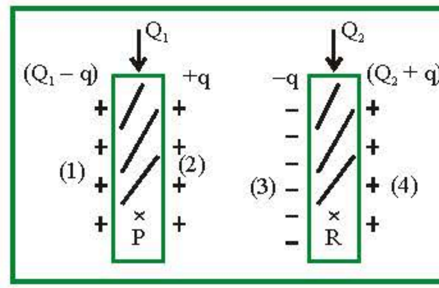


Two conducting plates having area 'A' (area is large as compared to distance, so that field is uniform) and the thickness of plates is small so that charge only appears on parallel faces.



Since the field lines are parallel, the net flux through the gaussian surface will be zero, surface (1) & (2) be inside the material of the conductor.

Hence it can be said that net charge enclosed will be zero which implies the charges appearing on the facing surfaces are equal & opposite to each other.



Net electric field at any point 'P' or 'R' has to be zero.

$(E_{net})_P = 0$

There are 4 distributions, the net field at P should be zero.

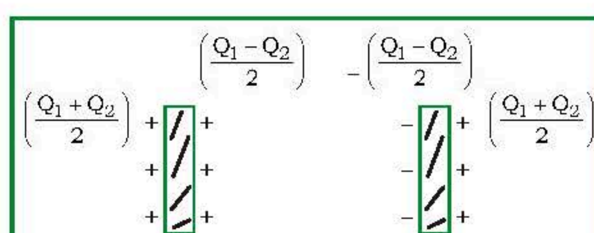
$$(E_P)_1 = \frac{\sigma}{2\varepsilon_0}(\rightarrow) = \frac{(Q_1 - q)}{2A\varepsilon_0}(\rightarrow) \quad (E_P)_2 = \frac{q}{2A\varepsilon_0}(\leftarrow)$$

$$(E_P)_3 = \frac{q}{2A\varepsilon_0}(\rightarrow) \quad (E_P)_4 = \frac{Q_2 + q}{2A\varepsilon_0}(\leftarrow)$$

$$\Rightarrow \left|(E_P)_1\right| + \left|(E_P)_3\right| = \left|(E_P)_2\right| + \left|(E_P)_4\right|$$

This shows $\dfrac{Q_1 - q}{2A\varepsilon_0} + \dfrac{q}{2A\varepsilon_0} = \dfrac{q}{2A\varepsilon_0} + \dfrac{Q_2 + q}{2A\varepsilon_0}$

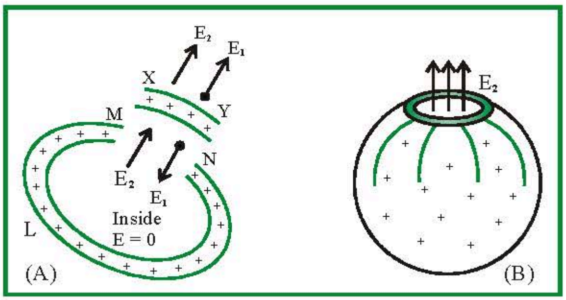
$\boxed{q = \dfrac{Q_1 - Q_2}{2}}$ so final distributions would be



> *Note..*
>
> When charged conducting plates are placed parallel to each other, the two outermost surfaces get equal charges and the facing surfaces get equal and opposite charges.

11.3 Force on a charged conductor

To find force on a charged conductor (due to repulsion of like charges) imagine a small part XY to be cut and just separated from the rest of the conductor MLN. The field in the cavity due to the rest of the conductor is E_2, while field due to small part is E_1. Then



Inside the conductor

$\ldots_1 - E_2 = 0$ or $E_1 = E_2$

Outside the conductor $E = E_1 + E_2 = \dfrac{\sigma}{\varepsilon_0}$. Thus,

$$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$$

To find force, imagine charged part XY (having charge σdA placed in the cavity MN having field E_2). Thus force

$dF = (\sigma dA)E_2$ or $dF = \dfrac{\sigma^2}{2\varepsilon_0} dA$. The force per unit area or

electric pressure is $P = \dfrac{dF}{dA} = \dfrac{\sigma^2}{2\varepsilon_0}$. (Electrostatic pressure)

The force is always outwards as $(\pm\sigma)^2$ is positive i.e., whether charged positively or negatively, this force will try to expand the charged body.

A soap bubble or rubber balloon expands on given charge to it (charge of any kind + or –).

CAPACITORS

1. CAPACITANCE

1.1 Definition

We know that charge given to a conductor increases it's potential

i.e., $Q \propto V \Rightarrow Q = CV$

Where *C* is a proportionality constant, called capacity or capacitance of conductor. Hence capacitance is the ability of conductor to hold the charge (and associated electrical energy).

1.2 Unit and dimensional formula

S.I. unit is $\dfrac{\text{Coulomb}}{\text{Volt}} = \text{Farad (F)}$

Smaller S.I. units are mF, μF, nF and pF

$\left(1\text{mF} = 10^{-3}\text{F},\ 1\mu\text{F} = 10^{-6}\text{F},\ 1\text{nF} = 10^{-9}\text{F},\ 1\text{pF} = 10^{-12}\text{F}\right)$

C.G.S. unit is Stat Farad. $1F = 9\times 10^{11}$ Stat Farad.

Dimension : $[C] = [M^{-1}L^{-2}T^4A^2]$.

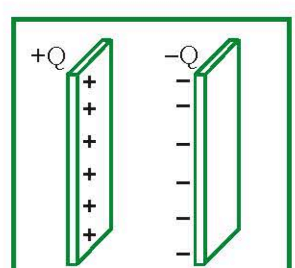
2. CAPACITOR

2.1 Definition

A capacitor is a device that stores electric energy. It is also named condenser.

or

A capacitor is a pair of two conductors of any shape, which are close to each other and have equal and opposite charge.



2.2 Symbol

The symbol of capacitor are shown below

or

2.3 Capacitance

The capacitance of a capacitor is defined as the magnitude of the charge Q on the positive plate divided by the magnitude of the potential difference V between the plates i.e., $C = Q/V$.

Note..

Capacitance of a capacitor is constant for the given dimensions & medium.

2.4 Charge on capacitor

Net charge on a capacitor is always zero, but when we speak of the charge Q on a capacitor, we are referring to the magnitude of the charge on each plate.

2.5 Energy stored

When a capacitor is charged by a voltage source (say battery) it stores the electric energy.

Energy density $= \frac{U}{\text{vol.}} = \frac{1}{2}\varepsilon_0 E^2$.

If C = Capacitance of capacitor; Q = Charge on capacitor and V = Potential difference across capacitor

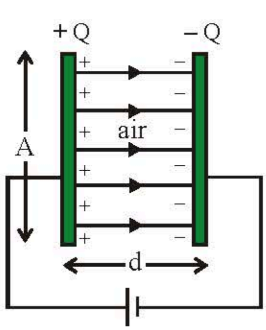
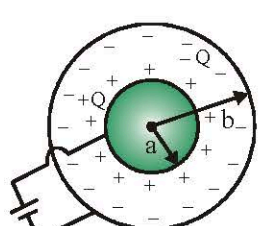
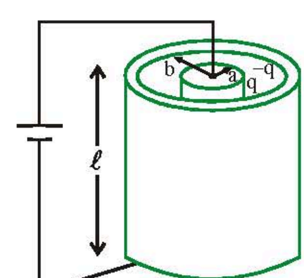
then energy stored in capacitor $U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$.

Note..

In charging capacitor by battery half the energy supplied is stored in the capacitor and remaining half energy (1/2 QV) is lost in the form of heat.

2.6 Types of capacitors

Capacitors are of mainly three types as described in given table :

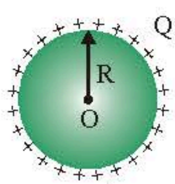
Parallel Plate Capacitor	Spherical Capacitor	Cylindrical Capacitor
It consists of two parallel metallic plates (may be circular, rectangular, square) separated by a small distance A = area of plate Q = Magnitude of charge 	It consists of two concentric conducting spheres of radii a and b ($a < b$). Inner sphere is given charge $+Q$, while outer sphere is given charge –Q [by battery] 	It consists of two concentric cylinders of radii a and b ($a < b$), inner cylinder is given charge $+Q$ while outer cylinder is given charge –Q. Common length of the cylinders is l then 

Capacitance : $C=\dfrac{\varepsilon_0 A}{d}$ σ =Surface charge density V = Potential difference E = Electric field between the plates the plates $(=\sigma/\varepsilon_0)$ In the presence of dielectric between plates $C=\dfrac{K\,\varepsilon_0\,A}{d}$	**Capacitance** $C=4\pi\varepsilon_0\dfrac{ab}{b-a}$ in C.G.S. $C=\dfrac{ab}{b-a}$. In the presence of dielectric medium (dielectric constant K) between the spheres $C'=4\pi\varepsilon_0 K\dfrac{ab}{b-a}$	**Capacitance** $C=\dfrac{2\pi\varepsilon_0\ell}{\ell n\left(\dfrac{b}{a}\right)}$ In the presence of dielectric medium (dielectric constant K) capacitance increases by K times and $C''=\dfrac{2\pi\varepsilon_0 K\ell}{\ln\left(\dfrac{b}{a}\right)}$

2.7 Capacity of an isolated spherical conductor

When charge Q is given to a spherical conductor of radius R, then potential at the surface of sphere is

$$V=\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$$



Hence it's capacity $C=\dfrac{Q}{V}=4\pi\varepsilon_0 R$

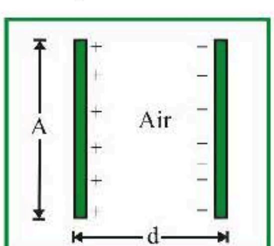
$$\Rightarrow\quad C=4\pi\varepsilon_0 R=\frac{1}{9\times10^9}\cdot R$$

in C.G.S. C = R

2.8 Force between the Plates of a Parallel Plate Capacitor

Field due to charge on one plate on the other is $E=\dfrac{\sigma}{2\varepsilon_0}$,

hence the force F = QE



$$F=-\sigma A\times\left(\frac{\sigma}{2\varepsilon_0}\right)=-\frac{\sigma^2}{2\varepsilon_0}A\Rightarrow|F|=\frac{\sigma^2 A}{2\varepsilon_0}=\frac{Q^2}{2\varepsilon_0 A}$$

3. PROPERTIES OF AN IDEAL BATTERY

(a) A battery has two terminals.

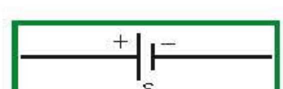
(b) The potential difference V between the terminals is constant for a given battery. The terminal with higher potential is called the positive terminal and that with lower potential is called the negative terminal.

(c) The value of this fixed potential difference is equal to the electromotive force or emf of the battery. If a conductor is connected to a terminal of a battery, the potential of the conductor becomes equal to the potential of the terminal. When the two plates of a capacitor are connected to the terminals of a battery, the potential difference between the plates of the capacitor becomes equal to the emf of the battery.

(d) The total charge in a battery always remains zero. If its positive terminal supplies a charge Q, its negative terminal supplies an equal, negative charge –Q.

(e) When a charge Q passes through a battery of emf $\mathcal{E}$ from the negative terminal to the positive terminal, an amount $Q\mathcal{E}$ of work is done by the battery.

An ideal battery is represented by the symbol shown in figure. The potential difference between the facing parallel lines is equal to the emf $\mathcal{E}$ of the battery. The longer line is at the higher potential.

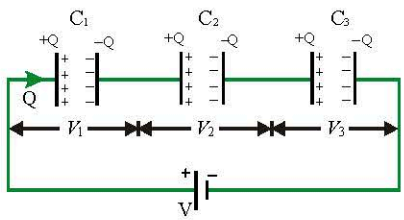


4. GROUPING OF CAPACITORS

4.1 Series grouping

(1) Charge on each capacitor remains same and equals to the main charge supplied by the battery

$V = V_1 + V_2 + V_3$



(2) Equivalent capacitance $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

(3) If two capacitors having capacitances C_1 and C_2 are connected in series then

$$V_1 = \left(\frac{C_2}{C_1 + C_2}\right).V \quad \text{and} \quad V_2 = \left(\frac{C_1}{C_1 + C_2}\right).V$$

(4) If n identical capacitors each having capacitances C are connected in series with supply voltage V then

Equivalent capacitance $C_{eq} = \frac{C}{n}$ and Potential difference across each capacitor $V' = \frac{V}{n}$.

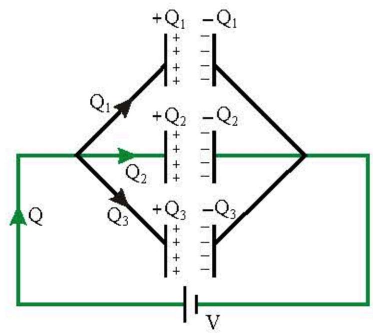
Note..

- Two capacitors are in series when charge leaving one capacitor directly enters into another capacitor, undivided and undisturbed.
- In series combination equivalent capacitance is always lesser than that of either of the individual capacitors.

4.2 Parallel grouping

(1) Potential difference across each capacitor remains same and equal to the applied potential difference

$Q = Q_1 + Q_2 + Q_3$



(2) $C_{-eq} = C_1 + C_2 + C_3$

(3) If two capacitors having capacitance C_1 and C_2 respectively are connected in parallel then

$$Q_1 = \left(\frac{C_1}{C_1 + C_2}\right).Q \quad \text{and} \quad Q_2 = \left(\frac{C_2}{C_1 + C_2}\right).Q$$

(4) If n identical capacitors are connected in parallel Equivalent capacitance $C_{eq} = nC$ and Charge on each capacitor $Q' = \frac{Q}{n}$.

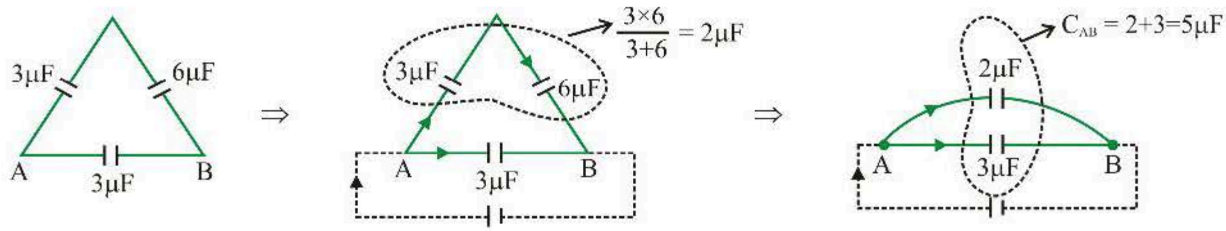
Note..

- Two capacitors are in parallel when their positive plates are connected and negative plates are also connected with each other.
- In parallel combination, equivalent capacitance is always greater than the individual capacitance.

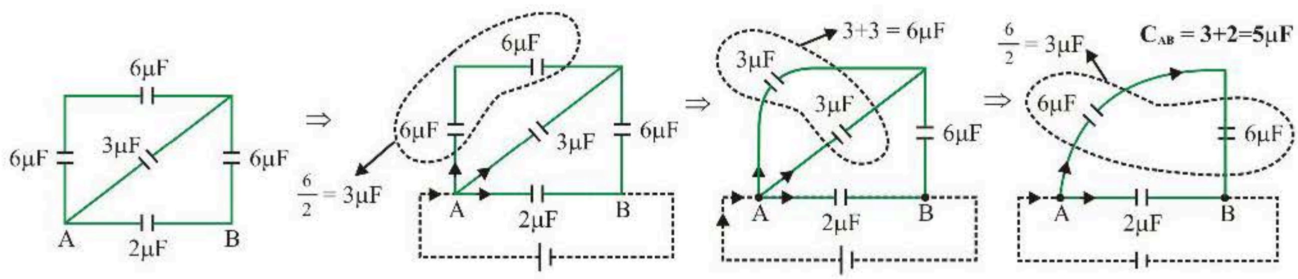
5. SIMPLE CIRCUITS (SERIES & PARALLEL)

Suppose equivalent capacitance is to be determined in the following networks between points A and B

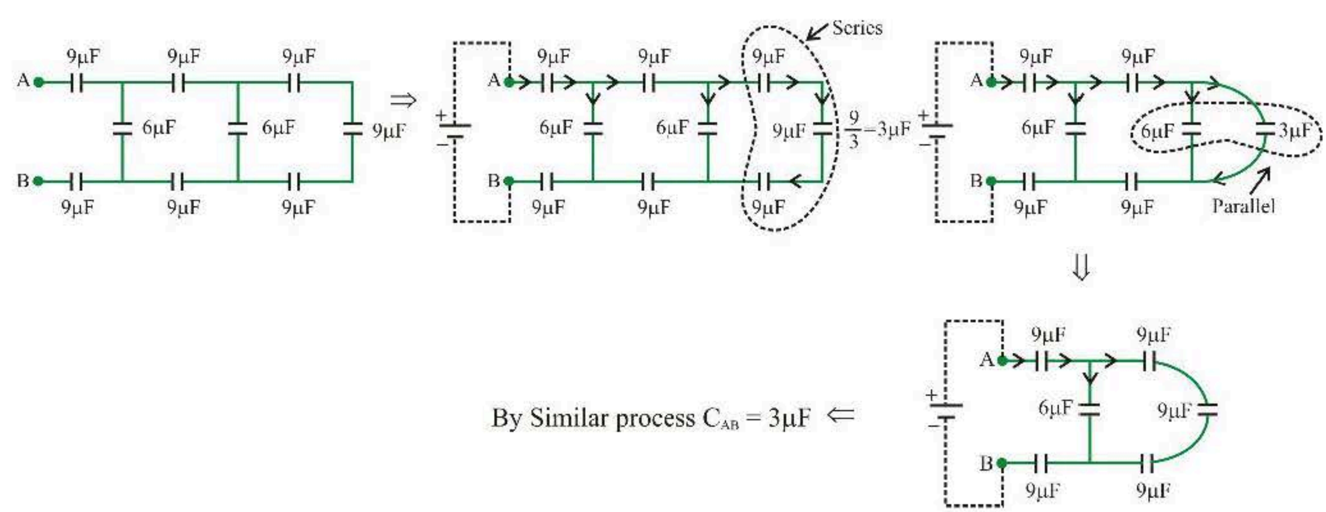
(i)



(ii)



(iii)

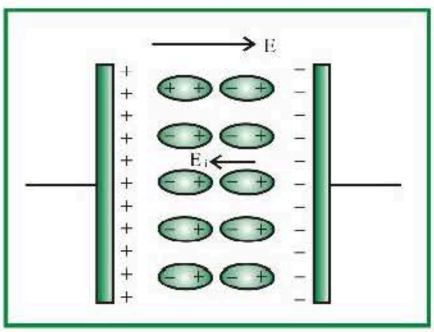


6. DIELECTRIC

Dielectrics are insulating (non-conducting) materials which transmits electric effect without conducting. We know that in every atom, there is a positively charged nucleus and a negatively charged electron cloud surrounding it. The two oppositely charged regions have their own centres of charge. The centre of positive charge is the centre of mass of positively charged protons in the nucleus. The centre of negative charge is the centre of mass of negatively charged electrons in the atoms/molecules.

6.1 Polarization of a dielectric slab

It is the process of inducing equal and opposite charges on the two faces of the dielectric on the application of electric field.



Suppose a dielectric slab is inserted between the plates of a capacitor. As shown in the figure.

Induced electric field inside the dielectric is E_i, hence this induced electric field decreases the main field E to $E - E_i$ i.e., New electric field between the plates will be $E_{net} = E - E_i$.

6.2 Dielectric constant

After placing a dielectric slab in an electric field. The net field is decreased in that region hence

If E = Original electric field and E_{net} = Net electric field. Then

$\frac{E}{E_{net}} = K$ where K is called dielectric constant K is also known as relative permittivity (ε_r) of the material.

The value of K is always greater than one. For vacuum there is no polarization and hence $E = E'$ and K = 1

$$E_i = E\left[1 - \frac{1}{k}\right] \quad \sigma_i = \sigma\left[1 - \frac{1}{k}\right]$$

6.3 Dielectric breakdown and dielectric strength

If a very high electric field is created in a dielectric, the outer electrons may get detached from their parent atoms. The dielectric then behaves like a conductor. This phenomenon is known as **dielectric breakdown.**

The maximum value of electric field (or potential gradient) that a dielectric material can tolerate without it's electric breakdown is called it's **dielectric strength.**

S.I. unit of dielectric strength of a material is V/m but practical unit is kV/mm.

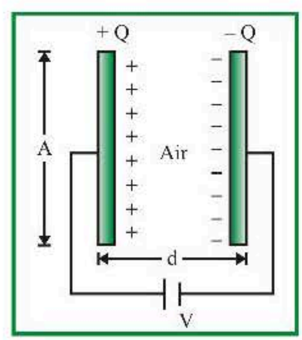
6.4 Variation of Different Variables (Q, C, V, E and U) of Parallel Plate Capacitor

Suppose we have an air filled charged parallel plate capacitor having variables as follows :

Charge : Q,

Surface charge density : $\sigma = \frac{Q}{A}$,

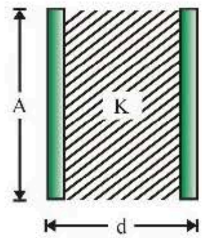
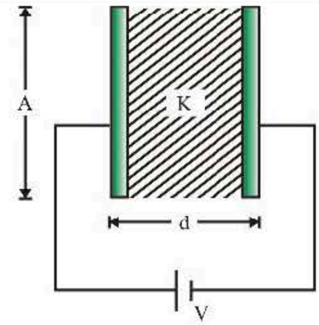
Capacitance : $C = \frac{\varepsilon_0 A}{d}$



Potential difference across the plates : V = E . d

Electric field between the plates : $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

Energy stored : $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV$

Quantity	Battery is Removed	Battery Remains connected
		
Capacity	. $'=KC$	$C'=KC$
Charge	$Q'=Q$ (Charge is conserved)	$Q'=KQ$
Potential	$V'=V/K$	$V'=V$ (Since Battery maintains the potential difference)
Intensity	$E'=E/K$	$E'=E$
Energy	$U'=U/K$	$U'=KU$

Note..

If nothing is said it is to be assumed that battery is disconnected.

7. VAN DE GRAFF ELECTROSTATIC GENERATOR

A van de graff generator is a device used for building up high potential differences of the order of a few million volts. Such high potential differences are used to accelerate charged particles like electrons, protons, ions etc. needed for various experiments of Nuclear Physics.

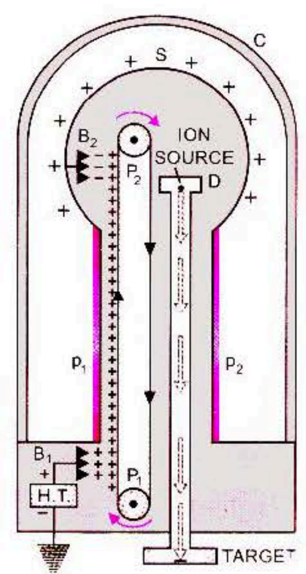
It was desighned by Van de graff in the year 1931.

Principle : This generator is based on

(i) the action of sharp points, i.e., the phenomenon of corona discharge.

(ii) the property that charge given to a hollow conductor is transferred to outer surface and is distributed uniformly over it.

Construction : The essential parts of Van de graff generator are shown in fig. S is large spherical conducting shell of radius equal to a few meters. This is supported by a conducting shell of radius equal to a few metres. This is supported at a suitable height (of several metres above the ground) over the insulating pillars p_1, p_2. A long narrow belt of insulating material like, silk, rubber or rayon is wrapped around two pulleys P_1 and P_2. P_1 is at the ground level and P_2 is at the centre of S. The belt is kept moving continuously over the pulleys with the help of a motor (not shown). B_1 and B_2 are two sharply pointed metal combs fixed as shown. B_1 is called the spray comb and B_2 is called the collecting comb.



The positive ions to be accelerated are produced in a discharge tube D. The ion source lies at the head of the tube inside the spherical shell. The other end of the tube carrying the target nucleus is earthed.

The generator is enclosed in a steel chamber C filled with nitrogen or methane at high pressure in order to minimise leakage in a steel spherical conductor.

Working : The spray comb is given a positive potential ($\approx 10^4$ volt) w.r.t. the earth by high tension source H.T. Due to discharging action of sharp points, a positively charged electric wind is set up, which sprays positive charge on the belt (corona discharge). As the belt moves, and reaches the comb., a negative charge is induced on the sharp ends of collecting comb B_2 and an equal positive charge is induced on the farther end of B_2. This positive charge shifts immediately to the outer surface of S. Due to discharging action of sharp points of B_2, a negatively charged electric wind is set up. This neutralises the positive charge on the belt. The uncharged belt returns down, collects the positive charge from B_1, which in turn is collected by B_2. This is repeated. Thus, the postive charge on S goes on accumulating.

Now, the capacity of spherical shell $c = 4\pi\varepsilon_0 R$, where R is radius of the shell.

As $$V = \frac{Q}{C} \therefore V = \frac{Q}{4\pi \epsilon_0 R}$$

Hence the potential V of the spherical shell goes on increasing with increase in Q.

The breakdown field of air is about 3×10^6 V/m. The moment the potential of spherical shell exceeds this value, air around S is ionised and leakage of charge starts. The leakage is minimised by housing the generator assembly inside a steel chamber filled with nitrogen or methane at high pressures.

If q is the charge on the ion to be accelerated and V is the potential difference developed across the ends of the discharge tube, then energy acquired by the ions = qV. The ions hit the target with this energy and carry out the artificial transmutation etc.

8. COMBINATION OF DROPS

Suppose we have *n* identical drops each having – Radius – r, Capacitance – c, Charge – q, Potential – v and Energy – u.

If these drops are combined to form a big drop of – Radius – R, Capacitance – C, Charge – Q, Potential – V and Energy – U then –

(i) Charge on big drop :

$$Q = nq$$

(ii) Radius of big drop : Volume of big drop = $n \times$ volume of a single drop *i.e.*,

$$\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3,\ R = n^{1/3}\ r$$

(iii) Capacitance of big drop :

$$C = n^{1/3}c$$

(iv) Potential of big drop :

$$V = \frac{Q}{C} = \frac{nq}{n^{1/3}c}$$

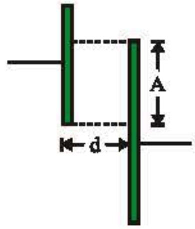
$$V = n^{2/3}v$$

(v) Energy of big drop : $U = \frac{1}{2}CV^2 = \frac{1}{2}\left(n^{1/3}c\right)\left(n^{2/3}v\right)^2$

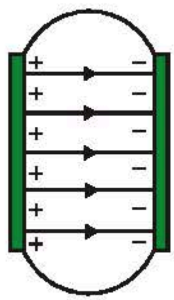
$$U = n^{5/3}\ u$$

Note..

- It is a very common misconception that a capacitor stores charge but actually a capacitor stores electric energy in the electrostatic field between the plates.
- Two plates of unequal area can also form a capacitor because effective overlapping area is considered.



- Capacitance of a parallel plate capacitor depends upon the effective overlapping area of plates $(C \propto A)$, separation between the plates $(C \propto 1/d)$ and dielectric medium filled between the plates. While it is independent of charge given, potential raised or nature of metals and thickness of plates.
- The distance between the plates is kept small to avoid fringing or edge effect (non-uniformity of the field) at the bounderies of the plates.



- Spherical conductor is equivalent to a spherical capacitor with it's outer sphere of infinite radius.
- A spherical capacitor behaves as a parallel plate capacitor if it's spherical surfaces have large radii and are close to each other.
- The intensity of electric field between the plates of a parallel plate capacitor $(E = \sigma/\varepsilon_0)$ does not depends upon the distance between them.
- Radial and non-uniform electric field exists between the spherical surfaces of spherical capacitor.

9. CHARGE DISTRIBUTION METHOD

(Circuit Solving method)

Sometimes it may not be easy to find the equivalent capacitance of a combination using the equations for series-parallel combinations. We may then use the general method as follows :

Step 1 :

Identify the two points between which the equivalent capacitance is to be calculated. Call any one of them as A and the other as B.

Step 2 :

Connect (mentally) a battery between A and B with the positive terminal connected to A and the negative terminal to B. Send a charge +Q from the positive terminal of the battery and –Q from the negative terminal of the battery.

Step 3 :

Write the charges appearing on each of the plates of the capacitors. The charge conservation principle may be used. The facing surfaces of a capacitor will always have equal and opposite charges. Assume variables Q_1, Q_2 ..... etc. for charges wherever needed. Mark the polarity across each circuit element corresponding to higher (+) & lower (–) potential ends.

Step 4 :

The algebraic sum of all the potential differences along a closed loop in a circuit is zero.

While using this rule, one starts from a point on the loop and goes along the loop, either clockwise or anticlockwise, to reach the same point again. Any potential difference encountered (from –ve to +ve) is taken to be positive and any potential drop (from +ve to –ve) is taken to be negative.

The net sum of all these potential differences should be zero.

The loop law follows directly from the fact that electrostatic force is a conservative force and the work done by it in any closed path is zero.

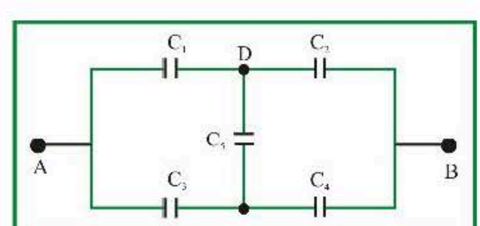
Step 5 :

Number of variables Q_1, Q_2, etc. must be the same as the number of equations obtained (loop equation). The equivalent capacitance $C_{eq} = \frac{Q}{V}$, where V is the potential difference across the assumed battery terminals.

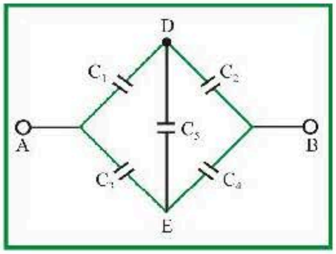
10. WHEATSTONE BRIDGE BASED CIRCUIT

If in a network five capacitors are arranged as shown in following figure, the network is called wheatstone bridge type circuit. If it is balanced then $\frac{C_1}{C_2} = \frac{C_3}{C_4}$ hence C_5 is removed and equivalent capacitance between A and B

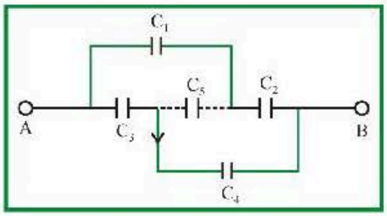
(i)



(ii)



(iii)



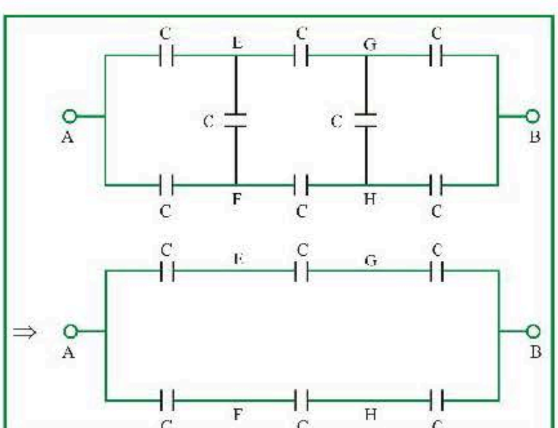
$$C_{AB} = \frac{C_1C_2}{C_1 + C_2} + \frac{C_3C_4}{C_3 + C_4}$$

11. EXTENDED WHEATSTONE BRIDGE

The given figure consists of two wheatstone bridge connected together. One bridge is connected between points AEGHFA and the other is connected between points EGBHFE.

This problem is known as extended wheatstone bridge problem, it has two branches EF and GH to the left and right of which symmetry in the ratio of capacities can be seen.

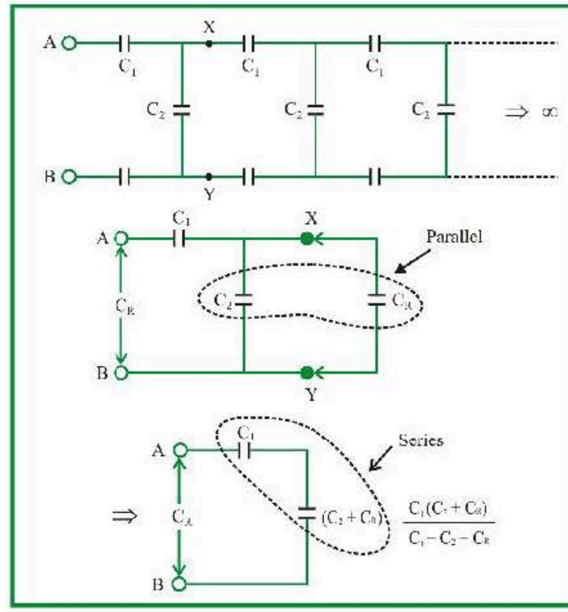
It can be seen that ratio of capacitances in branches AE and EG is same as that between the capacitances of the branches AF and FH. Thus, in the bridge AEGHFA; the branch EF can be removed. Similarly in the bridge EGBHFE branch GH can be removed



$$C_{AB} = \frac{2C}{3}$$

12. INFINITE NETWORK OF CAPACITORS

(i) Suppose the effective capacitance between A and B is C_R. Since the network is infinite, even if we remove one pair of capacitors from the chain, remaining network would still have infinite pair of capacitors, i.e., effective capacitance between X and Y would also be C_R

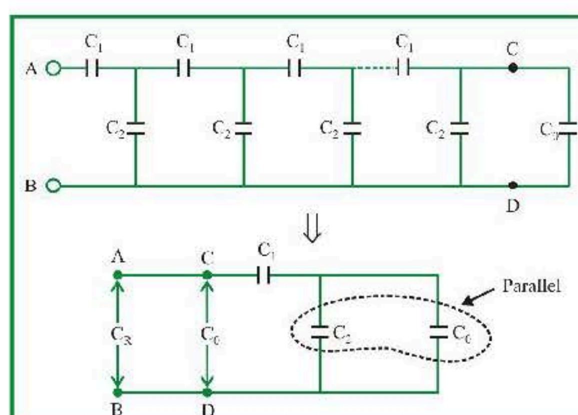


Hence equivalent capacitance between *A* and *B*

$$C_{AB} = \frac{C_1(C_2 + C_R)}{C_1 + C_2 + C_R} = C_R$$

$$\Rightarrow \quad C_{AB} = \frac{C_2}{2}\left[\sqrt{\left(1 + 4\frac{C_1}{C_2}\right)} - 1\right]$$

(ii) For what value of C_0 in the circuit shown below will the net effective capacitance between A and B be independent of the number of sections in the chain



Suppose there are n sections between A and B and the network is terminated by C_0 with equivalent capacitance C_R. Now if we add one more sections to the network between D and C (as shown in the following figure), the equivalent capacitance of the network C_R will be independent of number of sections if the capacitance between D and C still remains C_0 i.e.,

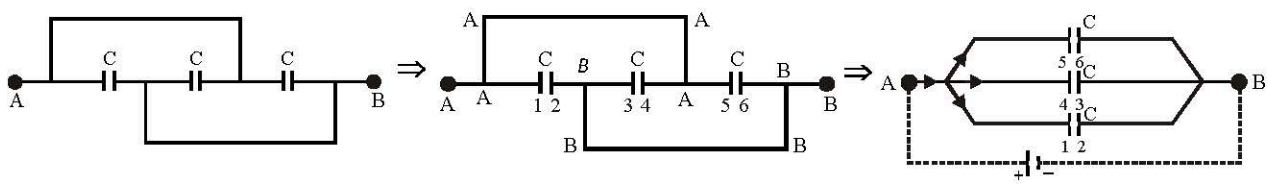
Hence $C_0 = \dfrac{C_1 \times (C_2 + C_0)}{C_1 + C_2 + C_0} \Rightarrow C_0^2 + C_2C_0 - C_1C_2 = 0$

On simplification $C_0 = \dfrac{C_2}{2}\left[\sqrt{\left(1 + 4\dfrac{C_1}{C_2}\right)} - 1\right]$

13. CIRCUITS WITH EXTRA WIRE (PLATE NUMBERING METHOD)

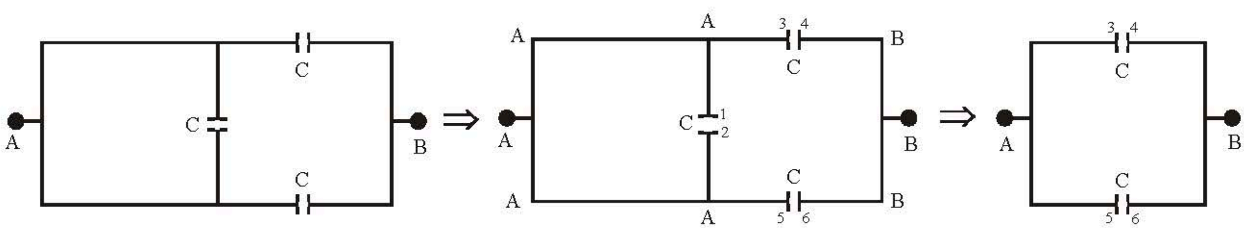
If there is no capacitor in any branch of a network then every point of this branch will be at same potential. Suppose equivalent capacitance is to be determine in following cases

(i)



$C_{AB} = 3C$

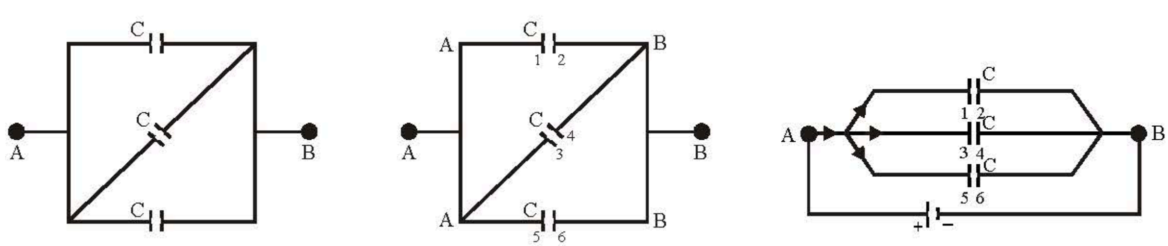
(ii)



No p.d. across vertical branch so it is removed

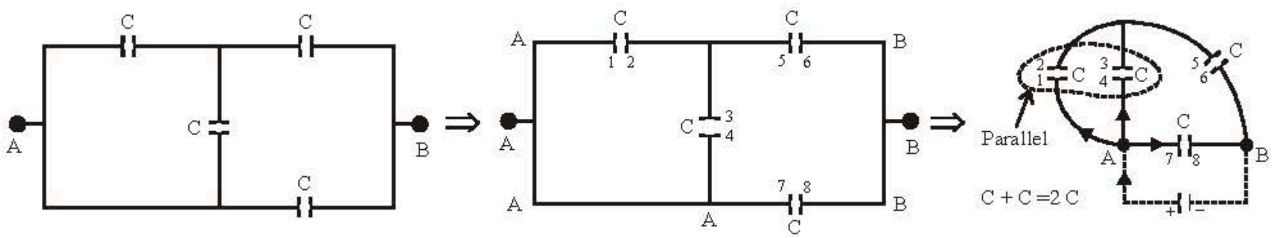
$C_{AB} = 2C$

(iii)

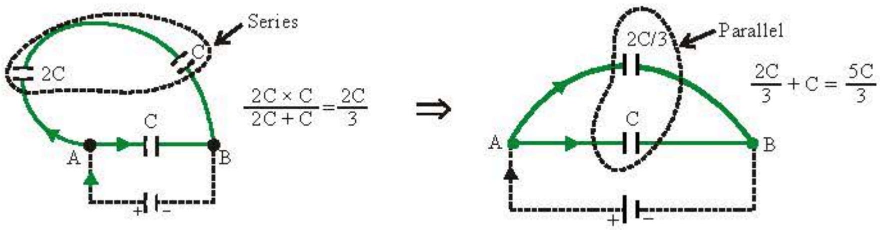


$C_{AB} = 3C$

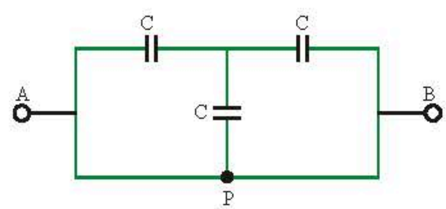
(iv)



Hence equivalent capacitance between A and B is 5C/3.



(v) . . . . *APB*, the points *A*, *P* and *B* are electrically same i.e., the input and output points are directly connected (short circuited).



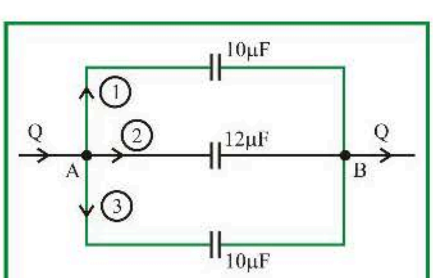
Thus, entire charge will prefer to flow along path *APB*. It means that the capacitors connected in the circuit will not receive any charge for storing. Thus equivalent capacitance of this circuit is zero.

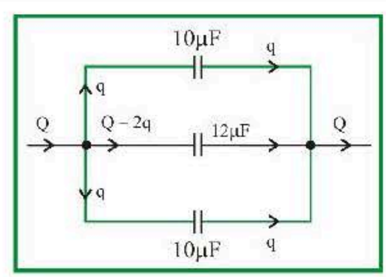
14. USING SYMMETRY BETWEEN TWO POINTS

1. Symmetry is always defined between 2 points.
2. Equivalent (symmetric) paths have same number, value and order of circuit elements along it.
3. When two or more paths in any network are equivalent, then charges flowing through those paths will be same.

   This technique makes the circuit easy to comprehend.

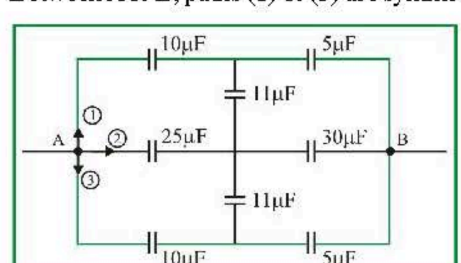
(i)





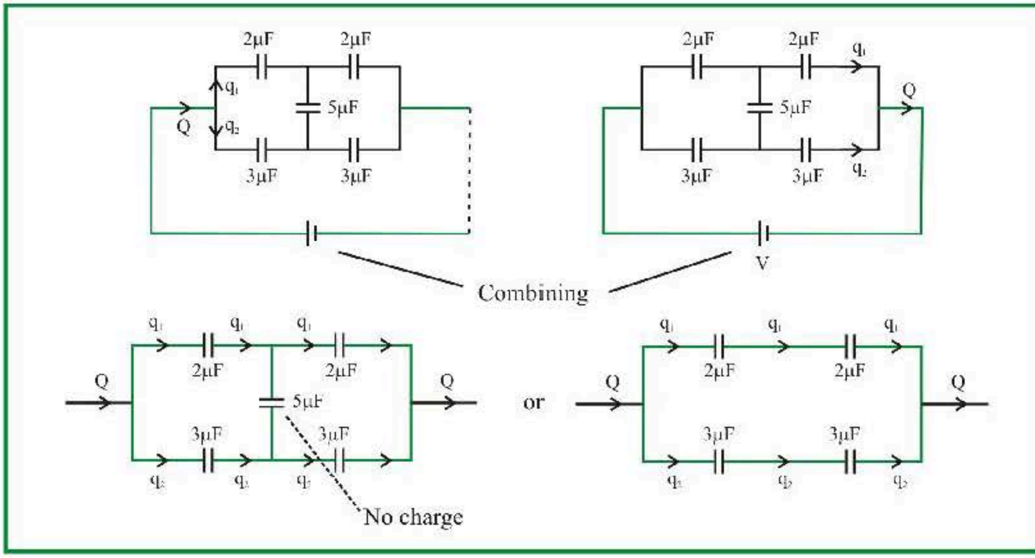
Between A & B, paths (1) & (3) are symmetric $\Rightarrow q_1 = q_3$

(ii)



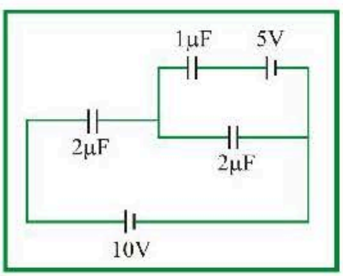
Paths (1) & (3) are symmetric between A & B thus equal charge will flow in them.

4. If the combination of elements in the network is symmetric w.r.t. battery ends, then the distribution of charge at one end will be same as is on the other end.

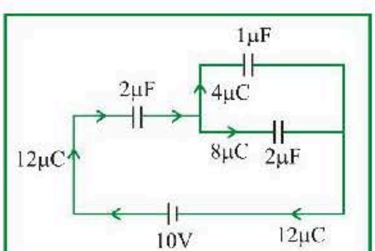


15. BATTERY SUPERPOSITION METHOD

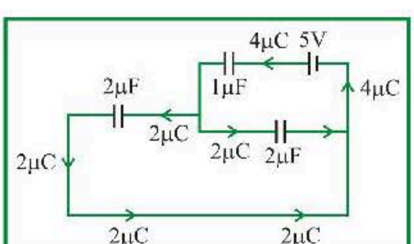
In a circuit involving multiple batteries, the charge flowing will be the superposition effect of each battery.



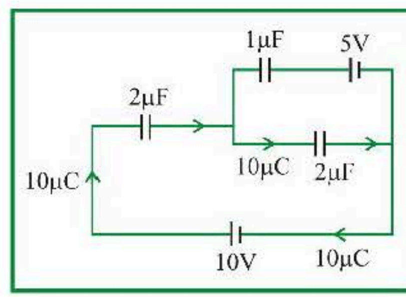
Effect of 10V battery



Effect of 5V battery



Combined effect



Note...

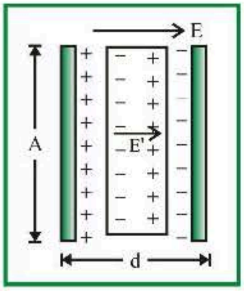
When more than 2 batteries are present, take individual effect of battery, assuming other batteries absent. And then superimpose to get total effect.

16. DIELECTRIC

16.1 Series and Parallel (with dielectrics)

(a) When dielectric is partially filled between the plates

If a dielectric slab of thickness t ($t < d$) is inserted between the plates as shown below, then E = Main electric field between the plates, E_i = Induced electric field in dielectric. $E' = (E - E_i)$ = The reduced value of electric field in the dielectric. Potential difference between the two plates of capacitor is given by

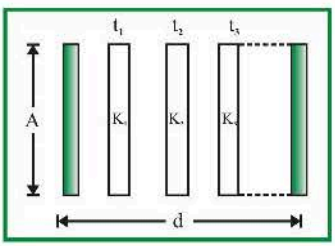


$$V' = E(d-t) + E't = E(d-t) + \frac{E}{K}\,.\,t$$

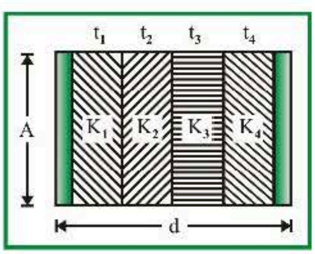
$$\Rightarrow \quad V' = E\left(d - t + \frac{t}{K}\right) = \frac{\sigma}{\varepsilon_0}\left(d - t + \frac{t}{K}\right) = \frac{Q}{A\varepsilon_0}\left(d - t + \frac{t}{K}\right)$$

Now capacitance of the capacitor

$$C' = \frac{Q}{V'} \Rightarrow C' = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

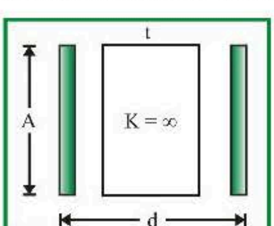


$$C' = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + t_3 + ...) + \left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} + ...\right)}$$



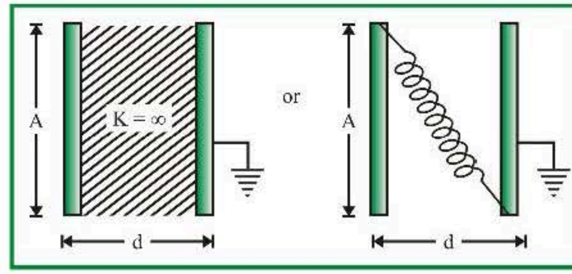
$$C' = \frac{\varepsilon_0 A}{\left(\frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} + \frac{t_4}{K_4}\right)}$$

(b) When a metallic slab is inserted between the plates



Capacitance

$$C' = \frac{\varepsilon_0 A}{(d-t)}$$

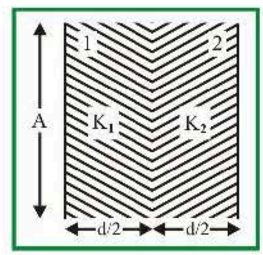


$C' = \infty$ (In this case capacitor is said to be short circuited)

(c) Advance case of compound dielectrics

If several dielectric medium filled between the plates of a parallel plate capacitor in different ways as shown.

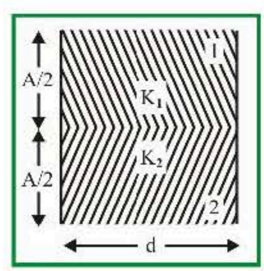
(i) The system can be assumed to be made up of two capacitors C_1 and C_2 which may be said to connected in series



$$C_1 = \frac{K_1\varepsilon_0 A}{\frac{d}{2}},\; C_2 = \frac{K_2\varepsilon_0 A}{\frac{d}{2}} \text{ and } \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\Rightarrow \quad C_{eq} = \left(\frac{2K_1K_2}{K_1 + K_2}\right).\frac{\varepsilon_0 A}{d} \quad \text{Also } K_{eq} = \frac{2K_1K_2}{K_1 + K_2}$$

(ii) In this case these two capacitors are in parallel and

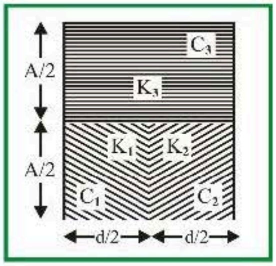


$$C_1 = \frac{K_1\varepsilon_0 A}{2d},\; C_2 = \frac{K_2\varepsilon_0 A}{2d}$$

Hence, $C_{eq} = C_1 + C_2 \Rightarrow C_{eq} = \left(\frac{K_1 + K_2}{2}\right).\frac{\varepsilon_0 A}{d}$

Also $K_{eq} = \frac{K_1 + K_2}{2}$

(iii) In this case C_1 and C_2 are in series while this combination is in parallel with C_3



$$C_1 = \frac{K_1\varepsilon_0 \frac{A}{2}}{\frac{d}{2}} = \frac{K_1\varepsilon_0 A}{d}, \quad C_2 = \frac{K_1\varepsilon_0 \frac{A}{2}}{\frac{d}{2}} = \frac{K_2\varepsilon_0 A}{d}$$

and $$C_3 = \frac{K_3\varepsilon_0 \frac{A}{2}}{d} = \frac{K_3\varepsilon_0 A}{2d}$$

Hence, $$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} + C_3 = \frac{\frac{K_1\varepsilon_0 A}{d} \times \frac{K_2\varepsilon_0 A}{d}}{\frac{k_1\varepsilon_0 A}{d} + \frac{k_2\varepsilon_0 A}{d}} + \frac{k_3\varepsilon_0 A}{2d}$$

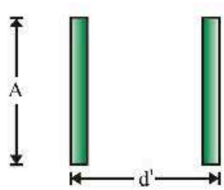
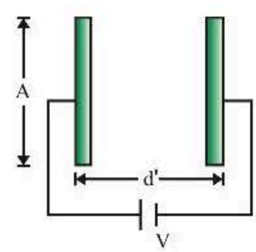
$$\Rightarrow \quad C_{eq} = \left(\frac{k_1 k_2}{k_1 + k_2} + \frac{k_3}{2}\right) \cdot \frac{\varepsilon_0 A}{d}$$

Also $$k_{eq} = \left(\frac{k_3}{2} + \frac{k_1 k_2}{k_1 + k_2}\right)$$

16.2 When separation between the plates is changing

If separation between the plates changes then it's capacitance also changes according to $C \propto \frac{1}{d}$. The effect on other variables depends on the fact that whether the charged capacitor is disconnected from the battery or battery is still connected.

(i) Separation is increasing

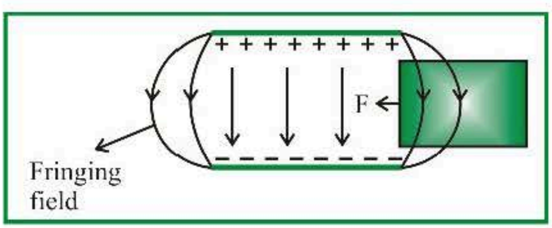
Quantity	Battery is removed	Battery remains connected
		
Capacity	Decreases because $C \propto \frac{1}{d}$ *i.e.*, $C' < C$	Decreases *i.e.*, $C' < C$
Charge	Remains constant because a battery is not present i.e., $Q' = Q$	Decreases because battery is present i.e., $Q' < Q$ Remaining charge $(Q - Q')$ goes back to the battery.
Potential difference	Increases because $V = \frac{Q}{C} \Rightarrow V \propto \frac{1}{C}$ i.e., $V' > V$	$V' = V$ (Since Battery maintains the potential difference)
Electric field	Remains constant because $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$ i.e., $E' = E$	Decrease because $E = \frac{Q}{A\varepsilon_0} \Rightarrow E \propto Q$ i.e., $E' < E$
Energy	Increases because $U = \frac{Q^2}{2C} \Rightarrow U \propto \frac{1}{C}$ i.e., $U' > U$	Decreases because $U = \frac{1}{2}CV^2 \Rightarrow U \propto C$ i.e., $U' < U$

(ii) Separation is decreasing

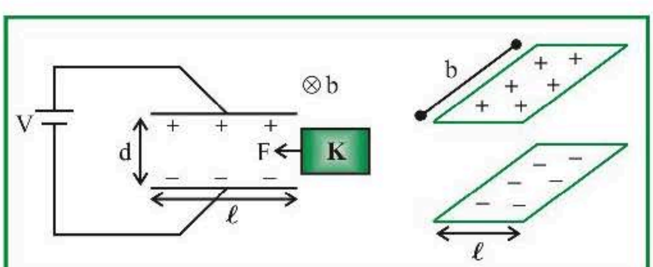
Quantity	Battery is removed	Battery remains connected
Capacity	Increases because $C \propto \frac{1}{d}$ i.e., $C' > C$	Increases i.e., $C' > C$
Charge	Remains constant because battery is not present i.e., $Q' = Q$	Increases because battery is present i.e., $Q' > Q$ Remaining charge $(Q' - Q)$ supplied from the battery.
Potential difference	Decreases because $V = \frac{Q}{C} \Rightarrow V \propto \frac{1}{C}$ i.e., $V' < V$	$V' = V$ (Since Battery maintains the potential difference)
Electric field	Remains constant because $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$ i.e., $E' = E$	Increases because $E = \frac{Q}{A\varepsilon_0} \Rightarrow E \propto Q$ i.e., $E' > E$
Energy	Decreases because $U = \frac{Q^2}{2C} \Rightarrow U \propto \frac{1}{C}$ i.e. $U' < U$	Increases because $U = \frac{1}{2}CV^2 \Rightarrow U \propto C$ i.e., $U' > U$

16.3 Force on dielectric

When dielectric is placed near the charged capacitor (rectangular plates), it experiences force towards the capacitor, due to fringing field just outside the plates.



(a) Battery connected (V remains same)

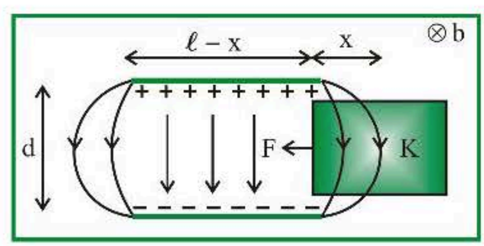


$$F = \frac{1}{2}\frac{\varepsilon_0 b V^2 (K-1)}{d} \text{ (towards capacitor)}$$

Note..

- Force doesn't depend on the amount of dielectric inside the plates.
- Force becomes zero when Dielectric is in middle of plates.

(b) Battery disconnected (Q remains same)



$$F = \frac{Q^2 d (K-1)}{2\varepsilon_0 b \left(\ell + x(K-1)\right)^2}$$

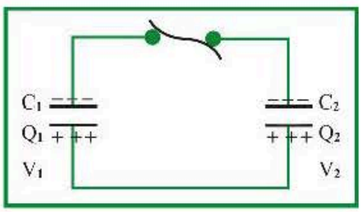
Note..

Force depends on x (amount of dielectric inside the capacitor plates).

17. REDISTRIBUTION OF CHARGE BETWEEN TWO CAPACITORS

When a charged capacitor is connected across an uncharged capacitor, then redistribution of charge occur to equalize the potential difference across each capacitor. Some energy is also wasted in the form of heat.

Suppose we have two charged capacitors C_1 and C_2 after disconnecting these two from their respective batteries. These two capacitors are connected to each other as shown below (positive plate of one capacitor is connected to positive plate of other while negative plate of one is connected to negative plate of other)



Charge on capacitors redistributed and new charge on them will be $Q'_1 = Q\left(\frac{C_1}{C_1+C_2}\right)$, $Q'_2 = Q\left(\frac{C_2}{C_1+C_2}\right)$

The common potential $V = \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1V_1+C_2V_2}{C_1+C_2}$ and loss of energy $\Delta U = \frac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2$

Note..

Two capacitors of capacitances C_1 and C_2 are charged to potential of V_1 and V_2 respectively. After disconnecting from batteries they are again connected to each other with reverse polarity i.e., positive plate of a capacitor connected to negative plate of other. So common potential $V = \frac{Q_1-Q_2}{C_1+C_2} = \frac{C_1V_1-C_2V_2}{C_1+C_2}$.