

## Chapter 10 Mechanical Properties of Fluids

### NCERT Exercises

#### Question 1:

Explain why

- (a) The blood pressure in human s is greater at the feet than at the brain
- (b) Atmospheric pressure at a height of about 6 km decreases to nearly half of its value at the sea level, though the height of the atmosphere is more than 100 km .
- (c) Hydrostatic pressure is a scalar quantity even though pressure is force divided by area.

#### Solution 1:

(a)

The pressure of a liquid is given by the relation:

$$P = h\rho g$$

Where,

$P$  = Pressure

$h$  = Height of the liquid column

$\rho$  = Density of the liquid

$g$  = Acceleration due to the gravity

It can be inferred that pressure is directly proportional to height. Hence, the blood pressure in human vessels depends on the height of the blood column in the body. The height of the blood column is more at the feet than it is at the brain. Hence, the blood pressure at the feet is more than it is at the brain.

(b)

Density of air is the maximum near the sea level. Density of air decreases with increase in height from the surface. At a height of about 6 km, density decreases to nearly half of its value at the sea level. Atmospheric pressure is proportional to density. Hence, at a height of 6 km from the surface, it decreases to nearly half of its value at the sea level.

(c)

When force is applied on a liquid, the pressure in the liquid is transmitted in all directions. Hence, hydrostatic pressure does not have a fixed direction and it is a scalar physical quantity.

#### Question 2:

Explain why

- (a) The angle of contact of mercury with glass is obtuse, while that of water with glass is acute.
- (b) Water on a clean glass surface tends to spread out while mercury on the same surface tends to form drops. (Put differently, water wets glass while mercury does not.)

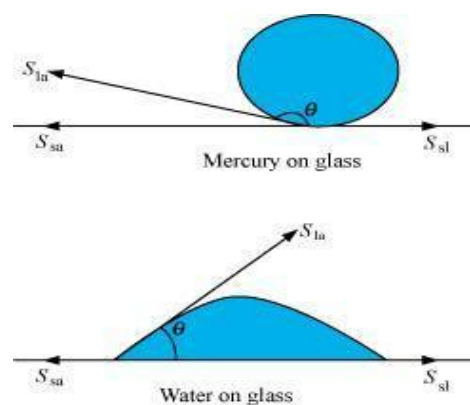
## Chapter 10

### Mechanical Properties of Fluids

- (c) Surface tension of a liquid is independent of the area of the surface  
 (d) Water with detergent dissolved in it should have small angles of contact.  
 (e) A drop of liquid under no external forces is always spherical in shape

#### Solution 2:

The angle between the tangent to the liquid surface at the point of contact and the surface inside the liquid is called the angle of contact ( $\theta$ ), as shown in the given figure.



$S_{la}$ ,  $S_{sa}$ , and  $S_{sl}$  are the respective interfacial tensions between the liquid-air, solid-air, and solid-liquid interfaces. At the line of contact, the surface forces between the three media must be in equilibrium, i.e.,

$$\cos \theta = \frac{S_{sa} - S_{sl}}{S_{la}}$$

The angle of contact  $\theta$ , is obtuse if  $S_{sa} < S_{la}$  (as in the case of mercury on glass). This angle is acute if  $S_{sl} < S_{la}$  (as in the case of water on glass).

(b)

Mercury molecules (which make an obtuse angle with glass) have a strong force of attraction between themselves and a weak force of attraction toward solids. Hence, they tend to form drops.

On the other hand, water molecules make acute angles with glass. They have a weak force of attraction between themselves and a strong force of attraction toward solids. Hence, they tend to spread out.

(c)

Surface tension is the force acting per unit length at the interface between the plane of a liquid and any other surface. This force is independent of the area of the liquid surface. Hence, surface tension is also independent of the area of the liquid surface.

(d)

## Chapter 10

### Mechanical Properties of Fluids

Water with detergent dissolved in it has small angles of contact ( $\theta$ ). This is because for a small  $\theta$ , there is a fast capillary rise of the detergent in the cloth. The capillary rise of a liquid is directly proportional to the cosine of the angle of contact ( $\theta$ ). If  $\theta$  is small, then  $\cos\theta$  will be large and the rise of the detergent water in the cloth will be fast.

(e)

A liquid tends to acquire the minimum surface area because of the presence of surface tension. The surface area of a sphere is the minimum for a given volume. Hence, under no external forces, liquid drops always take spherical shape.

#### Question 3:

Fill in the blanks using the word(s) from the list appended with each statement:

- (a) Surface tension of liquids generally . . . with temperatures (increases / decreases)
- (b) Viscosity of gases. with temperature, whereas viscosity of liquids . . . with temperature (increases / decreases)
- (c) For solids with elastic modulus of rigidity, the shearing force is proportional to . . . , while for fluids it is proportional to . . . (shear strain / rate of shear strain)
- (d) For a fluid in a steady flow, the increase in flow speed at a constriction follows (conservation of mass / Bernoulli's principle)
- (e) For the model of a plane in a wind tunnel, turbulence occurs at a ... speed for turbulence for an actual plane (greater / smaller)

#### Solution 3:

(a) decreases

The surface tension of a liquid is inversely proportional to temperature.

(b) increases; decreases

Most fluids offer resistance to their motion. This is like internal mechanical friction, known as viscosity. Viscosity of gases increases with temperature, while viscosity of liquids decreases with temperature.

(c) Shear strain; Rate of shear strain

With reference to the elastic modulus of rigidity for solids, the shearing force is proportional to the shear strain. With reference to the elastic modulus of rigidity for fluids, the shearing force is proportional to the rate of shear strain.

(d) Conservation of mass/Bernoulli's principle

For a steady-flowing fluid, an increase in its flow speed at a constriction follows the conservation of mass/Bernoulli's principle.

(e) Greater

For the model of a plane in a wind tunnel, turbulence occurs at a greater speed than it does for an actual plane. This follows from Bernoulli's principle and different Reynolds' numbers are

associated with the motions of the two planes.

#### Question 4:

Explain why

- (a) To keep a piece of paper horizontal, you should blow over, not under, it
- (b) When we try to close a water tap with our fingers, fast jets of water gush through the openings between our fingers
- (c) The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection
- (d) A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel
- (e) A spinning cricket ball in air does not follow a parabolic trajectory

#### Solution 4:

(a) When air is blown under a paper, the velocity of air is greater under the paper than it is above it. As per Bernoulli's principle, atmospheric pressure reduces under the paper. This makes the paper fall. To keep a piece of paper horizontal, one should blow over it. This increases the velocity of air above the paper. As per Bernoulli's principle, atmospheric pressure reduces above the paper and the paper remains horizontal.

(b) According to the equation of continuity:

$$\text{Area} \times \text{Velocity} = \text{Constant}$$

For a smaller opening, the velocity of flow of a fluid is greater than it is when the opening is bigger. When we try to close a tap of water with our fingers, fast jets of water gush through the openings between our fingers. This is because very small openings are left for the water to flow out of the pipe. Hence, area and velocity are inversely proportional to each other.

(c) The small opening of a syringe needle controls the velocity of the blood flowing out. This is because of the equation of continuity. At the constriction point of the syringe system, the flow rate suddenly increases to a high value for a constant thumb pressure applied.

(d) When a fluid flows out from a small hole in a vessel, the vessel receives a backward thrust. A fluid flowing out from a small hole has a large velocity according to the equation of continuity:

$$\text{Area} \times \text{Velocity} = \text{Constant}$$

According to the law of conservation of momentum, the vessel attains a backward velocity because there are no external forces acting on the system.

(e) A spinning cricket ball has two simultaneous motions – rotatory and linear. These two types of motion oppose the effect of each other. This decreases the velocity of air flowing below the ball. Hence, the pressure on the upper side of the ball becomes lesser than that on the lower side. An upward force acts upon the ball. Therefore, the ball takes a curved path. It does not follow a parabolic path.

**Question 5:**

A 50 kg girl wearing high heel shoes balances on a single heel. The heel is circular with a diameter 1.0 cm. What is the pressure exerted by the heel on the horizontal floor?

**Solution 5 :**

Mass of the girl,  $m = 50$  kg

Diameter of the heel,  $d = 1$  cm = 0.01 m

Radius of the heel,  $r = \frac{d}{2} = 0.005$  m

Area of the heel =  $\pi r^2$

$$= \pi (0.005)^2$$

$$= 7.85 \times 10^{-5} \text{ m}^2$$

Force exerted by the heel on the floor:

$$F = mg$$

$$= 50 \times 9.8$$

$$= 490 \text{ N}$$

Pressure exerted by the heel on the floor:

$$P = \frac{\text{Force}}{\text{Area}}$$

$$= \frac{490}{7.85 \times 10^{-5}}$$

$$= 6.24 \times 10^6 \text{ N m}^{-2}$$

Therefore, the pressure exerted by the heel on the horizontal floor is

$$6.24 \times 10^6 \text{ Nm}^{-2}.$$

**Question 6:**

Toricelli's barometer used mercury. Pascal duplicated it using French wine of density  $984 \text{ kg m}^{-3}$ . Determine the height of the wine column for normal atmospheric pressure.

**Solution 6:**

Density of mercury,  $\rho_1 = 13.6 \times 10^3 \text{ kg / m}^3$

Height of the mercury column,  $h_1 = 0.76 \text{ m}$

Density of French wine,  $\rho_2 = 984 \text{ kg / m}^3$

Height of the French wine column =  $h_2$

Acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$

The pressure in both the columns is equal, i.e.,

Pressure in the mercury column = Pressure in the French wine column

$$\rho_1 h_1 g = \rho_2 h_2 g$$

$$h_2 = \frac{\rho_1 h_1}{\rho_2}$$

$$= \frac{13.6 \times 10^3 \times 0.76}{984}$$

$$= 10.5 \text{ m}$$

Hence, the height of the French wine column for normal atmospheric pressure is 10.5 m.

#### Question 7 :

A vertical off-shore structure is built to withstand a maximum stress of  $10^9 \text{ Pa}$ . Is the structure suitable for putting up on top of an oil well in the ocean? Take the depth of the ocean to be roughly 3 km, and ignore ocean currents.

#### Solution 7:

**Answer:** Yes

The maximum allowable stress for the structure,  $P = 10^9 \text{ Pa}$

Depth of the ocean,  $d = 3 \text{ km} = 3 \times 10^3 \text{ m}$

Density of water,  $\rho = 10^3 \text{ kg/m}^3$

Acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$

The pressure exerted because of the sea water at depth,  $d = \rho dg$

$$= 3 \times 10^3 \times 10^3 \times 9.8$$

$$= 2.94 \times 10^7 \text{ Pa}$$

The maximum allowable stress for the structure ( $10^9 \text{ Pa}$ ) is greater than the pressure of the sea water ( $2.94 \times 10^7 \text{ Pa}$ ). The pressure exerted by the ocean is less than the pressure that the structure can withstand. Hence, the structure is suitable for putting up on top of an oil well in the ocean.

#### Question 8:

A hydraulic automobile lift is designed to lift cars with a maximum mass of 3000 kg. The area of cross-section of the piston carrying the load is  $425 \text{ cm}^2$ . What maximum pressure

would the smaller piston have to bear?

#### Solution 8:

The maximum mass of a car that can be lifted,  $m = 3000 \text{ kg}$

Area of cross-section of the load-carrying piston,  $A = 425 \text{ cm}^2 = 425 \times 10^{-4} \text{ m}^2$

The maximum force exerted by the load,  $F = mg$

$$= 3000 \times 9.8$$

$$= 29400 \text{ N}$$

The maximum pressure exerted on the load-carrying piston,  $P = \frac{F}{A}$

$$= \frac{29400}{425 \times 10^{-4}}$$

$$= 6.917 \times 10^5 \text{ Pa}$$

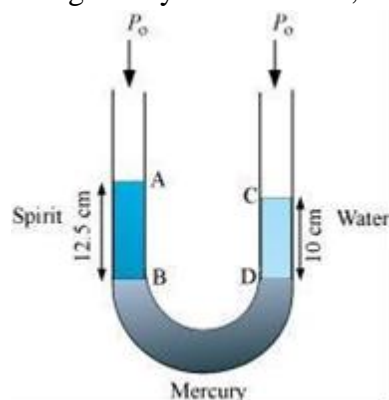
Pressure is transmitted equally in all directions in a liquid. Therefore, the maximum pressure that the smaller piston would have to bear is  $6.917 \times 10^5 \text{ Pa}$

#### Question 9:

A U-tube contains water and methylated spirit separated by mercury. The mercury columns in the two arms are in level with 10.0 cm of water in one arm and 12.5 cm of spirit in the other. What is the specific gravity of spirit?

#### Solution 9:

The given system of water, mercury, and methylated spirit is shown as follows:



Height of the spirit column,  $h_1 = 12.5 \text{ cm} = 0.125 \text{ m}$

Height of the water column,  $h_2 = 10 \text{ cm} = 0.1 \text{ m}$

$P_0$  = Atmospheric pressure

$\rho_1$  = Density of spirit

$\rho_2$  = Density of water

Pressure at point B =  $P_0 + h_1\rho_1g$

Pressure at point D =  $P_0 + h_2\rho_2g$

Pressure at points B and D is the same.

$$P_0 + h_1\rho_1g = h_2\rho_2g$$

$$\frac{\rho_1}{\rho_2} = \frac{h_2}{h_1}$$

$$= \frac{10}{12.5} = 0.8$$

Therefore, the specific gravity of spirit is 0.8.

**Question 10:**

In problem 10.9, if 15.0 cm of water and spirit each are further poured into the respective arms of the tube, what is the difference in the levels of mercury in the two arms? (Specific gravity of mercury = 13.6)

**Solution 10:**

Height of the water column,  $h_1 = 10 + 15 = 25 \text{ cm}$

Height of the spirit column,  $h_2 = 12.5 + 15 = 27.5 \text{ cm}$

Density of water,  $\rho_1 = 1 \text{ g cm}^{-3}$

Density of spirit,  $\rho_2 = 0.8 \text{ g cm}^{-3}$

Density of mercury =  $13.6 \text{ g cm}^{-3}$

Let  $h$  be the difference between the levels of mercury in the two arms.

Pressure exerted by height  $h$ , of the mercury column:

$$= h\rho g$$

$$= h \times 13.6g \dots (i)$$

Difference between the pressures exerted by water and spirit:

$$h_1\rho_1g - h_2\rho_2g$$

$$= g(25 \times 1 - 27.5 \times 0.8)$$



## Chapter 10

### Mechanical Properties of Fluids

$=3g \dots (ii)$

Equating equations (i) and (ii), we get:

$$13.6 hg = 3g$$

$$h = 0.220588 \approx 0.221 \text{ cm}$$

Hence, the difference between the levels of mercury in the two arms is 0.221 cm.

#### Question 11:

Can Bernoulli's equation be used to describe the flow of water through a rapid in a river? Explain.

#### Solution 11:

No

Bernoulli's equation cannot be used to describe the flow of water through a rapid in a river because of the turbulent flow of water. This principle can only be applied to a streamline flow.

#### Question 12:

Does it matter if one uses gauge instead of absolute pressures in applying Bernoulli's equation? Explain.

#### Solution 12:

No, It does not matter if one uses gauge pressure instead of absolute pressure while applying Bernoulli's equation. The two points where Bernoulli's equation is applied should have significantly different atmospheric pressures.

#### Question 13:

Glycerine flows steadily through a horizontal tube of length 1.5 m and radius 1.0 cm. If the amount of glycerine collected per second at one end is  $4.0 \times 10^{-3} \text{ kg s}^{-1}$ , what is the pressure difference between the two ends of the tube? (Density of glycerine =  $1.3 \times 10^3 \text{ kg m}^{-3}$  and viscosity of glycerine = 0.83 Pa s). [You may also like to check if the assumption of laminar flow in the tube is correct].

#### Solution 13:

Length of the horizontal tube,  $l = 1.5 \text{ m}$

Radius of the tube,  $r = 1 \text{ cm} = 0.01 \text{ m}$

Diameter of the tube,  $d = 2r = 0.02 \text{ m}$

Glycerine is flowing at a rate of  $4.0 \times 10^{-3} \text{ kg s}^{-1}$ .

$$M = 4.0 \times 10^{-3} \text{ kg s}^{-1}$$

Density of glycerine,  $\rho = 1.3 \times 10^3 \text{ kg m}^{-3}$

Viscosity of glycerine,  $\eta = 0.83 \text{ Pa s}$

Volume of glycerine flowing per sec:

$$\begin{aligned} V &= \frac{M}{\rho} \\ &= \frac{4.0 \times 10^{-3}}{1.3 \times 10^3} \\ &= 3.08 \times 10^{-6} \text{ m}^3 \text{ s}^{-1} \end{aligned}$$

According to Poiseville's formula, we have the relation for the rate of flow:

$$V = \frac{\pi p r^4}{8 \eta l}$$

Where,  $p$  is the pressure difference between the two ends of the tube

$$\begin{aligned} \therefore p &= \frac{V 8 \eta l}{\pi r^4} \\ &= \frac{3.08 \times 10^{-6} \times 8 \times 0.83 \times 1.5}{\pi \times (0.01)^4} \\ &= 9.8 \times 10^2 \text{ Pa} \end{aligned}$$

Reynolds' number is given by the relation:

$$\begin{aligned} R &= \frac{4 \rho V}{\pi d \eta} \\ &= \frac{4 \times 1.3 \times 10^3 \times 3.08 \times 10^{-6}}{\pi \times (0.02) \times 0.83} = 0.3 \end{aligned}$$

Reynolds' number is about 0.3. Hence, the flow is laminar.

#### Question 14:

In a test experiment on a model aero plane in a wind tunnel, the flow speeds on the upper and lower surfaces of the wing are  $70 \text{ m s}^{-1}$  and  $63 \text{ m s}^{-1}$  respectively. What is the lift on the wing if its area is  $2.5 \text{ m}^2$ ? Take the density of air to be  $1.3 \text{ kg m}^{-3}$ .

#### Solution 14:

Speed of wind on the upper surface of the wing,  $V_1 = 70 \text{ m/s}$

Speed of wind on the lower surface of the wing,  $V_2 = 63 \text{ m/s}$

Area of the wing,  $A = 2.5 \text{ m}^2$

Density of air,  $\rho = 1.3 \text{ kg m}^{-3}$

According to Bernoulli's theorem, we have the relation:

$$P_1 + \frac{1}{2} \rho V_1^2 = P_2 + \frac{1}{2} \rho V_2^2$$

$$P_2 - P_1 = \frac{1}{2} \rho (V_1^2 - V_2^2)$$

Where,

$P_1$  = Pressure on the upper surface of the wing

$P_2$  = Pressure on the lower surface of the wing

The pressure difference between the upper and lower surfaces of the wing provides lift to the aeroplane.

$$\text{Lift on the wing} = (P_2 - P_1)A$$

$$= \frac{1}{2} \rho (V_1^2 - V_2^2) A$$

$$= \frac{1}{2} \times 1.3 \left( (70)^2 - (63)^2 \right) \times 2.5$$

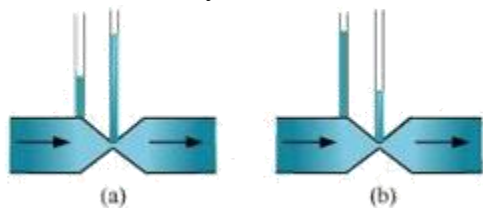
$$= 1512.87$$

$$= 1.51 \times 10^3 \text{ N}$$

Therefore, the lift on the wing of the aeroplane is  $1.51 \times 10^3 \text{ N}$ .

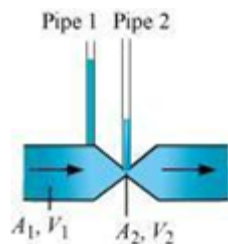
#### Question 15:

Figures (a) and (b) refer to the steady flow of a (non-viscous) liquid. Which of the two figures is incorrect? Why?



#### Solution 15:

Take the case given in figure (b).



Where,

$A_1$  = Area of pipe 1

$A_2$  = Area of pipe 2

$V_1$  = Speed of the fluid in pipe 1

$V_2$  = Speed of the fluid in pipe 2

From the law of continuity, we have:

$$A_1V_1 = A_2V_2$$

When the area of cross-section in the middle of the venturimeter is small, the speed of the flow of liquid through this part is more. According to Bernoulli's principle, if speed is more, then pressure is less. Pressure is directly proportional to height. Hence, the level of water in pipe 2 is less.

Therefore, figure (a) is not possible.

#### Question 16:

The cylindrical tube of a spray pump has a cross-section of  $8.0 \text{ cm}^2$  one end of which has 40 fine holes each of diameter  $1.0 \text{ mm}$ . If the liquid flow inside the tube is  $1.5 \text{ m min}^{-1}$ , what is the speed of ejection of the liquid through the holes?

#### Solution 16:

Area of cross-section of the spray pump,  $A_1 = 8 \text{ cm}^2 = 8 \times 10^{-4} \text{ m}^2$

Number of holes,  $n = 40$

Diameter of each hole,  $d = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$

Radius of each hole,  $r = d/2 = 0.5 \times 10^{-3} \text{ m}$

Area of cross-section of each hole,  $a = \pi r^2 = \pi (0.5 \times 10^{-3})^2 \text{ m}^2$

Total area of 40 holes,  $A_2 = n \times a$

$$= 40 \times \pi (0.5 \times 10^{-3})^2 \text{ m}^2$$

$$= 31.41 \times 10^{-6} \text{ m}^2$$

Speed of flow of liquid inside the tube,  $V_1 = 1.5 \text{ m/min} = 0.025 \text{ m/s}$

Speed of ejection of liquid through the holes =  $V_2$

According to the law of continuity, we have:

$$\begin{aligned}
 A_1 V_1 &= A_2 V_2 \\
 V_2 &= \frac{A_1 V_1}{A_2} \\
 &= \frac{8 \times 10^{-4} \times 0.025}{31.61 \times 10^{-6}} \\
 &= 0.633 \text{ m/s}
 \end{aligned}$$

Therefore, the speed of ejection of the liquid through the holes is 0.633 m/s.

#### Question 17:

A U-shaped wire is dipped in a soap solution, and removed. The thin soap film formed between the wire and the light slider supports a weight of  $1.5 \times 10^{-2} \text{ N}$ . (which includes the small weight of the slider). The length of the slider is 30 cm. What is the surface tension of the film?

#### Solution 17:

The weight that the soap film supports,  $W = 1.5 \times 10^{-2} \text{ N}$

Length of the slider,  $l = 30 \text{ cm} = 0.3 \text{ m}$

A soap film has two free surfaces.

∴ Total length =  $2l = 2 \times 0.3 = 0.6 \text{ m}$

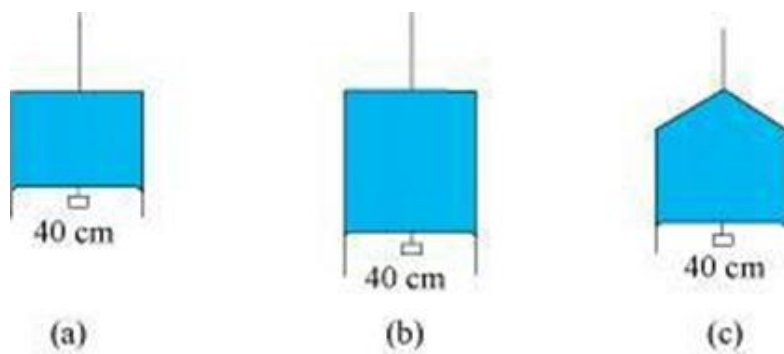
Surface tension,  $S = \frac{\text{Force or Weight}}{2l}$

$$= \frac{1.5 \times 10^{-2}}{0.6} = 2.5 \times 10^{-2} \text{ N/m}$$

Therefore, the surface tension of the film is  $2.5 \times 10^{-2} \text{ N m}^{-1}$ .

#### Question 18:

Figure 10.24 (a) shows a thin liquid film supporting a small weight =  $4.5 \times 10^{-2} \text{ N}$ . What is the weight supported by a film of the same liquid at the same temperature in Fig. (b) and (c)? Explain your answer physically.

**Solution 18:**

Take case (a):

The length of the liquid film supported by the weight,  $l = 40 \text{ cm} = 0.4 \text{ m}$

The weight supported by the film,  $W = 4.5 \times 10^{-2} \text{ N}$

A liquid film has two free surfaces.

$$\begin{aligned} \therefore \text{Surface tension} &= \frac{W}{2l} \\ &= \frac{4.5 \times 10^{-2}}{2 \times 0.4} = 5.625 \times 10^{-2} \text{ Nm}^{-1} \end{aligned}$$

In all the three figures, the liquid is the same. Temperature is also the same for each case.

Hence, the surface tension in figure (b) and figure (c) is the same as in figure

(a), i.e.,  $5.625 \times 10^{-2} \text{ Nm}^{-1}$ .

Since the length of the film in all the cases is 40 cm, the weight supported in each case is

$4.5 \times 10^{-2} \text{ N}$ .

**Question 19:**

What is the pressure inside the drop of mercury of radius 3.00 mm at room temperature?

Surface tension of mercury at that temperature ( $20^\circ\text{C}$ ) is  $4.65 \times 10^{-1} \text{ N m}^{-1}$ . The

atmospheric pressure is  $1.01 \times 10^5 \text{ Pa}$ . Also give the excess pressure inside the drop.

**Solution 19:**

Radius of the mercury drop,  $r = 3.00 \text{ mm} = 3 \times 10^{-3} \text{ m}$

Surface tension of mercury,  $S = 4.65 \times 10^{-1} \text{ N m}^{-1}$

Atmospheric pressure,  $P_0 = 1.01 \times 10^5 \text{ Pa}$

Total pressure inside the mercury drop

= Excess pressure inside mercury + Atmospheric pressure

$$= \frac{2S}{r} + P_0$$

$$= \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} + 1.01 \times 10^5$$

$$= 1.0131 \times 10^5$$

$$= 1.01 \times 10^5 \text{ Pa}$$

$$\text{Excess pressure} = \frac{2S}{r}$$

$$= \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} = 310 \text{ Pa}$$

#### Question 20:

What is the excess pressure inside a bubble of soap solution of radius 5.00 mm, given that the surface tension of soap solution at the temperature (20 °C) is  $2.50 \times 10^{-2} \text{ N m}^{-1}$ ? If an air bubble of the same dimension were formed at depth of 40.0 cm inside a container containing the soap solution (of relative density 1.20), what would be the pressure inside the bubble? (1 atmospheric pressure is  $1.01 \times 10^5 \text{ Pa}$ ).

#### Solution 20:

Excess pressure inside the soap bubble is 20 Pa;

Pressure inside the air bubble is  $1.06 \times 10^5 \text{ Pa}$

Soap bubble is of radius,  $r = 5.00 \text{ mm} = 5 \times 10^{-3} \text{ m}$

Surface tension of the soap solution,  $S = 2.50 \times 10^{-2} \text{ N m}^{-1}$

Relative density of the soap solution = 1.20

∴ Density of the soap solution,  $\rho = 1.2 \times 10^3 \text{ kg / m}^3$

Air bubble formed at a depth,  $h = 40 \text{ cm} = 0.4 \text{ m}$

Radius of the air bubble,  $r = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$

1 atmospheric pressure =  $1.01 \times 10^5 \text{ Pa}$

Acceleration due to gravity,  $g = 9.8 \text{ m / s}^2$

Hence, the excess pressure inside the soap bubble is given by the relation:

$$P = \frac{4S}{r}$$

$$= \frac{4 \times 2.5 \times 10^{-2}}{5 \times 10^{-3}}$$

$$= 20 Pa$$

Therefore, the excess pressure inside the soap bubble is 20 Pa. The excess pressure inside the air bubble is given by the relation:

$$P = \frac{2S}{r}$$

$$= \frac{2 \times 2.5 \times 10^{-2}}{5 \times 10^{-3}}$$

$$= 10 Pa$$

Therefore, the excess pressure inside the air bubble is 10 Pa.

At a depth of 0.4 m, the total pressure inside the air bubble

$$= \text{Atmospheric pressure} + h\rho g + P$$

$$= 1.01 \times 10^5 + 0.4 \times 1.2 \times 10^3 \times 9.8 + 10$$

$$= 1.057 \times 10^5 Pa$$

$$= 1.06 \times 10^5 Pa$$

Therefore, the pressure inside the air bubble is  $= 1.06 \times 10^5 Pa$

### Additional Exercise

#### Question 21:

A tank with a square base of area  $1.0 m^2$  is divided by a vertical partition in the middle. The bottom of the partition has a small-hinged door of area  $20 cm^2$ . The tank is filled with water in one compartment, and an acid (of relative density 1.7) in the other, both to a height of 4.0 m. compute the force necessary to keep the door close.

#### Solution 21:

Base area of the given tank,  $A = 1.0 m^2$

Area of the hinged door,  $a = 20 cm^2 = 20 \times 10^{-4} m^2$

Density of water,  $\rho_1 = 10^3 kg/m^3$

Density of acid,  $\rho_2 = 1.7 \times 10^3 kg/m^3$

Height of the water column,  $h_1 = 4 m$

Height of the acid column,  $h_2 = 4 m$

Acceleration due to gravity,  $g = 9.8$



Pressure due to water is given as:

$$\begin{aligned}
 P_1 &= h_1 \rho_1 g \\
 &= 4 \times 10^3 \times 9.8 \\
 &= 3.92 \times 10^4 \text{ Pa}
 \end{aligned}$$

Pressure due to acid is given as:

$$\begin{aligned}
 P_2 &= h_2 \rho_2 g \\
 &= 4 \times 1.7 \times 10^3 \times 9.8 \\
 &= 6.664 \times 10^4 \text{ Pa}
 \end{aligned}$$

Pressure difference between the water and acid columns:

$$\begin{aligned}
 \Delta P &= P_2 - P_1 \\
 &= 6.664 \times 10^4 - 3.92 \times 10^4 \\
 &= 2.744 \times 10^4 \text{ Pa}
 \end{aligned}$$

Hence, the force exerted on the door =  $P \times a$

$$\begin{aligned}
 &= 2.744 \times 10^4 \times 20 \times 10^{-4} \\
 &= 54.88 \text{ N}
 \end{aligned}$$

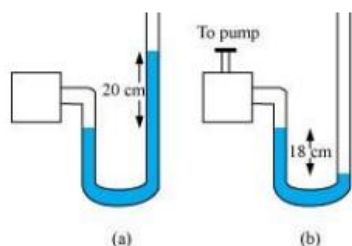
Therefore, the force necessary to keep the door closed is 54.88 N.

#### Question 22:

A manometer reads the pressure of a gas in an enclosure as shown in figure (a) When a pump removes some of the gas, the manometer reads as in figure (b) The liquid used in the manometers is mercury and the atmospheric pressure is 76 cm of mercury.

(a) Give the absolute and gauge pressure of the gas in the enclosure for cases (a) and (b), in units of cm of mercury.

(b) How would the levels change in case (b) if 13.6 cm of water (immiscible with mercury) are poured into the right limb of the manometer? (Ignore the small change in the volume of the gas).



#### Solution 22:

(a)

For figure (a)

Atmospheric pressure,  $P_0 = 76 \text{ cm of Hg}$

Difference between the levels of mercury in the two limbs gives gauge pressure.

Hence, gauge pressure is 20 cm of Hg.

Absolute pressure = Atmospheric pressure + Gauge pressure =  $76 + 20 = 96$   
cm of Hg

For figure (b)

Difference between the levels of mercury in the two limbs =  $-18 \text{ cm}$  Hence, gauge pressure is  $-18 \text{ cm of Hg}$ .

Absolute pressure = Atmospheric pressure + Gauge pressure =  $76 \text{ cm} - 18 \text{ cm} = 58 \text{ cm}$

(b)

13.6 cm of water is poured into the right limb of figure (b).

Relative density of mercury = 13.6

Hence, a column of 13.6 cm of water is equivalent to 1 cm of mercury.

Let  $h$  be the difference between the levels of mercury in the two limbs.

The pressure in the right limb is given as:

$$P_R = \text{Atmospheric pressure} + 1 \text{ cm of Hg} = 76 + 1 = 77 \text{ cm of Hg} \dots (i)$$

The mercury column will rise in the left limb.

$$\text{Hence, pressure in the left limb, } P_L = 58 + h \dots (ii)$$

Equating equations (i) and (ii), we get:

$$77 = 58 + h$$

$$\therefore h = 19 \text{ cm}$$

Hence, the difference between the levels of mercury in the two limbs will be 19 cm.

#### Question 23:

Two vessels have the same base area but different shapes. The first vessel takes twice the volume of water that the second vessel requires to fill up to a particular common height. Is the force exerted by the water on the base of the vessel the same in the two cases? If so, why do the vessels filled with water to that same height give different readings on a weighing scale?

#### Solution 23:

Yes, Two vessels having the same base area have identical force and equal pressure acting on their common base area. Since the shapes of the two vessels are different, the force exerted on the sides of the vessels has non-zero vertical components. When these vertical components are added, the total force on one vessel comes out to be greater than that on the other vessel.

Hence, when these vessels are filled with water to the same height, they give different readings on a weighing scale.

**Question 24:**

During blood transfusion the needle is inserted in a vein where the gauge pressure is 2000 Pa. At what height must the blood container be placed so that blood may just enter the vein? [Use the density of whole blood from Table 10.1].

**Solution 24:**

Gauge pressure,  $P = 2000 \text{ Pa}$

Density of whole blood,  $\rho = 1.06 \times 10^3 \text{ kg m}^{-3}$

Acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$

Height of the blood container =  $h$

Pressure of the blood container,  $P = h\rho g$

$$\begin{aligned}
 h &= \frac{P}{\rho g} \\
 &= \frac{2000}{1.06 \times 10^3 \times 9.8} \\
 &= 0.1925 \text{ m}
 \end{aligned}$$

The blood may enter the vein if the blood container is kept at a height greater than 0.1925 m, i.e., about 0.2 m.

**Question 25:**

In deriving Bernoulli's equation, we equated the work done on the fluid in the tube to its change in the potential and kinetic energy.

(a) What is the largest average velocity of blood flow in an artery of diameter  $2 \times 10^{-3} \text{ m}$  if the flow must remain laminar?

(b) Do the dissipative forces become more important as the fluid velocity increases? Discuss qualitatively.

**Solution 25:**

(a)

Diameter of the artery,  $d = 2 \times 10^{-3} \text{ m}$

Viscosity of blood,  $\eta = 2.084 \times 10^{-3} \text{ Pa s}$

Density of blood,  $\rho = 1.06 \times 10^3 \text{ kg/m}^3$

Reynolds' number for laminar flow,  $N_R = 2000$

The largest average velocity of blood is given as:

$$\begin{aligned}
 V_{\text{arg}} &= \frac{N_R \eta}{\rho d} \\
 &= \frac{2000 \times 2.084 \times 10^{-3}}{1.06 \times 10^3 \times 2 \times 10^{-3}} \\
 &= 1.966 \text{ m/s}
 \end{aligned}$$

Therefore, the largest average velocity of blood is 1.966 m/s.

(b)

As the fluid velocity increases, the dissipative forces become more important. This is because of the rise of turbulence. Turbulent flow causes dissipative loss in a fluid.

#### Question 26:

(a) What is the largest average velocity of blood flow in an artery of radius  $2 \times 10^{-3}$  m if the flow must remain laminar?

(b) What is the corresponding flow rate? (Take viscosity of blood to be  $2.084 \times 10^{-3}$  Pa s).

#### Solution 26:

(a)

Radius of the artery,  $r = 2 \times 10^{-3}$  m

Diameter of the artery,  $d = 2 \times 2 \times 10^{-3}$  m =  $4 \times 10^{-3}$  m

Viscosity of blood,  $\eta = 2.084 \times 10^{-3}$  Pa s

Density of blood,  $\rho = 1.06 \times 10^3$  kg/m<sup>3</sup>

Reynolds' number for laminar flow,  $N_R = 2000$

The largest average velocity of blood is given by the relation:

$$\begin{aligned}
 V_{\text{arg}} &= \frac{N_R \eta}{\rho d} \\
 &= \frac{2000 \times 2.084 \times 10^{-3}}{1.06 \times 10^3 \times 4 \times 10^{-3}} \\
 &= 0.983 \text{ m/s}
 \end{aligned}$$

Therefore, the largest average velocity of blood is 0.983 m/s.

(b)

Flow rate is given by the relation:

$$\begin{aligned}
 R &= \pi r^2 V_{\text{avg}} \\
 &= 3.14 \times (2 \times 10^{-3})^2 \times 0.983 \\
 &= 1.235 \times 10^{-5} \text{ m}^3 \text{ s}^{-1}
 \end{aligned}$$

Therefore, the corresponding flow rate is  $= 1.235 \times 10^{-5} m^3 s^{-1}$

#### Question 27:

A plane is in level flight at constant speed and each of its two wings has an area of  $25 m^2$ . If the speed of the air is  $180 km/h$  over the lower wing and  $234 km/h$  over the upper wing surface, determine the plane's mass. (Take air density to be  $1 kg m^{-3}$ ).

#### Solution 27:

The area of the wings of the plane,  $A = 2 \times 25 = 50 m^2$

Speed of air over the lower wing,  $V_1 = 180 km/h = 50 m/s$

Speed of air over the upper wing,  $V_2 = 234 km/h = 65 m/s$

Density of air,  $\rho = 1 kg m^{-3}$

Pressure of air over the lower wing  $= P_1$

Pressure of air over the upper wing  $= P_2$

The upward force on the plane can be obtained using Bernoulli's equation as:

$$P_1 + \frac{1}{2} \rho V_1^2 = P_2 + \frac{1}{2} \rho V_2^2$$

$$P_1 - P_2 = \frac{1}{2} \rho (V_2^2 - V_1^2) \quad \dots(i)$$

The upward force ( $F$ ) on the plane can be calculated as:

$$\begin{aligned} & (P_1 - P_2) a \\ &= \frac{1}{2} \rho (V_2^2 - V_1^2) a \quad \text{Using equation (i)} \\ &= \frac{1}{2} \times 1 \times ((65)^2 - (50)^2) \times 50 \\ &= 43125 N \end{aligned}$$

Using Newton's force equation, we can obtain the mass ( $m$ ) of the plane as:

$$\begin{aligned} F &= mg \\ \therefore m &= \frac{43125}{9.8} \\ &= 4400.51 kg \\ &= 4400 kg \end{aligned}$$

Hence, the mass of the plane is about 4400 kg.

#### Question 28:

In Millikan's oil drop experiment, what is the terminal speed of an uncharged drop of radius  $2.0 \times 10^{-5} m$  and density  $1.2 \times 10^3 kg m^{-3}$ ? Take the viscosity of air at the temperature of the experiment to be  $1.8 \times 10^{-5} Pa s$ . How much is the viscous force on the drop at that speed? Neglect buoyancy of the drop due to air.

#### Solution 28:

Terminal speed =  $5.8 cm/s$ ; Viscous force =  $3.9 \times 10^{-10} N$

Radius of the given uncharged drop,  $r = 2.0 \times 10^{-5} m$

Density of the uncharged drop,  $\rho = 1.2 \times 10^3 kg m^{-3}$

Viscosity of air,  $\eta = 1.8 \times 10^{-5} Pa s$

Density of air ( $\rho_0$ ) can be taken as zero in order to neglect buoyancy of air.

Acceleration due to gravity,  $g = 9.8 m/s^2$

Terminal velocity ( $v$ ) is given by the relation:

$$v = \frac{2r^2 \times (\rho - \rho_0) g}{9\eta}$$

$$= \frac{2 \times (2.0 \times 10^{-5})^2 (1.2 \times 10^3 - 0) \times 9.8}{9 \times 1.8 \times 10^{-5}}$$

$$= 5.807 \times 10^{-2} m s^{-1}$$

$$= 5.8 cm s^{-1}$$

Hence, the terminal speed of the drop is  $5.8 cm s^{-1}$ .

The viscous force on the drop is given by:

$$F = 6\pi\eta r v$$

$$\therefore F = 6 \times 3.14 \times 1.8 \times 10^{-5} \times 2.0 \times 10^{-5} \times 5.8 \times 10^{-2}$$

$$= 3.9 \times 10^{-10} N$$

Hence, the viscous force on the drop is  $3.9 \times 10^{-10} N$ .

**Question 29:**

Mercury has an angle of contact equal to  $140^\circ$  with soda lime glass. A narrow tube of radius 1.00 mm made of this glass is dipped in a trough containing mercury. By what amount does the mercury dip down in the tube relative to the liquid surface outside?

Surface tension of mercury at the temperature of the experiment is  $0.465 \text{ N m}^{-1}$ . Density of mercury =  $13.6 \times 10^3 \text{ kg m}^{-3}$ .

**Solution 29:**

Angle of contact between mercury and soda lime glass,  $\theta = 140^\circ$

Radius of the narrow tube,  $r = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$

Surface tension of mercury at the given temperature,  $s = 0.465 \text{ N m}^{-1}$

Density of mercury,  $\rho = 13.6 \times 10^3 \text{ kg m}^{-3}$

Dip in the height of mercury =  $h$

Acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$

Surface tension is related with the angle of contact and the dip in the height as:

$$s = \frac{h\rho gr}{2\cos\theta}$$

$$\therefore h = \frac{2s\cos\theta}{r\rho g}$$

$$= \frac{2 \times 0.465 \times \cos 140}{1 \times 10^{-3} \times 13.6 \times 10^3 \times 9.8}$$

$$= -0.00534 \text{ m}$$

$$= -5.34 \text{ mm}$$

Here, the negative sign shows the decreasing level of mercury. Hence, the mercury level dips by 5.34 mm.

**Question 30:**

Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is  $7.3 \times 10^{-2} \text{ N m}^{-1}$ . Take the angle of contact to be zero and density of water to be

$1.0 \times 10^3 \text{ kg m}^{-3}$  ( $g = 9.8 \text{ m s}^{-2}$ ).

**Solution 30:**

Diameter of the first bore,  $d_1 = 3.0 \text{ mm} = 3 \times 10^{-3} \text{ m}$

Hence the radius of the first bore,  $r_1 = \frac{d_1}{2} = 1.5 \times 10^{-3} \text{ m}$

Diameter of the second bore,  $d_2 = 6.0 \text{ mm}$

Hence, the radius of the second bore,  $r_2 = \frac{d_2}{2} = 3 \times 10^{-3} \text{ m}$

Surface tension of water,  $s = 7.3 \times 10^{-2} \text{ N m}^{-1}$

Angle of contact between the bore surface and water,  $\theta = 0$

Density of water,  $\rho = 1.0 \times 10^3 \text{ kg m}^{-3}$

Acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$

Let  $h_1$  and  $h_2$  be the heights to which water rises in the first and second tubes respectively.

These heights are given by the relations:

$$h_1 = \frac{2s \cos \theta}{r_1 \rho g} \quad \dots (i)$$

$$h_2 = \frac{2s \cos \theta}{r_2 \rho g} \quad \dots (ii)$$

The difference between the levels of water in the two limbs of the tube can be calculated as:

$$\begin{aligned} &= \frac{2s \cos \theta}{r_1 \rho g} - \frac{2s \cos \theta}{r_2 \rho g} \\ &= \frac{2s \cos \theta}{\rho g} \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] \\ &= \frac{2 \times 7.3 \times 10^{-2} \times 1}{1 \times 10^3 \times 9.8} \left[ \frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}} \right] \\ &= 4.966 \times 10^{-3} \text{ m} \\ &= 4.97 \text{ mm} \end{aligned}$$

Hence, the difference between levels of water in the two bores is 4.97 mm.

#### Question 31:

(a) It is known that density  $\rho$  of air decreases with height  $y$  as  $\rho = \rho_0 e^{-y/y_0}$

Where  $\rho_0 = 1.25 \text{ kg m}^{-3}$  is the density at sea level, and  $y_0$  is a constant. This density variation is called the law of atmospheres. Obtain this law assuming that the temperature of atmosphere remains a constant (isothermal conditions). Also assume that the value of  $g$  remains constant.

(b) A large He balloon of volume  $1425 \text{ m}^3$  is used to lift a payload of 400 kg. Assume that the balloon maintains constant radius as it rises. How high does it rise?

[Take  $y_0 = 8000 \text{ m}$  and  $\rho_{\text{He}} = 0.18 \text{ kg m}^{-3}$ ].

#### Solution 31:

(a)



Volume of the balloon,  $V = 1425 \text{ m}^3$

Mass of the payload,  $m = 400 \text{ kg}$

Acceleration due to gravity,  $g = 9.8 \text{ m/s}^2$

$$y_0 = 8000 \text{ m}$$

$$\rho_{\text{He}} = 0.18 \text{ kg m}^{-3}$$

$$\rho_0 = 1.25 \text{ kg / m}^3$$

Density of the balloon =  $\rho$

Height to which the balloon rises =  $y$

Density ( $\rho$ ) of air decreases with height ( $y$ ) as:

$$\rho = \rho_0 e^{-y/y_0}$$

$$\frac{\rho}{\rho_0} = e^{-y/y_0} \quad \dots(i)$$

This density variation is called the law of atmospherics.

It can be inferred from equation (i) that the rate of decrease of density with height is directly proportional to  $\rho$ , i.e.,

$$-\frac{d\rho}{dy} \propto \rho$$

$$\frac{d\rho}{dy} = -k\rho$$

$$\frac{d\rho}{\rho} = -kdy$$

Where,  $k$  is the constant of proportionality

Height changes from 0 to  $y$ , while density changes from  $\rho_0$  to  $\rho$ . Integrating the sides between these limits, we get:

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \int_0^y k dy$$

$$[\log_e \rho]_{\rho_0}^{\rho} = -ky$$

$$\log_e \frac{\rho}{\rho_0} = -ky$$

$$\frac{\rho}{\rho_0} = e^{-ky} \quad \dots(ii)$$

Comparing equations (i) and (ii), we get

$$y_0 = \frac{1}{k}$$

$$k = \frac{1}{y_0} \quad \dots(iii)$$

From equations (i) and (iii), we get :

$$\rho = \rho_0 e^{-y/y_0}$$

$$\begin{aligned} \text{(b) Density } \rho &= \frac{\text{Mass}}{\text{Volume}} \\ &= \frac{\text{Mass of the payload} + \text{Mass of helium}}{\text{Volume}} \end{aligned}$$

$$\begin{aligned} &= \frac{m + V \rho_{He}}{V} \\ &= \frac{400 + 1425 \times 0.18}{1425} \end{aligned}$$

$$= 0.46 \text{ kg / m}^3$$

From equations (ii) and (iii), we can obtain  $y$  as:

$$\rho = \rho_0 e^{-y/y_0}$$

$$\log_e \frac{\rho}{\rho_0} = -\frac{y}{y_0}$$

$$\therefore y = -8000 \times \log_e \frac{0.46}{1.25}$$

$$= -8000 \times -1$$

$$= 8000 \text{ m} = 8 \text{ km}$$

Hence, the balloon will rise to a height of 8 km.