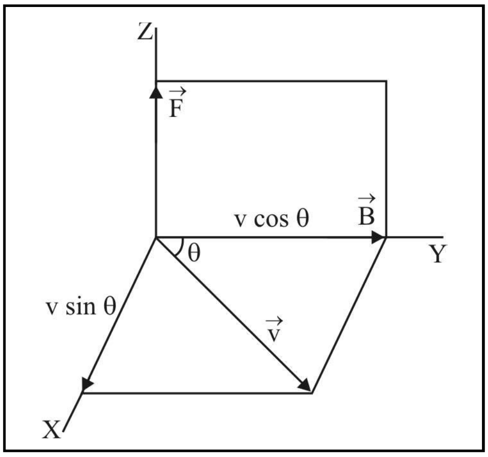
MAGNETIC

1. MAGNETIC FIELD AND FORCE

In order to define the magnetic field $\vec{B}$, we deduce an expression for the force on a moving charge in a magnetic field.

Consider a positive charge q moving in a uniform magnetic field $\vec{B}$, with a velocity $\vec{V}$. Let the angle between $\vec{V}$ and $\vec{B}$ be θ.



(i) The magnitude of force $\vec{F}$ experienced by the moving charge is directly proportional to the magnitude of the charge i.e.

$F \propto q$

(ii) The magnitude of force $\vec{F}$ is directly proportional to the component of velocity acting perpendicular to the direction of magnetic field, i.e.

$F \propto v \sin\theta$

(iii) The magnitude of force $\vec{F}$ is directly proportional to the magnitude of the magnetic field applied i.e.,

$F \propto B$

Combining the above factors, we get

$F \propto qv \sin\theta B$ or $F = kqv B \sin\theta$

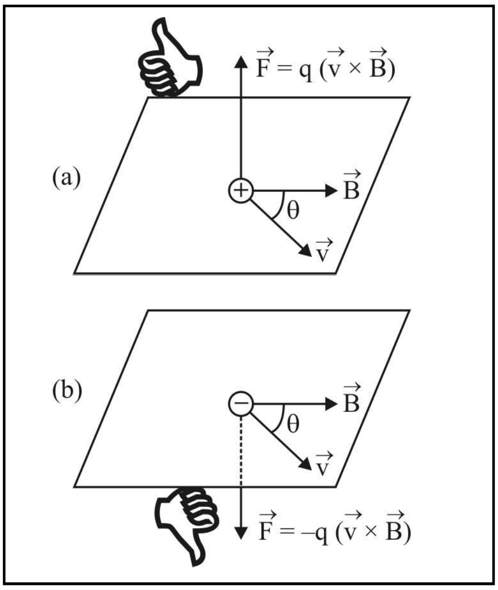
where k is a constant of proportionality. Its value is found to be one i.e. k = 1.

∴ $F = qv B \sin\theta$ ...(1)

$\vec{F} = q(\vec{v} \times \vec{B})$ ...(2)

The direction of $\vec{F}$ is the direction of cross-product of velocity $\vec{v}$ and magnetic field $\vec{B}$, which is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$. It is directed as given by the Right-handed-Screw Rule or Right-Hand Rule.

If $\vec{v}$ and $\vec{B}$ are in the plane of paper, then according to Right-Hand Rule, the direction of $\vec{F}$ on positively charged particle will be perpendicular to the plane of paper upwards as shown in figure (a), and on negatively charged particle will be perpendicular to the plane of paper downwards, figure (b).



Definition of $\vec{B}$

If v = 1, q = 1 and sin θ = 1 or θ = 90°, the nfrom (1),

$F = 1 \times 1 \times B \times 1 = B$.

Thus the magnetic field induction at a point in the magnetic field is equal to the force experienced by a unit charge moving with a unit velocity perpendicular to the direction of magnetic field at that point.

Special Cases

Case (i) If θ = 0° or 180°, then sin θ = 0.

∴ From (1),

$F = qv B (0) = 0$.

It means, a charged particle moving along or opposite to the direction of magnetic field, does not experience any force.

Case (ii) If v = 0, then $F = qv B \sin\theta = 0$.

It means, if a charged particle is at rest in a magnetic field, it experiences no force.

Case (iii) If θ = 90°, then sin θ = 1

∴ $F = qv B (1) = qv B$ (Maximum).

Unit of $\vec{B}$. SI unit of B is tesla (T) or weber/(metre)^2 i.e. (Wb/m^2) or Ns C^{-1} m^{-1}

Thus, the magnetic field induction at a point is said to be one tesla if a charge of one coulomb while moving at right angle to a magnetic field, with a velocity of 1 ms^{-1} experiences a force of 1 newton, at that point.

$$\text{Dimensions of } B = \frac{MLT^{-2}}{AT\left(LT^{-1}\right)} = \left[MA^{-1}T^{-2}\right]$$

2. LORENTZ FORCE

The force experienced by a charged particle moving in space where both electric and magnetic fields exist is called Lorentz force.

Force due to electric field. When a charged particle carrying charge +q is subjected to an electric field of strength $\vec{E}$, it experiences a force given by

$$\vec{F}_e = q\vec{E} \qquad ...(5)$$

whose direction is the same as that of $\vec{E}$.

Force due to magnetic field. If the charged particle is moving in a magnetic field $\vec{B}$, with a velocity $\vec{v}$ it experiences a force given by

$$\vec{F}_m = q\left(\vec{v} \times \vec{B}\right)$$

The direction of this force is in the direction of $\vec{v} \times \vec{B}$ i.e. perpendicular to the plane contaning $\vec{v}$ and $\vec{B}$ and is directed as given by Right hand screw rule.

Due to both the electric and magnetic fields, the total force experienced by the charged particle will be given by

$$\vec{F} = \vec{F}_e + \vec{F}_m = q\vec{E} + q\left(\vec{v} \times \vec{B}\right) = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

$$\boxed{\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)} \qquad ...(6)$$

This is called Lorentz force.

Special cases

Case I. When $\vec{v}, \vec{E}$ and $\vec{B}$, all the three are collinear. In this situation, the charged particle is moving parallel or antiparallel to the fields, the magnetic force on the charged particle is zero. The electric force on the charged particle will produce acceleration $\vec{a} = \frac{q\vec{E}}{m}$,

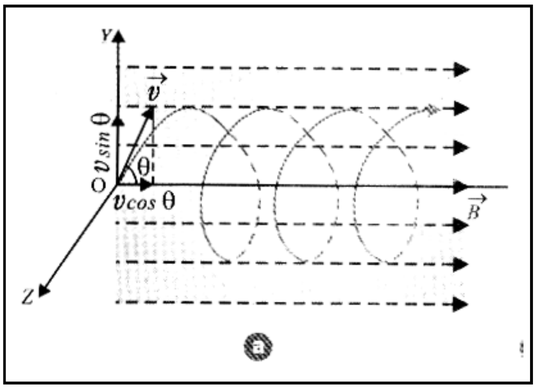
along the direction of electricl field. As a result of this, there will be change in the speed of charged particle along the direction of the field. In this situation there will be no change in the direction of motion of the charged particle but, the speed, velocity, momentum and kinetic energy of charged particle will change.

Case II. When $\vec{v}, \vec{E}$ and $\vec{B}$ are mutually perpendicular to each other. In this situation if $\vec{E}$ and $\vec{B}$ are such that $\vec{F} = \vec{F}_e + \vec{F}_m = 0$, then acceleration in the particle, $\vec{a} = \frac{\vec{F}}{m} = 0$. It means the particle will pass through the fields without any change in its velocity. Here, $F_e = F_m$ so $qE = q\,v\,B$ or $v = E/B$.

This concept has been used in **velocity-selector** to get a charged beam having a definite velocity.

3. MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

Suppose a particle of mass m and charge q, entering a uniform magnetic field induction $\vec{B}$ at O, with velocity $\vec{v}$, making an angle θ with the direction of magnetic field acting in the plane of paper as shown in figure



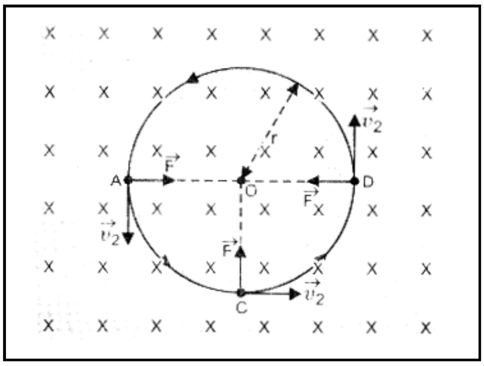
Resolving $\vec{v}$ into two rectangular components, we have : v cos θ (= v_1) acts in the direction of the magnetic field and v sin θ (= v_2) acts perpendicular to the direction of magnetic field.

For component velocity $\vec{v}_2$, the force acting on the charged particle due to magnetic field is

$$\vec{F} = q\left(\vec{v}_2 \times \vec{B}\right)$$

or $$F = q\left|\vec{v}_2 \times \vec{B}\right| = qv_2B\sin 90° = q\left(v\sin\theta\right)B \quad ...(1)$$

The direction of this force $\vec{F}$ is perpendicular to the plane containing $\vec{B}$ and $\vec{v}_2$ and is directed as given by Right hand rule. As this force is to remain always perpendicular to $\vec{v}_2$ it does not perform any work and hence cannot change the magnitude of velocity $\vec{v}_2$. It changes only the direction of motion of the particle. Due to it, the charged particle is made to move on a circular path in the magnetic field, as shown in figure



Here, magnetic field is shown perpendicular to the plane of paper directed inwards and particle is moving in the plane of paper. When the particle is at points A, C and D the direction of magnetic force on the particle will be along AO, CO and DO respectively, i.e., directed towards the centre O of the circular path.

The force F on the charged particle due to magnetic field provides the required centripetal force $= \left(mv_2^2/r\right)$ necessary for motion along a circular path of radius r.

$\therefore$ $Bqv_2 = mv_2^2/r$ or $v_2 = Bqr/m$

or $v\sin\theta = Bqr/m$ ...(2)

The angular velocity of rotation of the particle in magnetic field will be

$$\omega = \frac{v\sin\theta}{r} = \frac{Bqr}{mr} = \frac{Bq}{m}$$

The frequency of rotation of the particle in magnetic field will be

$$\nu = \frac{\omega}{2\pi} = \frac{Bq}{2\pi m} \quad ...(3)$$

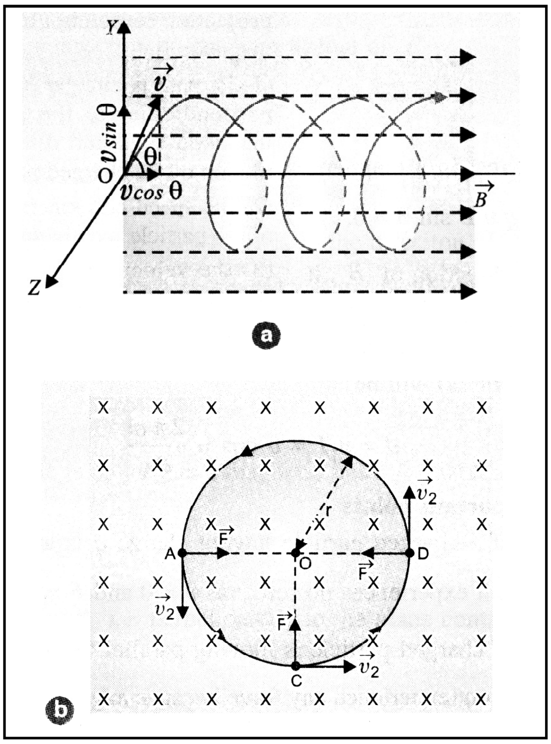
The time period of revolution of the particle in the magnetic field will be

$$T = \frac{1}{\nu} = \frac{2\pi m}{Bq} \quad ...(4)$$

From (3) and (4), we note that ν and T do not depend upon velocity $\vec{v}$ of the particle. It means, all the charged particles having the same specific charge (charge/mass) but moving with different velocities at a point, will complete their circular paths due to component velocities perpendicular to the magnetic fields in the same time.

For component velocity $v_1 \left(= v\cos\theta\right)$, there will be no force on the charged particle in the magnetic field, because the angle between $\vec{v}_1$ and $\vec{B}$ is zero. Thus the charged particle covers the linear distance in direction of the magnetic field with a constant speed v cos θ.

Therefore, under the combined effect of the two component velocities, the charged particle in magnetic field will cover linear path as well as circular path i.e. the path of the charged particle will be **helical**, whose axis is parallel to the direction of magnetic field, figure



The linear distance covered by the charged particle in the magnetic field in time equal to one revolution of its circular path **(known as pitch of helix)** will be

$$d = v_1T = v\cos\theta\frac{2\pi m}{Bq}$$

Important points

1. If a charged particle having charge q is at rest in a magnetic field $\vec{B}$, it experiences no force; as v = 0 and F = q v B sin θ = 0.
2. If charged particle is moving parallel to the direction of $\vec{B}$, it also does not experience any force because angle θ between $\vec{v}$ and $\vec{B}$ is 0° or 180° and sin 0° = sin 180° = 0. Therefore, the charged particle in this situation will continue moving along the same path with the same velocity.
3. If charged particle is moving perpendicular to the direction of $\vec{B}$, it experiences a maximum force which acts perpendicular to the direction $\vec{B}$ as well as $\vec{v}$. Hence this force will provide the required centripetal force and the

charged particle will describe a circular path in the magnetic field of radius r, given by $\frac{mv^2}{r} = Bqv$.

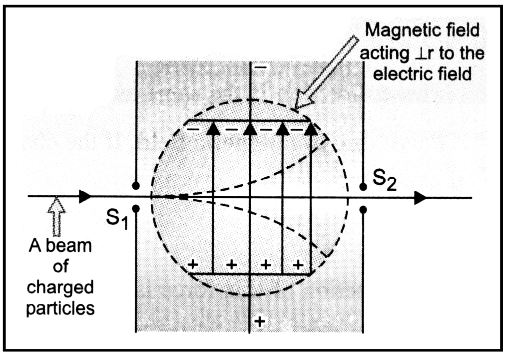
4. MOTION IN COMBINED ELECTRON AND MAGNETIC FIELDS

4.1 Velocity Filter

Velocity filter is an arrangement of cross electric and magnetic fields in a region which helps us to select from a beam, charged particles of the given velocity irrespective of their charge and mass.

A velocity selector consists of two slits S_1 and S_2 held parallel to each other, with common axis, some distance apart. In the region between the slits, uniform electric and magnetic fields are applied, perpendicular to each other as well as to the axis of slits, as shown in figure. When a beam of charged particles of different charges and masses after passing through slit S_1 enters the region of crossed electric field $\vec{E}$ and magnetic field $\vec{B}$, each particle experiences a force due to these fields. Those particles which are moving with the velocity v, irrespective of their mass and charge, the force on each such particle due to electric field (qE) is equal and opposite to the force due to magnetic field (q v B), then

$q E = q v B$ or $v = E/B$



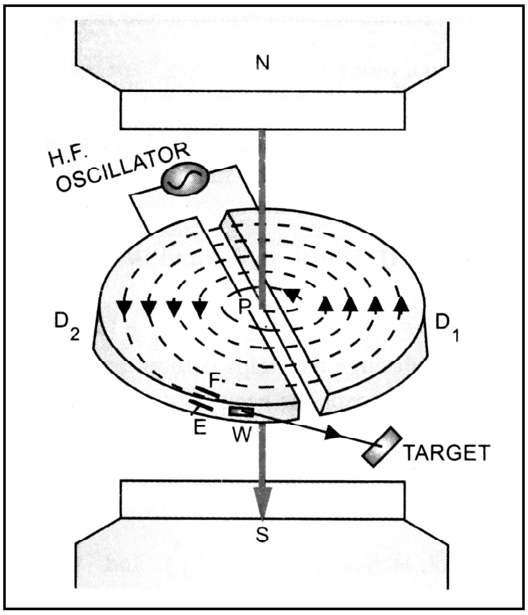
Such particles will go undeviated and filtered out of the region through the slit S_2. Therefore, the particles emerging from slit S_2 will have the same velocity even though their charge and mass may be different.

The velocity filter is used in mass spectrograph which helps to find the mass and specific charge (charge/mass) of the charged particle.

4.2 Cyclotron

A cyclotron is a device developed by Lawrence and Livingstone by which the positively charged particles like proton, deutron, alpha particle etc. can be accelerated.

Principle. The working of the cyclotron is based on the fact that a positively charged particle can be accelerated to a sufficiently high energy with the help of smaller values of oscillating electric field by making it to cross the same electric field time and again with the use of strong magnetic field.



Construction. It consists of two D-shaped hollow evacuated metal chambers D_1 and D_2 called the dees. These dees are placed horizontally with their diametric edges parallel and slightly separated from each other. The dees are connected to high frequency oscillator which can produce a potential difference of the order of 10^4 volts at frequency $\approx 10^7$ Hz. The two dees are enclosed in an evacuated steel box and are well insulated from it. The box is placed in a strong magnetic field produced by two pole pieces of strong electromagnets N, S. The magnetic field is perpendicular to the plane of the dees. P is a place of ionic source or positively charged particle figure.

Working and theory. The positive ion to be accelerated is produced at P. Suppose, at that instant, D_1 is at negative potential and D_2 is at positive potential. Therefore, the ion will be accelerated towards D_1. On reaching inside D_1, the ion will be in a field free space. Hence it moves with a constant speed in D_1 say v. But due to perpendicular magnetic field of strength B, the ion will describe a circular path of radius r (say) in D_1, given by $Bqv = \frac{mv^2}{r}$ where m and q are the mass and charge of the ion.

$$\therefore \quad r = \frac{mv}{Bq}$$

Time taken by ion to describe a semicircular path is given by, $t = \frac{\pi r}{v} = \frac{\pi m}{Bq} = \frac{\pi}{B(q/m)}$ = a constant.

This time is independent of both the speed of the ion and radius of the circular path. In case the time during which the positive ion describes a semicircular path is equal to the time during which half cycle of electric oscillator is completed, then as the ion arrives in the gap between the two dees, the polarity of the two dees is reversed i.e. D_1 becomes positive and D_2 negative. Then, the positive ion is accelerated towards D_2 and it enters D_2 with greater speed which remains constant in D_2. The ion will describe a semicircular path of greater radius due to perpendicular magnetic field and again will arrive in a gap between the two dees exactly at the instant, the polarity of the two dees is reversed. Thus, the positive ion will go on accelerating every time it comes into the gap between the dees and will go on describing circular path of greater and greater radius with greater and greater speed and finally acquires a sufficiently high energy. The accelerated ion can be removed out of the dees from window W, by applying the electric field across the deflecting plates E and F.

Maximum Energy of positive ion

Let v_0, r_0 = maximum velocity and maximum radius of the circular path followed by the positive ion in cyclotron.

Then, $\frac{mv_0^2}{r_0} = Bqv_0$ or $v_0 = \frac{Bqr_0}{m}$

$$\therefore \quad \text{Max. K.E.} = \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{Bqr_0}{m}\right)^2 = \frac{B^2q^2r_0^2}{2m}$$

Cyclotron Frequency

If T is the time period of oscillating electric field then

$T = 2t = 2\pi m/Bq$

The **cyclotron frequency** is given by $\boxed{\nu = \frac{1}{T} = \frac{Bq}{2\pi m}}$

It is also known as **magnetic resonance frequency**.

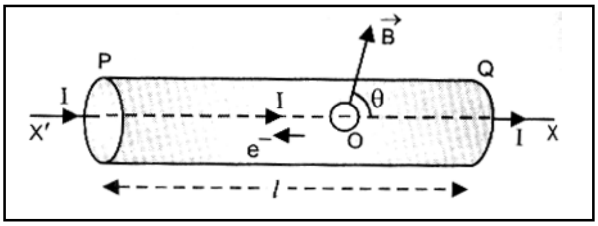
The **cyclotron angular frequency** is given by

$\omega_c = 2\pi\nu = Bq/m$

5. FORCE ON A CURRENT CARRYING CONDUCTOR PLACED IN A MAGNETIC FIELD

Expression for the force acting on the conductor carrying current placed in a magnetic field

Consider a straight cylindrical conductor PQ of length ℓ, area of cross-section A, carrying current I placed in a uniform magnetic field of induction, $\vec{B}$. Let the conductor be placed along X-axis and magnetic field be acting in XY plane making an angle θ with X-axis. Suppose the current I flows through the conductor from the end P to Q, figure. Since the current in a conductor is due to motion of electrons, therefore, electrons are moving from the end Q to P (along X′ axis).



Let, $\vec{v}_d$ drift velocity of electron

– e = charge on each electron.

Then magnetic Lorentz force on an electron is given by

$\vec{f} = -e\left(\vec{v}_d \times \vec{B}\right)$

If n is the number density of free electrons i.e. number of free electrons per unit volume of the conductor, then total number of free electrons in the conductor will be given by

$N = n(A\ell) = nA\ell$

∴ Total force on the conductor is equal to the force acting on all the free electrons inside the conductor while moving in the magnetic field and is given by

$$\vec{F} = N\vec{f} = nA\ell\left[-e\left(\vec{v}_d \times \vec{B}\right)\right] = -nA\ell e\left(\vec{v}_d \times \vec{B}\right) \quad ...(7)$$

We know that current through a conductor is related with drift velocity by the relation

$I = nAev_d$

∴ $I\ell = nAev_d.\ell$

We represent $I\vec{\ell}$ as current element vector. It acts in the direction of flow of current i.e. along OX. Since $I\vec{\ell}$ and $\vec{v}_d$ have opposite directions, hence we can write

$$I\vec{\ell} = -nA\ell e\vec{v}_d \quad ...(8)$$

From (7) and (8), we have

$$\vec{F} = I\vec{\ell} \times \vec{B} \quad ...(9)$$

$$\left|\vec{F}\right| = I\left|\vec{\ell} \times \vec{B}\right|$$

$$F = I\ell B\sin\theta \quad ...(10)$$

were θ is the smaller angle between $I\vec{\ell}$ and $\vec{B}$.

Special cases

Case I. If θ = 0° or 180°, sin θ = 0,

From (10), $F = I\ell B\,(0) = 0$ (Minimum)

It means a linear conductor carrying a current if placed parallel to the direction of magnetic field, it experiences no force.

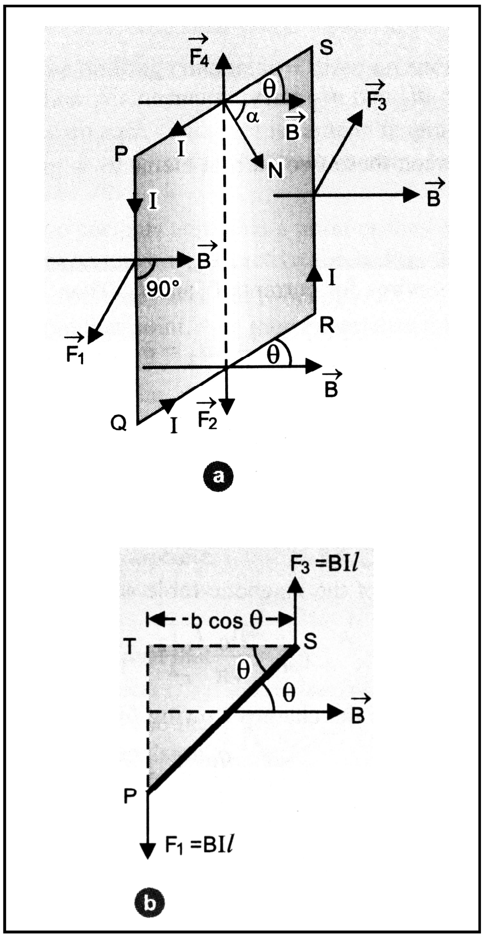
Case II. If θ = 90°, sin θ = q ;

From (10), $F = I\ell B \times 1 = I\ell B$ (Maximum)

It means a linear conductor carrying current if placed perpendicular to the direction of magnetic field, it experiences maximum force. The direction of which can be given by **Right handed screw rule**.

6. TORQUE ON A CURRENT CARRYING COIL IN A MAGNETIC FIELD

Consider a rectangular coil PQRS suspended in a uniform magnetic field of induction $\vec{B}$. Let PQ = RS = ℓ and QR = SP = b. Let I be the current flowing through the coil in the direction PQRS and θ be the angle which plane of the coil makes with the direction of magnetic field figure. The forces will be acting on the four arms of the coil.



Let $\vec{F}_1, \vec{F}_2, \vec{F}_3$ and $\vec{F}_4$ be the forces acting on the four current carrying arms PQ, QR, RS and SP of the coil.

The force on arm SP is given by,

$$\vec{F}_4 = I\left(\overrightarrow{SP} \times \vec{B}\right) \text{ or } F_4 = I\,(SP)\,B \sin(180° - \theta) = Ib\,B \sin\theta$$

The direction of this force is in the direction of $\left(\overrightarrow{SP} \times \vec{B}\right)$ i.e. in the plane of coil directed upwards.

The force on the arm QR is given by $\vec{F}_2 = I\left(\overrightarrow{QR} \times \vec{B}\right)$ or $F_2 = I\,(QR)\,B \sin\theta = I\,b\,B \sin\theta$

The direction of this force is in the plane of the coil directed downwards.

Since the forces $\vec{F}_2$ and $\vec{F}_4$ are equal in magnitude and acting in opposite directions along the same straight line, they cancel out each other i.e. their resultant effect on the coil is zero.

Now, the force on the arm PQ is given by

$$\vec{F}_1 = I\left(\overrightarrow{PQ} \times \vec{B}\right) \text{ or } F_1 = I\,(PQ)\,B \sin 90° = I\ell B \left(\because \overrightarrow{PQ} \perp \vec{B}\right)$$

Direction of this force is perpendicular to the plane of the coil directed outwards (i.e. perpendicular to the plane of paper directed towards the reader).

And, force on the arm RS is given by

$$\vec{F}_3 = I\left(\overrightarrow{RS} \times \vec{B}\right) \text{ or } F_3 = I\,(PQ)\,B \sin 90° = I\ell B \left(\because \overrightarrow{RS} \perp \vec{B}\right)$$

The direction of this force, is perpendicular to the plane of paper directed away from the reader i.e. into the plane of the coil.

The forces acting on the arms PQ and RS are equal, parallel and acting in opposite directions having different lines of action, form a **couple**, the effect of which is to rotate the coil in the anticlockwise direction about the dotted line as axis.

The torque on the coil (equal to moment of couple) is given by

τ = either force × arm of the couple

The forces F_1 and F_3 acting on the arms PQ and RS will be as shown in figure when seen from the top.

Arm of couple = ST = PS cos θ = b cos θ.

$\therefore$ $\tau = I\ell B \times b\cos\theta = IBA\cos\theta$ ($\because$ $\ell \times b = A$ = area of coil PQRS)

If the rectangular coil has n turns, then

$$\tau = nIBA\cos\theta$$

Note that if the normal drawn on the plane of the coil makes an angle α with the direction of magnetic field, then $\theta + \alpha = 90°$ or $\theta = 90° - \alpha$; And $\cos\theta = \cos(90° - \alpha) = \sin\alpha$

Then torque becomes,

$$\tau = nIBA\sin\alpha = MB\sin\alpha = \left|\vec{M} \times \vec{B}\right| = \left|nI\vec{A} \times \vec{B}\right|$$

where, nIA = M = magnitude of the magnetic dipole moment of the rectangular current loop

$$\therefore \quad \vec{\tau} = \vec{M} \times \vec{B} = nI\left(\vec{A} \times \vec{B}\right)$$

This torque tends to rotate the coil about its own axis. Its value changes with angle between plane of coil and direction of magnetic field.

Special cases 1.

If the coil is set with its plane parallel to the direction of magnetic field B, then

$\theta = 0°$ and $\cos\theta = 1$

$\therefore$ Torque, $\tau = nIBA(1) = nIBA$ (Maximum)

This is the case with a **radial field**.

2. **If the coil is set with its plane perpendicular to the direction of magentic field B,** then $\theta = 90°$ and $\cos\theta = 0$

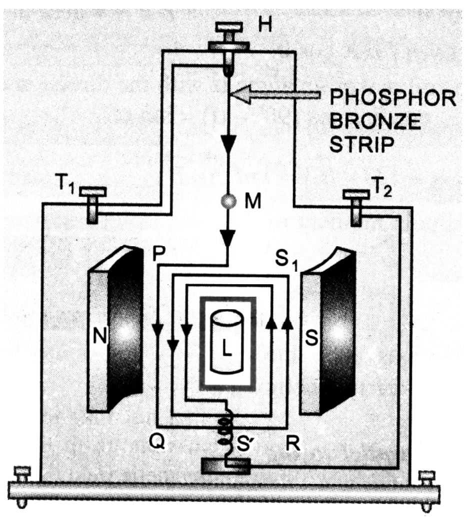
$\therefore$ Torque, $\tau = nIBA(0) = 0$ (Minimum)

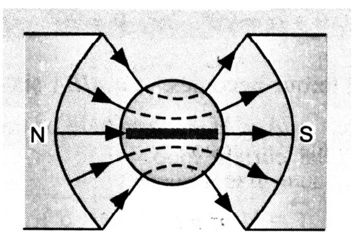
7. MOVING COIL GALVANOMETER

Moving coil galvanometer is an instrument used for detection and measurement of small electric currents.

Principle. Its working is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque.

Construction. It consists of a coil $PQRS_1$ having large number of turns of insulated copper wire, figure. The coil is wound over a non-magnetic metallic frame (usually brass) which may be rectangular or circular in shape. The coil is suspended from a movable torsion head H by means of phosphor bronze strip in a uniform magnetic field produced by two strong cylindrical magnetic pole pieces N and S.





RADIAL FIELD
(TOP VIEW OF
THE GALVANOMETER)

The lower end of the coil is connected to one end of a hair spring S′ of quartz or phosphor bronze. The other end of this highly elastic spring S′ is connected to a terminal T_2. L is soft iron core which may be spherical if the coil is circular and cylindrical, if the coil is rectangular. It is so held within the coil, that the coil can rotate freely without touching the iron core and pole pieces. This makes the magnetic field linked with coil to be **radial field** i.e. the plane of the coil in all positions remains parallel to the direction of magnetic field. M is concave mirror attached to the phosphor bronze strip. This helps us to note the deflection of the coil using lamp and scale arrangement. The whole arrangement is enclosed in a non-metallic case to avoid disturbance due to air etc. The case is provided with levelling screws at the base.

The spring S′ does three jobs for us : (i) It provides passage of current for the coil $PQRS_1$ (ii) It keeps the coil in position and (iii) generates the restoring torque on the twisted coil.

The torsion head is connected to terminal T_1. The galvanometer can be connected to the circuit through terminals T_1 and T_2.

Theory. Suppose the coil $PQRS_1$ is suspended freely in the magnetic field.

Let, ℓ = length PQ or RS_1 of the coil,

b = breadth QR or S_1P of the coil,

n = number of turns in the coil.

Area of each turn of the coil, $A = \ell \times b$.

Let, B = strength of the magnetic field in which coil is suspended.

I = current passing through the coil in the direction $PQRS_1$ as shown in figure.

Let at any instant, α be the angle which the normal drawn on the plane of the coil makes with the direction of magnetic field.

As already discussed, the rectangular coil carrying current when placed in the magnetic field experiences a torque whose magnitude is given by $\tau = nIBA \sin\alpha$.

If the magnetic field is radial i.e. the plane of the coil is parallel to the direction of the magnetic field then $\alpha = 90°$ and $\sin\alpha = 1$.

$\therefore \quad \tau = nIBA$

Due to this torque, the coil rotates. The phosphor bronze strip gets twisted. As a result of it, a restoring torque comes into play in the phosphor bronze strip, which would try to restore the coil back to its original position.

Let θ be the twist produced in the phosphor bronze strip due to rotation of the coil and k be the restoring torque per unit twist of the phosphor bronze strip, then total restoring torque produced = $k\,\theta$.

In equilibrium position of the coil, deflecting torque = restoring torque

$\therefore$ $nIBA = k\theta$

or $I = \frac{k}{nBA}\theta$ or $I = G\theta$

where $\frac{k}{nBA} = G =$ a constant for a galvanometer. It is known as galvanometer constant.

Hence, $I \propto \theta$

It means, the deflection produced is proportional to the current flowing through the galvanometer. Such a galvanometer has a linear scale.

Current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer when a unit current flows through it.

If θ is the deflection in the galvanometer when current I is passed through it, then

Current sensitivity,

$$I_s = \frac{\theta}{I} = \frac{nBA}{k} \qquad \left(\because I = \frac{k}{nBA}\theta\right)$$

The unit of current sensitivity is rad. A^{-1} or div. A^{-1}.

Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer when a unit voltage is applied across the two terminals of the galvanometer.

Let, V = voltage applied across the two terminals of the galvanometer,

θ = deflection produced in the galvanometer.

Then, voltage sensitivity, $V_S = \theta/V$

If R = resistance of the galvanometer, I = current through it. Then V = IR

$\therefore$ Voltage sensitivity,

$$V_S = \frac{\theta}{IR} = \frac{nBA}{kR} = \frac{I_S}{R}$$

the unit of V_S is rad V^{-1} or div. V^{-1}.

Conditions for a sensitive galvanometer

A galvanometer is said to be very sensitive if it shows large deflection even when a small current is passed through it.

From the theory of galvanometer, $\theta = \frac{nBA}{k}I$

For a given value of I, θ will be large if nBA/k is large. It is so if (a) n is large (b) B is large (c) A is large and (d) k is small.

(a) The value of n can not be increased beyond a certain limit because it results in an increase of the resistance of the galvanometer and also makes the galvanometer bulky. This tends to decrease the sensitivity. Hence n can not be increased beyond a limit.

(b) The value of B can be increased by using a strong horse shoe magnet.

(c) The value of A can not be increased beyond a limit because in that case the coil will not be in a uniform magnetic field. Moreover, it will make the galvanometer bulky and unmanageable.

(d) The value of k can be decreased. The value of k depends upon the nature of the material used as suspension strip. The value of k is very small for quartz or phosphor bronze. That is why, in sensitive galvanometer, quartz or phosphor bronze strip is used as a suspension strip.

8. AMMETER

An ammeter is a low resistance galvanometer. It is used to measure the current in a circuit in amperes.

A galvanometer can be converted into an ammeter by using a low resistance wire in parallel with the galvanometer. The resistance of this wire (called the shunt wire) depends upon the range of the ammeter and can be calculated as follows :

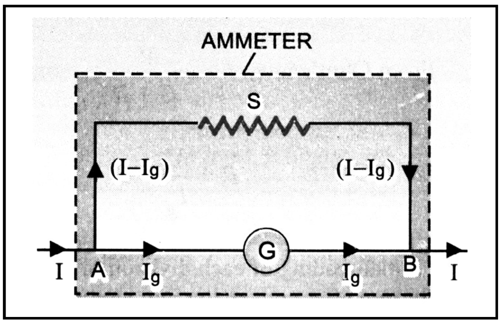
Let G = resistance of galvanometer, n = number of scale divisions in the galvanometer,

K = figure of merit or current for one scale deflection in the galvanometer.

Then current which produces full scale deflection in the galvanometer, $I_g = nK$.

Let I be the maximum current to be measured by galvanometer.

To do so, a shunt of resistance S is connected in parallel with the galvanometer so that out of the total current I, a part I_g should pass through the galvanometer and the remaining part $(I - I_g)$ flows through the shunt figure



$V_A - V_B = I_g G = (I - I_g) S$

or $$S = \left(\frac{I_g}{I - I_g}\right)G \qquad ...(20)$$

Thus S can be calculated.

If this value of shunt resistance S is connected in parallel with galvanometer, it works as an ammeter for the range 0 to I ampere. Now the same scale of the galvanometer which was recording the maximum current I_g before conversion into ammeter will record the maximum current I, after conversion into ammeter. It means each division of the scale in ammeter will be showing higher current than that of galvanometer.

Note...

Initial reading of each division of galvanometer to be used as ammeter is I_g/n and the reading of the same each division after conversion into ammeter is I/n.

The effective resistance R_P of ammeter (i.e. shunted galvanometer) will be

$$\frac{1}{R_P} = \frac{1}{G} + \frac{1}{S} = \frac{S+G}{GS} \text{ or } R_P = \frac{GS}{G+S}$$

As the shunt resistance is low, the combined resistance of the galvanometer and the shunt is very low and hence ammeter has a much lower resistance than galvanometer. An ideal ammeter has zero resistance.

9. VOLTMETER

A voltmeter is a high resistance galvanometer. It is used to measure the potential difference between two points of a circuit in volt.

A galvanometer can be converted into a voltmeter by connecting a high resistance in series with the galvanometer. The value of the resistance depends upon the range of voltmeter and can be calculated as follows :

Let, G = resistance of galvanometer,

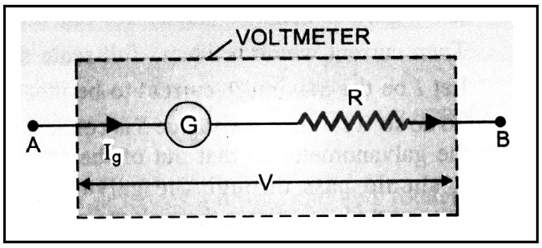
n = number of scale divisions in the galvanometer,

K = figure of merit of galvanometer i.e. current for one scale deflection of the galvanometer.

∴ Current which produces full scale deflection in the galvanometer, $I_g = nK$.

Let V be the potential difference to be measured by galvanometer.

To do so, a resistance R of such a value is connected in series with the galvanometer so that if a potential difference V is applied across the terminals A and B, a current I_g flows through the galvanometer. figure



Now, total resistance of voltmeter = G + R

From Ohm's law, $I_g = \frac{V}{G+R}$ or $G + R = \frac{V}{I_g}$

or $R = \frac{V}{I_g} - G$

If this value of R is connected in series with galvanometer, it works as a voltmeter of the range 0 to V volt. Now the same scale of the galvanometer which was recording the maximum potential I_g G before conversion will record and potential V after conversion in two voltmeter. It means each division of the scale in voltmeter will show higher potential than that of the galvanometer.

Effective resistance R_S of converted galvanometer into voltmeter is

$R_S = G + R$

For voltmeter, a high resistance R is connected in series with the galvanometer, therefore, the resistance of voltmeter is very large as compared to that of galvanometer. The resistance of an ideal voltmeter is infinity.

10. BIOT-SAVART'S LAW

According to Biot-Savart's law, the magnitude of the magnetic field induction dB (also called magnetic flux density) at a point P due to current element depends upon the factors at stated below :

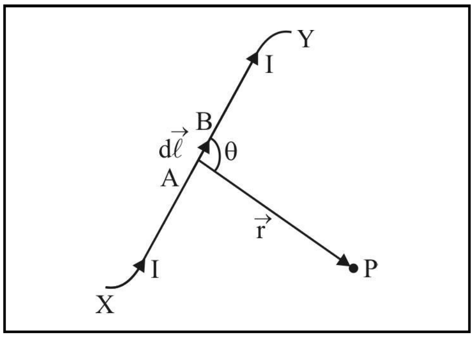
(i) $dB \propto I$ (ii) $dB \propto d\ell$

(iii) $dB \propto \sin\theta$ (iv) $dB \propto \frac{1}{r^2}$

Combining these factors, we get

$$dB \propto \frac{Id\ell \sin\theta}{r^2}$$

or $$dB = K\frac{Id\ell \sin\theta}{r^2}$$



where K is a constant of proportionality. Its value depends on the system of units chosen for the measurement of the various quantities and also on the medium between point P and the current element. When there is free space between current element and point, then

In SI units, $K = \frac{\mu_0}{4\pi}$ and In cgs system K = 1

where μ_0 is absolute magnetic permeability of free space

and $\mu_0 = 4\pi\times10^{-7}\,\text{Wb A}^{-1}\text{m}^{-1} = 4\pi\times10^{-7}\,\text{TA}^{-1}\text{m}$

($\because$ 1 T = 1 Wb m^{-2})

In SI units, $dB = \frac{\mu_0}{4\pi}\times\frac{Id\ell\sin\theta}{r^2}$ ...(3)

In cgs system, $dB = \frac{Id\ell\sin\theta}{r^2}$

In vector form, we may write

$$\left|d\vec{B}\right| = \frac{\mu_0}{4\pi}\frac{I\left|d\vec{\ell}\times\vec{r}\right|}{r^3} \text{ or } d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\left(d\vec{\ell}\times\vec{r}\right)}{r^3} \quad ...(4)$$

Direction of $d\vec{B}$. From (4), the direction of $d\vec{B}$ would obviously be the direction of the cross product vector, $d\vec{\ell}\times\vec{r}$. It is represented by the Right handed screw rule or Right Hand Rule. Here $d\vec{B}$ is perpendicular to the plane containing $d\vec{\ell}$ and $\vec{r}$ and is directed inwards. If the point P is to the left of the current element, $d\vec{B}$ will be perpendicular to the plane containing $d\vec{\ell}$ and $\vec{r}$, directed outwards.

Some important features of Biot Savart's law

1. Biot Savart's law is valid for a symmetrical current distribution.
2. Biot Savart's law is applicable only to very small length conductor carrying current.
3. This law can not be easily verified experimentally as the current carrying conductor of very small length can not be obtained practically.
4. This law is analogous to Coulomb's law in electrostatics.
5. The direction of $d\vec{B}$ is perpendicular to both $Id\vec{\ell}$ and $\vec{r}$.
6. If $\theta = 0°$ i.e. the point P lies on the axis of the linear conductor carrying current (or on the wire carrying current) then

$$dB = \frac{\mu_0}{4\pi}\frac{Id\ell\sin 0°}{r^2} = 0$$

It means there is no magnetic field induction at any point on the thin linear current carrying conductor.

7. If $\theta = 90°$ i.e. the point P lies at a perpendicular position w.r.t. current element, then

$$dB = \frac{\mu_0}{4\pi}\frac{Id\ell}{r^2}\text{, which is maximum.}$$

8. If $\theta = 0°$ or 180°, then dB = 0 i.e. minimum.

Similarities and Dis-similarities between the Biot-Savart's law for the magnetic field and coulomb's law for electrostatic field

Similarities

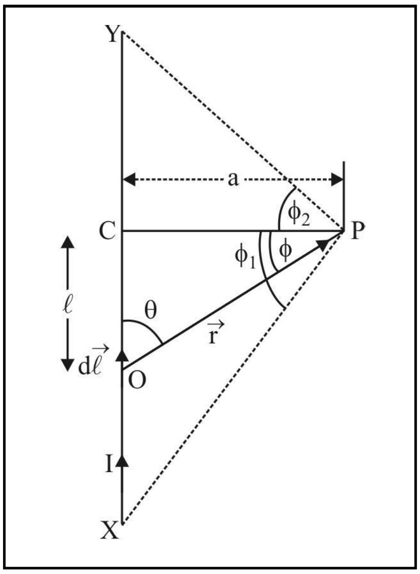
(i) Both the laws for fields are long range, since in both the laws, the field at a point varies inversely as the square of the distance from the source to point of observation.

(ii) Both the fields obey superposition principle.

(iii) The magnetic field is linear in the source $Id\vec{\ell}$, just as the electric field is linear in its source, the electric charge q.

11. MAGNETIC FIELD DUE TO A STRAIGHT CONDUCTOR CARRYING CURRENT

Consider a straight wire conductor XY lying in the plane of paper carrying current I in the direction X to Y, figure. Let P be a point at a perpendicular distance a from the straight wire conductor. Clearly, PC = a. Let the conductor be made of small current elements. Consider a small current element $Id\vec{\ell}$ of the straight wire conductor at O. Let $\vec{r}$ be the position vector of P w.r.t. current element and θ be the angle between $Id\vec{\ell}$ and $\vec{r}$. Let CO = ℓ.



According to Biot-Savart's law, the magnetic field $d\vec{B}$ (i.e. magnetic flux density or magnetic induction) at point P due to current element $Id\vec{\ell}$ is given by

$$d\vec{B} = \frac{\mu_0}{4\pi}\cdot\frac{Id\vec{\ell}\times\vec{r}}{r^3}$$

or $$dB = \frac{\mu_0}{4\pi}\times\frac{Id\ell\sin\theta}{r^2} \quad ...(5)$$

In rt. angled ΔPOC, $\theta + \phi = 90°$ or $\theta = 90° - \phi$

$\therefore \quad \sin\theta = \sin(90° - \phi) = \cos\phi \quad ...(6)$

Also, $\cos\phi = \dfrac{a}{r}$ or $r = \dfrac{a}{\cos\phi} \quad ...(7)$

And, $\tan\phi = \dfrac{\ell}{a}$ or $\ell = a\tan\phi$

Differentiating it, we get

$$d\ell = a\sec^2\phi\, d\phi \quad ...(8)$$

Putting the values in (5) from (6), (7) and (8), we get

$$dB = \frac{\mu_0}{4\pi}\frac{I\left(a\sec^2\phi\, d\phi\right)\cos\phi}{\left(\dfrac{a^2}{\cos^2\phi}\right)} = \frac{\mu_0}{4\pi}\frac{I}{a}\cos\phi\, d\phi \quad ...(9)$$

The direction of $d\vec{B}$, according to right hand thumb rule, will be perpendicular to the plane of paper and directed inwards. As all the current elements of the conductor will also produce magnetic field in the same direction, therefore, the total magnetic field at point P due to current through the whole straight conductor XY can be obtained by integrating Eq. (9) within the limits $-\phi_1$ and $+\phi_2$. Thus

$$B = \int_{-\phi_1}^{\phi_2} dB = \frac{\mu_0}{4\pi}\frac{I}{a}\int_{-\phi_1}^{\phi_2}\cos\phi\, d\phi = \frac{\mu_0}{4\pi}\frac{I}{a}\left[\sin\phi\right]_{-\phi_1}^{\phi_2}$$

$$= \frac{\mu_0}{4\pi}\frac{I}{a}\left[\sin\phi_2 - \sin(-\phi_1)\right] = \frac{\mu_0}{4\pi}\frac{I}{a}(\sin\phi_1 + \sin\phi_2) \quad ...(10)$$

Special cases. (i) When the conductor XY is of infinite length and the point P lies near the centre of the conductor then $\phi_1 = \phi_2 = 90°$

So, $B = \dfrac{\mu_0}{4\pi}\dfrac{I}{a}\left[\sin 90° + \sin 90°\right] = \dfrac{\mu_0}{4\pi}\dfrac{2I}{a} \quad ...(11)$

(ii) When the conductor XY is of infinite length but the point P lies near the end Y (or X) then $\phi_1 = 90°$ and $\phi_2 = 0°$.

So, $B = \dfrac{\mu_0}{4\pi}\dfrac{I}{a}\left[\sin 90° + \sin 0°\right] = \dfrac{\mu_0}{4\pi}\dfrac{I}{a} \quad ...(11\text{ a})$

Thus we note that the magnetic field due to an infinite long linear conductor carrying current near its centre is twice than that near one of its ends.

(iii) If length of conductor is finite, say L and point P lies on right bisector of conductor, then

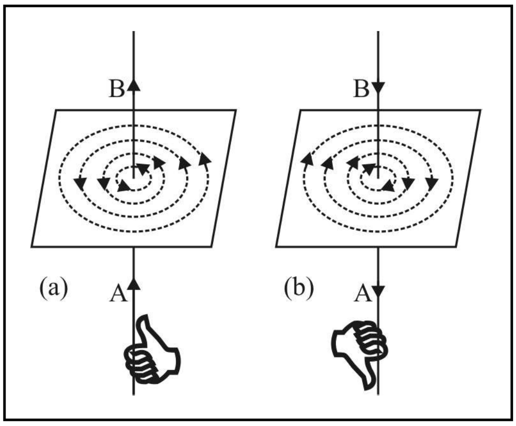
$$\phi_1 = \phi_2 = \phi \text{ and } \sin\phi = \frac{L/2}{\sqrt{a^2 + (L/2)^2}} = \frac{L}{\sqrt{4a^2 + L^2}}$$

Then, $B = \dfrac{\mu_0 I}{4\pi a}\left[\sin\phi + \sin\phi\right] = \dfrac{\mu_0}{4\pi}\dfrac{2I}{a}\sin\phi = \dfrac{\mu_0}{4\pi}\dfrac{2I}{a}\dfrac{L}{\sqrt{4a^2 + L^2}}$

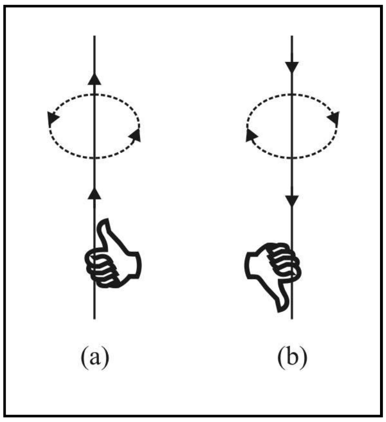
(iv) When point P lies on the wire conductor, then $d\vec{\ell}$ and $\vec{r}$ for each element of the straight wire conductor are parallel. Therefore, $d\vec{\ell} \times \vec{r} = 0$. So the magnetic field induction at P = 0.

Direction of magnetic field

The magnetic field lines due to straight conductor carrying current are in the form of concentric circles with the conductor as centre, lying in a plane perpendicular to the straight conductor. The direction of magnetic field lines is anticlockwise, if the current flows from A to B in the straight conductor figure (a) and is clockwise if the current flows from B to A in the straight conductor, figure (b). The direction of magnetic field lines is given by Right Hand Thumb Rule or Maxwell's cork screw rule.

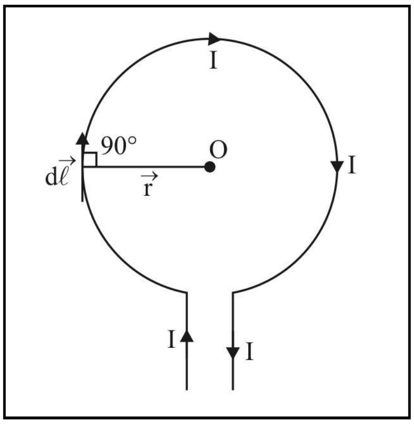


Right hand thumb rule. According to this rule, if we imagine the linear wire conductor to be held in the grip of the right hand so that the thumb points in the direction of current, then the curvature of the fingers around the conductor will represent the direction of magnetic field lines, figure (a) and (b).



12. MAGNETIC FIELD AT THE CENTRE OF THE CIRCULAR COIL CARRYING CURRENT

Consider a circular coil of radius r with centre O, lying with its plane in the plane of paper. Let I be the current flowing in the circular coil in the direction shown, figure (a). Suppose the circular coil is made of a large number of current elements each of length $d\ell$.



According to Biot-Savart's law, the magnetic field at the centre of the circular coil due to the current element $Id\vec{\ell}$ is given by

$$d\vec{B} = \frac{\mu_0}{4\pi} I\left(\frac{d\vec{\ell} \times \vec{r}}{r^3}\right)$$

or $$dB = \frac{\mu_0}{4\pi}\frac{Id\ell r\sin\theta}{r^3} = \frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r^2}$$

where $\vec{r}$ is the position vector of point O from the current element. Since the angle between $d\vec{\ell}$ and $\vec{r}$ is 90° (i.e., $\theta = 90°$), therefore,

$$dB = \frac{\mu_0}{4\pi}\frac{Id\ell\sin 90°}{r^2} \quad \text{or} \quad dB = \frac{\mu_0}{4\pi}\frac{Id\ell}{r^2} \quad ...(12)$$

In this case, the direction of $d\vec{B}$ is perpendicular to the plane of the current loop and is directed inwards. Since the current through all the elements of the circular coil will contribute to the magnetic feild in the same direction, therefore, the total magnetic field at point O due to current in the whole circular coil can be obtained by integrating eq. (12). Thus

$$B = \int dB = \int \frac{\mu_0}{4\pi}\frac{Id\ell}{r^2} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\int d\ell$$

But $\int d\ell$ = total length of the circular coil = circumference of the current loop = $2\pi r$

$\therefore$ $$B = \frac{\mu_0}{4\pi}\frac{I}{r^2}.2\pi r = \frac{\mu_0}{4\pi}\frac{2\pi I}{r}$$

If the circular coil consists of n turns, then

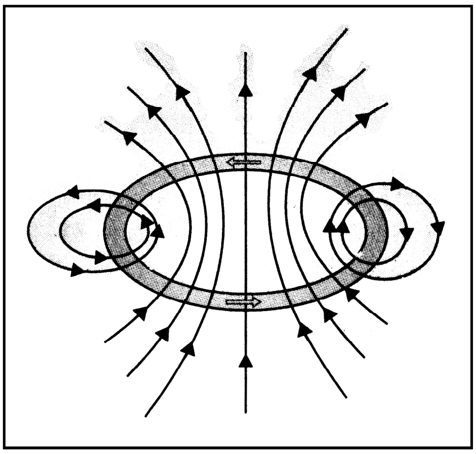
$$B = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r} = \frac{\mu_0}{4\pi}\frac{I}{r}\times 2\pi n \quad ...(13)$$

i.e. $B = \frac{\mu_0}{4\pi}\frac{I}{r}$ × angle subtended by coil at the centre.

Direction of $\vec{B}$

The direction of magnetic field at the centre of circular current loop is given by Right hand rule.

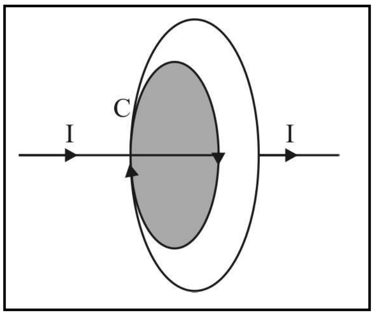
Right Hand rule. According to this rule, if we hold the thumb of right hand mutually perpendicular to the grip of the fingers such that the curvature of the fingers represent the direction of current in the wire loop, then the thumb of the right hand will point in the direction of magnetic field near the centre of the current loop.



13. AMPERE'S CIRCUITAL LAW

Consider an open surface with a boundary C, and the current I is passing through the surface. Let the boundary C be made of large number of small line elements, each of length $d\ell$. The direction of $d\vec{\ell}$ of small line element under study is acting tangentially to its length $d\ell$. Let B_t be the tangential component of the magnetic field induction at this element then $\vec{B}_t$ and $d\vec{\ell}$ are acting in the same direction, angle between them is zero. We take the product of B_t and $d\ell$ for that element. Then

$$B_t d\ell = \vec{B}.d\vec{\ell}$$



If length $d\ell$ is very small and products for all elements of closed boundary are added together, then sum tends to be an integral around the closed path or loop $(\text{i.e.}, \oint)$.

Therefore, Σ of $\vec{B}.d\vec{\ell}$ over all elements on a closed path $= \oint \vec{B}.d\vec{\ell} =$ Line integral of $\vec{B}$ around the closed path or loop whose boundary coincides with the closed path. According to Ampere's circuital law,

$$\oint \vec{B}.d\vec{\ell} = \mu_0 I \qquad ...(19)$$

where I is the total current threading the closed path or loop and μ_0 is the absolute permeability of the space. Thus,

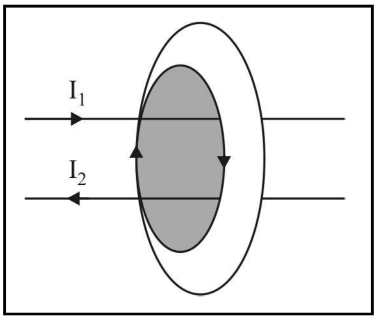
Ampere's circuital law states that the line integral of magnetic field induction $\vec{B}$ around a closed path in vacuum is equal to μ_0 times the total current I threading the closed path.

The relation (19) involves a sign convention, for the sense of closed path to be traversed while taking the line integral of magnetic field (i.e., direction of integration) and current threading it, which is given by Right Hand Rule. According to it, if curvature of the fingers is perpendicular to the thumb of right hand such that the curvature of the fingers represents the sense, the boundary is traversed in the closed path or loop for $\oint \vec{B}.d\vec{\ell}$, then the direction of thumb gives the sense in which the current I is regarded as positive.

According to sign convention, for the closed path as shown in figure, I_1 is positive and I_2 is negative. Then, according to Ampere's circuital law

$$\oint \vec{B}.d\vec{\ell} = \mu_0 (I_1 - I_2) = \mu_0 I_e$$

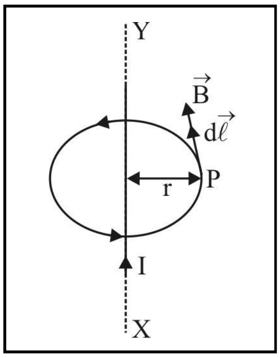
where I_e is the total current enclosed by the loop or closed path.



The relation (19) is independent of the size and shape of the closed path or loop enclosing the current.

14. MAGNETIC FIELD DUE TO INFINITE LONG STRAIGHT WIRE CARRYING CURRENT

Consider an infinite long straight wire lying in the plane of paper. Let I be the current flowing through it from X to Y. A magnetic field is produced which has the same magnitude at all points that are at the same distance from the wire, i.e. the magnetic field has cylindrical symmetry around the wire.



Let P be a point at a perpendicular distance r from the straight wire and $\vec{B}$ be the magnetic field at P. It will be acting tangentially to the magnetic field line passing through P. Consider an amperian loop as a circle of radius r, perpendicular to the plane of paper with centre on wire such that point P lies on the loop, figure. The magnitude of magnetic field is same at all points on this loop. The magnetic field $\vec{B}$ at P will be tangential to the circumference of the circular loop. We shall integrate the amperian path anticlockwise. Then $\vec{B}$ and $d\vec{\ell}$ are acting in the same direction. The line integral of $\vec{B}$ around the closed loop is

$$\oint \vec{B}.d\vec{\ell} = \oint B d\ell \cos 0° = B \oint d\ell = B 2\pi r$$

As per sign convention, here I is positive,

Using Ampere's circuital law

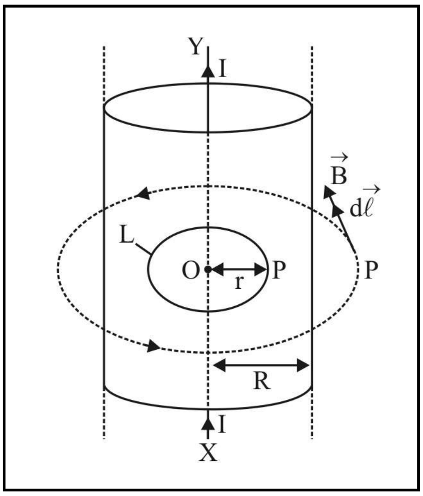
$$\oint \vec{B}.d\vec{\ell} = \mu_0 I \quad \text{or} \quad B 2\pi r = \mu_0 I$$

or

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0}{4\pi} \frac{2I}{r} \qquad ...(21)$$

15. MAGNETIC FIELD DUE TO CURRENT THROUGH A VERY LONG CIRCULAR CYLINDER

Consider an infinite long cylinder of radius R with axis XY. Let I be the current passing through the cylinder. A magnetic field is set up due to current through the cylinder in the form of circular magnetic lines of force, with their centres lying on the axis of cylinder. These lines of force are perpendicular to the length of cylinder.



Case I. Point P is lying outside the cylinder. Let r be the perpendicular distance of point P from the axis of cylinder, where r > R. Let $\vec{B}$ be the magnetic field induction at P. It is acting tangential to the magnetic line of force at P directed into the paper. Here $\vec{B}$ and $d\vec{\ell}$ are acting in the same direction.

Applying Ampere circuital law we have

$$\oint \vec{B}.d\vec{\ell} = \mu_0 I \quad \text{or} \quad \oint B d\ell \cos 0° = \mu_0 I$$

or $$\oint B d\ell = \mu_0 I \quad \text{or} \quad B 2\pi r = \mu_0 I$$

or $$B = \frac{\mu_0 I}{2\pi r}, \text{ i.e., } B \propto 1/r$$

Case II. Point P is lying inside cylinder. Here r < R. we may have two possibilities.

(i) If the current is only along the surface of cylinder which is so if the conductor is a cylindrical sheet of metal, then current through the closed path L is zero. Using Ampere circutal law, we have B = 0.

(ii) If the current is uniformly distributed throughout the cross-section of the conductor, then the current through closed path L is given by

$$I' = \frac{I}{\pi R^2} \times \pi r^2 = \frac{I r^2}{R^2}$$

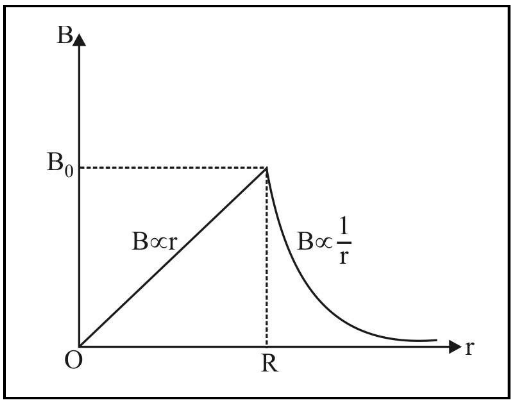
Applying Ampere's circuital law, we have

$$\oint \vec{B}.d\vec{\ell} = \mu_0 \mu_r I'$$

or $$2\pi r B = \mu_0 \mu_r I' = \frac{\mu_0 \mu_r I r^2}{R^2}$$

or $$B = \frac{\mu_0 \mu_r I r}{2\pi R^2} \text{ i.e., } B \propto r$$

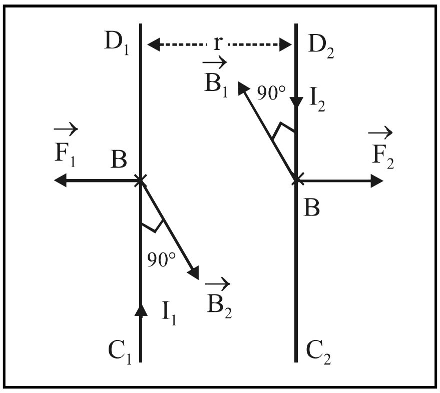
If we plot a graph between magnetic field induction B and distance from the axis of cylinder for a current flowing through a solid cylinder, we get a curve of the type as shown figure



Here we note that the magnetic field induction is maximum for a point on the surface of solid cylinder carrying current and is zero for a point on the axis of cylinder.

16. FORCE BETWEEN TWO PARALLEL CONDUCTORS CARRYING CURRENT

Consider C_1D_1 and C_2D_2, two infinite long straight conductors carrying currents I_1 and I_2 in the same direction. They are held parallel to each other at a distance r apart, in the plane of paper. The magnetic field is produced due to current through each conductor shown separately in figure. Since each conductor is in the magnetic field produced by the other, therefore, each conductor experiences a force.



Magnetic field induction at a point P on conductor C_2D_2 due to current I_1 passing through C_1D_1 is given by

$$B_1 = \frac{\mu_0}{4\pi} \frac{2I_1}{r} \qquad ...(12)$$

According to **right hand rule**, the direction of magnetic field $\vec{B}_1$ is perpendicular to the plane of paper, directed inwards.

As the current carrying conductor C_2D_2 lies in the magnetic field $\vec{B}_1$ (produced by the current through C_1D_1), therefore, the unit length of C_2D_2 will experience a force given by

$F_2 = B_1I_2 \times 1 = B_1I_2$

Putting the value of B_1, we have

$$F_2 = \frac{\mu_0}{4\pi}.\frac{2I_1I_2}{r} \quad ...(13)$$

It means the **two linear parallel conductors carrying currents in the same direction attract each other.**

Thus **one ampere is that much current which when flowing through each of the two parallel uniform long linear conductors placed in free space at a distance of one metre from each other will attract or repel each other with a force of 2×10^{-7} N per metre of their length.**

17. THE SOLENOID

A solenoid consists of an insulating long wire closely wound in the form of a helix. Its length is very large as compared to its diameter.

Magnetic field due to a solenoid

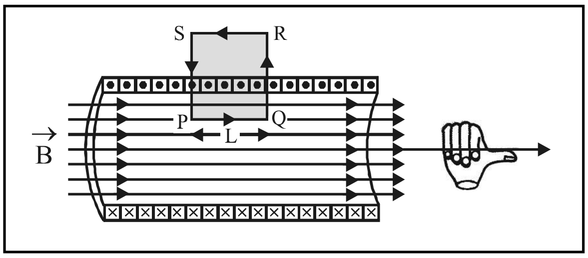
Consider a long straight solenoid of circular cross-section. Each two turns of the solenoid are insulated from each other. When current is passed through the solenoid, then each turn of the solenoid can be regarded as a circular loop carrying current and thus will be producing a magnetic field.

At a point outside the solenoid, the magnetic fields due to neighbouring loops oppose each other and at a point inside the solenoid, the magnetic fields are in the same direction. As a result of it, the effective magnetic field outside the solenoid becomes weak, whereas the magnetic field in the interior of solenoid becomes strong and uniform, acting along the axis of the solenoid.

Let us now apply Ampere's circuital law.

Let n be the number of turns per unit length of solenoid and I be the current flowing through the solenoid and the turns of the solenoid be closely packed.

Consider a rectangular amperian loop PQRS near the middle of solenoid as shown in figure



The line integral of magnetic field induction $\vec{B}$ over the closed path PQRS is

$$\oint_{PQRS} \vec{B}.d\vec{\ell} = \int_P^Q \vec{B}.d\vec{\ell} + \int_Q^R \vec{B}.d\vec{\ell} + \int_R^S \vec{B}.d\vec{\ell} + \int_S^P \vec{B}.d\vec{\ell}$$

Here, $\int_P^Q \vec{B}.d\vec{\ell} = \int_P^Q B d\ell \cos 0° = BL$

and $\int_Q^R \vec{B}.d\vec{\ell} = \int_Q^R B d\ell \cos 90° = 0 = \int_S^P \vec{B}.d\vec{\ell}$

Also, $\int_R^S \vec{B}.d\vec{\ell} = 0$ ($\because$ outside the solenoid, B = 0)

$$\oint_{PQRS} \vec{B}.d\vec{\ell} = BL + 0 + 0 + 0 = BL \quad ...(21)$$

From Ampere's circuital law

$$\oint_{PQRS} \vec{B}.d\vec{\ell} = \mu_0 \times \text{total current through the rectangle PQRS}$$

$= \mu_0 \times$ no. of turns in rectangle $\times$ current

$= \mu_0 n LI \quad ...(22)$

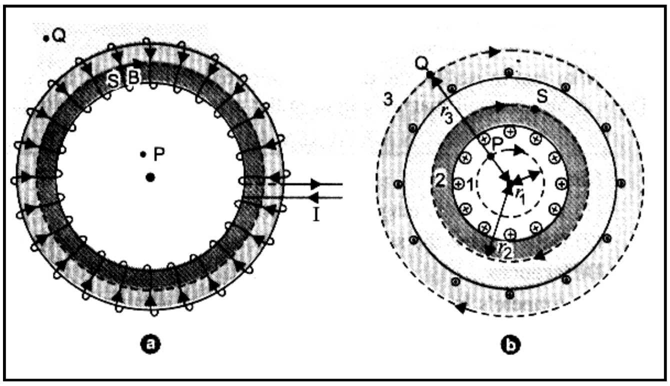
From (21) and (22), we have

$BL = \mu_0 n LI$ or $B = \mu_0 n I$

This relation gives the magnetic field induction at a point well inside the solenoid. At a point near the end of a solenoid, the magnetic field induction is found to be $\mu_0 n I/2$.

18. TOROID

The toroid is a hollow circular ring on which a large number of insulated turns of a metallic wire are closely wound. In fact, a toroid is an endless solenoid in the form of a ring, figure.



Magnetic field due to current in ideal toroid

Let n be the number of turns per unit length of toroid and I be the current flowing through it. In case of ideal toroid, the coil turns are circular and closely wound. A magnetic field of constant magnitude is set up inside the turns of toroid in the form of concentric circular magnetic field lines. The direction of the magnetic field at a point is given by the tangent to the magnetic field line at that point. We draw three circular amperian loops, 1, 2 and 3 of radii r_1, r_2 and r_3 to be traversed in clockwise direction as shown by dashed circles in figure, so that the points P, S and Q may lie on them. The circular area bounded by loops 2 and 3, both cut the toroid. Each turn of current carrying wire is cut once by the loop 2 and twice by the loop 3. Let B_1 be the magnitude of magnetic field along loop 1. Line integral of magnetic field B_1 along the loop 1 is

$$\oint_{\text{loop 1}} \vec{B}_1 . d\vec{\ell} = \oint_{\text{loop 1}} B_1 d\ell \cos 0° = B_1 2\pi r_1 \quad ...(i)$$

Loop 1 encloses no current.

According to Ampere's circuital law

$$\oint_{\text{loop 1}} \vec{B}_1 . d\vec{\ell} = \mu_0 \times \text{current enclosed by loop } 1 = \mu_0 \times 0 = 0$$

or $B_1 2\pi r_1 = 0$ or $B_1 = 0$

Let B_3 be the magnitude of magnetic field along the loop 3. The line integral of magnetic field B_3 along the loop 3 is

$$\oint_{\text{loop 3}} \vec{B}_3 . d\vec{\ell} = \oint_{\text{loop 3}} B_3 d\ell \cos 0° = B_3 2\pi r_3$$

From the sectional cut as shown in figure, we note that the current coming out of the plane of paper is cancelled exactly by the current going into it. Therefore, the total current enclosed by loop 3 is zero.

According to Ampere's circuital law

$$\oint_{\text{loop 3}} \vec{B}_3 . d\vec{\ell} = \mu_0 \times \text{total current through loop 3}$$

or $B_3 2\pi r_3 = \mu_0 \times 0 = 0$ or $B_3 = 0$

Let B the magnitude of magnetic field along the loop 2. Line integral of magnetic field along the loop 2 is

$$\oint_{\text{loop 2}} \vec{B} . d\vec{\ell} = B 2\pi r_2$$

Current enclosed by the loop 2 = number of turns × current in each turn = $2\pi r_2 n \times I$

According to Ampere's circuital law

$$\oint_{\text{loop 2}} \vec{B} . d\vec{\ell} = \mu_0 \times \text{total current}$$

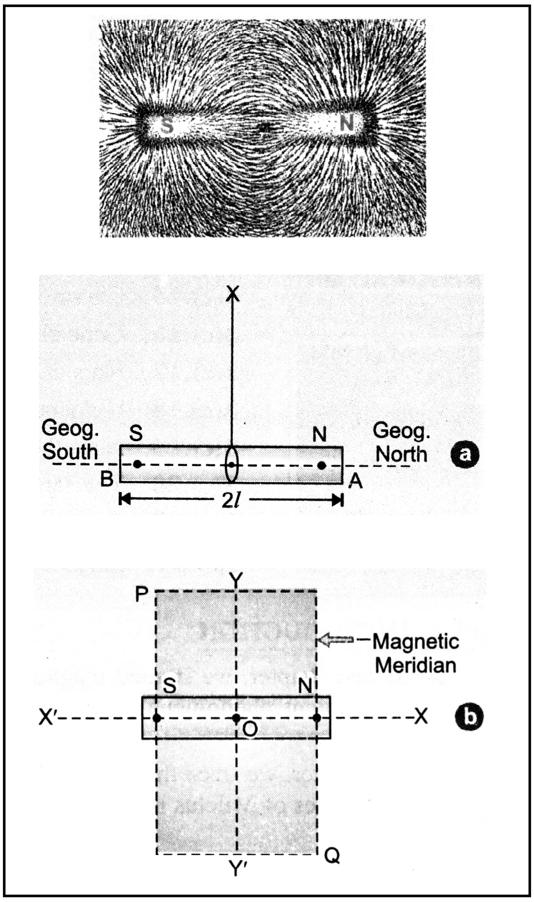
or $B 2\pi r_2 = \mu_0 \times 2\pi r_2 n I$ or $B = \mu_0 n I$

19. MAGNETISM & MATTER

19.1 The Bar Magnet

It is the most commonly used form of an artificial magnet.

When we hold a sheet of glass over a short bar magnet and sprinkle some iron filings on the sheet, the iron filings rearrange themselves as shown in figure. The pattern suggests that attraction is maximum at the two ends of the bar magnet. These ends are called poles of the magnet.



1. The earth behaves as a magnet.
2. Every magnet attracts small pieces of magnetic substances like iron, cobalt, nickel and steel towards it.
3. When a magnet is suspended freely with the help of an unspun thread, it comes to rest along the north south direction.
4. Like poles repel each other and unlike poles attract each other.
5. The force of attraction or repulsion F between two magnetic poles of strengths m_1 and m_2 separated by a distance r is directly proportional to the product of pole strengths and inversely proportional to the square of the distance between their centres, i.e.,

$F \propto \frac{m_1 m_2}{r^2}$ or $F = K\frac{m_1 m_2}{r^2}$, where K is magnetic force constant.

In SI units, $K = \frac{\mu_0}{4\pi} = 10^{-7}\,\text{Wb A}^{-1}\text{m}^{-1}$

where μ_0 is absolute magnetic permeability of free space (air/vacuum).

$\therefore \quad \boxed{F = \frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2}} \qquad ...(1)$

This is called Coulomb's law of magnetic force. However, in cgs system, the value of K = 1.

Note...

This corresponds to Coulomb's law in electrostatics.

SI Unit of magnetic pole strength

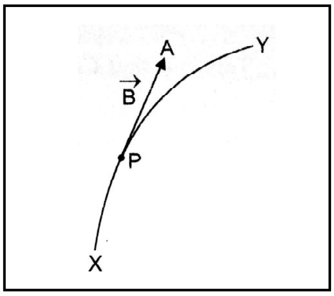
Suppose $m_1 = m_2 = m$ (say),

$r = 1$ m and $F = 10^{-7}$ N

From equation (1),

$10^{-7} = 10^{-7} \times \frac{(m)(m)}{1^2}$ or $m^2 = 1$ or $m = \pm 1$ ampere-metre (Am). Therefore, strength of a magnetic pole is said to be one ampere-metre, if it repels an equal and similar pole, when placed in vacuum (or air) at a distance of one metre from it, with a force of 10^{-7} N.

6. The magnetic poles always exist in pairs. The poles of a magnet can never be separated i.e. magnetic monopoles do not exist.

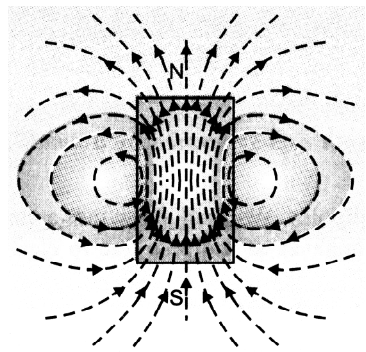


20. MAGNETIC FIELD LINES

Magnetic field line is an imaginary curve, the tangent to which at any point gives us the direction of magnetic field $\vec{B}$ at that point.

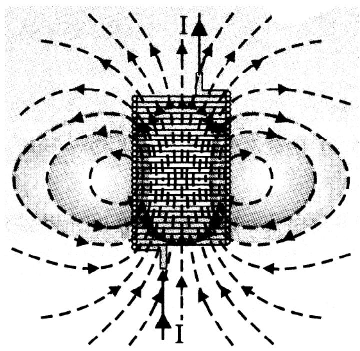
If we imagine a number of small compass needless around a magnet, each compass needle experiences a torque due to the field of the magnet. The torque acting on a compass needle aligns it in the direction of the magnetic field.

The path along which the compass needles are aligned is known as magnetic field line.



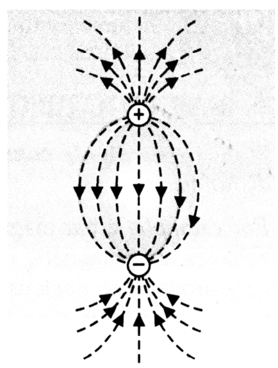
Magnetic field lines due to a bar magnet

(a)



Magnetic field lines due to a solenoid carrying current

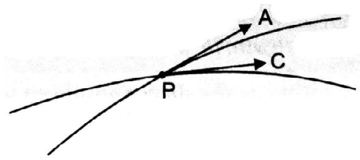
(b)


Electric filed lines due to an electric dipole

(c)

Properteis of magnetic field lines

1. The magnetic field lines of a magnet (or of a solenoid carrying current) form closed continuous loops.
2. Outside the body of the magnet, the direction of magnetic field lines is from north pole to south pole.
3. At any given point, tangent to the magnetic field line represents the direction of net magnetic field ($\vec{B}$) at that point.
4. The magnitude of magnetic field at any point is represented by the number of magnetic field lines passing normally through unit area around that point. Therefore, crowded lines represent a strong magnetic field and lines which are not so crowded represent a weak magnetic field.
5. No two magnetic field lines can intersect each other.



21. MAGNETIC DIPOLE

A magnetic dipole consists of two unlike poles of equal strength and separated by a small distance.

For example, a bar magnet, a compass needle etc. are magnetic dipoles. We shall show that a current loop behaves as a magnetic dipole. An atom of a magnetic material behaves as a dipole due to electrons revolving around the nucleus.

The two poles of a magnetic dipole (or a magnet), called north pole and south pole are always of equal strength, and of opposite nature. Further such two magnetic poles exist always in pairs and cannot be separated from each other.

The distance between the two poles of a bar magnet is called the **magnetic length** of the magnet. It is a vector directed from S-pole of magnet to its N-pole, and is represented by $2\vec{\ell}$.

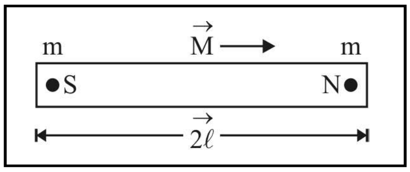
Magnetic dipole moment is the product of strength of either pole (m) and the magnetic length ($2\vec{\ell}$) of the magnet.

It is represented by $\vec{M}$.

Magnetic dipole moment = strength of either pole × magnetic length

$$\boxed{\vec{M} = m\left(2\vec{\ell}\right)}$$

Magnetic dipole moment is a vector quantity directed from South to North pole of the magnet, as shown in figure



We shall show that the SI unit of M is joule/tesla or ampere metre^2.

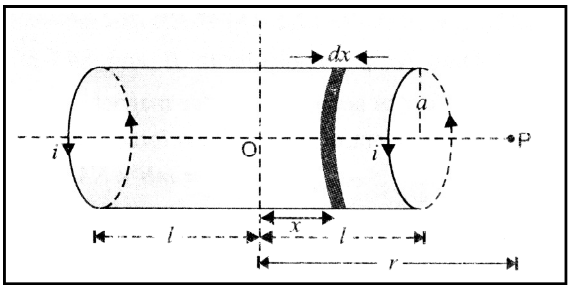
∴ SI unit of pole strength is Am.

Bar magnet as an equivalent solenoid

We know that a current loop acts as a magnetic dipole. According to Ampere's hypothesis, all magnetic phenomena can be explained in terms of circulating currents.

In figure magnetic field lines for a bar magnet and a current carrying solenoid resemble very closely. Therefore, a bar magnet can be thought of as a large number of circulating currents in analogy with a solenoid. Cutting a bar magnet is like cutting a solenoid. We get two smaller solenoids with weaker magnetic properties. The magnetic field lines remain continuous, emerging from one face of one solenoid and entering into other face of other solenoid. If we were to move a small compass needle in the neighbourhood of a bar magnet and a current carrying solenoid, we would find that the deflections of the needle are similar in both cases.

To demonstrate the similarity of a current carrying solenoid to a bar magnet, let us calculate axial field of a finite solenoid carrying current.



In figure, suppose

a = radius of solenoid,

2ℓ = length of solenoid with centre O

n = number of turns per unit length of solenoid,

i = strength of current passed through the solenoid

We have to calculate magnetic field at any point P on the axis of solenoid, where OP = r. Consider a small element of thickness dx of the solenoid, at a distance x from O.

Number of turns in the element = n dx.

Using equation, magnitude of magnetic field at P due to this current element is

$$dB = \frac{\mu_0 i a^2 (n\,dx)}{2\left[(r-x)^2 + a^2\right]^{3/2}} \quad ...(10)$$

If P lies at a very large distance from O, i.e., r >> a and r >> x, then $[(r-x)^2 + a^2]^{3/2} \approx r^3$

$$dB = \frac{\mu_0 i a^2 n dx}{2r^3} \quad ...(11)$$

As range of variation of x is from $x = -\ell$ to $x = +\ell$, therefore the magnitude of total magnetic field at P due to current carrying solenoid

$$B = \frac{\mu_0 n i a^2}{2r^3} \int_{x=-\ell}^{x=+\ell} dx = \frac{\mu_0 n i a^2}{2r^3} [x]_{x=-\ell}^{x=+\ell}$$

$$B = \frac{\mu_0 n i}{2} \frac{a^2}{r^3} (2\ell) = \frac{\mu_0}{4\pi} \frac{2n(2\ell) i \pi a^2}{r^3} \quad ...(12)$$

If M is magnetic moment of the solenoid, then

M = total no. of turns × current × area of cross section

$M = n(2\ell) \times i \times (\pi a^2)$

$$\therefore \quad \boxed{B = \frac{\mu_0}{4\pi} \frac{2M}{r^3}} \quad ...(13)$$

This is the expression for magnetic field on the axial line of a short bar magnet.

Thus, the axial field of a finite solenoid carrying current is same as that of a bar magnet. Hence, for all practical purposes, a finite solenoid carrying current is equivalent to a bar magnet.

Potential energy of a magnetic dipole in a magnetic field

Potential energy of a magnetic dipole in a magnetic field is the energy possessed by the dipole due to its particular position in the field.

When a magnetic dipole of moment $\vec{M}$ is held at an angle θ with the direction of a uniform magnetic field $\vec{B}$, the magnitude of the torque acting on the dipole is

$$\tau = MB \sin\theta \quad ...(16)$$

This torque tends to align the dipole in the direction of the field. Work has to be done in rotating the dipole against the action of the torque. This work done is stored in the magnetic dipole as potential energy of the dipole.

Now, small amount of work done in rotating the dipole through a small angle dθ against the restoring torque is

$dW = \tau\, d\theta = MB \sin\theta\, d\theta$

Total work done in rotating the dipole from $\theta = \theta_1$ to $\theta = \theta_2$ is

$$W = \int_{\theta_1}^{\theta_2} MB \sin\theta\, d\theta = MB[-\cos\theta]_{\theta_1}^{\theta_2} = -MB[\cos\theta_2 - \cos\theta_1]$$

∴ Potential energy of the dipole is

$$\boxed{U = W = -MB(\cos\theta_2 - \cos\theta_1)} \quad ...(17)$$

When $\theta_1 = 90°$, and $\theta_2 = \theta$, then

$U = W = -MB(\cos\theta - \cos 90°)$

$W = -MB\cos\theta \quad ...(18)$

In vector notation, we may rewrie (18) as

$$\boxed{U = -\vec{M}.\vec{B}} \quad ...(19)$$

Particular Cases

1. When θ = 90°

   $U = -MB\cos\theta = -MB\cos 90° = 0$

i.e., when the dipole is perpendicular to magnetic field its potential energy is zero.

Hence to calculate potential energy of diole at any position making angle θ with B, we use

$U = -MB(\cos\theta_2 - \cos\theta_1)$ and take $\theta_1 = 90°$ and $\theta_2 = \theta$. Therefore,

$U = -MB(\cos\theta - \cos 90°) = -MB\cos\theta$

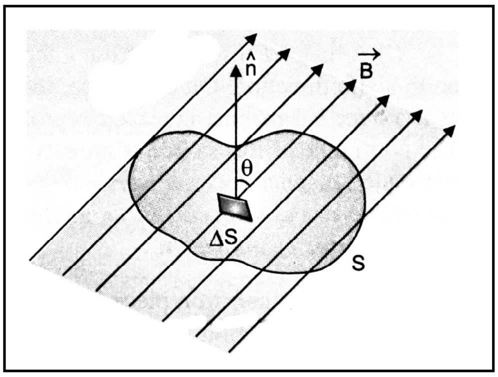
2. When θ = 0°

   $U = -MB\cos 0° = -MB$

   which is minimum. This is the position of stable equilibrium, i.e., when the magnetic dipole is aligned along the magnetic field, it is in stable equilibrium having minimum P.E.

3. When θ = 180°

   $U = -MB\cos 180° = MB$, which is maximum. This is the position of unstable equilibrium.



22. MAGNETISM AND GAUSS'S LAW

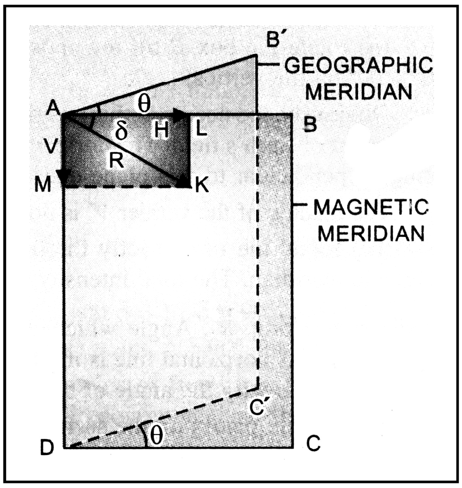
According to Gauss's law for magnetism, the net magnetic flux (ϕ_B) through any closed surface is always zero.

23. EARTH'S MAGNETISM

Magnetic elements of earth at a place are the quantities which describe completely in magnitude as well as direction, the magnetic field of earth at that place.

23.1 Magnetic declination

Magnetic declination at a place is the angle between magnetic meridian and geographic meridian at that place.



Retain in Memory

1. The earth's magnetic poles are not at directly opposite positions on globe. Current magnetic south is farther from geographic south than magnetic north is from geographic north.
2. Infact, the magnetic field of earth varies with position and also with time. For example, in a span of 240 years from 1580 to 1820 A.D., the magnetic declination at London has been found to change by 3.5° – suggesting that magnetic poles of earth change their position with time.
3. The magnetic declination in India is rather small. At Delhi, declination is only 0° 41′ East and at Mumbai, the declination is 0° 58′ West. Thus at both these places, the direction of geographic north is given quite accurately by the compass needle (within 1° of the actual direction).

23.2 Magnetic Dip or Magnetic Inclination

Magnetic dip or magnetic inclination at a place is defined as the angle which the direction of total strength of earth's magnetic field makes with a horizontal line in magnetic meridian.

23.3 Horizontal Component

It is the component of total intensity of earth's magnetic field in the horizontal direction in magnetic meridian. It is represented by H.

In figure, AK represents the total intensity of earth's magnetic field, $\angle BAK = \delta$. The resultant intensity R along AK is resolved into two rectangular components :

Horizontal component along AB is

$AL = H = R\cos\delta$ ...(23)

Vertical component along AD is

$AM = V = R\sin\delta$ ...(24)

Square (23) and (24), and add

$H^2 + V^2 = R^2(\cos^2\delta + \sin^2\delta) = R^2$

$\therefore$ $\boxed{R = \sqrt{H^2 + V^2}}$ ...(25)

Dividing (24) by (23), we get

$\frac{R\sin}{R\cos} = \frac{V}{H}$ or $\boxed{\tan = \frac{V}{H}}$ ...(26)

The value of horizontal component $H = R\cos\delta$ is different at different places. At the magnetic poles, $\delta = 90°$

$\therefore$ $H = R\cos 90° =$ zero

At the magnetic equator, $\delta = 0°$

$\therefore$ $H = R\cos 0° = R$

Horizontal component (H) can be measured using both, a vibration magnetometer and a deflection magnetometer.

The value of H at a place on the surface of earth is of the order of 3.2×10^{-5} tesla.

Memory note

Note that the direction of horizontal component H of earth's magnetic field is from geographic south to geographic north above the surface of earth. (if we ignore declination).

24. MAGNETIC PROPERTIES OF MATTER

To describe the magnetic properties of materials, we define the following few terms, which should be clearly understood

24.1 Magnetic Permeability

It is the ability of a material to permit the passage of magnetic lines of force through it i.e. the degree or extent to which magnetic field can penetrate or permeate a material is called relative magnetic permeability of the material. It is represented by μ_r.

Relative magnetic permeability of a mterial is defined as the ratio of the number of magnetic field lines per unit area (i.e. flux density B) in that material to the number of magnetic field lines per unit area that would be present, if the medium were replaced by vacuum. (i.e. flux density B_0).

i.e., $\boxed{\mu_r = \frac{B}{B_0}}$

Relative magnetic permeability of a material may also be defined as the ratio of magnetic permeability of the material (μ) and magnetic permeability of free space (μ_0)

$\therefore$ $\boxed{\mu_r = \frac{\mu}{\mu_0}}$ or $\mu = \mu_r\mu_0$

We know that $\mu_0 = 4\pi \times 10^{-7}$ weber/amp-metre ($Wb\,A^{-1}\,m^{-1}$) or henry/metre (Hm^{-1})

∴ SI units of permeability (μ) are

$Hm^{-1} = Wb\,A^{-1}\,m^{-1} = (T\,m^2)\,A^{-1}\,m^{-1} = T\,m\,A^{-1}$

24.2 Magnetic Intensity ($\vec{H}$)

The degree to which a magnetic field can magnetise a material is represented in terms of magnetising force or magnetise intensity ($\vec{H}$).

24.3 Magnetisation or Intensity of Magnetisation 'I'

It represents the extent to which a specimen is magnetised, when placed in a magnetising field. Quantitatively,

The magnetisation of a magnetic material is defined as the magnetic moment per unit volume of the material.

$$(M) = \frac{\text{Magnetic moment}}{\text{volume}} = \frac{m}{V}$$

There are SI unit of I, which are the same as SI units of H.

Magnetic susceptibility (χ_m) of a magnetic material is defined as the ratio of the intensity of magnetisation (I) induced in the material to the magnetising force (H) applied on it. Magnetic susceptibility is represented by χ_m .

Thus $\boxed{\chi_m = \frac{I}{H}}$

Relation between magnetic permeability and magnetic susceptibility

When a magnetic material is placed in a magnetising field of magnetising intensity H, the material gets magnetised. The total magnetic induction B in the material is the sum of the magnetic induction B_0 in vacuum produced by the magnetic intensity and magnetic induction B_m, due to magnetisation of the material. Therefore,

$B = B_0 + B_m$

But $B_0 = \mu_0$ H and $B_m = m_0$ I, where I is the intensity of magnetisation induced in the magnetic material. Therefore, from above

$$B = \mu_0 H + \mu_0 I = \mu_0 (H + I),$$

i.e., $\boxed{B = \mu_0 (H + I)}$

Now as $\chi_m = \frac{I}{H}$ ∴ $I = \chi_m H$

From above, $B = \mu_0 (H + \chi_m H) = \mu_0 H(1 + \chi_m)$

But $B = \mu H$

∴ $\mu H = \mu_0 H(1 + \chi_m)$ or $\frac{\mu}{\mu_0} = 1 + \chi_m$

or $\boxed{\mu_r = 1 + \chi_m}$

This is the relation between relative magnetic permeability and magnetic susceptibility of the material.

25. CLASSIFICATION OF MAGNETIC MATERIALS

There is a large variety of elements and compounds on earth. Some new elements, alloys and compounds have been synthesized in the laboratory. **Faraday** classified these substances on the basis of their magnetic properties, into the following three categories :

(i) Diamagnetic substances,

(ii) Paramagnetic substances, and

(iii) Ferromagnetic substances

Their main characteristics are discussed below :

25.1 Diamagnetic Substances

The diamagnetic substances are those in which the individual atoms/molecules/ions do not possess any net magnetic moment on their own. When such substances are placed in an external magnetising field, they get feebly magnetised in a direction opposite to the magnetising field.

when placed in a non-uniform magnetic field, these substances have a tendency to move from stronger parts of the field to the weaker parts.

When a specimen of a diamagnetic material is placed in a magnetising field, the magnetic field lines prefer not to pass through the specimen.

Relative magnetic permeability of diamagnetic substances is always less than unity.

From the relation $\mu_r = (1 + \chi_m)$, a $\mu_r < 1, \chi_m$ is negative.

Hence susceptibility of diamagnetic substances has a small negative value.

A superconductor repels a magnet and in turn, is repelled by the magnet.

The phenomenon of perfect diamagnetism in superconductors is called **Meissner effect**. Superconducting magnets have been used for running magnetically leviated superfast trains.

25.2 Paramagnetic substances

Paramagnetic substacnes are those in which each individual atom/molecule/ion has a net non zero magnetic moment of its own. When such substances are placed in an external magnetic field, they get feebly magnetised in the direction of the magnetising field.

When placed in a non-uniform magnetic field, they tend to move from weaker parts of the field to the stronger parts.

When a specimen of a paramagnetic substance is placed in a magnetising field, the magnetic field lines prefer to pass through the specimen rather than through air.

From the SI relation, $\mu_r = 1 + \chi_m$, as $\mu_r > 1$, therefore, χ_m must be positive. Hence, susceptibility of paramagnetic substances is positive, though small.

Susceptibility of paramagnetic substances varies inversely as the temperature of the substance i.e. $\chi_m \propto \frac{1}{T}$ i.e. they lose their magnetic character with rise in temperature.

25.3 Ferromagnetic substances

Ferromagnetic substances are those in which each individual atom/molecule/ion has a non zero magnetic moment, as in a paramagnetic substance.

When such substances are placed in an external magnetising field, they get strongly magnetised in the direction of the field.

The ferromagnetic materials show all the properties of paramagnetic substances, but to a much greater degree. For example,

(i) They are strongly magnetised in the direction of external field in which they are placed.

(ii) Relative magnetic permeability of ferromagnetic materials is very large ($\approx 10^3$ to 10^5)

(iii) The susceptibility of ferromagnetic materials is also very large. $(\because \chi_m = \mu_r - 1)$

That is why they can be magnetised easily and strongly.

(iv) With rise in temperature, susceptibility of ferromagnetics decreases. At a certain temperature, ferromagnetics change over to paramagnetics. This transition temperature is called curie temperature. For example, curie temperature of iron is about 1000 K.

Substance	χ_m	μ_r	μ
1. Diamagnetic	$-1 \le \chi_m < 0$	$0 \le \mu_r < 1$	$\mu < \mu_0$
2. Paramagnetic	$0 < \chi_m < \in *$	$1 < \mu_r < (1 + \in)$	$\mu > \mu_0$
3. Ferromagnetic	$\chi_m >> 1$	$\mu_r >> 1$	$\mu >> \mu_0$

25.4 Curie Law in Magnetism

According to Curie law,

Intensity of magnetisation (I) of a magnetic material is (i) directly proportional to magnetic induction (B), and (ii) inversely proportional to the temperature (T) of the material.

i.e., $I \propto B$, and $I \propto \frac{1}{T}$

Combining these factors, we get $I \propto \frac{B}{T}$

As $B \propto H$, magnetising intensity

$\therefore \quad I \propto \frac{H}{T}$ or $\frac{I}{H} \propto \frac{1}{T}$

But $\frac{I}{H} = \chi_m$

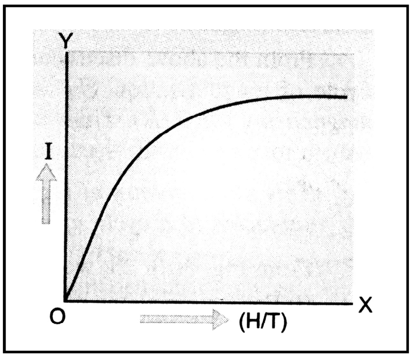
$\therefore \quad \chi_m \propto \frac{1}{T}$ or $\boxed{\chi_m = \frac{C}{T}}$

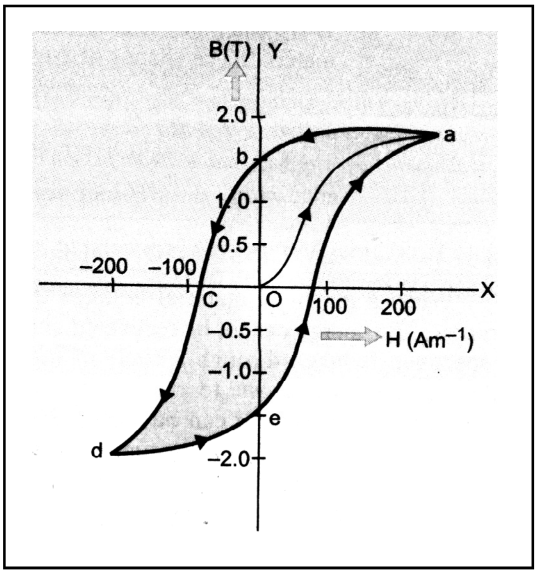
where C is a constant of proportionality and is called Curie constant.

26. HYSTERISIS CURVE

The hysterisis curve represents the relation between magnetic induction $\vec{B}$ (or intensity of magnetization $\vec{I}$) of a ferromagnetic material with magnetiziing force or magnetic intensity $\vec{H}$. The shape of the hysterisis curve is shown in figure. It represents the behaviour of the material as it is taken through a cycle of magnetization.

Suppose the material is unmagnetised initially i.e., $\vec{B} = 0$ and $\vec{H} = 0$. This state is represented by the origin O. We place the material in a solenoid and increase the current through the solenoid gradually. The magnetising force $\vec{H}$ increases. The magnetic induction $\vec{B}$ in the material increases and saturates as depicted in the curve oa. This behaviour represents alignment and merger of the domains of ferromagnetic material until no further enhancement in $\vec{B}$ is possible. Therefore, there is no use of inreasing solenoid current and hence magnetic intensity beyond this.





Next, we decrease the solenoid current and hence magnetic intensity $\vec{H}$ till it reduces to zero. The curve follows the path ab showing that when $\vec{H}=0$, $\vec{B}\neq 0$. Thus, some magnetism is left in the specimen.

The value of magnetic induction $\vec{B}$ left in the specimen when the magnetising force is reduced to zero is called Retentivity or Remanence or Residual magnetism of the material.

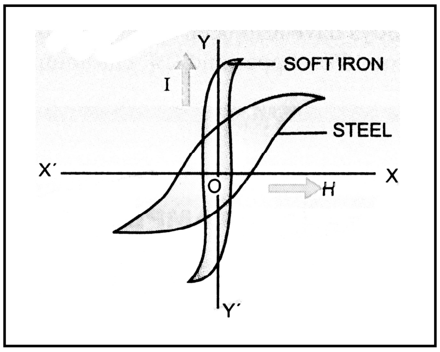
It shows that the domains are not completely randomised even when the magnetising force is removed. Next, the current in the solenoid is reversed and increased slowly. Certain domains are flipped until the net magnetic induction $\vec{B}$ inside is reduced to zero. This is represented by the curve bc. It means to reduce the residual magnetism or retentivity to zero, we have to apply a magnetising force = OC in opposite direction. This value of magnetising force is called coercivity of the material.

As the reverse current in solenoid is increased in magnitude, we once again obtain saturation in the reverse direction at d. The variation is represented by the curve cd. Next, the solenoid current is reduced (curve de), reversed and increased (curve ea). The cycle repeats itself. From figure, we find that saturated magnetic induction B_S is of the order of 1.5 T and coercivity is of the order of –90 Am^{-1}.

From the above discussion, it is clear that when a specimen of a magnetic material is taken through a cycle of magnetisation, the intensity of magnetisation (I) and magnetic induction (B) lag behind the magnetising force (H). Thus, even if the magnetising force H is made zero, the values of I and B do not reduce to zero i.e., the specimen tends to retain the magnetic properties.

This phenomenon of lagging of I or B behind H when a specimen of a magnetic material is subjected to a cycle of magnetisation is called hysteresis.

For example, hysteresis loop for soft iron is narrow and large, whereas the hysteresis loop for steel is wide and short, figure



The hysterisis loops of soft iron and steel reveal that

(i) The retentivity of soft iron is greater than the retentivity of steel,

(ii) Soft iron is more strongly magnetised than steel,

(iii) Coercivity of soft iron is less than coercivity of steel. It means soft iron loses its magnetism more rapidly than steel does.

(iv) As area of I-H loop for soft iron is smaller than the area of I-H loop for steel, therefore, hysterisis loss in case of soft iron is smaller than the hysterisis loss in case of steel.

(a) Permanent Magnets

Permanent magnets are the materials which retain at room temperature, their ferromagnetic properties for a long time. The material chosen should have

(i) high retentivity so that the magnet is strong,

(ii) high coercivity so that the magnetisation is not erased by stray magnetic fields, temperature changes or mechanical damage due to rough handling etc.

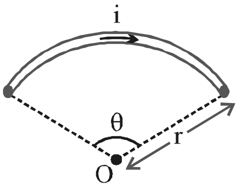
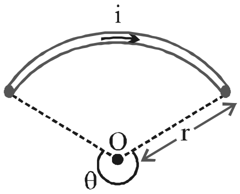
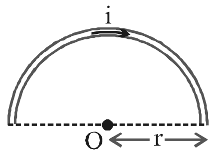
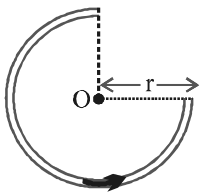
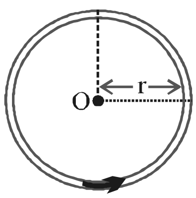
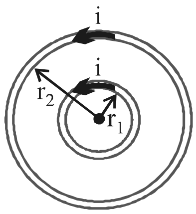
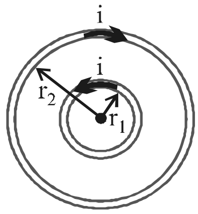
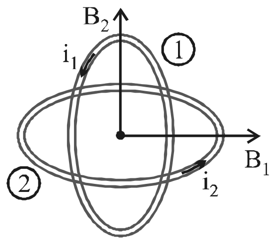
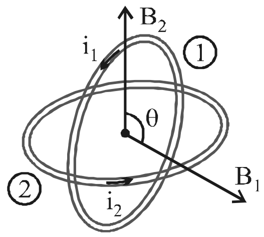
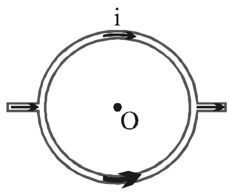
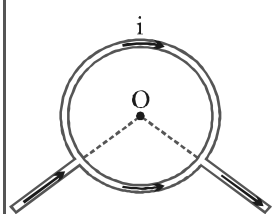
(iii) high permeability so that it can be magnetised easily.

Steel is preferred for making permanent magnets.

(b) Electromagnets

The core of electromagnets are made of ferromagnetic materials, which have high permeability and low retentivity. Soft iron is a suitable material for this purpose. When a soft iron rod is placed in a solenoid and current is passed through the solenoid, magnetism of the solenoid is increased by a thousand fold. When the solenoid current is switched off, the magnetism is removed instantly as retentivity of soft iron is very low. Electromagnets are used in electric bells, loudspeakers and telephone diaphragms. Giant electromagnets are used in cranes to lift machinery etc.

Magnetic Field at Centre O in different conditions of Circular Current

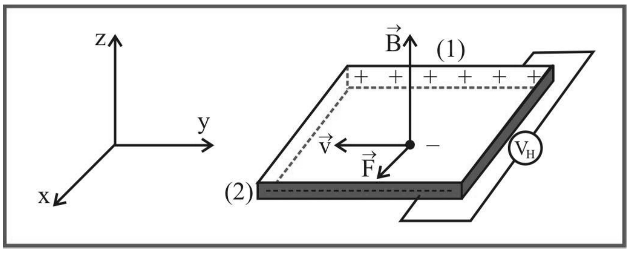
Condition	Figure	Magnetic field
Arc subtends angle θ at the centre		$B = \frac{\mu_0}{4\pi} \cdot \frac{\theta i}{r}$
Arc subtends angle $(2\pi - \theta)$ at the centre		$B = \frac{\mu_0}{4\pi} \cdot \frac{(2\pi - \theta) i}{r}$
Semi-circular arc		$B = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{r} = \frac{\mu_0 i}{4r}$
Three quarter semi-circular current carrying arc		$B = \frac{\mu_0}{4\pi} \cdot \frac{\left(2\pi - \frac{\pi}{2}\right) i}{r}$ $= \frac{3\mu_0 i}{8r}$
Circular current carrying arc		$B = \frac{\mu_0}{4\pi} \frac{2\pi i}{r}$ $= \frac{\mu_0 i}{2r}$
Concentric co-planer circular loops carries current in the same direction		$B_1 = \frac{\mu_0}{4\pi} 2\pi i \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$
Concentric co-planer circular loops carries current in the opposite direction		$B_2 = \frac{\mu_0}{4\pi} 2\pi i \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$
Concentric loops but their planes are perpendicular to each other		$B = \sqrt{B_1^2 + B_2^2}$ $= \frac{\mu_0}{2r} \sqrt{i_1^2 + i_2^2}$
Concentric loops but their planes are at an angle θ with each other		$B = \sqrt{B_1^2 + B_2^2 + 2B_1B_2 \cos\theta}$
Distribution of current across the diameter		$B = 0$
Distribution of current between any two points on the circumference		$B = 0$

27. HALL EFFECT

The Phenomenon of producing a transverse emf in a current carrying conductor on applying a magnetic field perpendicular to the direction of the current is called Hall effect.

Hall effect helps us to know the nature and number of charge carriers in a conductor.

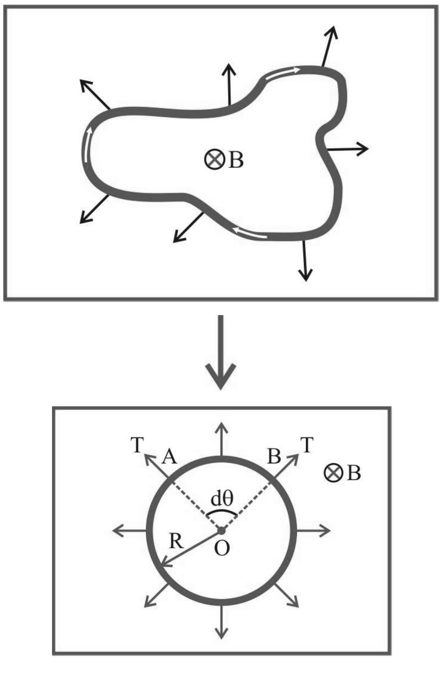
Consider a conductor having electrons as current carriers. The electrons move with drift velocity $\vec{v}$ opposite to the direction of flow of current



Force acting on electron $F_m = -e(\vec{v}\times\vec{B})$. This force acts along x-axis and hence electrons will move towards face (2) and it becomes negatively charged.

28. STANDARD CASES FOR FORCE ON CURRENT CARRYING CONDUCTORS

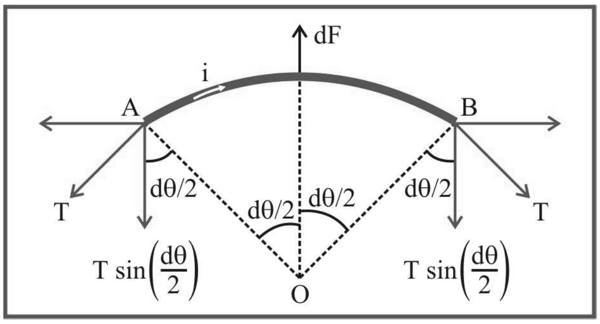
Case 1 : When an arbitrary current carrying loop placed in a magnetic field ($\perp$ to the plane of loop), each element of loop experiences a magnetic force due to which loop stretches and open into circular loop and tension developed in it's each part.



Specific example

In the above circular loop tension in part A and B.

In balanced condition of small part AB of the loop is shown below



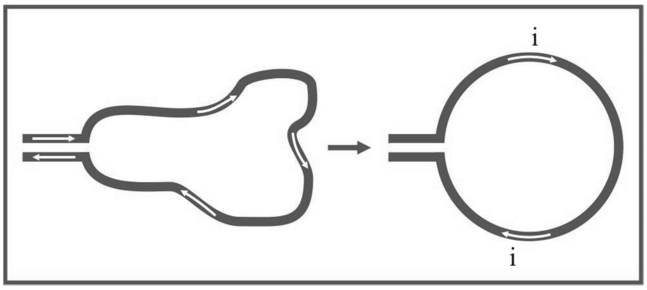
$$2T\sin\frac{d\theta}{2} = dF = Bid\ell \Rightarrow 2T\sin\frac{d\theta}{2} = BiRd\theta$$

If dθ is small so, $\sin\frac{d\theta}{2} \approx \frac{d\theta}{2} \Rightarrow 2T.\frac{d\theta}{2} = BiRd\theta$

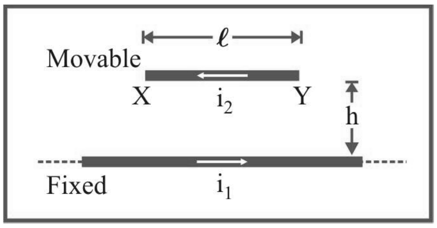
$$T = BiR, \text{ if } 2\pi R = L \text{ so } T = \frac{BiL}{2\pi}$$

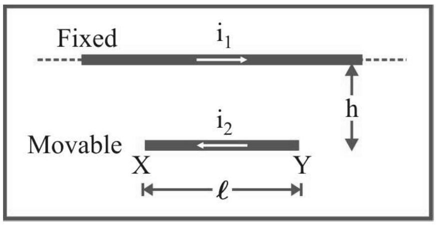
Note...

If no magnetic field is present, the loop will still open into a circle as in it's adjacent parts current will be in opposite direction and opposite currents repel each other.



Case 2 : Equilibrium of a current carrying conductor : When a finite length current carrying wire is kept parallel to another infinite length current carrying wire, it can suspend freely in air as shown below





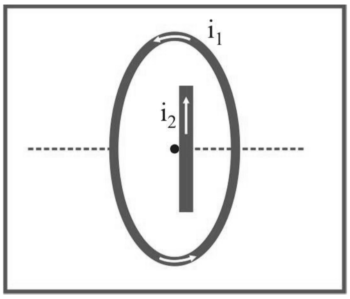
In both the situations for equilibrium of XY it's downward weight = upward magnetic force i.e. $mg = \frac{\mu_0}{4\pi}.\frac{2i_1i_2}{h}.\ell$

Note..

In the first case if wire XY is slightly displaced from its equilibrium position, it executes SHM and it's time period is given by $T = 2\pi\sqrt{\frac{h}{g}}$.

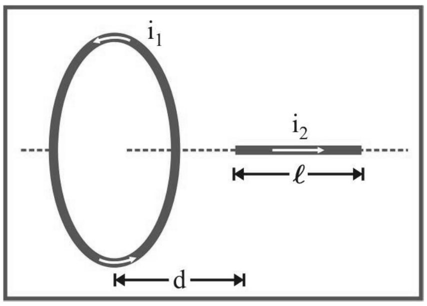
If direction of current in movable wire is reversed then it's instantaneous acceleration produced is 2g↓.

Case 3 : Current carrying wire and circular loop : If a current carrying straight wire is placed in the magnetic field of current carrying circular loop.



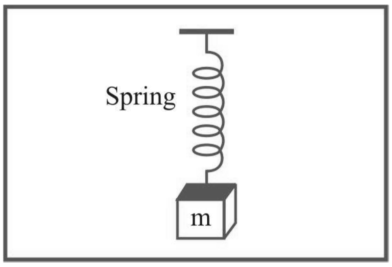
Wire is placed in the perpendicular magnetic field due to coil at it's centre, so it will experience a maximum force

$$F = Bi\ell = \frac{\mu_0 i_1}{2r} \times i_2\ell$$

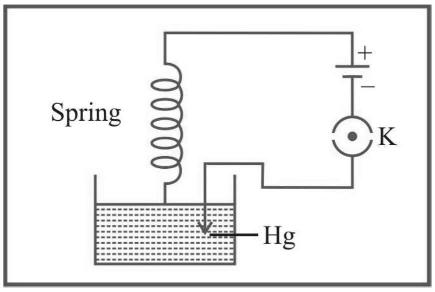


Wire is placed along the axis of coil so magnetic field produced by the coil is parallel to the wire. Hence it will not experience any force.

Case 4 : Current carrying spring : If current is passed through a spring, then it will contract because current will flow through all the turns in the same direction.



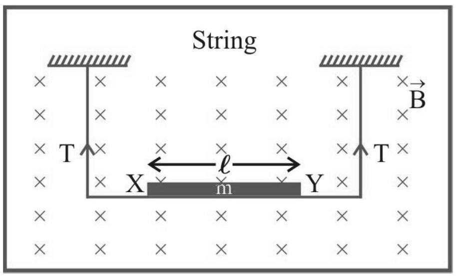
If current makes to flow through spring, then spring will contract and weight lift up.

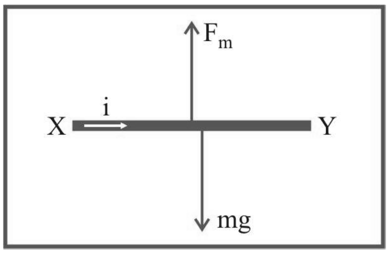


If switch is closed then current start flowing, spring will execute oscillation in vertical plane.

Case 5 : Tension less strings : In the following figure the value and direction of current through the conductor XY so that strings becomes tensionless ?

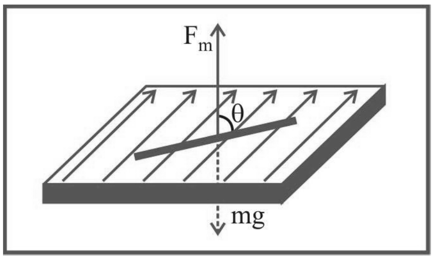
Strings becomes tensionless if weight of conductor XY balanced by magnetic force (F_m).





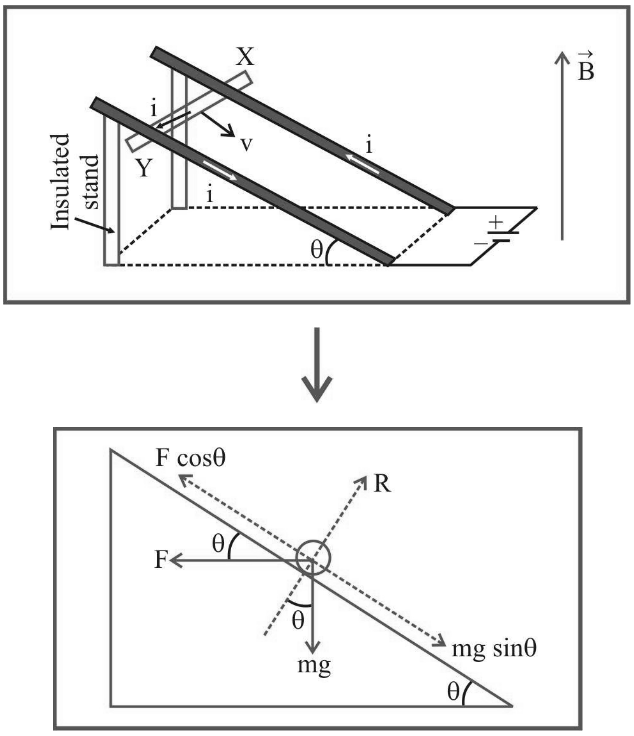
Hence direction of current is from X → Y and in balanced condition $F_m = mg \Rightarrow Bi\ell = mg \Rightarrow i = \frac{mg}{B\ell}$

Case 6 : A current carrying conductor floating in air such that it is making an angle θ with the direction of magnetic field, while magnetic field and conductor both lies in a horizontal plane.



In equilibrium $mg = Bi\ell \sin\theta \Rightarrow i = \frac{mg}{B\ell \sin\theta}$

Case 7 : Sliding of conducting rod on inclined rails : When a conducting rod slides on conducting rails.

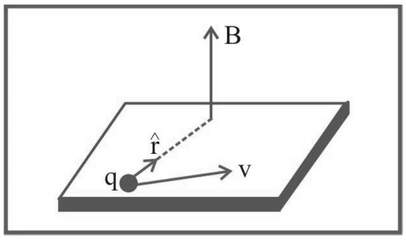


In the following situation conducting rod (X, Y) slides at constant velocity if

$$F\cos\theta = mg\sin\theta \Rightarrow Bi\ell\cos\theta = mg\sin\theta \Rightarrow B = \frac{mg}{i\ell}\tan\theta$$

TIPS & TRICKS

1. The device whose working principle based on Halmholtz coils and in which uniform magnetic field is used called as "Halmholtz galvanometer".
2. The value of magnetic field induction at a point, on the centre of separation of two linear parallel conductors carrying equal currents in the same direction is zero.
3. If a current carrying circular loop (n = 1) is turned into a coil having n identical turns then magnetic field at the centre of the coil becomes n^2 times the previous field i.e. $B_{(n\ turn)} = n^2\, B_{(single\ turn)}$
4. When a current carrying coil is suspended freely in earth's magnetic field, it's plane stays in East-West direction.
5. Magnetic field $(\vec{B})$ produced by a moving charge q is given by $\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\hat{r})}{r^2}$; where v = velocity of charge and v < < c (speed of light).



6. If an electron is revolving in a circular path of radius r with speed v then magnetic field produced at the centre of circular path $B = \frac{\mu_0}{4\pi}\frac{ev}{r^2} \Rightarrow r \propto \sqrt{\frac{v}{B}}$
7. The line integral of magnetising field $(\vec{H})$ for any closed path called magnetomotive force (MMF). It's S.I. unit is amp.
8. Ratio of dimension of e.m.f. to MMF is equal to the dimension of resistance.
9. The positive ions are produced in the gap between the two dees by the ionisation of the gas. To produce proton, hydrogen gas is used; while for producing alpha-particles, helium gas is used.

10. Cyclotron frequency is also known as magnetic resonance frequency.

11. Cyclotron can not accelerate electrons because they have very small mass.

12. The energy of a charged particle moving in a uniform magnetic field does not change because it experiences a force in a direction, perpendicular to it's direction of motion. Due to which the speed of charged particle remains unchanged and hence it's K.E. remains same.

13. Magnetic force does no work when the charged particle is displaced while electric force does work in displacing the charged particle.

14. Magnetic force is velocity dependent, while electric force is independent of the state of rest or motion of the charged particle.

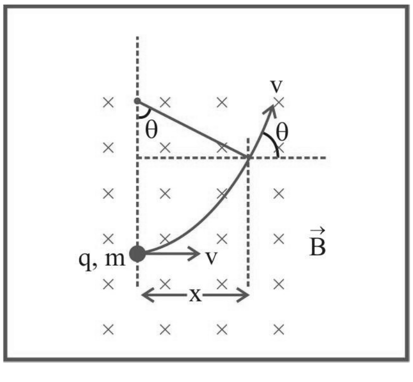
15. If a particle enters a magnetic field normally to the magnetic field, then it starts moving in a circular orbit. The point at which it enters the magnetic field lies on the circumference. (Most of us confuse it with the centre of the orbit)

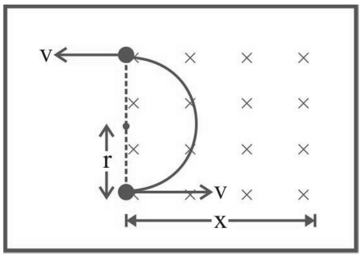
16. Deviation of charged particle in magnetic field : If a charged particle (q, m) enters a uniform magnetic field $\vec{B}$ (extends upto a length x) at right angles with speed v as shown in figure. The speed of the particle in magnetic field does not change. But it gets deviated in the magnetic field.

Deviation in terms of time t ; $\theta = \omega t = \left(\frac{Bq}{m}\right)t$

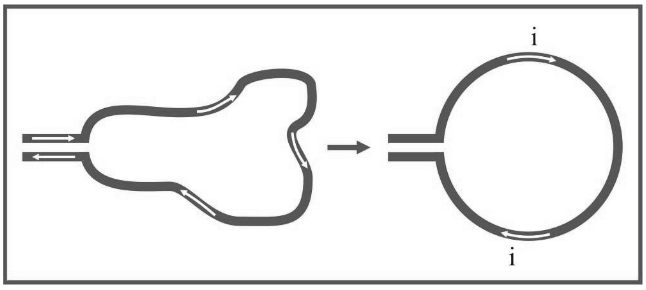
Deviation in terms of length of the magnetic field ; $\theta = \sin^{-1}\left(\frac{x}{r}\right)$. This relation can be used only when $x \le r$.

For $x > r$, the deviation will be 180° as shown in the following figure

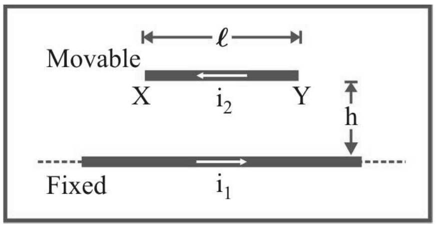




17. If no magnetic field is present, the loop will still open into a circle as in it's adjacent parts current will be in opposite direction and opposite currents repel each other.



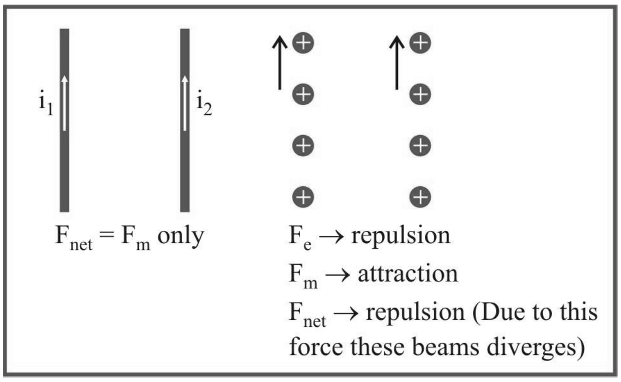
18. In the following case if wire XY is slightly displaced from its equilibrium position, it executes SHM and it's time period is given by $T = 2\pi\sqrt{\frac{h}{g}}$.



19. In the previous case if direction of currnet in movable wire is reversed then it's instantaneous acceleration produced is 2g↓.

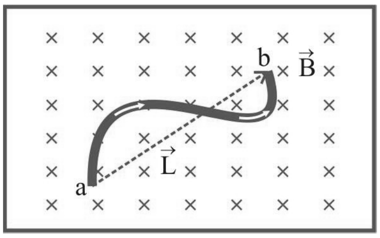
20. Electric force is an absolute concept while magnetic force is a relative concept for an observer.

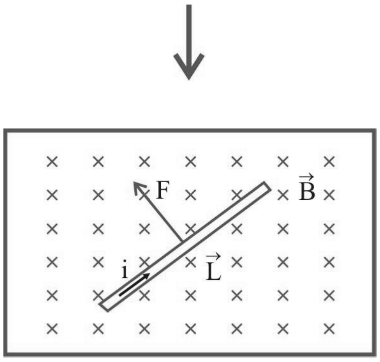
21. The nature of force between two parallel charge beams decided by electric force, as it is dominator. The nature of force between two parallel current carrying wires decided by magnetic force.



22. If a straight current carrying wire is placed along the axis of a current carrying coil then it will not experience magnetic force because magnetic field produced by the coil is parallel to the wire.

23. The force acting on a curved wire joining points a and b as shown in the figure is the same as that on a straight wire joining these points. It is given by the expression $\vec{F} = i\vec{L} \times \vec{B}$





24. If a current carrying conductor AB is placed transverse to a long current carrying conductor as shown then force.

Experienced by wire AB

$$F = \frac{\mu_0 i_1 i_2}{2\pi} \log_e \left(\frac{x + \ell}{x} \right)$$