1 Mark Questions

1. A metallic wire coil is stationary in a non – uniform magnetic field. What is the emf. Induced in the coil?

Ans. NO emf is induced in the coil as there is no change in the magnetic flux linked with the secondary coil.

2. Why does metallic piece becomes very hot when it is surrounded by a coil carrying high frequency (H.F) alternating current?

Ans. When a metallic piece is surrounded by a coil carrying high frequency (H.F) alternating current, it becomes hot because eddy currents are produced which in turn produces joule's heating effect.

3. An electrical element X when connected to an alternating voltage source has current through it leading the voltage by $\dfrac{\pi}{2}$ radian. Identify X and write expression for its reactance?

Ans. X is a purely capacitive circuit

$$X_c = \frac{1}{2\pi \upsilon c} = \frac{1}{wC}$$

4. A transformer steps up 220 V to2200 V. What is the transformation ratio?

Ans. $K = \dfrac{Ns}{Np} = \dfrac{Es}{Ep} = \dfrac{2200}{220} = 10$

5. The induced emf is also called back emf. Why?

Ans. it is because induced emf produced in a circuit always opposes the cause which produces it.

6. Why the oscillations of a copper disc in a magnetic field are lightly damped?

Ans. Copper disc oscillates because of the production of eddy currents which opposes its oscillating motion and as a result the motion gets damped.

7.A metallic wire coil is stationary in a non – uniform magnetic field. What is the emf. Induced in the coil?

Ans.NO emf is induced in the coil as there is no change in the magnetic flux linked with the secondary coil.

2 Mark Questions

1. IF the rate of change of current of 2A/s induces an emf of 10mV in a solenoid. What is the self-inductance of the solenoid?

Ans. $L = \dfrac{\epsilon}{dI / dt} = \dfrac{10 \times 10^{-3}}{2} = 5 \times 10^{-3} \, Henry$

$\Rightarrow L = 5 \times 10^{-3} H$

2. A circular copper disc. 10 cm in radius rotates at a speed of 2π rad/s about an axis through its centre and perpendicular to the disc. A uniform magnetic field of 0.2T acts perpendicular to the disc.

1) Calculate the potential difference developed between the axis of the disc and the rim.

2) What is the induced current if the resistant of the disc is $2\,\Omega$?

Ans. (1) Radius = 10cm, B = 0.2T w = 2π rad/s

$$\epsilon = \frac{1}{2}Bwr^2$$

$$\epsilon = \frac{1}{2} \times 0.2 \times 2\pi \times (0.1)^2$$

$$\epsilon = 0.00628 \ volts$$

$$I = \frac{\epsilon}{R} = \frac{0.0628}{2}$$

I = 0.0314 A

3. An ideal inductor consumes no electric power in a.c. circuit. Explain?

Ans. P = E rms I rms cos ϕ

But for an ideal inductor $\phi = \dfrac{\pi}{2}$

$$\Rightarrow \cos\phi = \cos\frac{\pi}{2} = 0$$

$$\Rightarrow P = 0$$

4. Capacitor blocks d.c. why?

Ans. The capacitive reactance

$$X_c = \frac{1}{wC} = \frac{1}{2\pi Vc}$$

For d.c. $V = 0$

$$\Rightarrow X_c = \infty$$

Since capacitor offers infinite resistance to the flow of d.c. so d.c. cannot pass through the capacitor.

5. Why is the emf zero, when maximum number of magnetic lines of force pass through the coil?

Ans. The magnetic flux will be maximum in the vertical position of the coil. But as the coil

rotates $\dfrac{d\phi}{dt} = 0$

Hence produced emf $\varepsilon = \dfrac{d\phi}{dt} = 0$

6. An inductor L of reactance X_L is connected in series with a bulb B to an a.c. source as shown in the figure.
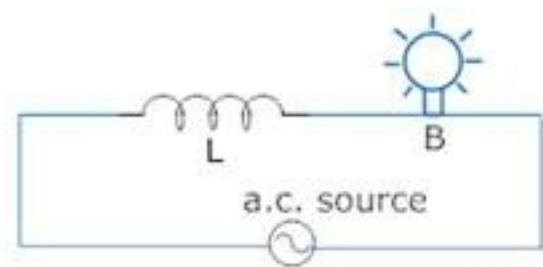
Briefly explain how does the brightness of the bulb change when

(a) Number of turns of the inductor is reduced.

(b) A capacitor of reactance $X_C = X_L$ is included in series in the same circuit.

Ans. (a) Since Z = $\sqrt{R^2 + X_L{}^2}$

When number of turns of the inductor gets reduced X_L and Z decreases and in turn current increases

Hence the bulb will grow more brightly

(b) When capacitor is included in the circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

But $X_L = X_C$ (given)

$\Rightarrow$ Z = R (minimum)

Hence brightness of the bulb will become maximum.

7. A jet plane is travelling towards west at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the Earth's magnetic field at the location has a magnitude of $5 \times 10^{-4} T$ and the dip angle is $30°$.

Ans. Speed of the jet plane, v = 1800 km/h = 500 m/s

Wing span of jet plane, l = 25 m

Earth's magnetic field strength, $B = 5.0 \times 10^{-4} \ T$

Angle of dip, $\delta = 30^0$ s

Vertical component of Earth's magnetic field,

$BV = B \sin \delta$

$= 5 \times 10 - 4 \ sin 30°$

$= 2.5 \times 10 - 4 \ T$

Voltage difference between the ends of the wing can be calculated as:

$e = (B_v) \times l \times v$

$= 2.5 \times 10^{-4} \times 25 \times 500$

= 3.125 V

Hence, the voltage difference developed between the ends of the wings is 3.125 V.

---

8. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil?

Ans. Mutual inductance of a pair of coils, μ = 1.5 H

Initial current, I_1 = 0 A

Final current I_2 = 20 A

Change in current, $dl = 1_2 - 1_1 = 20 - 0 = 20 A$

Induced emf, $e = \dfrac{d\phi}{dt}$ ...(i)

Where $d\phi$ is the change in the flux linkages with the coil.

Emf is related with mutual inductance as:

Equating equations (1) and (2), we get

$$\dfrac{d\phi}{dt} = \mu \dfrac{dI}{dt}$$

$$d\phi = 1.5 \times (20)$$

$$= 30 Wb$$

Hence, the change in the flux linkage is 30 Wb.

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9. A horizontal straight wire 10 m long extending from east to west is falling with a speed of $5.0 \ m \ s^{-1}$, at right angles to the horizontal component of the earth's magnetic field, $0.30 \times 10^{-4} Wb \ m^{-2}$.

(a) What is the instantaneous value of the emf induced in the wire?

(b) What is the direction of the emf?

(c) Which end of the wire is at the higher electrical potential?

Ans. Length of the wire, l = 10 m

Falling speed of the wire, v = 5.0 m/s

Magnetic field strength, B = $0.3 \times 10^{-4} \ Wb \ m^{-2}$

(a) Emf induced in the wire,

$e = Blv$

$$= 0.3 \times 10^{-4} \times 5 \times 10$$

$$= 1.5 \times 10^{-3} V$$

(b) Using Fleming's right hand rule, its can be inferred that the direction of the induced emf is from West to East.

(c) The eastern end of the wire is at a higher potential.

---

10. A 1.0 m long metallic rod is rotated with an angular frequency of 400 rad s^{-1} about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring.

Ans. Length of the rod, l = 1 m

Angular frequency, ω = 400 rad/s

Magnetic field strength, B = 0.5 T

One end of the rod has zero linear velocity, while the other end has a linear velocity of $l_{(i)}$.

Average linear velocity of the rod, $v = \dfrac{1_{(i)} + 0}{2} = \dfrac{l_{(i)}}{2}$

Emf developed between the centre and the ring,

$$e = Blv = Bl\left(\dfrac{l_{(i)}}{2}\right) = \dfrac{Bl^2 (i)}{2}$$

$$= \dfrac{0.5 \times (1)^2 \times 400}{2} = 100V$$

Hence, the emf developed between the centre and the ring is 100 V.

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11. Use Lenz's law to determine the direction of induced current in the situations described by Fig. 6.19:

(a)   (b)
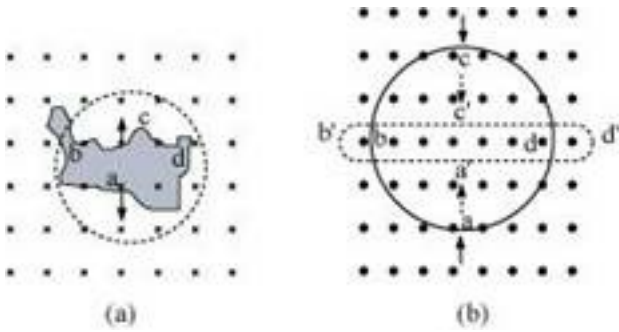
(a) A wire of irregular shape turning into a circular shape;

(b) A circular loop being deformed into a narrow straight wire.

Ans. According to Lenz's law, the direction of the induced *emf* is such that it tends to produce a current that opposes the change in the magnetic flux that produced it.

(a) When the shape of the wire changes, the flux piercing through the unit surface area increases. As a result, the induced current produces an opposing flux. Hence, the induced current flows along adcb.

(b) When the shape of a circular loop is deformed into a narrow straight wire, the flux piercing the surface decreases. Hence, the induced current flows along

$$e = \frac{4\pi \times 10^{-7} \times 50 \times 0.1 \times 10}{2\pi \times 0.2} \qquad e = 5 \times 10^{-5} V \ a'd'c'b'.$$

3 Mark Questions

1. How is the mutual inductance of a pair of coils affected when

(1) Separation between the coils is increased.

(2) The number of turns of each coil is increased.

(3) A thin iron sheet is placed between two coils, other factors remaining the same. Explain answer in each case.

Ans. (1) When the Separation between the coils is increased, the flux linked with the

secondary coils decreases, hence mutual induction decreases.

(2) Since m = $\dfrac{\mu_0 N_1 N_2 A}{l}$, so when $N_1 and\ N_2$ increases, mutual induction increase.

(3) Mutual induction will increase because

$M \propto \mu r$ (Relative permeability of material)

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2. Distinguish between resistances, reactance and impedance of an a.c. circuit?

Ans.

	Resistance	Reactance	Impedance
1	Opposition offered by the resistor to the flow of current	Opposition offered by the inductor or capacitor to the flow of current	Opposition offered by the combination of resistor, inductor or capacitor
2	It is independent of the frequency of the source.	It depends on the frequency of the source	It depends on the frequency of the source

3. A sinusoidal voltage V = 200 sin 314t is applied to a resistor of 10 Ω resistance. Calculate

(1) rms value of the voltage

(2) rms value of the current

(3) Power dissipated as heat in watt.

Ans. V = 200 sin 314t

V = Vo sin wt

Vo = 200V, w = 314 rad/s.

R = 10 Ω

(1) V rms = $\sqrt{2}Vo$

V rms = $\sqrt{2} \times 200$ = 282.8 V

(2) $I \text{ rms} = \dfrac{V \text{ rms}}{R} = \dfrac{282.8}{10}$

I rms = 28.28 A

(3) Since circuit is purely resistive

$\therefore \phi = 0$

$\Rightarrow P = EvIv\cos\phi$

$P = EvIv$

$P = 282.8 \times 28.28$

P = 7.998 watt

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4. Obtain an expression for the self inductance of a long solenoid? Hence define one
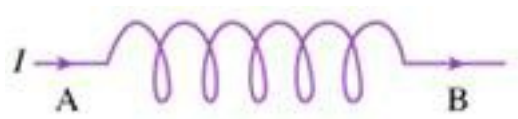
Henry?

Ans. Consider a long solenoid of area A through which current I is flowing

Let N be the total number of turns in the solenoid

Total flux ϕ = NBA

Here = B = $\mu_0 nI$

Where n is no. of turns per unit length of the solenoid

N = nl

$\Rightarrow \phi = nl \times \mu_0 nIA$

$\phi = \mu_0 n^2 AlI$ ------(1)

Also ϕ=LI------------(2)

Equation (1) & (2)

$\mu_0 n^2 AlI = LI$

$L = \mu_0 n^2 Al$

$\Rightarrow L = \dfrac{\mu_0 N^2 A}{l}$ [n = N/ ℓ]

One Henry – if current is changing at a rate of 1 A/s in a coil induces an emf. of 1 volt in it then the inductance of the coil is one henry.

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5. A conducting rod rotates with angular speed w with one end at the centre and other end at circumference of a circular metallic ring of radius R, about an axis passing through the centre of the coil perpendicular to the plane of the coil A constant magnetic field B parallel to the axis is present everywhere. Show that the emf. between
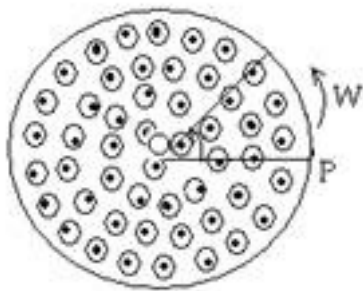
the centre and the metallic ring is $\frac{1}{2} B w R^2$.

Ans. Consider a circular loop connect the centre with point P with a resistor.

The potential difference across the resistor = induced emf.

$\in = \vec{B} \times$ Rate of change of area of loop.

If the resistor QP is rotated with angular velocity w and turns by an angle θ in time t then

Area swept $A = \frac{1}{2} \times R \times R\theta$

$A = \frac{1}{2} R^2 \theta$

$\phi = BA \cos \theta^\circ = BA$

$\varphi = B \times \frac{1}{2} R^2 \theta$

$\in = \frac{d\phi}{dt} = \frac{d}{dt}\left(\frac{1}{2} BR^2 \theta\right) = \frac{1}{2} BR^2 \left(\frac{d\theta}{dt}\right)$

$\in = \frac{1}{2} B w R^2$

---

6. (a) At a very high frequency of a.c., capacitor behaves as a conductor. Why?

(b) Draw the graph showing the variation of reactance of

(i) A capacitor

(ii) An inductor with a frequency of an a.c. circuit.

Ans. (a) $X_C = \dfrac{1}{2\pi \upsilon c}$
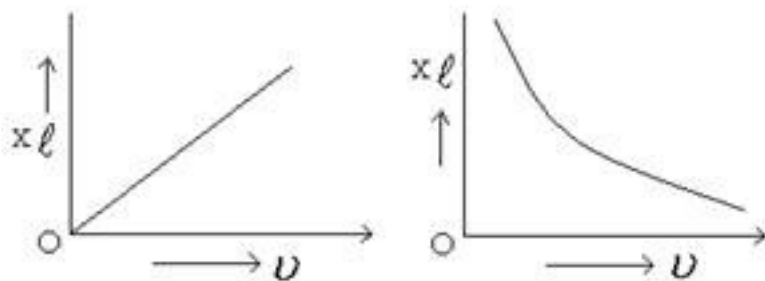
For a.c. when $\upsilon \to \infty \; X_C = 0$

Thus at a very high frequency of a.c. capacitor behaves as a conductor

(b) $X_C = \dfrac{1}{2\pi \upsilon c}$

$\Rightarrow X_C \propto \dfrac{1}{\upsilon}$

$X_L = WL = 2\pi \upsilon L$

$XL \propto \upsilon$

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7. Calculate the current drawn by the primary of a transformer which steps down 200 V to 20 V to operate a device of resistance 20 Ω . Assume the efficiency of the transformer to be 80%?

Ans. η = 80% Ep = 200V Es = 20V Z = 20 Ω

$Is = \dfrac{Es}{Z} = \dfrac{20}{20} = 1A$

Now $\eta = \dfrac{E_sI_s}{E_pI_p}$

$\eta = \dfrac{80}{100} = \dfrac{20 \times 1}{200 \times I_p}$
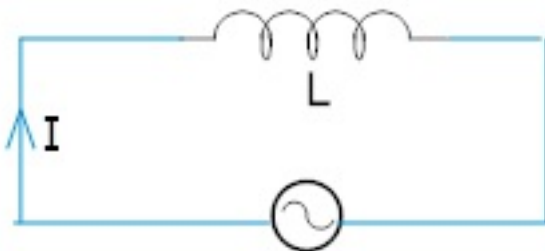
$\Rightarrow I_p = \dfrac{2000}{80 \times 200}$

I_p = 0.125 A

---

8. An a.c. voltage E = Eo sin wt is applied across an inductance L. obtain the expression for current I?

Ans.E = Eo sin wt (Given)

Emf produced across L = $\dfrac{-LdI}{dt}$

Total emf of the circuit = $E + \left(\dfrac{-LdI}{dt}\right)$

Since there is no circuit element across which potential drop may occur

$\therefore E + \left(\dfrac{-LdI}{dt}\right) = 0$

$\Rightarrow E = \dfrac{LdI}{dt}$

$dI = \dfrac{E}{L}dt$

$$dI \cdot \frac{Eo}{L} \sin wt \ dt$$

Integrating

$$I = \frac{Eo}{L} \int \sin wt \ dt$$

$$I = \frac{Eo}{L} \left(\frac{-\cos wt}{w} \right)$$

$$I = \frac{-Eo}{wL} \cos wt$$

$$I = \frac{Eo}{wL} \sin \left(wt - \frac{\pi}{2} \right)$$

$$I = \frac{Eo}{X_L} \sin \left(wt - \frac{\pi}{2} \right)$$

$where \dfrac{Eo}{X_L} = Io$ (peak vlue of current)

$$\Rightarrow I = Io \sin \left(wt - \frac{\pi}{2} \right)$$

---

9. A series circuit with L = 0.12H, C = 0.48mF and R = 25 Ω is connected to a 220V variable frequency supply. At what frequency is the circuit current maximum?

Ans.L = 0.12H, C = 0.48mF = $0.48 \times 10^{-3} F$

R = 25 Ω Ev = 220V

$$Iv = \frac{Ev}{\sqrt{R^2 + (X_L - X_c)^2}}$$

In the circuit when I is maximum, R will be minimum

$$\Rightarrow X_L = X_C$$

$$I = \frac{Ev}{R}$$

and $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\times3.14\sqrt{0.12\times0.48\times10^{-3}}}$

f = 21 Hz

---

10. A rectangular wire loop of sides 8 cm and 2 cm with a small cut is moving out of a region of uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the emf developed across the cut if the velocity of the loop is 1 cm s^{-1} in a direction normal to the (a) longer side, (b) shorter side of the loop? For how long does the induced voltage last in each case?

Ans. Length of the rectangular wire, l = 8 cm = 0.08 m

Width of the rectangular wire, b = 2 cm = 0.02 m

Hence, area of the rectangular loop,

$A = lb$

$= 0.08\times0.02$

$= 16\times10-4\,m^2$

Magnetic field strength, B = 0.3 T

Velocity of the loop, v = 1 cm/s = 0.01 m/s

(a) Emf developed in the loop is given as:

$e = Blv$

$= 0.3 \times 0.08 \times 0.01 = 2.4 \times 10^{-4}\,V$

Time taken to travel along the width. ts

$$\frac{Dis\tan ce\,travelled}{Velocity} = \frac{b}{v} = \frac{0.02}{0.01} = 2s$$

Hence, the induced voltage is 2.4×10^{-4} V which lasts for 2 s.

(b) Emf developed, *e = Bbv*

$= 0.3 \times 0.02 \times 0.01 = 0.6 \times 10^{-4} V$

Time taken to travel along the length,

$$t = \frac{Dis\tan ce\,travelled}{Velocity} = \frac{1}{v} = \frac{0.08}{0.01} = 8s$$

Hence, the induced voltage is 0.6×10^{-4} V which lasts for 8 s.

---

11. Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V induced, give an estimate of the self-inductance of the circuit.

Ans. Initial current, $I_1 = 5.0\ A$

Final current, $I_2 = 0.0\ A$

Change in current, $dI = I_1 - I_2 = 5A$

Time taken for the change, *t* = 0.1

Average emf, *e* = 200 V

For self-inductance (*L)* of the coil, we have the relation for average emf as:

$e = L \dfrac{di}{dt}$ s

$L = \dfrac{r}{\left(\dfrac{di}{dt}\right)}$

$$d\phi$$

$$=\frac{200}{\dfrac{5}{0.1}}=4H$$

$$L=\left(\dfrac{\overset{e}{\frac{di}{dt}}}{}\right)_{S}$$

Hence, the self-induction of the coil is 4 H.

---

12. Suppose the loop in Exercise 6.4 is stationary but the current feeding the electromagnet that produces the magnetic field is gradually reduced so that the field decreases from its initial value of 0.3 T at the rate of $0.02\ T\ s^{-1}$. If the cut is joined and the loop has a resistance of 1.6 Ω how much power is dissipated by the loop as heat? What is the source of this power?

Ans. Sides of the rectangular loop are 8 cm and 2 cm.

Hence, area of the rectangular wire loop,

$$A = length \times width$$

$$= 8 \times 2 = 16 cm^2$$

$$= 16 \times 10 - 4\, m^2$$

Initial value of the magnetic field, $B' = 0.3T$

Rate of decrease of the magnetic field, $\dfrac{dB}{dt} = 0.02T\,/\,s$

Emf developed in the loop is given as:

$$e = \dfrac{d\phi}{dt}$$

Where,

$d\phi$ = Change in flux through the loop area

= AB

$$\therefore e = \frac{d(AB)}{dt} = \frac{AdB}{dt}$$

$$= 16 \times 10^{-4} \times 0.02 = 0.32 \therefore 10^{-4} V$$

Resistance of the loop, R = 1.6 Ω

The current induced in the loop is given as:

$$i = \frac{r}{R}$$

$$= \frac{0.32 \times 10^{-4}}{1.6} = 2 \times 10^{-5} A$$

Power dissipated in the loop in the form of heat is given as:

$$P = i^2 R$$

$$= (2 \times 10^{-5})^2 \times 1.6$$

$$= 6.4 \times 10^{-10} W$$

The source of this heat loss is an external agent, which is responsible for changing the magnetic field with time.

---

13. An air-cored solenoid with length 30 cm, area of cross-section $25\ cm^2$ and number of turns 500, carries a current of 2.5 A. The current is suddenly switched off in a brief time of 10^{-3} s . How much is the average back emf induced across the ends of the open switch in the circuit? Ignore the variation in magnetic field near the ends of the solenoid.

Ans. Length of the solenoid, l = 30 cm = 0.3 m

Area of cross-section, A = $25 \, cm^2$ = $25 \times 10^{-4} \, m^2$

Number of turns on the solenoid, N = 500

Current in the solenoid, I = 2.5 A

Current flows for time, t = 10^{-3} s

Average back emf, $e = \dfrac{d\phi}{dt}$ ...(1)

Where,

$d\phi$ = Change in flux

= NAB ......(2)

Where,

B = Magnetic field strength

$= \mu_0 \dfrac{NI}{l}$ ...(3)

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7} \, T \, m \, A^{-1}$

Using equations (2) and (3) in equation (1), we get

$e = \dfrac{\mu_0 N^2 I A}{lt}$

$= \dfrac{4\pi \times 10^{-7} \times (500)^2 \times 2.5 \times 25 \times 10^{-4}}{0.3 \times 10^{-3}} = 6.5V$

Hence, the average back emf induced in the solenoid is 6.5 V.

14. A line charge λ **per unit length is lodged uniformly onto the rim of a wheel of mass** M **and radius** R**. The wheel has light non-conducting spokes and is free to rotate without friction about its axis (Fig. 6.22). A uniform magnetic field extends over a circular region within the rim. It is given by,**
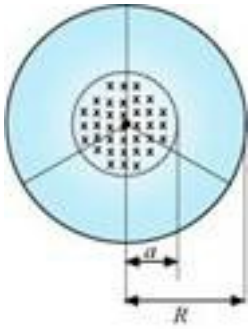
$B = -B_0\, k \;(r \le a; a < R)$ = 0 (otherwise) **What is the angular velocity of the wheel after the field is suddenly switched off?**

Ans. Line charge per unit length

$$= \lambda = \frac{Total\ ch\arg e}{Length} = \frac{Q}{2\pi r}$$

Where,

r = Distance of the point within the wheel

Mass of the wheel = M

Radius of the wheel = R

Magnetic field, $B = -\overrightarrow{B} = B_0\, \hat{k}$

At distance r, the magnetic force is balanced by the centripetal force i.e.,

$$BQv = \frac{Mv^2}{r}$$

Where

V=linear velocity of the wheel

$$\therefore B2\pi r\lambda r^2 = \frac{Mv}{r}$$

$$v = \frac{B2\pi r\lambda r^2}{M}$$

$$BQv = \frac{Mv^2}{r}$$

$\therefore$ Angular velocity, $(i) = \dfrac{v}{R} = \dfrac{B2\pi r\lambda r^2}{M}$

$For\ r \leq and\ a < R,\ we\ get:$

$$(i) = -\frac{2B_0 a^2 \lambda}{MR}k$$

5 Mark Questions

1. (a) State the condition under which the phenomenon of resonance occurs in a series LCR circuit. Plot a graph showing the variation of current with frequency of a.c. sources in a series LCR circuit.

(b) Show that in a series LCR circuit connected to an a.c. source exhibits resonance at its natural frequency equal to $\dfrac{1}{\sqrt{LC}}$?

Ans. (a) In a series LCR circuit

Resonance occurs when $X_L = X_C$.
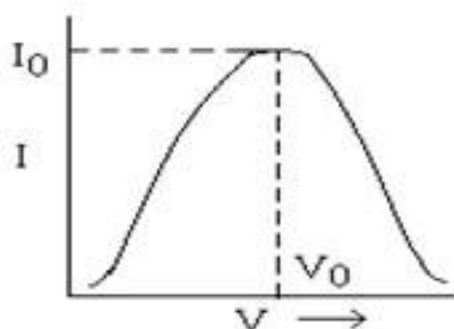
The variation of current with frequency of a.c. source in series LCR circuit

(b) Electrical resonance takes place in a series LCR circuit when circuit allows maximum alternating current for which

$$X_L = X_C.$$

Impedance Z = $\sqrt{R^2 + (X_L - X_C)^2}$

$$I = \frac{E}{Z} - \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

For electrical resonance

$$X_L = X_C.$$

$$wL = \frac{1}{wC} \ or \ w^2 = \frac{1}{LC}$$

$$w = \frac{1}{\sqrt{LC}}$$

Where w is the natural frequency of the circuit.

---

2. In a step up transformer, transformation ratio is 100. The primary voltage is 200 V and input is 1000 watt. The number of turns in primary is 100. Calculate

(1) Number of turns in the secondary

(2) Current in the primary

(3) The voltage across the secondary

(4) Current in the secondary

(5) Write the formula for transformation ratio?

Ans. (1) k = 100, Ep = 200V

EpIp = 1000 W, Np = 100

$$K = 100 = \frac{Ns}{Np}$$

$$\Rightarrow Ns = 100 \times Np$$

$$Ns = 100 \times 100$$

NS = 10,000

(2) EpIp = 1000W

$$Ip = \frac{1000}{Ep}$$

$$Ip = \frac{1000}{200} = 5A$$

Ip = 5A

(3) $\dfrac{Es}{Ep} = \dfrac{Ns}{Np}$

$$\therefore Es = Ep \times \frac{Ns}{Np}$$

$$Es = 200 \times 100$$

Es = 20000 Volt

(4) $\dfrac{Es}{Ep} = \dfrac{Ip}{Is}$

$Is = \dfrac{IpEp}{Es}$

$Is = \dfrac{1000}{20000} = \dfrac{1}{20}$

Is = 0.05 A

(5) For step up Trans former k > 1 $K = \dfrac{Ns}{Np}$

---

3. Drive an expression for the average power consumed in a.c. series LCR circuit. Hence define power factor?

Ans. For an a.c. series circuit

E = Eo sin wt

And I = Io sin (wt + ϕ)

Where ϕ is the phase angle by which current leads the emf.

Now using dw = EIdt

dw = (Eo sin wt) (Io sin (wt + ϕ))dt

dw = EoIo sin wt (sin wt cos ϕ + cos wt sin ϕ)dt

dw = EoIo (sin^2 wt cos ϕ + sin wt cos wt sin ϕ)dt

$$\left[\because \sin 2wt = 2 \sin wt \cos wt \right]$$

$$\Rightarrow dw = EoIo \left(\dfrac{1 - \cos 2wt}{2} \cos \phi + \dfrac{\sin 2wt}{2} \sin \phi \right)$$

$$\left[\therefore \sin^2 wt = \frac{1-\cos 2wt}{2}\right]$$

$$dw = \frac{EoIo}{2}(\cos\phi - \cos\phi\cos 2wt + \sin\phi\sin wt)dt$$

Integrating within limits t = o to t = T

$$W = \frac{EoIo}{2}\left[\cos\phi\int_0^T dt - \cos\phi\int_0^T \cos 2wtdt + \sin\phi\int_0^T \sin 2wtdt\right]$$

$$\Rightarrow W = \frac{EoIo}{2}\cos\phi\int_0^T dt \left[\because \int_0^T \sin 2wtdt = \int_0^T \cos 2wtdt = 0\right]$$

$$W = \frac{EoIo}{2}T\cos\phi$$

Hence average power consumed in a.c circuit is given by

$$Pav = \frac{W}{T} = \frac{EoIo}{2}\cos\phi$$

Pav = EvIv cos ϕ -----(1)

Power factor – In the above expression

Cos ϕ is termed as power factor

When cos ϕ = 1 ϕ = 0°

It means circuit is purely resistive and Pav = EvIv
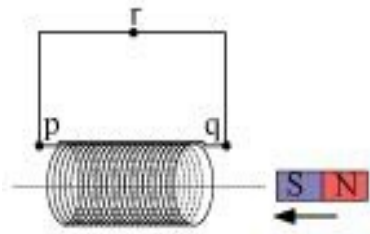
When cos ϕ = 0 ϕ = 90°
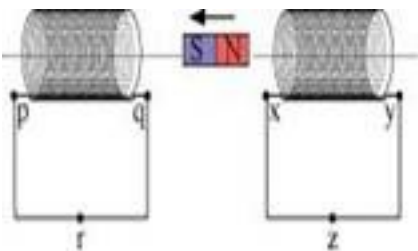
It means circuit is purely capacitive or inductive.

Pav = 0

Hence

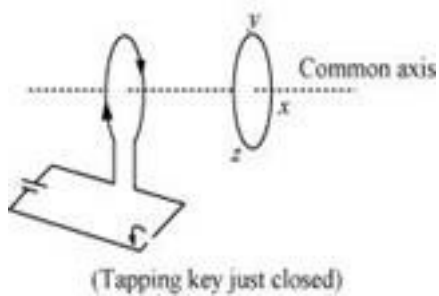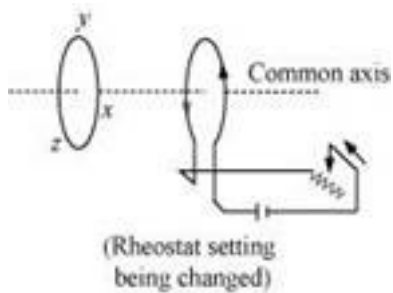**4. Predict the direction of induced current in the situations described by the following Figs. 6.18(a) to (f).**

(a)

(b)

(c)

(Tapping key just closed)

(d)

(Rheostat setting being changed)

(e)

(Tapping key just released)

(f)

Current (I) decreasing
at a steady rate
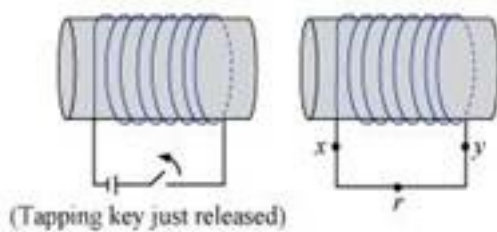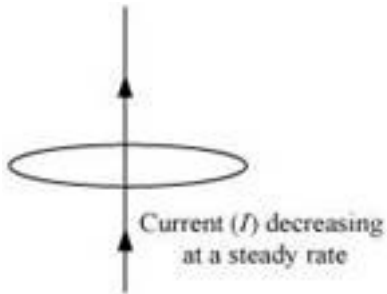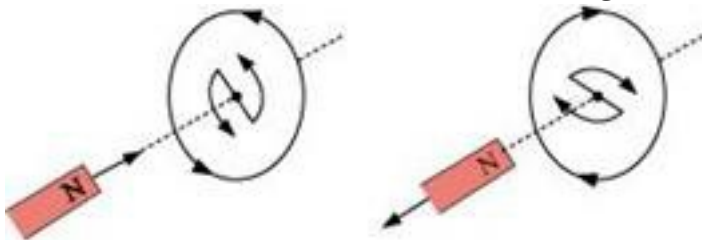
Ans. The direction of the induced current in a closed loop is given by Lenz's law. The given pairs of figures show the direction of the induced current when the North pole of a bar magnet is moved towards and away from a closed loop respectively. Using Lenz's rule, the direction of the induced current in the given situations can be predicted as follows:

(a) The direction of the induced current is along **qrpq**.

(b) The direction of the induced current is along **prqp**.

(c) The direction of the induced current is along *yzxy*.

(d) The direction of the induced current is along *zyxz*.

(e) The direction of the induced current is along *xryx*.

(f) No current is induced since the field lines are lying in the plane of the closed loop.

5. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside the solenoid normal to its axis. If the current carried by the solenoid changes steadily from 2.0 A to 4.0 A in 0.1 s, what is the induced emf in the loop while the current is changing?

Ans. Number of turns on the solenoid = 15 turns/cm = 1500 turns/m

Number of turns per unit length, n = 1500 turns

The solenoid has a small loop of area, $A = 2.0 \ cm^2 = 2 \times 10 \, m^2$

Current carried by the solenoid changes from 2 A to 4 A.

∴ Change in current in the solenoid, di = 4 - 2 = 2 A

Change in time, dt = 0.1 s

Induced *emf* in the solenoid is given by Faraday's law as:

$$e = \frac{d\phi}{dt} \quad ......(i)$$

Where,

ϕ = Induced flux through the small loop

= BA ...(*ii*)

B = Magnetic field

= $\mu_0 ni$ \qquad ...(*iii*)

μ_0 = Permeability of free space

$= 4n \times 10^{-7} \ H/m$

Hence, equation (*i*) reduces to:

$$e = \frac{d}{dt}\left(\frac{di}{dt}\right)$$

$$= A\mu_0 n \times \left(\frac{di}{dt}\right)$$

$$= 2\times 10^{-4} \times 4\pi \times 10^{-7} \times 1500 \times \frac{2}{0.1}$$

$$= 7.54 \times 10^{-6}V$$

Hence, the induced voltage in the loop is $7.54 \times 10^{-6}V$

---

6. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 $\mathrm{rad\ s^{-1}}$ in a uniform horizontal magnetic field of magnitude 3.0×10^{-2} T . Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10Ω, calculate the maximum value of current in the coil. Calculate the average power loss due to Joule heating. Where does this power come from?

Ans. Max induced *emf* = 0.603 V

Average induced *emf* = 0 V

Max current in the coil = 0.0603 A

Average power loss = 0.018 W

(Power comes from the external rotor)

Radius of the circular coil, r = 8 cm = 0.08 m

Area of the coil, $A = \pi r^2 = \pi \times (0.08)^2 \ m^2$

Number of turns on the coil, N = 20

Angular speed, ω = 50 rad/s

Magnetic field strength, $B = 3 \times 10^{-2} \ T$

Resistance of the loop, R = 10 Ω

Maximum induced *emf* is given as:

$$e = N\omega AB$$

$$= 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^{-2} = 0.603 \ V$$

The maximum *emf* induced in the coil is 0.603 V.

Over a full cycle, the average *emf* induced in the coil is zero.

Maximum current is given as:

$$I = \frac{e}{R}$$

$$= \frac{0.603}{10} = 0.0603 A$$

Average power loss due to joule heating:

$$P = \frac{eI}{2}$$

$$= \frac{0.603 \times 0.0603}{2} = 0.018W$$

The current induced in the coil produces a torque opposing the rotation of the coil. The rotor is an external agent. It must supply a torque to counter this torque in order to keep the coil rotating uniformly. Hence, dissipated power comes from the external rotor.

---

7. A square loop of side 12 cm with its sides parallel to X and Y axes is moved with a velocity of $8 \, cm \, s^{-1}$ in the positive *x*-direction in an environment containing a magnetic field in the positive *z*-direction. The field is neither uniform in space nor constant in time. It has a gradient of 10^{-3} T cm^{-1} along the negative *x*-direction (that is it increases by 10^{-3} T cm^{-1} as one moves in the negative *x*-direction), and it is decreasing in time at the rate of 10^{-3} T s^{-1}. Determine the direction and magnitude of the induced current in the loop if its resistance is 4.50 Ω .

Ans. Side of the square loop, *s* = 12 cm = 0.12 m

Area of the square loop, $A = 0.12 \times 0.12 = 0.0144 \, m^2$

Velocity of the loop, *v* = 8 cm/s = 0.08 m/s

Gradient of the magnetic field along negative *x*-direction,

$$\frac{dB}{dx} = 10^{-3} Tcm^{-1} = 10^{-1} Tm^{-1}$$

And, rate of decrease of the magnetic field,

$$\frac{dB}{dx} = 10^{-3} Ts^{-1}$$

Resistance of the loop, $R = 4.5 m\Omega = 4.5 \times 10^{-3} \Omega$

Rate of change of the magnetic flux due to the motion of the loop in a non-uniform magnetic

field is given as:

$$\frac{d\phi}{dt} = A \times \frac{dB}{dx} \times v$$

$$= 144 \times 10^{-4} m^2 \times 10^{-1} \times 0.08$$

$$= 11.52 \times 10^{-5} Tm^2 s^{-1}$$

Rate of change of the flux due to explicit time variation in field B is given as:

$$\frac{d\phi'}{dt} = A \times \frac{dB}{dx}$$

$$= 144 \times 10^{-4} \times 10^{-3}$$

$$= 1.44 \times 10^{-5} Tm^2 s^{-1}$$

Since the rate of change of the flux is the induced *emf*, the total induced *emf* in the loop can be calculated as:

$$e = 144 \times 10^{-5} + 11.52 \times 10^{-5}$$

$$= 12.96 \times 10^{-5} V$$

∴Induced current, $i = \dfrac{e}{R}$ s

$$s = \frac{12.96 \times 10^{-5}}{1.5 \times 10^{-3}}$$

$$i = 2.88 \times 10^{-2} A$$

Hence, the direction of the induced current is such that there is an increase in the flux through the loop along positive z-direction.

---

8. It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area 2 cm^2 with 25 closely wound turns, is

positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick $90°$ turn to bring its plane parallel to the field direction). The total charge flown in the coil (measured by a ballistic galvanometer connected to coil) is 7.5 mC. The combined resistance of the coil and the galvanometer is 0.50 Ω . Estimate the field strength of magnet.

Ans. Area of the small flat search coil, A = $2\ cm^2 = 2 \times 10^{-4}\ m^2$

Number of turns on the coil, N = 25

Total charge flowing in the coil, Q = 7.5 mC = $7.5 \times 10^{-3}\ C$

Total resistance of the coil and galvanometer, R = 0.50 Ω

Induced current in the coil,

$$I = \frac{Induced\ emf\ (e)}{R} \qquad ...(i)$$

Induced emf is given as:

$$e = -N\frac{d\phi}{dt} \qquad ...(2)$$

Where,

$d\phi$ = Charge in flux

Combining equations (1) and (2), we get

$$1 = -\frac{N\frac{d\phi}{dt}}{R}$$

$$Idt = -\frac{N}{R}d\phi \qquad ...(3)$$

Initial flux through the coil, $\phi_i = BA$

Where,

B = Magnetic field strength

Final flux through the coil, $\phi_f = 0$

Integrating equation (3) on both sides, we have

$$\left(\int I dt = \frac{-N^{\phi_f}}{R_{\phi_f}} \right) d\phi$$

But total charge, $Q = \int I dt.$

$$\therefore Q = \frac{-N}{R-}(\phi_f - \phi_i) = \frac{-N}{R}(-\phi_i) = +\frac{N\phi_i}{R}$$

$$Q = \frac{NBA}{R}$$

$$\therefore B = \frac{QR}{NA}$$

$$= \frac{7.5 \times 10^{-3} \times 0.5}{25 \times 2 \times 10^{-4}} = 0.75T$$

Hence, the field strength of the magnet is 0.75 T.

---

9. Figure 6.20 shows a metal rod PQ resting on the smooth rails AB and positioned between the poles of a permanent magnet. The rails, the rod, and the magnetic field are in three mutual perpendicular directions. A galvanometer G connects the rails through a switch K. Length of the rod = 15 cm, B = 0.50 T, resistance of the closed loop containing the rod = 9.0 m Ω . Assume the field to be uniform.

(a) Suppose K is open and the rod is moved with a speed of 12 **cm** s^{-1} **in the direction shown. Give the polarity and magnitude of the induced emf.**
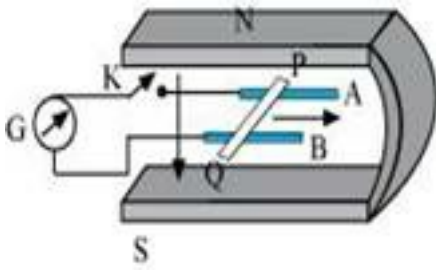
(b) Is there an excess charge built up at the ends of the rods when K is open? What if K is closed?

(c) With K open and the rod moving uniformly, there is *no* experience magnetic force due to the motion of the rod. Explain.

(d) What is the retarding force on the rod when K is closed?

(e) How much power is required (by an external agent) to keep the rod moving at the same speed (=12 cm s^{-1}) when K is closed? How much power is required when K is open?

(f) How much power is dissipated as heat in the closed circuit? What is the source of this power?

(g) What is the induced emf in the moving rod if the magnetic field is parallel to the rails instead of being perpendicular?

Ans. Length of the rod, l = 15 cm = 0.15 m

Magnetic field strength, B = 0.50 T

Resistance of the closed loop, R = 9 Ω = 9×10^{-3} Ω

(a) Induced emf = 9 mV; polarity of the induced emf is such that end P shows positive while end Q shows negative ends.

Speed of the rod, v = 12 cm/s = 0.12 m/s

Induced emf is given as:

$e = Bvl$

$= 0.5 \times 0.12 \times 0.15$

$= 9 \times 10^{-3}$ v

= 9 mV

The polarity of the induced emf is such that end *P* shows positive while end *Q* shows negative ends.

(b) Yes; when key K is closed, excess charge is maintained by the continuous flow of current.

When key K is open, there is excess charge built up at both ends of the rods.

When key K is closed, excess charge is maintained by the continuous flow of current.

(c) Magnetic force is cancelled by the electric force set-up due to the excess charge of opposite nature at both ends of the rod.

There is no net force on the electrons in rod PQ when key K is open and the rod is moving uniformly. This is because magnetic force is cancelled by the electric force set-up due to the excess charge of opposite nature at both ends of the rods.

(d) Retarding force exerted on the rod, *F* = *IBl*

Where,

I = Current flowing through the rod

$$= \frac{e}{R} = \frac{9 \times 10^{-3}}{9 \times 10^{-3}} = 1A$$

$$\therefore F = 1 \times 0.5 \times 0.15$$

$$= 75 \times 10^{-3} N$$

(e) 9 mW; no power is expended when key K is open.

Speed of the rod, *v* = 12 cm/s = 0.12 m/s

Hence, power is given as:

$P = Fv$

$= 75 \times 10^{-3} \times 0.12$

$= 9 \times 10^{-3} W$

$= 9\, mW$

When key K is open, no power is expended.

(f) 9 mW; power is provided by an external agent.

Power dissipated as heat $= I2\ R$

$= (1)2 \times 9 \times 10^{-3}$

= 9 mW

The source of this power is an external agent.

(g) Zero In this case, no emf is induced in the coil because the motion of the rod does not cut across the field lines.

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10. (a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown in Fig. 6.21.
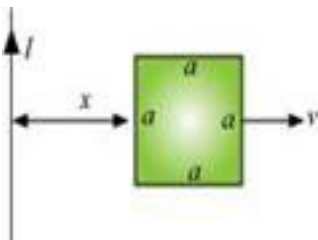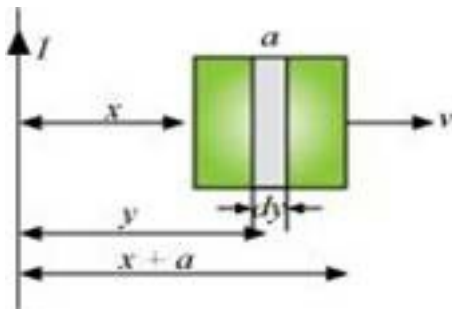

(b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, v = 10 m/s.

Calculate the induced emf in the loop at the instant when x = 0.2 m.

Take a = 0.1 m and assume that the loop has a large resistance.

Ans.

(a) Take a small element dy in the loop at a distance y from the long straight wire (as shown in the given figure).

Magnetic flux associated with element dy, $d\phi = BdA$

Where,

dA = Area of element dy = $a\,dy$

B = Magnetic field at distance y

$$= \frac{\mu_0 I}{2\pi y} \text{ s}$$

I = Current in the wire

μ_0 = Permeability of free space = $4n \times 10^{-7}\ T\ m\ A^{-1}$

$$\therefore d\phi = \frac{\mu_0 I a}{2\pi} \frac{dy}{y}$$

$$\phi = \frac{\mu_0 I a}{2\pi} \int \frac{dy}{y}$$

y tends from x to a+x.s

$$\therefore \phi = \frac{\mu_0 I a}{2\pi} \int_{x}^{a+x} \frac{dy}{y}$$

$$= \frac{\mu_0 I a}{2\pi} \left[\log_e y\right]_{x}^{a+x}$$

$$= \frac{\mu_0 Ia}{2\pi}\log_e\left(\frac{a+x}{x}\right)$$

For mutual inductance M, the flux is given as:

$$\phi = mI$$

$$\therefore mI = \frac{\mu_0 Ia}{2\pi}\log\left(\frac{a}{x}+1\right)$$

$$m = \frac{\mu_0 Ia}{2\pi}\log\left(\frac{a}{x}+1\right)$$

(b) Emf induced in the loop, $e = B'av\left(\frac{\mu_0 I}{2\pi}\right)av$ Given,

$I = 50$ A
$x = 0.2$ m
$a = 0.1$ m

$v = 10$ m/s

$$BQv = \frac{Mv^2}{r}$$

$$\therefore B2\pi r\lambda r^2 = \frac{Mv}{r}$$

$$v = \frac{B2\pi r\lambda r^2}{M}$$

For $r <$ and $a < R$, we get:

$$(i) = -\frac{2B_0 a^2\lambda}{MR}k$$

$$e = 5\times10^{-5}V$$