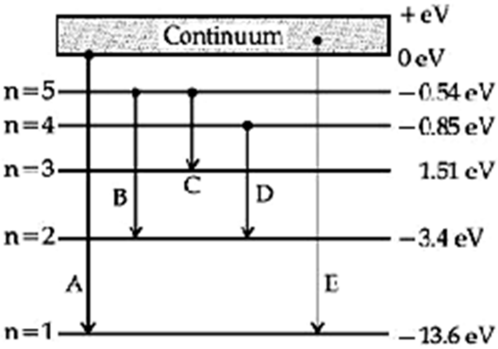
PHYSICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. In the given figure, the energy levels of hydrogen atom have been shown along with some transitions marked A, B, C, D and E. The transitions A, B and C respectively represent :



1) The first member of the Lyman series, third member of Balmer series and second member of Paschen series.

2) The series limit of Lyman series, second member of Balmer series and second member of Paschen series.

3) The ionization potential of hydrogen, second member of Balmer series and third member of Paschen series

4) The series limit of Lyman series, third member of Balmer series and second member of Paschen series

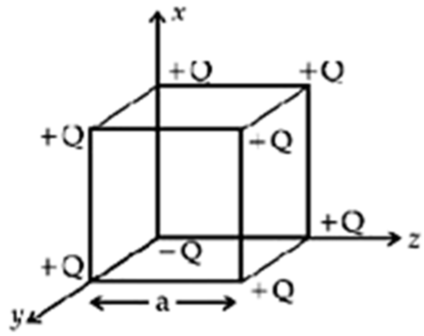
Key: 4

Sol: $A \rightarrow$ Series limit of laymen

$B \rightarrow 3^{rd}$ line of Balmer

$C \rightarrow 2^{nd}$ line of paschan

2. A cube of side 'a' has point charges +Q located at each of its vertices except at the origin where the charge is – Q. The field at the centre of cube is :



1) $\frac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$ **2)** $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

3) $\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$ **4)** $\frac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

Key: 2

Sol: If we consider two point charges +q and –q at position of –q charge, then after interchanging –q charge with +q charge, net electric field at centre of cube is zero due to symmetry. Now remaining charges are –2q so net electric field at centre is $\left(\frac{-8kq}{3a^2}\right)$.

3. A current through a wire depends on time as $i=\alpha_0 t+\beta t^2$ where $\alpha_0 = 20A/s$ and $\beta = 8As^{-2}$. Find the charge crossed through a section of the wire in 15 s.

1) 2100 C **2)** 2250 C **3)** 260 C **4)** 11250 C

Key: 4

Sol: $\frac{dq}{dt}=\left(20t+8t^2\right)$

$$\int dq = \int_0^{15}\left(20t+8t^2\right)dt$$

$$\Delta q = \left[20\frac{t^2}{2}+\frac{8t^3}{3}\right]_0^{15} = \frac{20\times(15)^2}{2}+\frac{8\times(15)^3}{3} \quad \Delta q = 11250C$$

4. Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8 hr respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to be

1) 2 : 1 **2)** 1 : 8 **3)** 1 : 4 **4)** 8 : 1

Key: 1

Sol: ratio of time period

$$\frac{T_1}{T_2}=\frac{1}{8}$$

$$\frac{\frac{2\pi}{\omega_1}}{\frac{2\pi}{\omega_2}}=\frac{1}{8} \qquad \frac{\omega_1}{\omega_2}=8$$

5. **Match List I with List II.**

List I	**List II**
a) Isothermal	i) Pressure constant
b) Isochoric	ii) Temperature constant
c) Adiabatic	iii) Volume constant
d) Isobaric	iv) Heat content is constant

Choose the correct answer from the options given below :

1) $(a)\to(ii),(b)\to(iv),(c)\to(iii),(d)\to(i)$

2) $(a)\to(iii),(b)\to(ii),(c)\to(i),(d)\to(iv)$

3) $(a)\to(i),(b)\to(iii),(c)\to(ii),(d)\to(iv)$

4) $(a)\to(ii),(b)\to(iii),(c)\to(iv),(d)\to(i)$

Key: 4

Sol: P-iv ; Q-iii ; R-ii ; S-i

6. Given below are two statements :

Statement I : Two photons having equal linear momenta have equal wavelengths.

Statement II : If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below

1) Both Statement I and Statement II are false

2) Statement I is false but Statement II is true

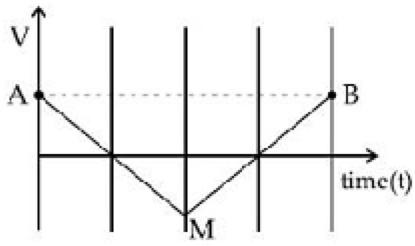
3) Both Statement I and Statement II are true

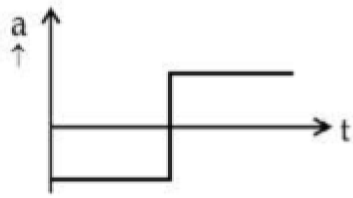
4) Statement I is true but Statement II is false

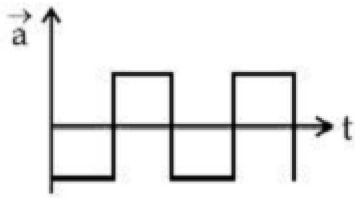
Key: 4

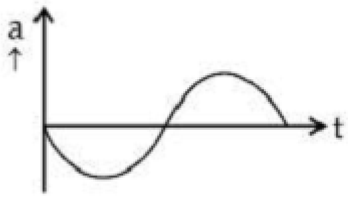
Sol: $\lambda = \dfrac{h}{p}$

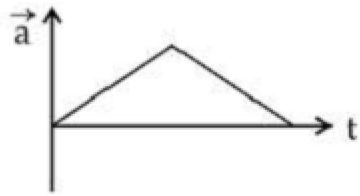
7. If the velocity time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph?



1) 

2) 

3) 

4) 

Key: 1

Sol: $v = -mt + C$

$\dfrac{dv}{dt} = -m$

$$\left| \begin{array}{l} v = mt - C \\ \dfrac{dv}{dt} = m \end{array} \right|$$

8. In a Young's double slit experiment, the width of the one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit width. Find the ratio of the maximum to the minimum intensity in the interference pattern

1) 2 : 1 **2)** 1 : 4 **3)** 3 : 1 **4)** 4 : 1

Key: 4

Sol: $\dfrac{A_1}{A_2} = \dfrac{1}{3}$ $A_1 = x, A_2 = 3x$

$$\frac{I_{max}}{I_{min}} = \left(\frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}\right) = \frac{(4x)^2}{(2x)^2} = \frac{16}{4} = 4:1$$

9. Each side of a box made of metal sheet in cubic shape is ‘a’ room temperature ‘T’, the coefficient of linear expansion of the metal sheet is ‘a’. The metal sheet is heated uniformly, by a small temperature ΔT, so that its temperature is $T + \Delta T$. Calculate the increase in the volume of the metal box.

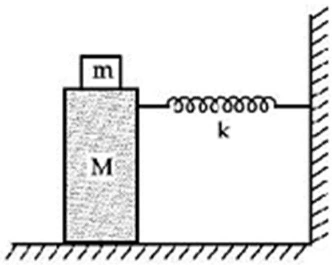
1) $\frac{4}{3}\pi a^3 \alpha \Delta T$ **2)** $3a^3 \alpha \Delta T$ **3)** $4\pi a^3 \alpha \Delta T$ **4)** $4a^3 \alpha \Delta T$

Key: 2

Sol: $\frac{\Delta V}{V} = \gamma \Delta T$

$= 3\alpha \Delta T$ $\Delta V = 3a^3 \alpha \Delta T$

10. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be :



1) $A\sqrt{\frac{M - m}{M}}$ **2)** $A\sqrt{\frac{M}{M + m}}$ **3)** $A\sqrt{\frac{M}{M - m}}$ **4)** $A\sqrt{\frac{M + m}{M}}$

Key: 2

Sol: Velocity at mean position is $= A\omega$

Conserving momentum $MA\omega_0 = (M + m)V^1$

$$V^1 = \frac{MA\omega_0}{M + m} = (A')\sqrt{\frac{K}{M + m}} \qquad A' = \frac{MA\sqrt{\frac{K}{M}}}{M + m} \times \sqrt{\frac{M + m}{K}} = \sqrt{\frac{M}{(M + m)}}A$$

11. The work done by a gas molecule in an isolated system is given by, $W = \alpha\beta^2 e \frac{x^2}{\alpha KT}$, where x is the displacement k is the Boltzmann constant and T is the temperature, α and β are constants. Then the dimensions of β will be :

1) $\left[M^0LT^0\right]$ **2)** $\left[M^2LT^2\right]$ **3)** $\left[MLT^{-2}\right]$ **4)** $\left[ML^2T^{-2}\right]$

Key: 3

Sol: $\frac{x^2}{\alpha KT} =$ dimensions

$\frac{L^2}{KT} \Rightarrow \alpha$

$\alpha \Rightarrow \frac{L^2}{v^2M} = L^2M^{-1}L^{-2}T^2 = M^{-1}T^2$

work = $\alpha.\beta^2$. (dimensionless)

$M^1L^1T^{-2}.L^1 = M^{-1}T^2\beta^2$

$v = \sqrt{\frac{3KT}{M}}$

$\frac{v^2.M}{3} = KT$

$\beta = M^1LT^{-2}$

12. The focal length f is related to the radius of curvature r of the spherical convex mirror by :

1) $f = -\frac{1}{2}r$ **2)** $f = +\frac{1}{2}r$ **3)** $f = r$ **4)** $f = -r$

Key: 2

Sol: $\frac{R}{2} = f$

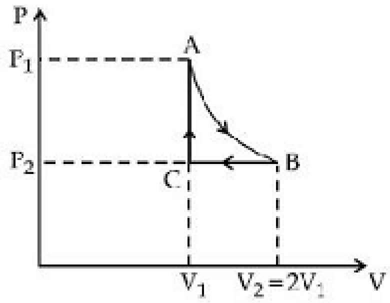
13. n mole of a perfect gas and undergoes a cyclic process ABCA (see figure) consisting of the following processes.

$A \rightarrow B$: Isothermal expansion at temperature T so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure changes from P_1 to P_2.

$B \rightarrow C$: Isobaric compression at pressure P_2 to initial volume V_1

$C \rightarrow A$: Isochoric change leading to change of pressure from P_2 to P_1.

Total work done in the complete cycle ABCA is :



1) $nRT\left(\ln 2+\frac{1}{2}\right)$ **2)** 0 **3)** $nRT\ln 2$ **4)** $nRT\left(\ln 2-\frac{1}{2}\right)$

Key: 4

Sol: $W_{AB} = 2P_1V_1\ell n2$

$W_{BC} = -P_1V_1$

$W_{CA} = 0$

$W_{ABCA} = \left(2P_1V_1\ell n2 - P_1V_1\right)$

$= nRT\left(2\ell n2 - 1\right)$

14. If Y, K and η are the values of Young's modulus, bulk and modulus of rigidly of any material respectively. Choose the correct relation for those parameters.

1) $Y = \frac{9K\eta}{2\eta + 3K} N/m^2$ **2)** $\eta = \frac{3YK}{9K + Y} N/m^2$

3) $K = \frac{Y\eta}{9\eta - 3Y} N/m^2$ **4)** $Y = \frac{9K\eta}{3K - \eta} N/m^2$

Key: 3

Sol:

15. Moment of inertia (M.I.) of four bodies, having same mass and radius, are reported as;

$I_1 =$ M.I. of thin circular ring about its diameter,

$I_2 =$ M.I. of circular disc about an axis perpendicular to disc and going through the centre,

$I_3 =$ M.I of solid cylinder about its axis and

$I_4 =$ M.I. of solid sphere about its diameter.

1) $I_1 + I_3 < I_2 + I_4$ 2) $I_1 + I_2 = I_3 + \frac{5}{2}I_4$

3) $I_1 + I_2 = I_3 < I_4$ 4) $I_1 = I_2 = I_3 > I_4$

Key: 4

Sol: $I_1 = \frac{MR^2}{2}$

$I_2 = \frac{MR^2}{2}$

$I_3 = \frac{MR^2}{2}$

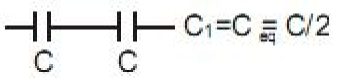
$I_4 = \frac{2}{5}MR^2$

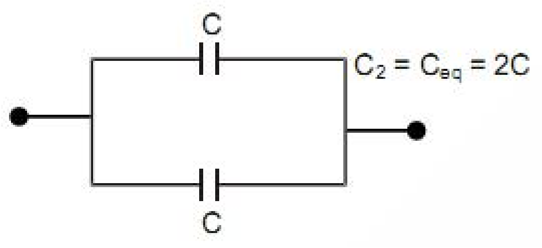
16. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be:

1) 2 : 1 **2)** 1 : 2 **3)** 4 : 1 **4)** 1 : 4

Key: 4

Sol:





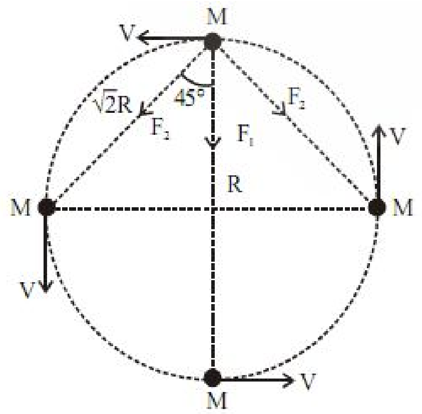
$\frac{C_1}{C_2} = \frac{1}{4}$

17. Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be :

1) $\sqrt{G\left(1+2\sqrt{2}\right)}$ **2)** $\frac{\sqrt{\left(1+2\sqrt{2}\right)G}}{2}$ **3)** $\sqrt{\frac{G}{2}\left(2\sqrt{2}-1\right)}$ **4)** $\sqrt{\frac{G}{2}\left(1+2\sqrt{2}\right)}$

Key: 2

Sol:

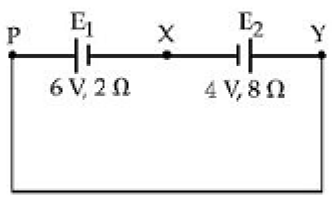


Net force on one particle $F_{net} = F_1 + 2F_2 \cos 45^o =$ centripetal force

$$\Rightarrow \frac{GM^2}{(2R)^2} + \left[\frac{2GM^2}{(\sqrt{2}R)^2}\cos 45^o\right] = \frac{MV^2}{R}$$

$$V = \frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)} V = \frac{1}{2}\sqrt{G\left(1+2\sqrt{2}\right)}$$

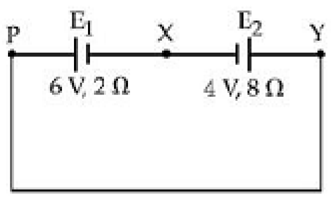
18. A cell E_1 of emf 6V and internal resistance 2Ω is connected with another cell E_2 of emf 4 V and internal resistance 8Ω (as shown in the figure). The potential difference across points X and Y is :



1) 10.0 V **2)** 3.6 V **3)** 5.6 V **4)** 2.0 V

Key: 3

Sol: Current $I = \frac{6-4}{10} = \frac{1}{5}A$



$$v_x + 4 + 8 \times \frac{1}{5} = v_y$$

$$v_x + v_y = -5.6v$$

19. If an emitter current is changed by 4 mA, the collector current changes by 3.5 mA. The value of β will be :

1) 0.875 **2)** 3.5 **3)** 7 **4)** 0.5

Key: 3

Sol: $\Delta I_E = 4$

$\Delta I_C = 3.5$

$$\alpha = \frac{\Delta I_C}{\Delta I_E} = \left(\frac{3.5}{4}\right) = \left(\frac{7}{8}\right)$$

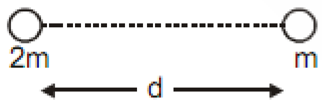
$$\beta = \frac{\alpha}{1-\alpha} = \frac{\frac{7}{8}}{1-\frac{7}{8}} = 7$$

20. Two stars of masses m and 2m at a distance d rotate about their common centre of mass in free space. The period of revolution is:

1) $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$ **2)** $2\pi\sqrt{\frac{3Gm}{d^3}}$ **3)** $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$ **4)** $2\pi\sqrt{\frac{d^3}{3Gm}}$

Key: 4

Sol:



$$\frac{G(m)(2m)}{d^2} = m\omega^2 \times \frac{2d}{3}$$

$$\frac{2Gm}{d^2} = \omega^2 \times \frac{2d}{3}$$

$$\omega^2 = \frac{3Gm}{d^3} \qquad \omega = \sqrt{\frac{3Gm}{d^3}}; T = 2\pi\sqrt{\frac{d^3}{3Gm}}$$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. A ball with a speed of 9 m/s collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is x : y, where x is ______

Key: 1

Sol: Using linear momentum conservation in y- direction $P_i = 0$

$$P_f = m \times \frac{1}{2} v_1 - m \times \frac{1}{2} v_2$$

$$v_1 = v_2$$

22. A common transistor radio set requires 12 V (D.C) for its operation. The D.C. source is constructed by using a transformer and a rectifier circuit, which are operated at 220V (A.C) on standard domestic A.C. supply. The number of turns of secondary coil are 24, then the number of turns of primary are _______

Key: 440

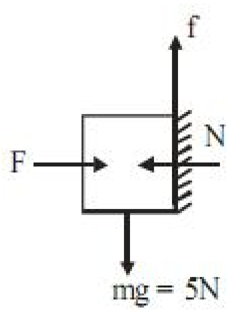
Sol: $$\frac{N_P}{N_S} = \frac{V_P}{V_S}$$

$$\frac{N_P}{24} = \frac{220}{12}; N_P = 440$$

23. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be _______ N. $\left[g = 10ms^{-2}\right]$

Key: 25

Sol: F = N, f = 0.2× N



$$0.2N \le 5$$

$$N \le 25$$

24. An unpolarized light beam is incident on the polarizer of a polarization experiment and the intensity of light beam emerging from the analyzer is measured as 100 Lumens. Now, if the analyzer is rotated around the horizontal axis (direction of light) by 30º in clockwise direction the intensity of emerging light will be ______Lumens.

Key: 75

Sol: $I = I_o \cos^2\theta = I_o \cos^2 30^0 \quad = I_o\left(\frac{3}{4}\right) = 75$

25. A hydraulic press can lift 100 kg when a mass 'm' is placed on the smaller piston. It can lift ________ kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass 'm' on the smaller piston.

Key: 25600

Sol: Initially $\frac{100g}{A_1} = \frac{mg}{A_2}$..........(i)

Initially $\frac{Mg}{16A_1} = \frac{mg}{\left(\frac{A_2}{16}\right)}$..........(ii)

$\frac{100\times 16}{M} = \frac{1}{16} = M = 25600\text{kg}$

26. A resonance circuit having inductance and resistance $2\times 10^{-4}\text{H}$ and 6.28Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is ______. $[\pi = 3.14]$

Key: 200 **Our key is : 2000**

Sol: $Q = \frac{x_L}{R} = \frac{\omega L}{R} = \frac{2\pi f L}{R}$

$Q = \frac{2\pi \times 10^6 \times 10 \times 2 \times 10^{-4}}{6.28} = 2000$

$Q = 2000$

27. An electro magnetic wave of frequency 5 GHz is travelling in a medium whose relative electric permittivity and relative magnetic permeability both are 2. Its velocity in this medium is________ $\times 10^7 \text{m/s}$.

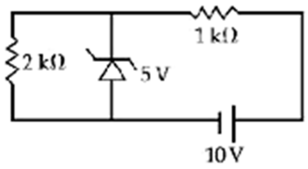
Key: 15

Sol: $n = \sqrt{\mu_r \varepsilon_r} = 2$

$v = \frac{C}{n} = \frac{3\times 10^8}{2} = 15\times 10^7 \text{m/s}$

$x = 15$

28. In connection with the circuit drawn below, the value of current flowing, through $2k\Omega$ resistor is ________ $\times 10^{-4}$ A.



Key: 25

Sol: Zener diode breakdown

$$\Rightarrow i = \frac{5}{2\times10^{-3}} = 2.5\times10^{-3}$$

$$x\times10^{-4} = 2.5\times10^{-3}$$

$$x = 2.5mA$$

29. An inclined plane is bent in such a way that the vertical cross–section is given by $y-\frac{x^2}{4}$ where y is in vertical and *x* in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction $\mu = 0.5$, the maximum height in cm at which a stationary block will not slip downward is ________cm.

Key: 25

Sol: $\mu \ge \tan\theta = \frac{dy}{dx} = \frac{dx}{4} = \frac{x}{2}$

$0.5 \ge \frac{x}{2}$ $\qquad\qquad$ $x \le 1$

$\sqrt{4y \le 1}$

$2\sqrt{y} \le 1$

$y \le \frac{1}{4}$

30. An audio signal $v_m = 20\sin 2\pi(1500t)$ amplitude modulates a carrier $v_c = 80\sin 2\pi(100{,}000t)$.

The value of percent modulation is ___________

Key: 25

Sol: $m\% = \frac{A_m}{A_c}\times100 = \frac{20}{80}\times100 = 25\%$

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. Which of the following ore is concentrated using group 1 cyanide salt ?

1) Calamine **2)** Siderite **3)** Sphalerite **4)** Malachite

Key: 3

Sol: NaCN is used as depressant in froth flotation process for the separation of galena & Sphalerite

32. Al_2O_3 was leached with alkali to get X. The solution of X on passing of gas Y, forms Z. X, Y and Z respectively are :

1) $X = Al(OH)_3, Y = SO_2, Z = Al_2O_3.xH_2O$

2) $X = Na\left[Al(OH)_4\right], Y = CO_2, Z = Al_2O_3.xH_2O$

3) $X = Na\left[Al(OH)_4\right], Y = SO_2, Z = Al_2O_3.xH_2O$

4) $X = Al(OH)_3, Y = CO_2, Z = Al_2O_3$

Key: 2

Sol: $Al + 4NaOH \longrightarrow Na\left(Al(OH)_4\right) \xrightarrow{CO_2} Na_2CO_3 + Al_2O_3.xH_2O$

33. Out of the following, which type of interaction for the stabilisation of $\alpha-$helix structure of protons ?

1) Covalent bonding **2)** Hydrogen bonding

3) Ionic bonding **4)** vander Waals forces

Key: 2

Sol: Hydrogen bond is responsible for the stacking of $\alpha-$ helix structure of protein.

34. Match List – I with List – II.

List - I (Monomer Unit)	List – II (Polymer)
(a) Caprolactum	(i) Natural rubber
(b) 2-Chloro-1, 3-butadiene	(ii) Buna – N
(c) Isoprene	(iii) Nylon 6
(d) Acrylonitrile	(iv) Neoprene

Choose the correct answer from the options given below :

1) $a \rightarrow i, b \rightarrow ii, c \rightarrow iii, d \rightarrow iv$ **2)** $a \rightarrow iv, b \rightarrow iii, c \rightarrow ii, d \rightarrow i$

3) $a \rightarrow ii, b \rightarrow i, c \rightarrow iv, d \rightarrow iii$ **4)** $a \rightarrow iii, b \rightarrow iv, c \rightarrow i, d \rightarrow ii$

Key: 4

Sol: $a \to iii, b \to iv, c \to i, d \to ii$

35. $(A)\quad HOCl + H_2O_2 \to H_3O^+ + Cl^- + O_2$

$(B)\quad I_2 + H_2O_2 + 2OH^- \to 2I^- + 2HO + O_2$

Choose the correct option

1) H_2O_2 acts as reducing and oxidizing agent respectively in equations (A) and (B)

2) H_2O_2 acts as reducing agent in equations (A) and (B)

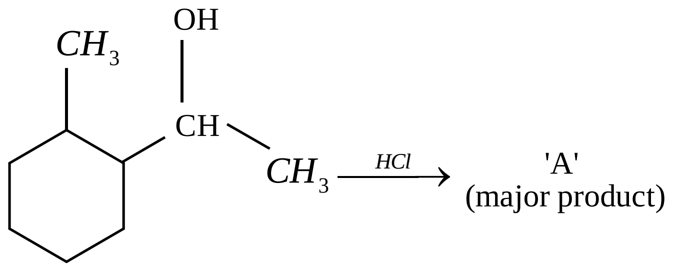
3) H_2O_2 acts as oxidixing agent in equations (A) and (B)

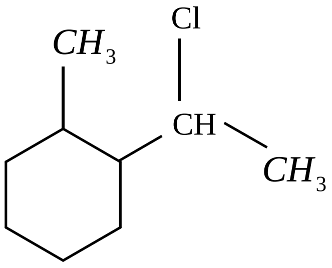
4) H_2O_2 acts as oxidixing and reducing agent respectively in equations (A) and (B)

Key: 2

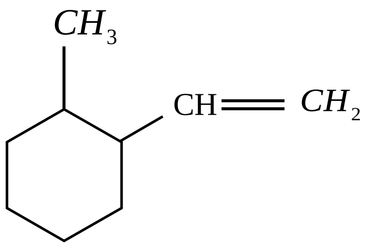
Sol: In 1 & 2 H_2O_2 acts as reducing agent

36. What is the final product (major) 'A' in the given reaction ?

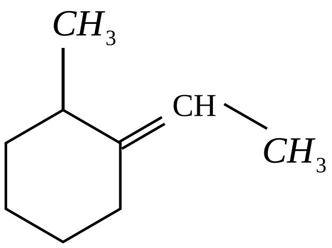




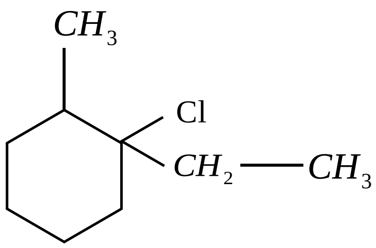
1)



2)



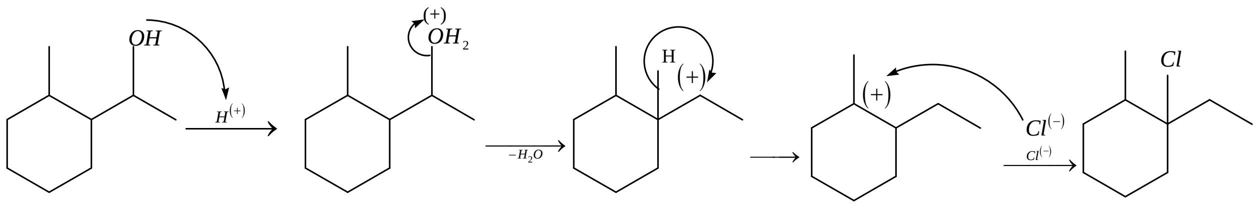
3)

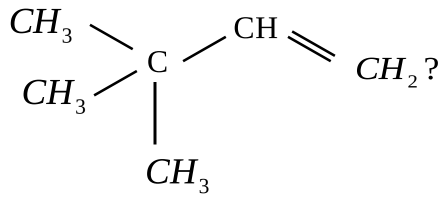


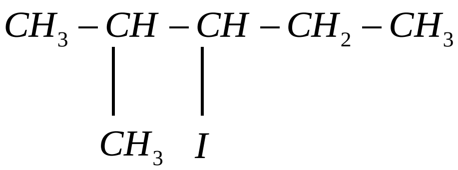
4)

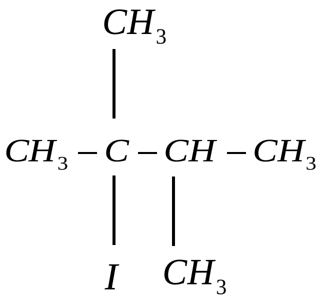
Key:

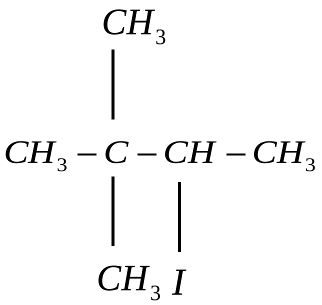
Sol:

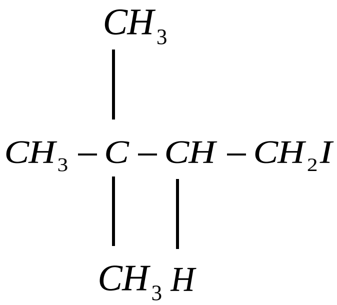


37. What is the product formed by HI on reaction with 

1) 

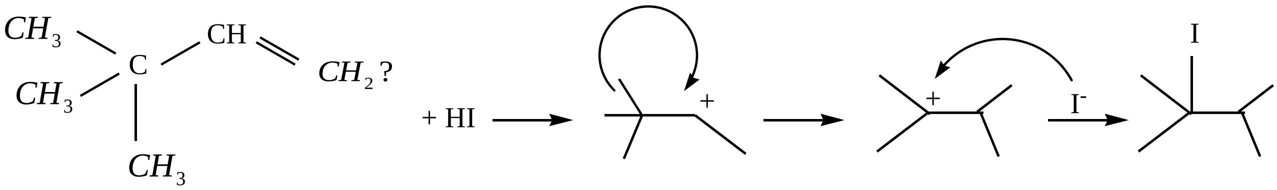
2) 

3) 

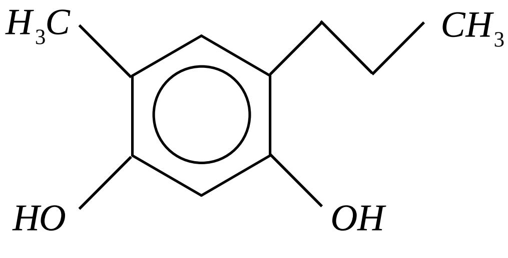
4) 

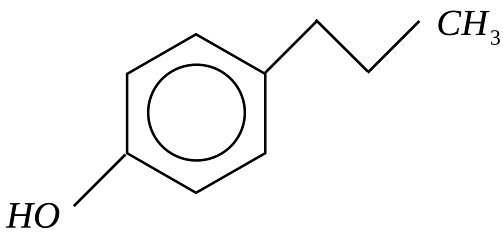
Key: 2

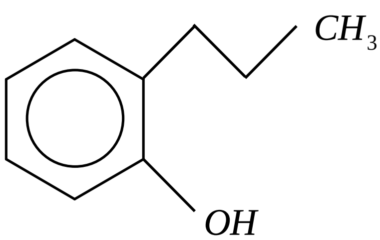
Sol:

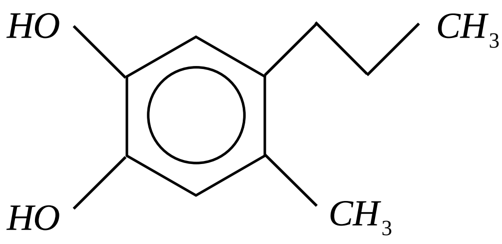


38. Which of the following compound gives pink colour on reaction with phthalic anhydride in conc. H_2SO_4 followed by treatment with NaOH ?

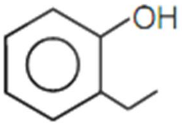
1) 

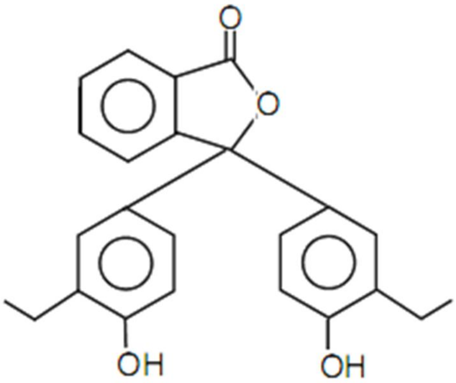
2) 

3) 

4) 

Key: 3

P is 



Sol:

39. Which of the following are isostructural pairs ?

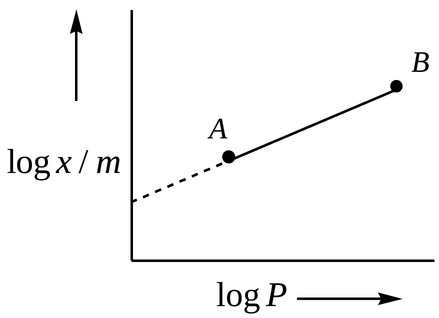
A) SO_4^{2-} & CrO_4^{2-} B) $SiCl_4$ & $TiCl_4$ C) NH_3 & NO_3^- D) BCl_3 & $BrCl_3$

1) A and B only **2)** B and C only **3)** A and C only D) C and D only

Key: 1

Sol: SO_4^{2-} & CrO_4^{2-}, $SiCl_4$ & $TiCl_4$ The two pairs are isostructural, all are tetrahedral

40. In Freundlich adsorption isotherm, slope of AB line is :



1) $\frac{1}{n} with \left(\frac{1}{n} = 0 to 1\right)$ **2)** log n with (n > 1)

3) n with (n, 0.1 to 0.5) **4)** $\log \frac{1}{n} with (n < 1)$

Key: 1

Sol: $\frac{x}{m} = kp^{1/n}$

$$\log \frac{x}{m} = \log k + \frac{1}{n} \log p$$

$$Slope = \frac{1}{n}(0 to 1)$$

41. The gas released during anaerobic degradation of vegetation may lead to :

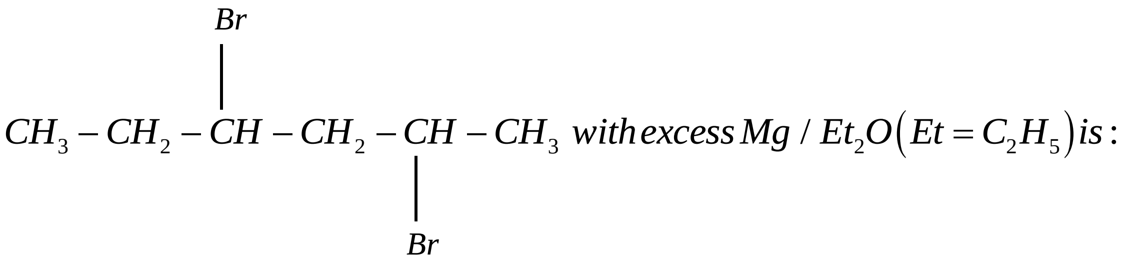
1) Corrosion metals **2)** Ozone hole

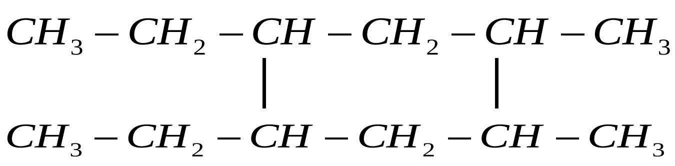
3) Global warming and cancer **4)** Acid rain

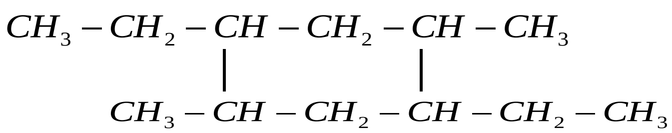
Key: 3

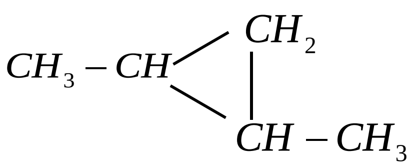
Sol: The gas CH_4 evolved due to anaerobic degradation of vegetation which causes global warming and cancer.

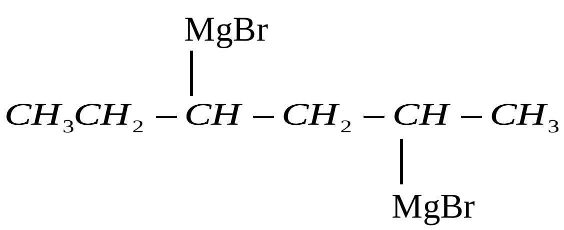
42. The product formed in the first step of the reaction of



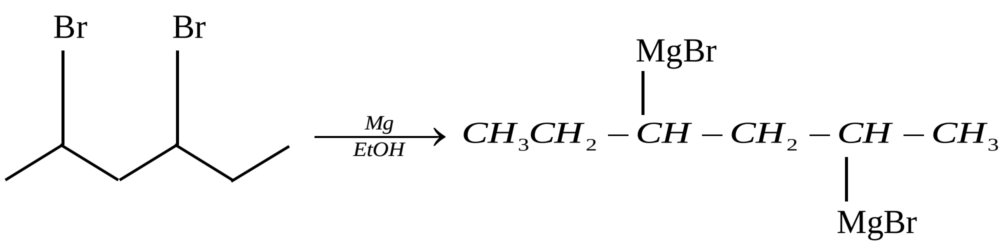
1) 

2) 

3) 

4) 

Key: 4

Sol: 

43. Which of the following regents is used for the following reaction ?

$$CH_3CH_2CH_3 \xrightarrow{?} CH_3CH_2CHO$$

1) Potassium permanganate

2) Molybdenum oxide

3) Manganese acetate

4) Copper at high temperature and pressure

Key: 2

Sol: Control oxidizing reagent is molybdenum oxide

44. Consider the elements Mg, Al, S, P ad Si, the correct increasing order of their first ionization enthalpy is :

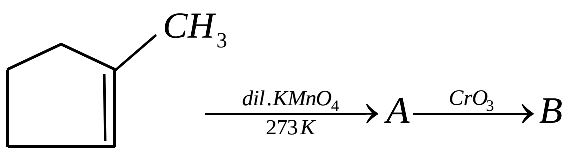
1) Mg < Al < Si < S < P **2)** Mg < Al < Si < P < S

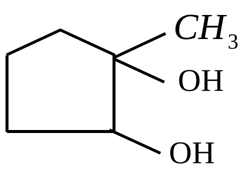
3) Al < Mg < S < Si < P **4)** Al < Mg < Si < S < P

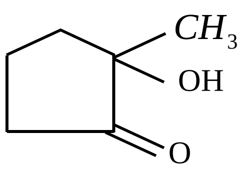
Key: 4

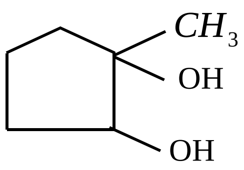
Sol: Al < Mg < Si < S < P

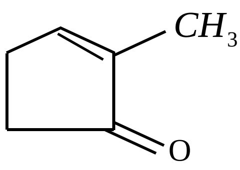
45. Identify products A and B.



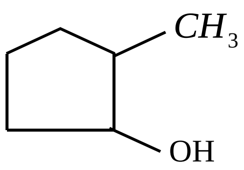
1) A : 

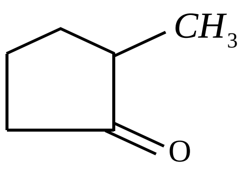
B : 

2) A : 

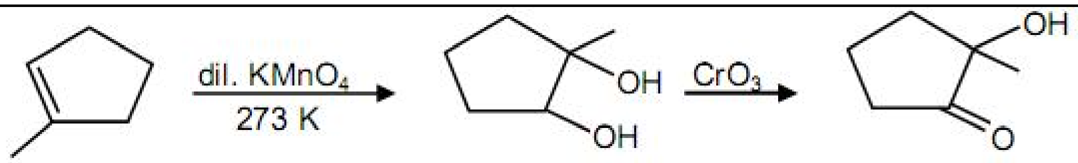
B : 

3) $A: OHC-CH_2CH_2CH_2-\overset{O}{\overset{\|}{C}}-CH_3$ $B: HOOC-CH_2CH_2CH_2-\overset{O}{\overset{\|}{C}}-CH_3$

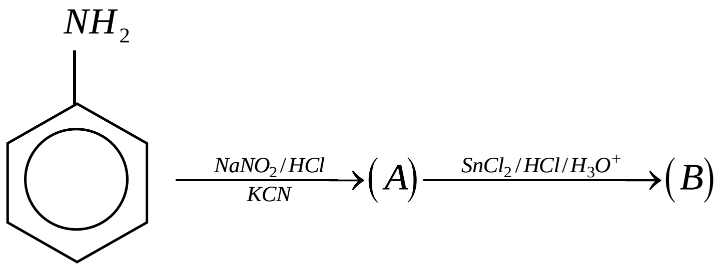
4) A : 

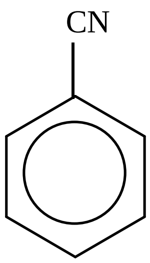
B : 

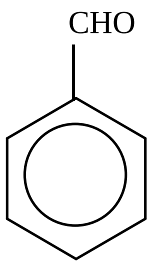
Key: 1

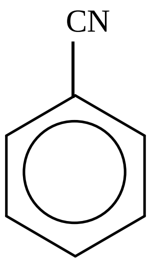
Sol: 

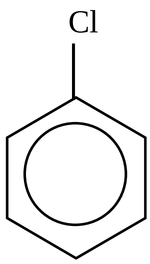
46. ‘A’ and ‘B’ in the following reactions are :

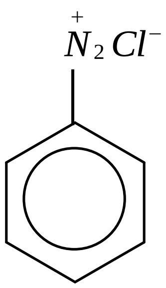


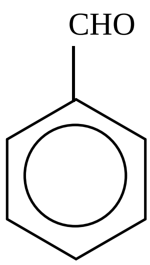
1) A : 

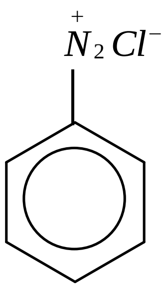
B : 

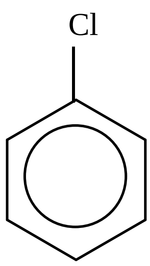
2) A : 

B : 

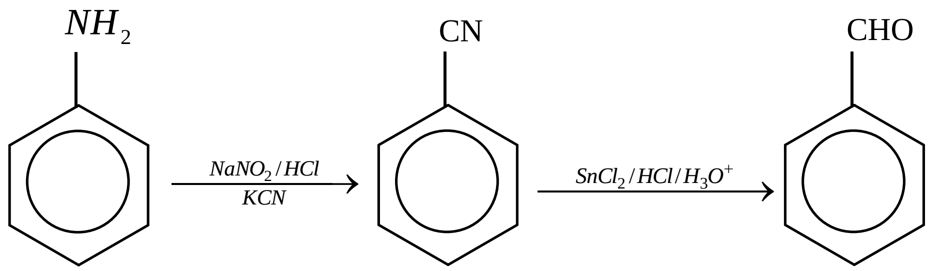
3) A : 

B : 

4) A : 

B : 

Key: 1

Sol: 

47. The electrode potential of M^{2+}/M of 3d – series elements shows positive value for :

1) Co **2)** Zn **3)** Fe **4)** Cu

Key: 4

Sol: 'Cu' has reduction potential value +0.34

48. Consider below are two statements :

Statement – I : Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

Statement – II : Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminus flame.

In the light of the above statements, choose the most appropriate answer from the options given below.

1) Both Statement I and Statement II are false

2) Statement I is false but Statement II is true

3) Statement I is true but Statement II is false

4) Both Statement I and Statement II are true

Key: 1

Sol: S – 1 Cupric metaborate is Blue – green colour

S – 2 Cupric metaborate is formed by heating $CuSO_4, B_2O_3$ in non luminous flame.

49. The major components in "Gun Metal" are :

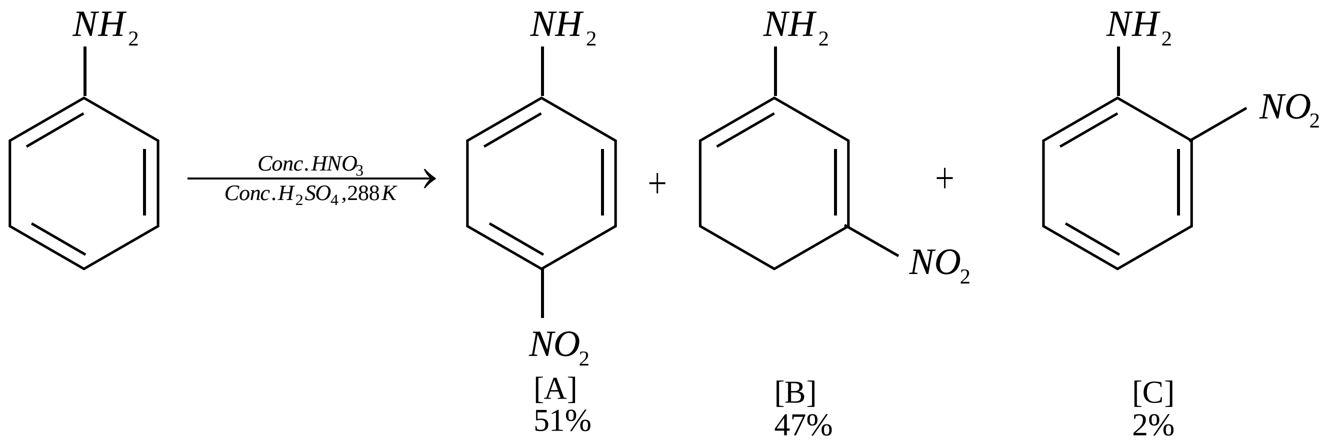
1) Cu, Zn and Ni **2)** Al , Cu, Mg and Mn

3) Cu, Sn and Zn **4)** Cu, Ni and Fe

Key: 3

Sol: Composition of gunmetal is Cu + Sn + Zn

50. In the following reaction the reason why meta-nitro product also formed is :



1) – NH_2 group is highly meta-directive

2) – NO_2 substitution always takes place at meta-position

3) Formation of anilinium ion

4) Low temperature

Key: 3

Sol: In acidic medium, aniline is converted into anilinium ion which is meta directing

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

51. 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution in M is $x \times 10^{-1}$. The value of x is ____. (Rounded off to the nearest integer)

Key: 2

Sol: $M = \frac{n}{V} = \frac{4.5/90}{250/1000} = 0.2 = 2 \times 10^{-1}$

52. The reaction of sulphur in alkaline medium is given below :

$S_{8(s)} + a\,OH^-_{(aq)} \rightarrow b\,S^{2-}_{(aq)} + c\,S_2O^{2-}_{3(aq)} + dH_2O_{(l)}$

The value of 'a' is _____ (Integer answer)

Key: 12

Sol: $S_8 + 12OH^- \longrightarrow cS^{2-} + dS_2O_3^{2-} + H_2O$

53. At 1990 K and 1 atm pressure there are equal number of Cl_2 molecules and Cl atoms in the reaction mixture. The value of K_p for the reaction $Cl_{2(g)} \rightleftharpoons 2Cl_{(g)}$ under the above conditions is $x \times 10^{-1}$. The value of x is _____ (Rounded off to the nearest integer).

Key: 5

Sol: $Cl_2 \rightleftharpoons 2Cl$

Moles at equilibrium x x

P.P. $\frac{1}{2}$ $\frac{1}{2}$

$$K_p = \frac{P_{Cl}^2}{P_{Cl}}$$

$$= \frac{\left(\frac{1}{2}\right)^2}{\frac{1}{2}} = \frac{1}{2} = 0.5$$

$$= 5 \times 10^{-1} = 5$$

54. The stepwise formation of $\left[Cu\left(NH_3\right)_4\right]^{2+}$ is given below

$$Cu^{2+} + NH_3 \xrightleftharpoons{K_1} \left[Cu\left(NH_3\right)\right]^{2+}$$

$$\left[Cu\left(NH_3\right)\right]^{2+} + NH_3 \xrightleftharpoons{K_2} \left[Cu\left(NH_3\right)_2\right]^{2+}$$

$$\left[Cu\left(NH_3\right)_2\right]^{2+} + NH_3 \xrightleftharpoons{K_3} \left[Cu\left(NH_3\right)_3\right]^{2+}$$

$$\left[Cu\left(NH_3\right)_2\right]^{2+} + NH_3 \xrightleftharpoons{K_1} \left[Cu\left(NH_3\right)_4\right]^{2+}$$

The value of stability constants K_1, K_2, K_3 and K_4 are 10^4, 1.58×10^3, 5×10^2 and 10^2 respectively. The overall equilibrium constants for dissociation of $\left[Cu\left(NH_3\right)_4\right]^{2+}$ is $x \times 10^{-12}$. The value of x is ________. (Rounded off to the nearest integer).

Key: 1

Sol: $\left[Cu\left(NH_3\right)_4\right]^{2+} \rightleftharpoons Cu^{2+} + 4NH_3$

$$K = \frac{1}{K_1K_2K_3K_4} = \frac{1}{10^4 \times 1.58 \times 10^3 \times 5 \times 10^2 \times 10^2}$$

$$= 1.26 \times 10^{-12} = 1.26$$

55. A proton and a Li^{3+} molecule are accelerated by the same potential. If λ_{Li} and λ_p denote the de Broglie wavelength of Li^{3+} and proton respectively, then the value of $\frac{\lambda_{Li}}{\lambda_p}$ is $x \times 10^{-1}$. The value of x is ________. (Rounded off to the nearest integer).

[Mass of Li^{3+} = 8.3 mass of proton]

Key: 2

Sol: $\lambda_{DB} \alpha \frac{1}{\sqrt{m.K.E}}$

$$\frac{\lambda_{Li^{3+}}}{\lambda_p} = \sqrt{\frac{m_p \times e_p V}{8.33 m_p \times 3 e_p V}}$$

$$\sqrt{\frac{1}{25}} = \frac{1}{5} = 0.2 = 2 \times 10^{-1}$$

56. Number of amphoteric compounds among the following is ____

i) BeO ii) BaO iii) $Be(OH)_2$ iv) $Sr(OH)_2$

Key: 2

Sol: $Be(OH)_2$, BeO

57. For the reaction $A_{(g)} \rightarrow B_{(g)}$ the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of $\Delta_r G$ for the reaction at 300 K and 1 atm in $J\,mol^{-1}$ is – xR, where x is ________. (Rounded off to the nearest integer).

$\left(R = 8.31 J\,mol^{-1} K^{-1} \text{and} \ in 10 = 23\right)$

Key: 1380

Sol: $\Delta_r G^{o} = -RT \ln K_{eq}$

$= -R \times 300 \times 2 \times 2.3$

$= -1380R$

58. When 9.45 g of $ClCH_2COOH$ is added to 500 mL of water, its freezing point drops by 0.5°C. The dissociation constant of $ClCH_2COOH$ is $x \times 10^{-3}$. The value of x is ________. (Rounded off to the nearest integer).

$\left[K_f\left(H_2O\right) = 1.86 K\,kg\,mol^{-1}\right]$

Key: 7.5

Sol: $\Delta T_f = i \times k_f \times m$

$$0.5 = (1+\alpha) \times (1.86) \times \frac{9.45 \times 1000}{94.5 \times 500}$$

$$0.5 = (1+\alpha) \times 1.86 \times \frac{1}{5}$$

$$(1+\alpha) = \frac{0.5}{1.86} \alpha = 0.344$$

$$= 344 \times 10^{-3}$$

59. Gaseous cycobutene isomerizes to butadiene in a first order process which has a 'k' value of $3.3 \times 10^{-4} s^{-1}$ at 153°C. The time in minutes it takes for the isomerization to proceed 40% to completion at this temperature is ________. (Rounded off to the nearest integer).

Key: 26

Sol: $t = \frac{2.303}{K} \log \frac{100}{100-x}$

$$= \frac{2.303}{3.3 \times 10^{-4}} \log \frac{100}{100-40}$$

$$= \frac{2.303}{3.3 \times 10^{-4}} \times 0.22$$

$$= 1535.3 \sec$$

$$= \frac{1535.3}{60} = 25.58 = 26 \min$$

60. The coordination number of an atom in a body-centered cubic structure is ____. [Assume that the lattice is made up of atoms].

Key: 8

Sol: Theory

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.
Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases. 1.

61. The population P = P(t) at time 't' of a certain spectes follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If $P(0) - 850$, then the time at which population becomes zero is:

1) $\frac{1}{2}\log_e 18$ **2)** $\log_e 9$ **3)** $\log_e 18$ **4)** $2\log_e 18$

Key: 4

Sol: $\frac{dP(t)}{dt} = \frac{P(t) - 900}{2}$

$\int_0^t \frac{dP(t)}{P(t) - 900} = \int_0^t \frac{dt}{2}$

$\left\{\ln\left|P(t) - 900\right|\right\}_0^t = \left\{\frac{t}{2}\right\}_0^t$

$\ln\left|P(t) - 900\right| - \ln\left|P(0) - 900\right| = \frac{t}{2}$

$\ln\left|P(t) - 900\right| - \ln 50 = \frac{t}{2}$

$\text{Let at } t = t_1, P(t) = 0 \text{hence}$

$\ln\left|P(t) - 900\right| - \ln 50 = \frac{t_1}{2}$

$t_1 = 2\ln 18$

62. Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as

$g(r) = \frac{x - \frac{1}{2}}{x - 1}$

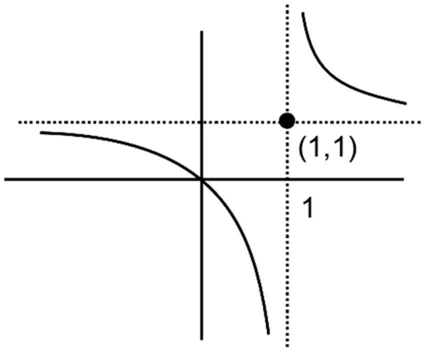
Then the composition function f (g(x)) is :

1) Both one-one and onto **2)** Neither one-one nor onto

3) One-one but not onto **4)** Onto but not one-one

Key: 3

Sol:



f(g(x)) = 2g(x) – 1

$$= 2\frac{\left(x-\frac{1}{2}\right)}{x-1} = \frac{x}{x-1}$$

$$f\left(g\left(x\right)\right) = 1 + \frac{x}{x-1}$$

one-one, into

Can challenge as co domain of fog(x) is NOT given

63. If the tangent to the curve $y = x^3$ at the point $P\left(t, t^3\right)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

1) 0 **2)** $2t^3$ **3)** $-2t^3$ **4)** $-t^3$

Key: 3

Sol: Equation of tangent at $P\left(t, t^3\right)$

$\left(y - t^3\right) = 3t^2\left(x - t\right)$_____$\left(1\right)$

now solve the above equation with

$y = x^3$ ______$\left(2\right)$

By (**1)** & (**2)**

$x^3 - t^3 = 3t^2\left(x - t\right)$

$x^2 + xt + t^2 = 3t^2$

$x^2 + xt - 2t^2 = 0$

(x – t) (x + 2t) = 0

$\Rightarrow x = -2t \Rightarrow Q\left(-2t, -8t^3\right)$

Ordinate of required point = $\dfrac{2t+\left(-8t^3\right)}{3}=-2t^3$

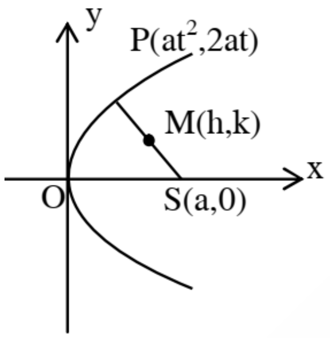
64. The locus of the mid-point of the line segment joining the focus of the parabola $y^2=4ax$ to a moving point of the parabola, is another parabola whose directrix is:

1) $x = a$ **2)** $x=-\dfrac{a}{2}$ **3)** $x = 0$ **4)** $x=\dfrac{a}{2}$

Key: 3

Sol: $h=\dfrac{at^2+a}{2}, k=\dfrac{2at+0}{2}$

$\Rightarrow t^2=\dfrac{2h-a}{a} \,\&\, t=\dfrac{k}{a}$



$\Rightarrow \dfrac{k^2}{a^2}=\dfrac{2h-a}{a}$

$\Rightarrow$ Locus of (h, k) is $y^2=a\left(2x-a\right)$

$\Rightarrow y^2=2a\left(x-\dfrac{a}{2}\right)$

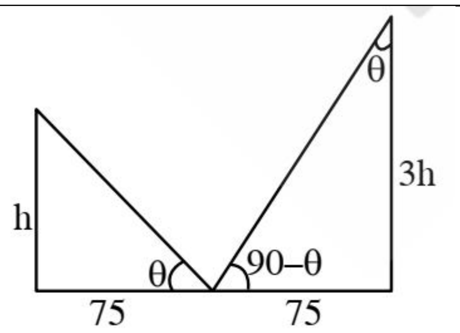
Its directrix is $x-\dfrac{a}{2}=-\dfrac{a}{2}\Rightarrow x=0$

65. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is :

1) $20\sqrt{3}$ **2)** 25 **3)** 30 **4)** $25\sqrt{3}$

Key: 4

Sol:



$$\tan\theta = \frac{h}{75} = \frac{75}{3h}$$

$$\Rightarrow h^2 = \frac{(75)^2}{3}$$

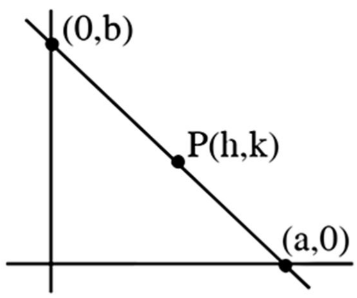
$$h = 25\sqrt{3}m$$

66. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, **1)**, (2, **2)** and (4, **4)** respectively. Then which of these stones is/are on the path of the man?

1) B only **2)** A only **3)** C only **4)** All the three

Key: 1

Sol:



$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{h}{a} + \frac{k}{b} = 1 \_\_\_\_\_ (i)$$

$$\frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{1}{4}$$

$$\therefore \frac{1}{a} + \frac{1}{b} = \frac{1}{2} \_\_\_\_\_ (ii)$$

$\therefore$ Line passes through fixed point (2, 2)

(from (1) and (2))

67. The function $f(x)=\frac{4x^3-3x^2}{6}-2\sin x+(2x-1)\cos x$:

1) Increases in $\left[\frac{1}{2},\infty\right)$ **2)** Increases in $\left(-\infty,\frac{1}{2}\right]$

3) Decreases in $\left[\frac{1}{2},\infty\right)$ **4)** Decreases in $\left(-\infty,\frac{1}{2}\right]$

Key: 1

Sol: $f'(x)=(2x-1)(x-\sin x)$

$\Rightarrow f'(x)\geq 0 \text{in} x\in\left[\frac{1}{2},\infty\right)$

$\text{and} f'(x)\leq 0 \text{in } x\in\left(\infty,\frac{1}{2}\right]$

68. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed is:

1) 1625 **2)** 560 **3)** 575 **4)** 1050

Key: 1

Sol: (2I , 4F) + (3I , 6F) + (4I , 8F)

$={}^6C_2{}^8C_4+{}^6C_3{}^8C_6+{}^6C_4{}^8C_8$

= 15 × 70 + 20 × 28 + 15 × 1

= 1050 + 560 + 15 = 1625

69. The system of linear equations

$3x-2y-kz=10$

$2x-4y-2z=6$

$x+2y-z=5m$

is inconsistent if :

1) $k\neq 3, m\in R$ **2)** $k=3, m\neq\frac{4}{5}$ **3)** $k\neq 3, m\neq\frac{4}{5}$ **4)** $k=3, m=\frac{4}{5}$

Key: 2

Sol: $\Delta = \begin{vmatrix} 3 & 2 & -k \\ 1 & -2 & 3 \\ 1 & 2 & -3 \end{vmatrix} = 0 \Rightarrow k = 3$

$\Delta_x = \begin{vmatrix} 10 & 2 & -3 \\ 3 & -2 & 3 \\ 5m & 2 & -3 \end{vmatrix} = 0$

$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 1 & 3 & 3 \\ 1 & 5m & -3 \end{vmatrix} = 6(7-10m)$

$\Delta_z = \begin{vmatrix} 3 & 2 & 10 \\ 1 & -2 & 3 \\ 1 & 2 & 5m \end{vmatrix} = 4(7-10m)$

Hence, $k = 3$ and $m \neq \frac{7}{10}$

70. If $e^{\left(\cos^2 x+\cos^4 x+\cos^6 x+.....\infty\right)\log_{e^2}}$ satisfies the equation $t^2 - 9t + 8 = 0,$ then the value of $\frac{2\sin x}{\sin x + \sqrt{3}\cos x}\left(0 < x < \frac{\pi}{2}\right)$ is:

1) $\frac{1}{2}$ **2)** $\frac{3}{2}$ **3)** $\sqrt{3}$ **4)** $2\sqrt{3}$

Key: 1

Sol: $e^{\left(\cos^2\theta+\cos^4\theta+.....\infty\right)\ln 2} = 2^{\cos^2\theta+\cos^4\theta+....\infty}$

$= 2^{\cot^2\theta}$

$t^2 - 9t + 8 = 0 \Rightarrow t = 1,8$

$\Rightarrow 2^{\cot^2\theta} = 1,8 \Rightarrow \cot^2\theta = 0,3$

$0 < \theta < \frac{\pi}{2} \Rightarrow \cot\theta = \sqrt{3}$

$\Rightarrow \frac{2\sin\theta}{\sin\theta + \sqrt{3}\sin\theta} = \frac{2}{1+\sqrt{3}\cot\theta} = \frac{2}{4} = \frac{1}{2}$

71. Let p and q be two positive numbers such that p + q = 2 and $p^4 + q^4 = 272.$ Then p and q are roots of the equation:

1) $x^2 - 2x + 8 = 0$ **2)** $x^2 - 2x + 16 = 0$ **3)** $x^2 - 2x + 2 = 0$ **4)** $x^2 - 2x + 136 = 0$

Key:2

Sol: $\left(p^2+q^2\right)^2-2p^2q^2=272$

$$\left(\left(p^2+q^2\right)^2-2pq\right)^2-2p^2q^2=272$$

$$16-16pq+2p^2q^2=272$$

$$\left(pq\right)^2-8pq-128=0$$

$$pq=\frac{8\pm 24}{2}=16,-8$$

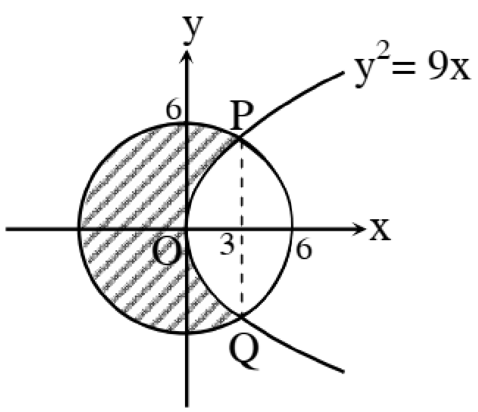
$$pq=16$$

72. The area (in sq. units) of the part of the circle $x^2+y^2=36,$ which is outside the parabola $y^2=9x,$ is:

1) $24\pi+3\sqrt{3}$ **2)** $12\pi+3\sqrt{3}$ **3)** $12\pi-3\sqrt{3}$ **4)** $24\pi-3\sqrt{3}$

Key: 4

Sol: The curves intersect at points $\left(3,\pm 3\sqrt{3}\right)$



Required area

$$=\pi r^2-2\left[\int_0^3\sqrt{9x}\,dx+\int_3^6\sqrt{36-x^2}\,dx\right]$$

$$=36\pi-12\sqrt{3}-2\left|\frac{x}{2}\sqrt{36-x^2}+18\sin^{-1}\left(\frac{x}{6}\right)\right|_3^6$$

$$=36p-12\sqrt{3}-2\left(9\pi-\left(\frac{9\sqrt{3}}{2}+3\pi\right)\right)=24\pi-3\sqrt{3}$$

73. If $f : R \to R$ is a function defined by $f(x) = [x-1]\cos\left(\frac{2x-1}{2}\right)\pi,$, where [.] denotes the greatest integer function, then f is :

1) Discontinuous at all integral values of *x* except at *x* = 1

2) Continuous only at *x* = 1

3) Continuous for every real *x*

4) Discontinuous only at *x* = 1

Key: 3

Sol: Doubtful points are $x = n, n \in I$

$$\text{L.H.L} = \lim_{x \to n^-}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi = (n-2)\cos\left(\frac{2x-1}{2}\right)\pi = 0$$

$$\text{R.H.L} = \lim_{x \to n^-}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi = (n-1)\cos\left(\frac{2x-1}{2}\right)\pi = 0$$

f(n) = 0

Hence continuous

74. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z+5}{2}$ and the plane x + y + z – 17 is:

1) $2\sqrt{19}$ **2)** $\sqrt{38}$ **3)** 38 **4)** $19\sqrt{2}$

Key: 2

Sol: $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2} = \lambda$

$\Rightarrow x = \lambda + 3, y = 2\lambda + 4, z = 2\lambda + 5$

Which lies on given plane hence

$\Rightarrow \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$\Rightarrow \lambda = \frac{5}{5} = 1$

Hence, point of intersection is Q (4, 6, 7)

$\therefore$ Required distance = PQ

$= \sqrt{9 + 25 + 4} = \sqrt{38}$

75. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is:

1) $\frac{1}{32}$ **2)** $\frac{3}{16}$ **3)** $\frac{1}{2}$ **4)** $\frac{5}{16}$

Key: 3

Sol: P(odd no. twice) = P(even no. thrice)

$$\Rightarrow {}^nC_2\left(\frac{1}{2}\right)^n = {}^nC_3\left(\frac{1}{2}\right)^n \Rightarrow n = 5$$

success is getting an odd number then P(odd successes) = P(**1)** + P(**3)** + P(5)

$$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$$

$$= \frac{16}{2^5} = \frac{1}{2}$$

76. The statement among the following that is a tautology is:

1) $\left[A \wedge (A \rightarrow B)\right] \rightarrow B$ **2)** $A \wedge (A \vee B)$

3) $A \vee (A \wedge B)$ **4)** $B \rightarrow \left[A \wedge (A \rightarrow B)\right]$

Key: 1

Sol: $A \wedge (\sim A \vee B) \rightarrow B$

$$= \left[(A \wedge \sim A) \vee (A \wedge B)\right] \rightarrow B$$

$$= (A \wedge B) \rightarrow B$$

$$= \sim A \vee \sim B \vee B$$

77. The equation of the plane passing through the point (1, 2, **- 3)** and perpendicular to the planes 3x + y – 2z = 5 and 2x – 5y – z = 7, is :

1) 6x – 5y + 2z + 10 = 0 **2)** 3x – 10y - 2z + 11 = 0

3) 11x + y + 17z + 38 = 0 **4)** 6x – 5y - 2z - 2 = 0

Key: 3

Sol: Normal vector of required plane is $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$

$\therefore$ +11 (x – **1)** + (y – **2)** + 17 (z + **3)** = 0

11x + y + 17z + 38 = 0

78. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a \sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to :

1) (1, - 3) **2)** (- 1, 3) **3)** (3, 1) **4)** (1, 3)

Key:4

Sol: Put $\sin\theta + \cos\theta = t \Rightarrow 1 + \sin 2\theta = t^2$

$\Rightarrow (\cos\theta - \sin\theta) d\theta = dt$

$\therefore \; I = \int \frac{dt}{\sqrt{8 - (t^2 - 1)}} = \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{=1}\left(\frac{t}{3}\right) + C = \sin^{-1}\left(\frac{\sin\theta + \cos\theta}{3}\right) + C$

$\Rightarrow \; a = 1 \;\&\; b = 3$

79. $\lim_{x \to 0} \frac{\int_0^{x^2} \left(\sin\sqrt{t}\right) dt}{x^3}$ is equal to :

1) $\frac{3}{2}$ **2)** 0 **3)** $\frac{1}{15}$ **4)** $\frac{2}{3}$

Key: 4

Sol: $\lim_{x \to 0} \frac{\int_0^{x^2} \sin\sqrt{t}\, dt}{x^3} = \lim_{x \to 0} \frac{(\sin|x| 2x)}{3x^2} = \lim_{x \to 0} \left(\frac{\sin x}{x}\right) \times \frac{2}{3} = \frac{2}{3}$

80. The value of

$-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ... - 15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + ... + {}^{14}C_{11}$ is

1) $2^{13} - 14$ **2)** 2^{14} **3)** $2^{13} - 13$ **4)** $2^{16} - 1$

Key: 1

Sol: $S_1 = -{}^{15}C_1 + 2.{}^{15}C_2 - ...... - 15.{}^{15}C_{15} \qquad = \sum_{r=1}^{15} (-1)^r .r.{}^{15}C_r = \sum_{r=1}^{15} (-1)^r .r.{}^{14}C_{r-1}$

$= 15 - \left({}^{14}C_0 + {}^{14}C_1 - .... - {}^{14}C_{14}\right) = 15(0) = 0$

$S_2 = {}^{14}C_1 + {}^{14}C_3 + ... + {}^{14}C_{11}$

$= \left({}^{14}C_1 + {}^{14}C_3 + ... + {}^{14}C_{11} + {}^{14}C_{13}\right) - {}^{14}C_{13} = 2^{13} - 14 \qquad S_1 + S_2 = 2^{13} - 14$

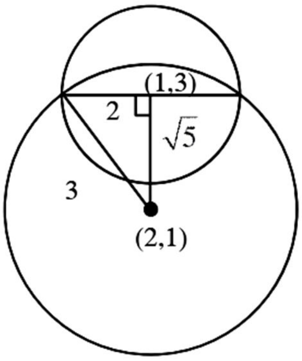
(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.

Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

81. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose centre is at (2, 1), then is radius is _________

Key: 3



Sol:

distance between (1, 3) and (2, 1) is $\sqrt{5}$

$\therefore \left(\sqrt{5}\right)^2 + (2)^2 = r^2$

$\Rightarrow r = 3$

82. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1-\sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is _______.

Key: 9

Sol: Let $f(x) = \frac{4}{\sin x} + \frac{1}{1-\sin x}$

$y = \frac{4-3\sin x}{\sin x(1-\sin x)}$

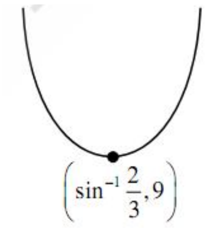
Let sin x = t when $t \in (0,1)$

$y = \frac{4-3t}{t-t^2}$

$\frac{dy}{dt} = \frac{-3\left(t-t^2\right)-(1-2t)(4-3t)}{\left(t-t^2\right)^2} = 0$

$\Rightarrow 3t^2 - 3t - \left(4 - 11t + 6t^2\right) = 0$

$\Rightarrow 3t^2 - 8t + 4 = 0$



$\Rightarrow t=\frac{2}{3}$

$\Rightarrow \alpha \geq 9$

Least α is equal to 9

83. Let three vectors $\vec{a},\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}, \vec{a}.\vec{c}=7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a}=-\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}-2\hat{i}+\hat{k}$, then the value of $2\left|\vec{a}+\vec{b}+\vec{c}\right|^2$ is _______

Key: 75

Sol: $\vec{c}=\lambda\left(\vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)$

$=\lambda\left(\left(\vec{b}.\vec{b}\right)\vec{b}-\left(\vec{b}.\vec{a}\right)\vec{b}\right)$

$=\lambda\left(5\left(-\hat{i}+\hat{j}+\hat{k}\right)+2\hat{i}+\hat{k}\right)$

$=\lambda\left(-3\hat{i}+5\hat{j}+6\hat{k}\right)$

$\vec{c}.\vec{a}=7 \Rightarrow 3\lambda+5\lambda+6\lambda=7$

$\lambda=\frac{1}{2}$

$\therefore 2\left|\left(\frac{-3}{2}-1+2\right)\hat{i}+\left(\frac{5}{2}+1\right)\hat{j}+(3+1+1)\hat{k}\right|^2$

$=2\left(\frac{1}{4}+\frac{49}{4}+25\right)=25+50=75$

84. If $\int_{-a}^{a}\left(|x|+|x-2|\right)dx-22,\left(a>2\right)$ and [x]denotes the greatest integer $\leq x$, , then $\int_{a}^{-a}\left(x+\left[x\right]\right)dx$ is equal to ______.

Key: 3

Sol: $\int_{-a}^{0}(-2x+2)dx+\int_{0}^{2}(x+2+2)dx+\int_{2}^{a}(2x-2)dx=22$

$x^2-2x\big|_0^{-a}+2x\big|_0^{2}+x^2-2x\big|_2^{a}=22$

$a^2+2a+4+a^2-2a-(4-4)=22$

$2a^2=18\Rightarrow a=3$

$\int_{-3}^{3}(x+[x])dx=-3-2-1+1+2=-3$

85. Let $P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$, where $\alpha\in R$. Suppose $Q=[q_{if}]$ is a matrix satisfying $PQ=kl_3$ for some non-zero $k\in R$. If $q_{23}=-\frac{k}{8}$ and $|Q|=\frac{k^2}{2}$, then α^2+k^2 is equal to ________.

Key: 17

Sol: As PQ = kI $\Rightarrow Q=kP^{-1}I$

now $Q=\frac{k}{|P|}(\text{adj}P)I\Rightarrow Q=\frac{k}{(20+12\alpha)}\begin{bmatrix}- & - & -\\ - & - & (-3\alpha-4)\\ - & - & -\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$\because q_{23}=-\frac{k}{8}\Rightarrow\frac{k}{(20+12\alpha)}(-3\alpha-4)=-\frac{k}{8}\Rightarrow 2(3\alpha+4)=5+3\alpha$

$3\alpha=-3\Rightarrow\alpha=-1$

also $|Q|=\frac{k^3|I|}{|P|}\Rightarrow\frac{k^2}{2}=\frac{k^3}{(20+12\alpha)}$

$(20+12\alpha)=2k\Rightarrow 8=2k\Rightarrow k=4$

86. Let $B_f\,(i=1,2,3)$ be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_f occurs and these 4 probabilities satisfy the equations $(\alpha-2\beta)p=\alpha\beta$ and $(\beta-3\gamma)p=-2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1). Then $\frac{P(B_1)}{P(B_3)}$ is equal to _________.

Key: 6

Sol: Let x, y, z be probability of B_1, B_2, B_3 respectively

$\Rightarrow x(1-y)(1-z)=\alpha \Rightarrow y(1-x)(1-z)=\beta$

$\Rightarrow z(1-x)(1-y)=\gamma \Rightarrow (1-x)(1-z)=P$

Putting in the given relation we get x = 2y and y = 3z $\Rightarrow x=6z \Rightarrow \frac{x}{z}=6$

87. Let $A=\{n \in N: n \text{ is a } 3-\text{digit number}\}$ $B-\{9k+2; k \in N\}$ and $C-\{9k+l; k \in N\}$ for some l ($0 < l < 9$)

If the sum of all elements of the set $A \cap (B \cap C)$ is 274 × 400, then l is equal to ____

Key: 5

Sol: 3 digit number of the form 9K + 2 are {101,109,...............,992}

$\Rightarrow$ Sum equal to $\frac{100}{2}(1093)$

Similarly sum of 3 digit number of the form 9K + 5 is $\frac{100}{2}(1099)$

$\frac{100}{2}(1093)+\frac{100}{2}(1099)=100\times(1096)$

$=400\times 274 \Rightarrow l=5$

88. If the least and the largest real values of α, for which the equation

$z+\alpha|z-1|+2i=0\left(z \in C \text{ and } i=\sqrt{-1}\right)$ has a solution, are p and q respectively, then

$4\left(p^2+q^2\right)$ is equal to_________.

Key: 10

Sol: $x+iy+\alpha\sqrt{(x-1)^2+y^2}+2i=0$

$\therefore\ y+2=0 \,\&\, x+\alpha\sqrt{(x-1)^2+y^2}=0$

$y=-2 \,\&\, x^2=\alpha^2(x-2x+1+4)$

$\alpha^2=\frac{x^2}{x^2-2x+5}$

$\therefore\ \alpha^2 \in \left[0,\frac{5}{4}\right]$

$\therefore\ \alpha^2 \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$

Then $4\left[\left(\alpha_{max}\right)^2 + \left(\alpha_{min}\right)^2\right] = 4\left[\frac{5}{4} + \frac{5}{4}\right] = 10$

89. Let M be any 3 × 3 matrix with entries from the set {0, 1, 2}. Then maximum number of such matrices, for which the sum of diagonal elements of $M^T M$ is seven, is ______

Key: 540

Sol: $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case I : Seven (1's) and two (0's)

$^9C_2 = 36$

Case- II : One (2) and three (1’s) and five (0’s

$\frac{9!}{5!3!} = 504$

$\therefore\ Total = 540$

90. $\lim_{n\to\infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to _____.

Key: 1

Sol: $\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\left[\tan^{-1}(r+1) - \tan^{-1}(r)\right]\right)$

$= \tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1) - \frac{\pi}{4}\right)\right)$

$= \tan\left(\frac{\pi}{4}\right) = 1$