SOLVED PAPERS 2021

JEE MAIN SESSION-4 ADVANCED

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Class XI-XII **CBSE warm up! TERM-I**

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BRUSH UP for NEET/JEE
Class XI-XII

Thinking of taking a break for JEE and NEET?

Before you decide on taking a year off to study exclusively for JEE or NEET, you need to weigh the pros and cons carefully.

To drop a year or not to, can ring in a Shakespearean dilemma for you. Taking a year's break to prepare for competitive exams such as the Joint Entrance Exam (JEE) and National Eligibility cum Entrance Test (UG) (NEET) can be a monumental decision. One needs to evaluate the pros and cons before taking a call on this. Let us help you.

Let's look at the benefits that dropping a year after your Class 12 board exams may bring.

The Pros

Shot at a better college : This may be the sole reason for taking a break for preparation, so a break can provide you the time to work hard and aim for your dream college in the next attempt.

Complete focus : This break also means you have no school or practicals to worry about. With boards out of the way, you can dedicate yourself fully to JEE or NEET preparations.

Condensed syllabus : You do not have to divert your attention to topics that may not be important for JEE or NEET. You can concentrate exclusively on the exam syllabus.

Clearing Concepts : Being clear about concepts is an important factor to do well in exams such as JEE and NEET. If you need to clear your concepts after your Class 11 and 12, dropping a year might help you.

More practice : You can devote more time to practice and go through all previous years' question papers, mock tests and chapter-wise tests that you missed before.

Working on areas to improve and time management: Taking the JEE or NEET once would have given you a good understanding about what are your strong subjects and topics. It would have also highlighted the areas you would want to improve. You can take the year to address these. You can also further improve on your time management and plan your study schedule thoroughly.

The Cons

With the extra time comes extra stress. It's possibly the first big decision you make as a young adult. Hence, you must understand what it really means and what you can expect in the coming year.

Absence of a routine : While the absence of regular school gives you more time to prepare, it also means a deviation from a daily routine. It's easy to become unproductive without supervision. If you are dropping a year, make sure to carve your own schedule and adhere to it religiously. It becomes easier if you join a coaching program and have scheduled classes. If you are struggling to follow the routine you have created, enlist parents or friends to help you stick to your schedule.

Repetitive syllabus : Picking up the same books and studying the topics you have been studying in the previous years may become tiresome. Don't forget that understanding and grasping the topics well are the reasons you are taking this break. Stay focused. Don't let lethargy set in.

Bearing boredom : With no external routines such as school or extracurriculars to follow, you may also be prone to boredom. Online games or social media can be addictive and may end up being a distraction. Cultivate healthy outlets such as outdoor sports, reading or music.

Performance pressure : Don't get bogged down by curious relatives, pressing parents or peers. Take it in your stride. Don't worry about things you cannot control. Remember: If the end result is not success, then at least you will have the satisfaction of knowing it wasn't because you didn't try.

The burden of peers : Several of your friends and schoolmates may be joining colleges and moving on to other things. Learning about their new colleges and exciting days may waver your determination and induce fear of missing out. This may cause unnecessary stress. Keep your focus on yourself. Don't lose sight of the goal.

Mental health : All the above factors may contribute to your mental stress. Falling a year behind your peers, missing out on life in college with your peers, the vast syllabus, incessant studying and the endless questions may lead you to doubt your own decision. It's natural. But you need to be strong mentally.

Know this : The long year ahead will throw up more challenges but you must tower over self-doubts to conquer what you set out to do. We hope these pros and cons help you in making a decision that's best for you. Keep your eye on the goal, everything else is a momentary distraction.

Disclaimer : The information provided in this article is of a general nature and should not be considered a substitute for professional advice.

PHYSICS for you

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Editor : Anil Ahlawat

JEE ADVANCED 2021 SOLVED PAPER

Held on 3rd October

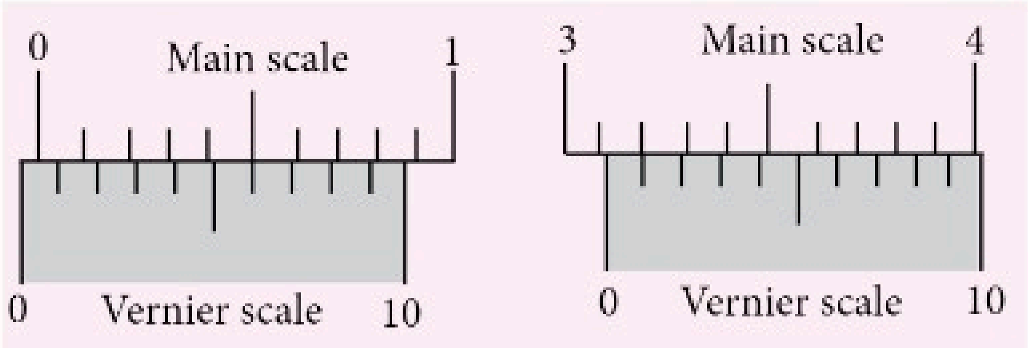
PAPER - I

SECTION 1

- *This section contains FOUR (04) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.*
- *For each question, choose the correct option corresponding to the correct answer.*
- *Answer to each question will be evaluated according to the following marking scheme:*

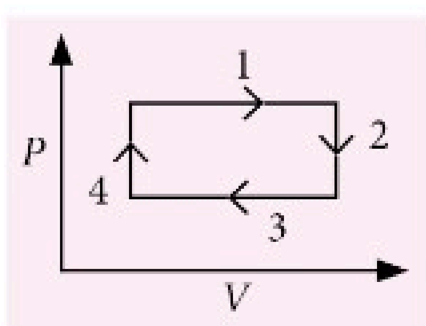
Full Marks : +3 *If ONLY the correct option is chosen;*
Zero Marks : 0 *If none of the options is chosen (i.e. the question is unanswered);*
Negative Marks : −1 *In all other cases.*

1. The smallest division on the main scale of a Vernier calipers is 0.1 cm. Ten divisions of the Vernier scale correspond to nine divisions of the main scale. The figure below on the left shows the reading of this calipers with no gap between its two jaws. The figure on the right shows the reading with a solid sphere held between the jaws. The correct diameter of the sphere is



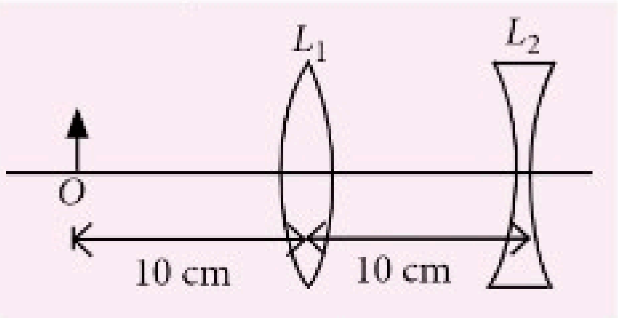
(a) 3.07 cm (b) 3.11 cm
(c) 3.15 cm (d) 3.17 cm

2. An ideal gas undergoes a four step cycle as shown in the P-V diagram. During this cycle, heat is absorbed by the gas in



(a) steps 1 and 2 (b) steps 1 and 3
(c) steps 1 and 4 (d) steps 2 and 4

3. An extended object is placed at point O, 10 cm in front of a convex lens L_1 and a concave lens L_2 is placed 10 cm behind it, as shown in the figure. The radii of curvature of all the curved surfaces in both the lenses are 20 cm. The refractive index of both the lenses is 1.5. The total magnification of this lens system is



(a) 0.4
(b) 0.8
(c) 1.3
(d) 1.6

4. A heavy nucleus Q of half-life 20 minutes undergoes alpha-decay with probability of 60% and beta-decay with probability of 40%. Initially, the number of Q nuclei is 1000. The number of alpha-decays of Q in the first one hour is

(a) 50 (b) 75 (c) 350 (d) 525

SECTION 2

- *This section contains THREE (03) question stems.*
- *There are TWO (02) questions corresponding to each question stem.*
- *The answer to each question is a NUMERICAL VALUE.*
- *For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.*
- *If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +2 *If ONLY the correct numerical value is entered at the designated place*
Zero Marks : 0 *In all other cases.*

Question Stem for Question Nos. 5 and 6

Question Stem

A projectile is thrown from a point O on the ground at an angle 45° from the vertical and with a speed $5\sqrt{2}$ m s^{-1}. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down to the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x meters from the point O. The acceleration due to gravity $g = 10$ m s^{-2}.

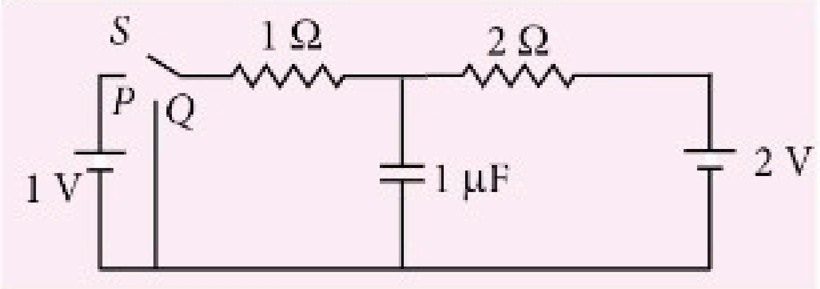
5. The value of t is _____.

6. The value of x is ______.

Question Stem for Question Nos. 7 and 8

Question Stem

In the circuit shown below, the switch S is connected to position P for a long time so that the charge on the capacitor becomes q_1 μC. Then S is switched to position Q. After a long time, the charge on the capacitor is q_2 μC.



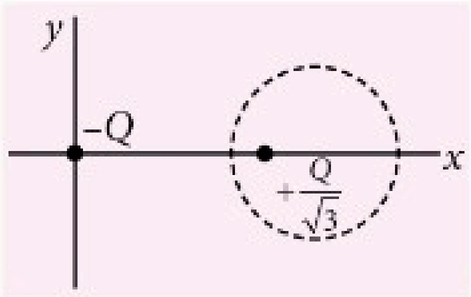
7. The magnitude of q_1 is ______.

8. The magnitude of q_2 is ______.

Question Stem for Question Nos. 9 and 10

Question Stem

Two point charges $-Q$ and $+\frac{Q}{\sqrt{3}}$ are placed in the xy-plane at the origin (0, 0) and a point (2, 0), respectively, as shown in the figure. This results in an equipotential circle of radius R and potential $V = 0$ in the xy-plane with its center at $(b, 0)$. All lengths are measured in meters.



9. The value of R is ______ meter.

10. The value of b is ______ meter.

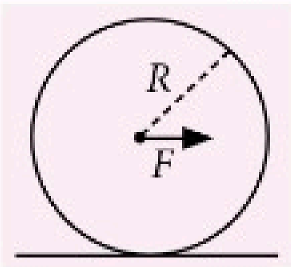
SECTION 3

- *This section contains SIX (06) questions.*
- *Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).*
- *For each question, choose the option(s) corresponding to (all) the correct answer(s).*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks :	*+4*	*If only (all) the correct option(s) is(are) chosen;*
Partial Marks :	*+3*	*If all the four options are correct but ONLY three options are chosen;*
Partial Marks :	*+2*	*If three or more options are correct but ONLY two options are chosen, both of which are correct;*
Partial Marks :	*+1*	*If two or more options are correct but ONLY one option is chosen and it is a correct option;*
Zero Marks :	*0*	*If unanswered;*
Negative Marks :	*−2*	*In all other cases.*

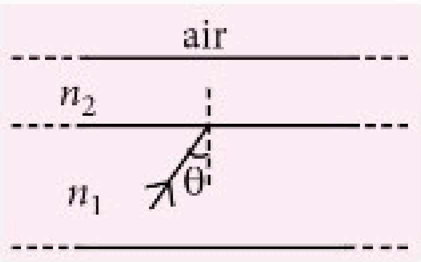
For example, in a question, if (a), (b) and (d) are the ONLY three options corresponding to correct answers, then choosing ONLY (a), (b) and (d) will get +4 marks; choosing ONLY (a) and (b) will get +2 marks; choosing ONLY (a) and (d) will get +2 marks; choosing ONLY (b) and (d) will get +2 marks; choosing ONLY (a) will get +1 mark; choosing ONLY (b) will get +1 mark; choosing ONLY (d) will get +1 mark; choosing no option(s) (i.e. the question is unanswered) will get 0 marks and choosing any other option(s) will get −2 marks.

11. A horizontal force F is applied at the center of mass of a cylindrical object of mass m and radius R, perpendicular to its axis as shown in the figure. The coefficient of friction between the object and the ground is μ. The center of mass of the object has an acceleration a. The acceleration due to gravity is g. Given that the object rolls without slipping, which of the following statement(s) is(are) correct?



(a) For the same F, the value of a does not depend on whether the cylinder is solid or hollow.
(b) For a solid cylinder, the maximum possible value of a is $2\mu g$.
(c) The magnitude of the frictional force on the object due to the ground is always μmg.
(d) For a thin-walled hollow cylinder, $a = F/2m$.

12. A wide slab consisting of two media of refractive indices n_1 and n_2 is placed in air as shown in the figure. A ray of light is incident from medium n_1 to n_2 at an angle θ, where sinθ is slightly larger than $1/n_1$. Take refractive index of air as 1. Which of the following statement(s) is(are) correct?



(a) The light ray enters air if $n_2 = n_1$.
(b) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 < n_1$.
(c) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 > n_1$.
(d) The light ray is reflected back into the medium of refractive index n_1 if $n_2 = 1$.

13. A particle of mass $M = 0.2$ kg is initially at rest in the xy-plane at a point ($x = -l$, $y = -h$), where $l = 10$ m and $h = 1$ m. The particle is accelerated at time $t = 0$ with a constant acceleration $a = 10$ m s^{-2} along the positive x-direction. Its angular momentum and torque with respect to the origin, in SI units, are represented by $\vec{L}$ and $\vec{\tau}$, respectively. $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along the positive x, y and z-directions, respectively. If $\hat{k} = \hat{i} \times \hat{j}$ then which of the following statement(s) is(are) correct?

(a) The particle arrives at the point ($x = l$, $y = -h$) at time $t = 2$ s.

(b) $\vec{\tau} = 2\hat{k}$ when the particle passes through the point ($x = l$, $y = -h$).

(c) $\vec{L} = 4\hat{k}$ when the particle passes through the point ($x = l$, $y = -h$).

(d) $\vec{\tau} = \hat{k}$ when the particle passes through the point ($x = 0$, $y = -h$).

14. Which of the following statement(s) is(are) correct about the spectrum of hydrogen atom?

(a) The ratio of the longest wavelength to the shortest wavelength in Balmer series is 9/5.

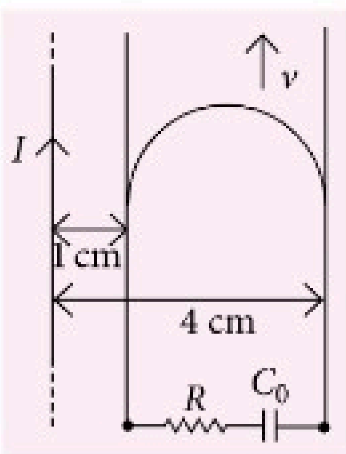
(b) There is an overlap between the wavelength ranges of Balmer and Paschen series.

(c) The wavelengths of Lyman series are given by $\left(1+\frac{1}{m^2}\right)\lambda_0$, where λ_0 is the shortest wavelength of Lyman series and m is an integer.

(d) The wavelength ranges of Lyman and Balmer series do not overlap.

15. A long straight wire carries a current, $I = 2$ ampere. A semi-circular conducting rod is placed beside it on two conducting parallel rails of negligible resistance. Both the rails are parallel to the wire. The wire, the rod and the rails lie in the same horizontal plane, as shown in the figure. Two ends of the semi-circular rod are at distances 1 cm and 4 cm from the wire. At time $t = 0$, the rod starts moving on the rails with a speed $v = 3.0$ m s^{-1} (see the figure).

A resistor $R = 1.4\ \Omega$ and a capacitor $C_0 = 5.0\ \mu$F are connected in series between the rails. At time $t = 0$, C_0 is uncharged. Which of the following statement(s) is(are) correct?



[$\mu_0 = 4\pi \times 10^{-7}$ SI units. Take ln2 = 0.7]

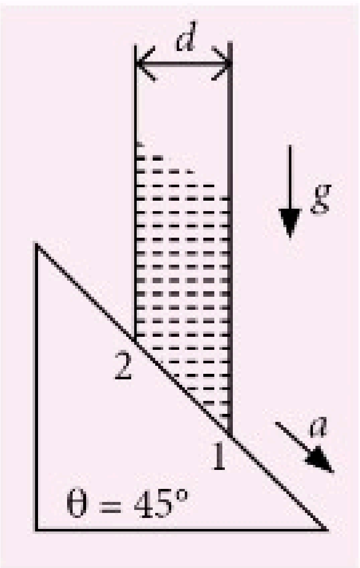
(a) Maximum current through R is 1.2×10^{-6} ampere.

(b) Maximum current through R is 3.8×10^{-6} ampere.

(c) Maximum charge on capacitor C_0 is 8.4×10^{-12} coulomb.

(d) Maximum charge on capacitor C_0 is 2.4×10^{-12} coulomb.

16. A cylindrical tube, with its base as shown in the figure, is filled with water. It is moving down with a constant acceleration a along a fixed inclined plane with angle $\theta = 45°$. P_1 and P_2 are pressures at points 1 and 2, respectively, located at the base of the tube. Let $\beta = \frac{P_1 - P_2}{\rho g d}$, where ρ is density of water, d is the inner diameter of the tube and g is the acceleration due to gravity. Which of the following statement(s) is(are) correct?



(a) $\beta = 0$ when $a = \frac{g}{\sqrt{2}}$ (b) $\beta > 0$ when $a = \frac{g}{\sqrt{2}}$

(c) $\beta = \frac{\sqrt{2}-1}{\sqrt{2}}$ when $a = \frac{g}{2}$

(d) $\beta = \frac{1}{\sqrt{2}}$ when $a = \frac{g}{2}$

SECTION 4

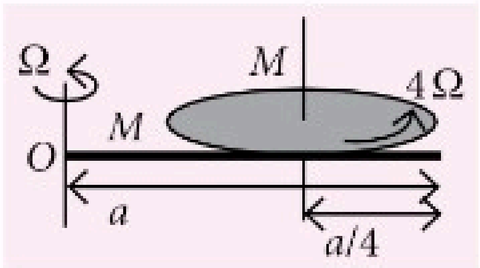
- *This section contains THREE (03) questions.*
- *The answer to each question is a NON-NEGATIVE INTEGER.*
- *For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.*
- *Answer to each question will be evaluated according to the following marking scheme:*

Full Marks : +4 *If ONLY the correct integer is entered;*

Zero Marks : 0 *In all other cases.*

17. An α-particle (mass 4 amu) and a singly charged sulfur ion (mass 32 amu) are initially at rest. They are accelerated through a potential V and then allowed to pass into a region of uniform magnetic field which is normal to the velocities of the particles. Within this region, the α-particle and the sulfur ion move in circular orbits of radii r_α and r_S, respectively. The ratio (r_s/r_α) is ____.

18. A thin rod of mass M and length a is free to rotate in horizontal plane about a fixed vertical axis passing through point O. A thin circular disc of mass M and of radius $a/4$ is pivoted on this rod with its center at a distance $a/4$ from the free end so that it can rotate freely about its vertical axis, as shown in the figure. Assume that both the rod and the disc have uniform density and they remain horizontal during the motion. An outside stationary observer finds the rod rotating with an angular velocity Ω and the disc rotating about its vertical axis with angular velocity 4Ω. The total angular momentum of the system about the point O is $\left(\frac{Ma^2\Omega}{48}\right)n$. The value of n is ______.



19. A small object is placed at the center of a large evacuated hollow spherical container. Assume that the container is maintained at 0 K. At time $t = 0$, the temperature of the object is 200 K. The temperature of the object becomes 100 K at $t = t_1$ and 50 K at $t = t_2$. Assume the object and the container to be ideal black bodies. The heat capacity of the object does not depend on temperature. The ratio $\left(\frac{t_2}{t_1}\right)$ is ____.

PAPER - II

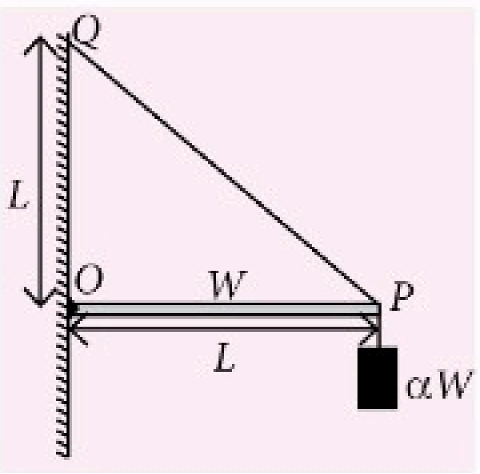
SECTION 1

- ***This section contains SIX (06) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).***
- ***For each question, choose the option(s) corresponding to (all) the correct answer(s).***
- ***Answer to each question will be evaluated according to the following marking scheme:***

Full Marks :	***+4***	***If only (all) the correct option(s) is(are) chosen;***
Partial Marks :	***+3***	***If all the four options are correct but ONLY three options are chosen;***
Partial Marks :	***+2***	***If three or more options are correct but ONLY two options are chosen, both of which are correct;***
Partial Marks :	***+1***	***If two or more options are correct but ONLY one option is chosen and it is a correct option;***
Zero Marks :	***0***	***If unanswered;***
Negative Marks :	***−2***	***In all other cases.***

- ***For example, in a question, if (a), (b) and (d) are the ONLY three options corresponding to correct answers, then choosing ONLY (a), (b) and (d) will get +4 marks; choosing ONLY (a) and (b) will get +2 marks; choosing ONLY (a) and (d) will get +2 marks; choosing ONLY (b) and (d) will get +2 marks; choosing ONLY (a) will get +1 mark; choosing ONLY (b) will get +1 mark; choosing ONLY (d) will get +1 mark; choosing no option(s) (i.e. the question is unanswered) will get 0 marks and choosing any other option(s) will get −2 marks.***

1. One end of a horizontal uniform beam of weight W and length L is hinged on a vertical wall at point O and its other end is supported by a light inextensible rope. The other end of the rope is fixed at point Q, at a height L above the hinge at point O. A block of weight αW is attached at the point P of the beam, as shown in the figure (not to scale). The rope can sustain a maximum tension of $(2\sqrt{2})W$.



Which of the following statement(s) is(are) correct?

(a) The vertical component of reaction force at O does not depend on α.

(b) The horizontal component of reaction force at O is equal to W for $\alpha = 0.5$

(c) The tension in the rope is $2W$ for $\alpha = 0.5$

(d) The rope breaks if $\alpha > 1.5$

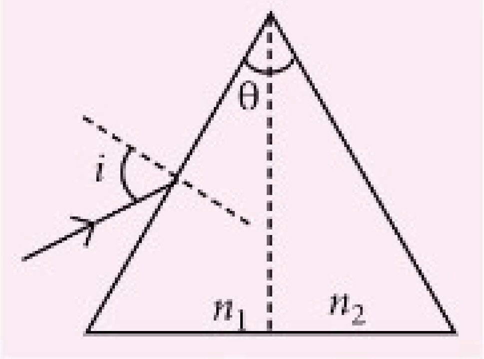
2. A source, approaching with speed u towards the open end of a stationary pipe of length L, is emitting a sound of frequency f_s. The farther end of the pipe is closed. The speed of sound in air is v and f_0 is the fundamental frequency of the pipe. For which of the following combination(s) of u and f_s, will the sound reaching the pipe lead to a resonance?

(a) $u = 0.8v$ and $f_s = f_0$ (b) $u = 0.8v$ and $f_s = 2f_0$

(c) $u = 0.8v$ and $f_s = 0.5f_0$

(d) $u = 0.5v$ and $f_s = 1.5f_0$

3. For a prism of prism angle $\theta = 60°$, the refractive indices of the left half and the right half are, respectively, n_1 and n_2 ($n_2 \geq n_1$) as shown in the figure. The angle of



incidence i is chosen such that the incident light rays will have minimum deviation if $n_1 = n_2 = n = 1.5$. For the case of unequal refractive indices, $n_1 = n$ and $n_2 = n + \Delta n$ (where $\Delta n << n$), the angle of emergence $e = i + \Delta e$. Which of the following statement(s) is(are) correct?

(a) The value of Δe (in radians) is greater than that of Δn.

(b) Δe is proportional to Δn.

(c) Δe lies between 2.0 and 3.0 milliradians, if $\Delta n = 2.8 \times 10^{-3}$.

(d) Δe lies between 1.0 and 1.6 milliradians, if $\Delta n = 2.8 \times 10^{-3}$.

4. A physical quantity $\vec{S}$ is defined as $\vec{S} = \dfrac{\vec{E} \times \vec{B}}{\mu_0}$ where $\vec{E}$ is electric field, $\vec{B}$ is magnetic field and μ_0 is the permeability of free space. The dimensions of $\vec{S}$ are the same as the dimensions of which of the following quantity(ies) ?

(a) $\dfrac{\text{Energy}}{\text{Charge} \times \text{Current}}$ (b) $\dfrac{\text{Force}}{\text{Length} \times \text{Time}}$

(c) $\dfrac{\text{Energy}}{\text{Volume}}$ (d) $\dfrac{\text{Power}}{\text{Area}}$

5. A heavy nucleus N, at rest, undergoes fission $N \rightarrow P + Q$, where P and Q are two lighter nuclei. Let $\delta = M_N - M_P - M_Q$, where M_P, M_Q and M_N are the masses of P, Q and N respectively. E_P and E_Q are the kinetic energies of P and Q respectively. The speeds of P and Q are v_P and v_Q, respectively. If c is the speed of light, which of the following statement(s) is(are) correct?

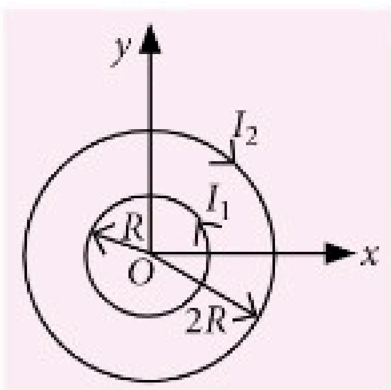
(a) $E_P + E_Q = c^2\delta$

(b) $E_P = \left(\dfrac{M_P}{M_P + M_Q}\right)c^2\delta$

(c) $\dfrac{v_P}{v_Q} = \dfrac{M_Q}{M_P}$

(d) The magnitude of momentum for P as well as Q is $c\sqrt{2\mu\delta}$, where $\mu = \dfrac{M_P M_Q}{(M_P + M_Q)}$

6. Two concentric circular loops, one of radius R and the other of radius $2R$, lie in the xy-plane with the origin as their common center, as shown in the figure. The smaller loop carries current I_1 in the anti-clockwise direction and the larger loop carries current I_2 in the clockwise direction, with $I_2 > 2I_1$. $\vec{B}(x, y)$ denotes the magnetic field at a point (x, y) in the xy-plane. Which of the following statement(s) is(are) correct?



(a) $\vec{B}(x, y)$ is perpendicular to the xy-plane at any point in the plane.

(b) $|\vec{B}(x, y)|$ depends on x and y only through the radial distance $r = \sqrt{x^2 + y^2}$.

(c) $|\vec{B}(x, y)|$ is non-zero at all points for $r < R$.

(d) $\vec{B}(x, y)$ points normally outward from the xy-plane for all the points between the two loops.

SECTION 2

- ***This section contains THREE (03) question stems.***
- ***There are TWO (02) questions corresponding to each question stem.***
- ***The answer to each question is a NUMERICAL VALUE.***
- ***For each question, enter the correct numerical value corresponding to the answer in the designated place using the mouse and the on-screen virtual numeric keypad.***
- ***If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

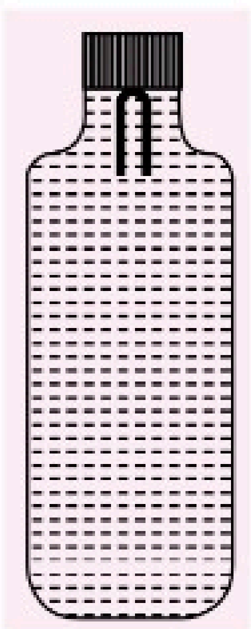
Full Marks : ***+2*** ***If ONLY the correct numerical value is entered at the designated place***

Zero Marks : ***0*** ***In all other cases.***

Question Stem for Question Nos. 7 and 8

Question Stem

A soft plastic bottle, filled with water of density 1 gm cc^{-1}, carries an inverted glass test-tube with some air (ideal gas) trapped as shown in the figure. The test-tube has a mass of 5 gm, and it is made of a thick glass of density 2.5 gm cc^{-1}. Initially the bottle is sealed at atmospheric pressure $p_0 = 10^5$ Pa so that the volume of the trapped air is $v_0 = 3.3$ cc. When the bottle is squeezed from outside at constant temperature, the pressure inside rises and the volume of the trapped air reduces. It is found that the test tube begins to sink at pressure $p_0 + \Delta p$ without changing its orientation. At this pressure, the volume of the trapped air is $v_0 - \Delta v$.



Let $\Delta v = X$ cc and $\Delta p = Y \times 10^3$ Pa.

7. The value of X is ______ .

8. The value of Y is ______.

Question Stem for Question Nos. 9 and 10

Question Stem

A pendulum consists of a bob of mass $m = 0.1$ kg and a massless inextensible string of length $L = 1.0$ m. It is suspended from a fixed point at height $H = 0.9$ m above a frictionless horizontal floor. Initially, the bob of the pendulum is lying on the floor at rest vertically below the point of suspension. A horizontal impulse $P = 0.2$ kg m s^{-1} is imparted to the bob at some instant. After the bob slides for some distance, the string becomes taut and the bob lifts off the floor. The magnitude of the angular momentum of the pendulum about the point of suspension just before the bob lifts off is J kg m^2 s^{-1}. The kinetic energy of the pendulum just after the lift-off is K Joules.

9. The value of J is ______ .

10. The value of K is _____.

Question Stem for Question Nos. 11 and 12

Question Stem

In a circuit, a metal filament lamp is connected in series with a capacitor of capacitance C μF across a 200 V, 50 Hz supply. The power consumed by the lamp is 500 W while the voltage drop across it is 100 V. Assume that there is no inductive load in the circuit. Take rms values of the voltages. The magnitude of the phase-angle (in degrees) between the current and the supply voltage is ϕ. Assume $\pi\sqrt{3} \approx 5$.

11. The value of C is ______.

12. The value of ϕ is ______.

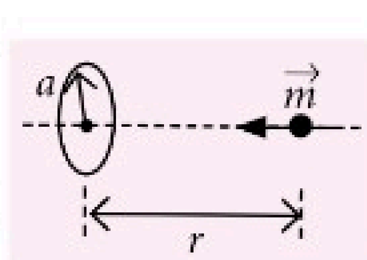
SECTION 3

- ***This section contains TWO (02) paragraphs. Based on each paragraph, there are TWO (02) questions.***
- ***Each question has FOUR options (a), (b), (c) and (d). ONLY ONE of these four options is the correct answer.***
- ***For each question, choose the option corresponding to the correct answer.***
- ***Answer to each question will be evaluated according to the following marking scheme:***

Full Marks :	*+3*	*If ONLY the correct option is chosen;*
Zero Marks :	*0*	*If none of the options is chosen (i.e. the question is unanswered);*
Negative Marks :	*–1*	*In all other cases.*

Paragraph-1

A special metal S conducts electricity without any resistance. A closed wire loop, made of S, does not allow any change in flux through itself by inducing a suitable current to generate a compensating flux. The induced current in the loop cannot decay due to its zero resistance. This current gives rise to a magnetic moment which in turn repels the source of magnetic field or flux. Consider such a loop, of radius a, with its center at the origin. A magnetic dipole of moment m is brought along the axis of this loop from infinity to a point at distance r (>> a) from the center of the loop with its north pole always facing the loop, as shown in the figure below.



The magnitude of magnetic field of a dipole m, at a point on its axis at distance r, is $\frac{\mu_0}{2\pi}\frac{m}{r^3}$, where μ_0 is the permeability of free space. The magnitude of the force between two magnetic dipoles with moments, m_1 and m_2, separated by a distance r on the common axis, with their north poles facing each other, is $\frac{km_1m_2}{r^4}$, where k is a constant of appropriate dimensions. The direction of this force is along the line joining the two dipoles.

13. When the dipole m is placed at a distance r from the center of the loop (as shown in the figure), the current induced in the loop will be proportional to

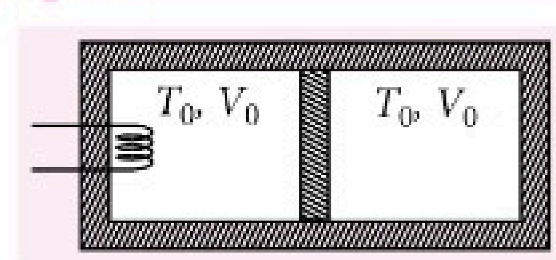
(a) $\frac{m}{r^3}$ (b) $\frac{m^2}{r^2}$ (c) $\frac{m}{r^2}$ (d) $\frac{m^2}{r}$

14. The work done in bringing the dipole from infinity to a distance r from the center of the loop by the given process is proportional to

(a) $\frac{m}{r^5}$ (b) $\frac{m^2}{r^5}$ (c) $\frac{m^2}{r^6}$ (d) $\frac{m^2}{r^7}$

Paragraph-2

A thermally insulating cylinder has a thermally insulating and frictionless movable partition in the middle, as shown in the figure below.



On each side of the partition, there is one mole of an ideal gas, with specific heat at constant volume, $C_V = 2R$. Here, R is the gas constant. Initially, each side has a volume V_0 and temperature T_0. The left side has an electric heater, which is turned on at very low power to transfer heat Q to the gas on the left side. As a result the partition moves slowly towards the right reducing the right side volume to $V_0/2$. Consequently, the gas temperatures on the left and the right sides become T_L and T_R, respectively. Ignore the changes in the temperatures of the cylinder, heater and the partition.

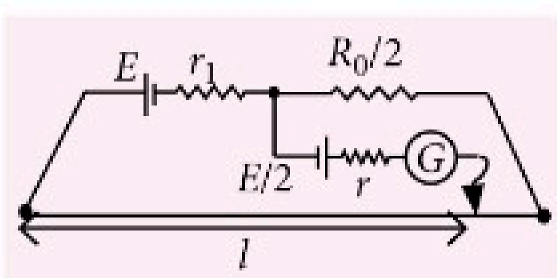
15. The value of T_R/T_0 is
(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 2 (d) 3

16. The value of Q/RT_0 is
(a) $4(2\sqrt{2}+1)$ (b) $4(2\sqrt{2}-1)$
(c) $(5\sqrt{2}+1)$ (d) $(5\sqrt{2}-1)$

SECTION 4

- ***This section contains THREE (03) questions.***
- ***The answer to each question is a NON-NEGATIVE INTEGER.***
- ***For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.***
- ***Answer to each question will be evaluated according to the following marking scheme:***
Full Marks : +4 If ONLY the correct integer is entered;
Zero Marks : 0 In all other cases.

17. In order to measure the internal resistance r_1 of a cell of emf E, a meter bridge of wire resistance R_0 = 50 Ω, a resistance $R_0/2$, another cell of emf $E/2$ (internal resistance r) and a galvanometer G are used in a circuit, as shown in figure. If the null point is found at l = 72 cm, then the value of r_1 = ____ Ω.



18. The distance between two stars of masses $3M_S$ and $6M_S$ is $9R$. Here R is the mean distance between the centers of the Earth and the Sun, and M_S is the mass of the Sun. The two stars orbit around their common center of mass in circular orbits with period nT, where T is the period of Earth's revolution around the Sun. The value of n is _____.

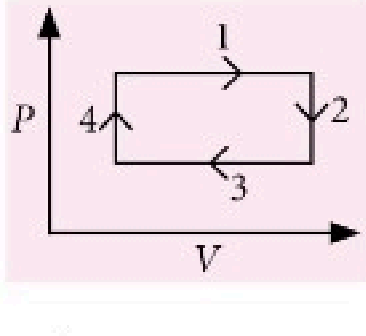
19. In a photoemission experiment, the maximum kinetic energies of photoelectrons from metals P, Q and R are E_P, E_Q and E_R respectively, and they are related by $E_P = 2E_Q = 2E_R$. In this experiment, the same source of monochromatic light is used for metals P and Q while a different source of monochromatic light is used for the metal R. The work functions for metals P, Q and R are 4.0 eV, 4.5 eV and 5.5 eV, respectively. The energy of the incident photon used for metal R, in eV, is _____.

SOLUTIONS

PAPER - I

1. **(c) :** Smallest division on Main scale = 0.1 cm
10 div on V.S = 9 div on M.S
10 VSD = 9 MSD
1 VSD = 9/10 MSD = 9/10 × 0.1 cm
Least count L.C. = 1 MSD – 1 VSD
= MSD – 9/10 MSD = 1/10 × 0.1 = 0.01 cm
Main scale reading MSR = 3.1 cm
n = (division coincide) = 1
zero error is negative zero error as vernier scale is on left of 0 of main scale.
Zero error = –(10 – 6) × L.C = –4 × 0.01 = – 0.04 cm
Reading = MSR + n × LC – Zero error
= 3.1 + 1 × 0.01 – (–0.04) = 3.15 cm

2. **(c) :** During process 1, pressure is constant, volume increases, so temperature also increases.



So, ΔW = +ve, ΔU = +ve
So, by first law of thermodynamics
$\Delta Q = \Delta W + \Delta U$
ΔQ = +ve (heat is absorbed by the gas)
During process 2, volume is constant and pressure decreases so temperature also decreases.
$W = \int p.dv = 0$
ΔT is negative. So, ΔU = –ve
$\Delta Q = \Delta U + \Delta W$
ΔQ = –ve so heat is rejected by gas
During process 3, pressure constant, volume decreases, temperature also decreases.
So, ΔU = –ve, ΔW = –ve
Heat is rejected by gas.
During process 4, volume is constant, pressure increases and hence temperature increases.
So, ΔW = 0, ΔU = +ve
$\Delta Q = \Delta U + \Delta W$ = +ve, heat is absorbed by gas

3. **(b) :** For lens L_1 :
R_1 = 20 cm ; R_2 = –20 cm ; μ = 1.5
Let the focal length is f_1.

$$\frac{1}{f_1} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_1} = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{-20}\right) = 0.5 \times \frac{2}{20}$$

f_1 = 20 cm
For lens L_2: R_1 = –20 cm ; R_2 = 20 cm; μ = 1.5
Let the focal length is f_2.

$$\frac{1}{f_2}=(1.5-1)\left(\frac{1}{(-20)}-\frac{1}{20}\right)=-0.5\times\frac{2}{20}$$

$f_2 = -20$ cm

For lens L_1 : $u = -10$ cm ; $f_1 = 20$ cm ; $v = ?$

$$\frac{1}{v}=\frac{1}{20}-\frac{1}{10}=-\frac{1}{20}$$

$$v = -20 \text{ cm} \;;\; m_1=\frac{v}{u}=\frac{-20}{-10}=2$$

For lens L_2:

$u' = -30$ cm ; $f_2 = -20$ cm ; $v' = ?$

$$\frac{1}{-20}=\frac{1}{v'}+\frac{1}{30} \Rightarrow \frac{1}{v'}=\frac{-1}{20}-\frac{1}{30}$$

$$v' = -12 \text{ cm} \;;\; m_2=\frac{v'}{u'}=\frac{-12}{-30}=\frac{2}{5}$$

so, total magnification $m_1m_2 = 2\times\frac{2}{5}=\frac{4}{5}=0.8$

4. (d) : Half life, $T_{1/2} = 20$ min

Given : Alpha decay = 60%, Beta decay = 40%

Initial amount $N_0 = 1000$

Time, $t = 1$ hr = 60 min

The decay constant is $\lambda=\frac{ln2}{T_{1/2}}=\frac{ln2}{20}$

Using the formula of radioactivity for α decay,

$$N = N_0\, e^{-\lambda t} \;;\; N = 600\; e^{-\frac{ln2}{20}\times 60}$$

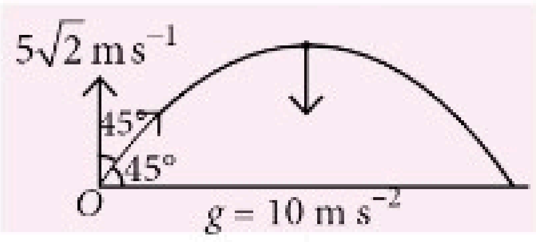
$$N = 600\, e^{-3ln\,2} = 600\, e^{ln\,2^{-3}}$$

$$N=\frac{600}{8}=75$$

so, number of a decays = 600 – 75 = 525

5. (0.5)

6. (7.5) : The horizontal range

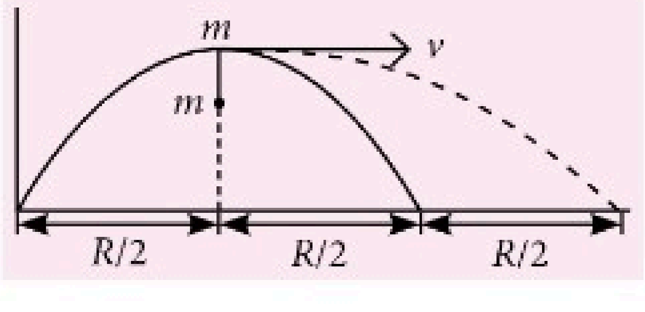


$$R=\frac{2u_x\,u_y}{g}$$

$$=\frac{2\times5\sqrt{2}\cos45^\circ\times5\sqrt{2}\sin45^\circ}{10}$$

$R = 5$ m

The time of flight is, $T=\frac{2u_y}{g}=\frac{2\times5\sqrt{2}\sin45^\circ}{10}=1\text{ s}$



The time of motion for first part to reach to maximum height $t=\frac{T}{2}=\frac{1}{2}=0.5$ s

Using conservation of momentum along x-axis,

$2\,m \times 5 = m \times v + m \times 0$

$v = 10$ m s^{-1}

Displacement of other part in 0.5 s along x-axis

$S_x = 10 \times t = 10 \times 0.5 = 5$ m

Total distance of second part from O is,

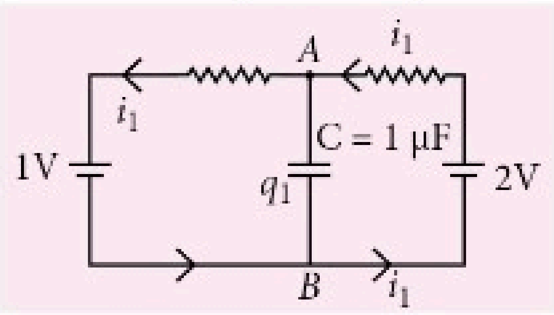
$$x=\frac{R}{2}+\frac{R}{2}+\frac{R}{2}=3\frac{R}{2} \;;\; x=\frac{3\times5}{2}=7.5\text{ m}$$

7. (1.33)

8. (0.67) : When the switch S is closed at P.

Let the current from dominant battery 2V is i_1.

Using KVL,



$V_A - 1 \times i_1 - 1$

$+ 2 - 2i_1 = V_A$

$-i_1 - 2i_1 = -1$

$i_1 = 1/3$A

Now, $V_A - 1 \times i_1 - 1 = V_B$

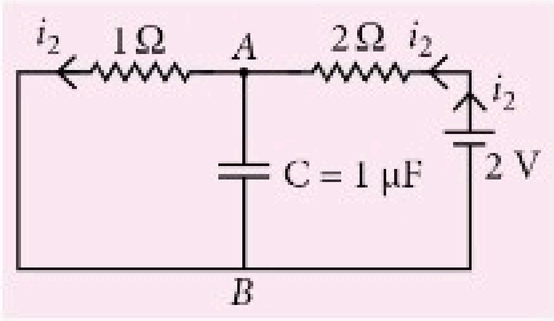
$V_A - V_B = 1 + i_1 = 1 + 1/3 = 4/3$ V

So, potential difference across capacitor, $\Delta V = 4/3$ V

$$q_1 = \text{C}.\,\Delta\text{V} = 1\times\frac{4}{3}=\frac{4}{3}\,\mu\text{C}=1.33\,\mu\text{C}$$

When the switch closed at Q

Using KVL,



$V_A - 1 \times i_2 + 2 - 2i_2 = V_A$

$3i_2 = 2$

$\Rightarrow i_2 = 2/3$A

And $V_A - i_2 \times 1 = V_B$

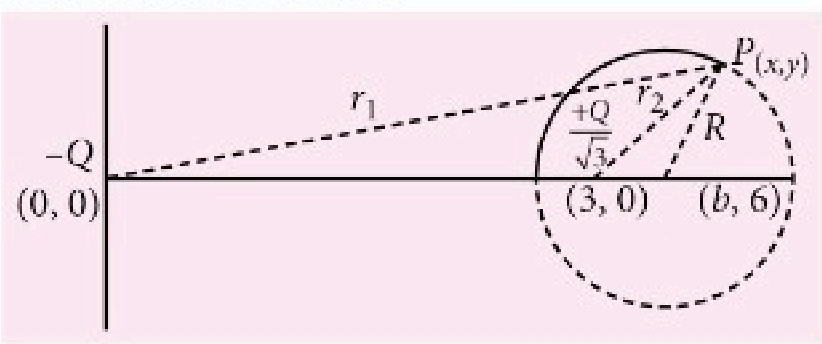
$$V_A - V_B = \frac{2}{3}\text{V}$$

P.d across capacitor, $\Delta\text{V}=\frac{2}{3}\text{V}$

$$q_2 = \text{C}.\Delta\text{V} = 1\times\frac{2}{3}=0.67\,\mu\text{C}$$

9. (1.73)

10. (3) : Let there is a point P on the circle. The potential at P is zero.



$$\text{So, } \frac{K(-Q)}{r_1}+\frac{KQ/\sqrt{3}}{r_2}=0$$

$$\Rightarrow \frac{Q}{\sqrt{x^2+y^2}}=\frac{Q}{\sqrt{3}\left(\sqrt{(x-2)^2+y^2}\right)}$$

$\Rightarrow\ 3(x-2)^2+3y^2=x^2+y^2\ ;\ 3(x^2+4-4x)+3y^2=x^2+y^2$

$\Rightarrow\ (x-3)^2+y^2=(\sqrt{3})^2$

So, $R=\sqrt{3}=1.73\ ;\ b=3$

11. (b,d) : Here, f is the friction force and α is the angular acceleration.

Using Newton's second law,

$F-f=ma$ ...(i)

$\tau=R\times f=\tau.\alpha$ ...(ii)

$a=R\alpha$ ...(iii)

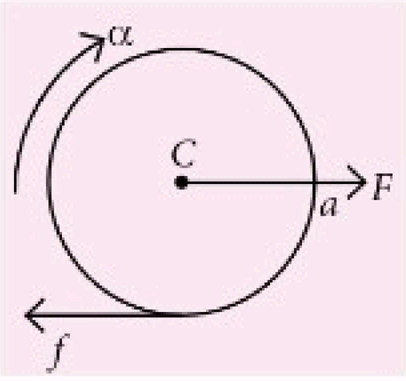
So, $F-I\cdot\dfrac{\alpha}{R}=ma\ ;\ F=ma+\dfrac{I}{R}\cdot\dfrac{a}{R}$

$a=\dfrac{F}{m+\dfrac{I}{R^2}}$

So, $f=\dfrac{I\alpha}{R}=\dfrac{I}{R}\cdot\dfrac{a}{R}=\dfrac{Ia}{R^2}=\dfrac{F}{m+\dfrac{I}{R^2}}\times\dfrac{I}{R^2}$

Now for hollow cylinder,

$I=mR^2$



so, $a=\dfrac{F}{2m}$ and $f<\mu mg$

$\dfrac{Ia}{R^2}<\mu mg\ \Rightarrow\ a<\dfrac{\mu mgR^2}{I}$

For solid cylinder, $I=\dfrac{mR^2}{2}\ ;a<2\mu g$

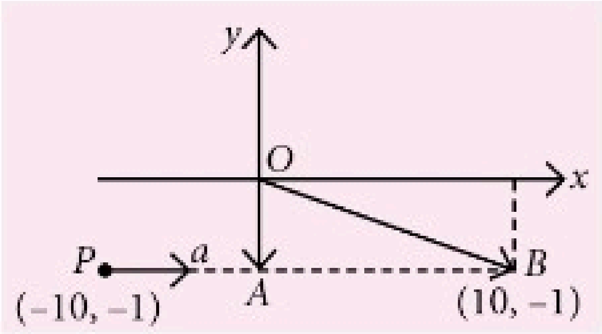
So maximum value of acceleration is 2 μg

12. (b,c,d)

13. (a,b,c) : Mass, $M=0.2$ kg $(x=-l,\ y=-h)$

$l=10$ m ; $h=1$ m ; $a=10$ m s^{-2}

Vector $\vec{r}_A=-\hat{j}$



$S=ut+\dfrac{1}{2}at^2$

$20=0+\dfrac{1}{2}\times10\ t^2$

$t=2$ s

Torque at B, $\vec{\tau}_0=\vec{r}\times\vec{F}$

$\vec{\tau}_B=+10\hat{i}-\hat{j}$

$\vec{F}=m\vec{a}=0.2\times10\hat{i}=2\hat{i}$

$\vec{\tau}_0=(10\hat{i}-\hat{j})\times(2\hat{i})=2\hat{k}\ ;\ \vec{\tau}_0=\vec{r}_B\times\vec{p}=\vec{r}_B\times m\vec{v}$

$\vec{\tau}_0=\vec{r}_B\times m(u+\vec{a}t)=\vec{r}_B\times m(\vec{a}\times2)$

$\vec{\tau}_0=0.2(10\hat{i}-\hat{j})\times2\times10\hat{i}\ ;\ \vec{\tau}_0=4\hat{k}$

Torque at A, (0, –1), $\vec{\tau}_0=\vec{r}_A\times\vec{F}-=(-\hat{j})\times(2\hat{i})=2\hat{k}$

14. (a, d) : (A) For Balmer series,

$n_1=2,\ n_2=3,4,5\ .....\ \infty$

for longest wavelength, $n_1=2,\ n_2=3$

Let λ_{max} is longest wavelength and λ_{min} is shortest wavelength

for shortest wavelength, $n_1=2,\ n_2=\infty$

So, $\dfrac{1}{\lambda_{max}}=R\left[\dfrac{1}{2^2}-\dfrac{1}{3^2}\right]=\dfrac{R\times5}{36}$

$\lambda_{max}=\dfrac{36}{5R}$ ...(i)

$\dfrac{1}{\lambda_{min}}=R\left[\dfrac{1}{2^2}-\dfrac{1}{\infty}\right]=\dfrac{R}{4}\Rightarrow\lambda_{min}=\dfrac{4}{R}$ ...(ii)

so, $\dfrac{\lambda_{max}}{\lambda_{min}}=\dfrac{36\times R}{5R\times4}=\dfrac{9}{5}$

(B) λ longest of Balmer $=\dfrac{36}{5R}$

λ shortest for Paschen, $n_1=3,\ n_2=\infty$

$\dfrac{1}{\lambda_{shortest}}=R\left[\dfrac{1}{9}-\dfrac{1}{\infty}\right]$

λ shortest for Paschen $=\dfrac{9}{R}$

So, they do not overlap.

(C) For lyman series, $\dfrac{1}{\lambda}=R\left[\dfrac{1}{1^2}-\dfrac{1}{m^2}\right]$

Also, $\dfrac{1}{\lambda_0}=R\ \Rightarrow\ \dfrac{1}{\lambda}=\dfrac{1}{\lambda_0}\left[1-\dfrac{1}{m^2}\right]\ ;\ \lambda=\dfrac{\lambda_0}{1-1/m^2}$

(D) λ longest for lyman

$\dfrac{1}{\lambda_{longest}}=R\left[\dfrac{1}{1^2}-\dfrac{1}{2^2}\right]=\dfrac{3R}{4}$

λ longest for lyman $=\dfrac{4}{3R}$; λ shortest for Balmer $=\dfrac{4}{R}$

So, they do not overlap.

15. (a,c) : Current $I=2$A

Speed, $v=3$ m s^{-1}

Resistance, $R=1.4\ \Omega\ ;\ C_0=5\mu$F

The induced emf across the semi circular rod is

$E=\int_1^4\dfrac{\mu_0 i}{2\pi r}v\,dr=\dfrac{\mu_0 iv}{2\pi}\int_1^4\dfrac{1}{r}\cdot dr$

$E=\dfrac{\mu_0 iv}{2\pi}[ln\,r]_1^4=\dfrac{\mu_0 iv}{2\pi}(ln4-ln1)\ ;\ E=\dfrac{\mu_0 iv\ ln2}{\pi}$

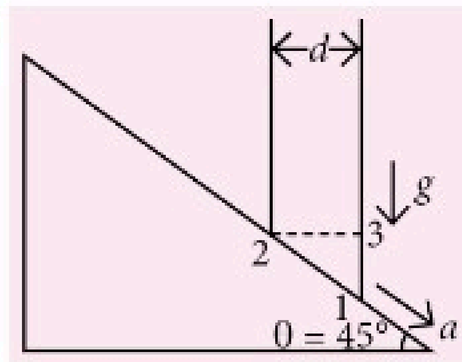
$E=\dfrac{4\pi\times10^{-7}\times2\times3\times0.7}{\pi}=24\times7\times10^{-8}$ V

$i_{max}=\dfrac{E}{R}=\dfrac{24\times7\times10^{-8}}{1.4}=1.2\times10^{-6}$ A

$Q_{max}=C_0E=24\times7\times10^{-8}\times5\times10^{-6}=8.4\times10^{-12}$ C

16. (a,c): $\theta = 45°$

$$\beta = \frac{P_1 - P_2}{\rho g d}$$



The component of acceleration along downward is $a \sin 45°$.

So, $P_1 - P_3 = \rho(g - a \sin 45°)d$

$P_2 - P_3 = \rho\, a \sin 45° \times d$

Also, $P_1 - P_2 = \rho d\left[g - \frac{2a}{\sqrt{2}}\right]$

$$\frac{P_1 - P_2}{\rho g d} = \left[1 - \frac{\sqrt{2}a}{g}\right] = \beta \quad \text{(given)}$$

When, $\beta = 0 \Rightarrow 1 = \sqrt{2}\,a/g$; $a = g/\sqrt{2}$

If $\beta = \frac{\sqrt{2}-1}{2}$, then $a = \frac{g}{2}$

17. (4) : Mass on α-particle, $m_\alpha = 4$ amu

Mass on sulphur ion, $m_s = 32$ amu

Potential = V

The KE is given by, $\frac{1}{2}mv^2 = qv \Rightarrow v = \sqrt{\frac{2qv}{m}}$

Let the radius is r.

$$r = \frac{mv}{Bq} = \frac{\sqrt{2mqV}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

so, $\frac{r_s}{r_\alpha} = \sqrt{\frac{m_s}{m_\alpha}\cdot\frac{q_\alpha}{q_s}} = \sqrt{\frac{32}{4}\times\frac{2}{1}}$; $\frac{r_s}{r_\alpha} = 4$

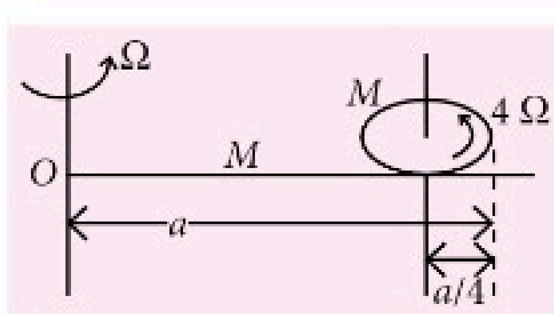
18. (49) : Mass of rod = Mass of disc = m

Angular velocity of rod = Ω

Angular velocity of disc = $4\,\Omega$

Radius of disc, $R = a/4$

Angular momentum of system,



$L = (I\Omega)_{Rod} + (I\Omega)_{disc}$

$$L = \left[\frac{Ma^2}{3} + M\left(\frac{3a}{4}\right)^2\right]\Omega + \frac{M\left(\frac{a}{2}\right)^2}{2}\cdot 4\Omega$$

$$L = \frac{49}{48}Ma^2\Omega$$

By comparing with $L = \left(\frac{Ma^2\Omega}{48}\right)n$,

we get $n = 49$.

19. (9) : Temperature of container = 0 K

Temperature of object = 200 K

According to the Newton's law and stefan's law

$$\sigma A T^4 = -\frac{msdT}{dt}$$

where, s is the specific heat and A is area, σ is stefan's constant

$$\int_{200}^{100}\frac{dT}{T^4} = \int_0^{t_1} kdt \;;\; \left[\frac{1}{3T^3}\right]_{200}^{100} = kt_1$$

$$kt_1 = \frac{1}{3}\left[\frac{1}{100^3} - \frac{1}{200^3}\right] \quad \ldots(i)$$

Similarly,

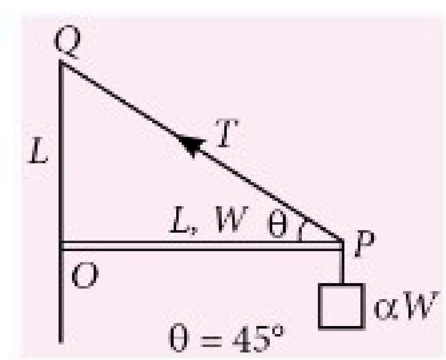
$$kt_2 = \frac{1}{3}\left[\frac{1}{50^3} - \frac{1}{200^3}\right] \quad \ldots(ii)$$

From equation (i) and (ii),

$$\frac{t_2}{t_1} = \frac{\frac{1}{50^3} - \frac{1}{200^3}}{\frac{1}{100^3} - \frac{1}{200^3}} = \frac{(200^3 - 50^3)}{(200^3 - 100^3)} \times \frac{100^3}{50^3} \;;\; \frac{t_2}{t_1} = 9$$

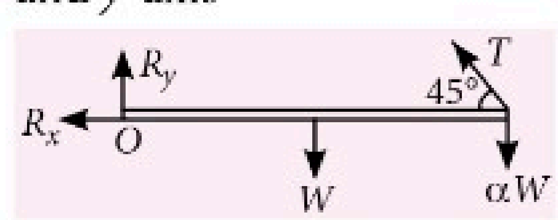
PAPER - II

1. (a,b,d) : Max. tension in rope $T = 2\sqrt{2}\,W$



Free body diagram of rope

Balance the forces along x and y-axis



$$R_x = T\cos 45° = \frac{T}{\sqrt{2}} \quad \text{...(i)}$$

$$R_y + T\sin 45° = W + \alpha W;\; R_y + \frac{T}{\sqrt{2}} = W + \alpha W \quad \text{...(ii)}$$

Take the moments about O.

$$W \times \frac{l}{2} + \alpha W L = T\sin 45° \times L$$

$$T = \sqrt{2}\left(\frac{W}{2} + \alpha W\right) \quad \text{...(iii)}$$

Take the moments about P

$$R_y L = W\frac{L}{2} \Rightarrow R_y = \frac{W}{2}$$

When tension is maximum, $T = T_{max}$

From equation (iii), $2\sqrt{2}\,W = \sqrt{2}\left(\frac{W}{2} + \alpha W\right)$

$$\Rightarrow \quad 2\sqrt{2}\,W = W\sqrt{2}\left(\frac{1}{2} + \alpha\right) \Rightarrow 2 - \frac{1}{2} = \alpha \Rightarrow \alpha = \frac{3}{2}$$

$$R_x = \frac{T}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \sqrt{2}\left(\frac{W}{2} + \alpha W\right)$$

If $\alpha = 0.5$

$$R_x = \frac{W}{2} + 0.5\,W = W$$

2. **(a, d)** : Speed of source = u
Open end pipe length = L
Speed of sound in air = v
Fundamental frequency = f_0
According to the formula of Doppler's effect

$$f = f_0 = \left(\frac{v}{v-u}\right)$$

(a) $u = 0.8\,v, f_s = f_0$. So, $f = f_0\left(\frac{v}{v-0.8v}\right) = 5\,f_0$

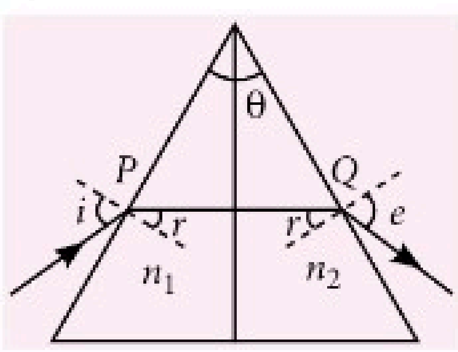
(b) $u = 0.8\,v, f_s = f_0\,;\; f = 2f_0\left(\frac{v}{v-0.8v}\right) = 10\,f_0$

(c) $f = u = 0.8\,v, f_s = 0.5 f_0;\; f = 0.5\,f_0\left(\frac{v}{v-0.8v}\right) = 2.5\,f_0$

(d) $u = 0.8\,u, f_s = 1.5\,f_0\,;\; f = 1.5\,f_0\left(\frac{v}{v-0.8v}\right) = 3\,f_0$

As the pipe is closed at one end so all odd harmonics are present. So option (a) and (d) is correct.

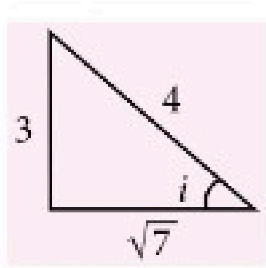
3. **(b,c)** : $\theta = 60°$



Use Snell's law at P
$1 \times \sin i = n_1 \sin \theta/2$
$\sin i = n_1 \sin 30°$

$$\sin i = \frac{n_1}{2} = \frac{3}{4} \qquad \text{...(i)}$$

$n_1 = n_2 = n = 1.5$



Use Snell's law at Q
$n_2 \sin 30° = 1 \times \sin e$
Differentiate both sides
$dn = \sin 30° = de$ (cone)

$$de = \frac{dn}{2\cos e} \qquad (i = e)$$

$$de = \frac{dn \times 4}{2 \times \sqrt{7}} = \frac{2}{\sqrt{7}}dn$$

So, $de < dn$. If $dn = 2.8 \times 10^{-3}$

$$de = \frac{2.8 \times 10^{-3} \times 2}{\sqrt{7}} = 2.11 \text{ m rad}$$

4. **(b,d)** : $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$

$\vec{S}$ denotes flow of energy per unit area per unit time.

$$\text{Dim. of } \vec{S} = \frac{\text{Watt}}{m^2} = \frac{\text{ML}^2\,\text{T}^{-2}}{\text{TL}^2} = [\text{MT}^{-3}]$$

$$\text{Dim. of } \frac{\text{Force}}{\text{Length} \times \text{Time}} = \frac{[\text{MLT}^{-2}]}{\text{LT}} = [\text{MT}^{-3}]$$

$$\text{Dim. of } \frac{\text{Power}}{\text{Area}} = \frac{[\text{ML}^2\text{T}^{-2}]}{[\text{T}][\text{L}^2]} = [\text{MT}^{-3}]$$

So the correct option is (b) and (d).

5. **(a,c,d) :** $N \longrightarrow P + Q$
The amount of energy released in the reaction is
$E = (M_N - M_P - M_Q)C^2 = \delta C^2$ (given)
This energy is redistributed in the form of KE of P and Q.
$E_P + E_Q = \delta C^2$ ...(i)
Use the conservation of momentum
$M_P v_P = M_Q v_Q$

$$\frac{v_P}{v_Q} = \frac{M_Q}{M_P} \qquad \text{...(ii)}$$

The relation of KE and momentum is, $\text{KE} = \frac{P^2}{2m}$

$$\text{So, } E_P = \frac{M_Q}{M_p + M_q} \cdot C^2\delta \qquad \text{...(iii)}$$

$$\text{From eqn. (i), } \frac{P^2}{2M_P} + \frac{P^2}{2M_Q} = C^2\delta$$

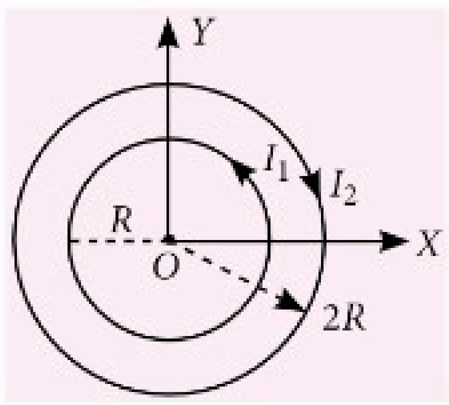
$$\Rightarrow \frac{P^2\,(M_Q + M_P)}{2(M_P \times M_Q)} = C^2\delta$$

$$\text{Now } \mu = \frac{M_P M_Q}{M_P + M_Q}. \text{ So, } \frac{P^2}{2\mu} = \delta C^2 \Rightarrow P = C\sqrt{2\mu\,\delta}$$

So option (a), (c) and (d) are correct.

6. **(a,b) :** (a) From Biot Savart's law,

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i\,d\vec{l} \times \vec{r}}{r^3}$$



Here, $d\vec{l}$ is in x-y plane and $\vec{r}$ also in xy plane.
(b) Due to the symmetry, it depends on r.

$$r = \sqrt{x^2 + y^2}$$

(c) The magnetic field at centre due to current I_1 is

$$B_1 = \frac{\mu_0 I_1}{2R_1} = \frac{\mu_0\,I_1}{2R}$$

and due to current I_2 is, $B_2 = \frac{\mu_0 I_2}{2R_2} = \frac{\mu_0\,I_2}{4R}$

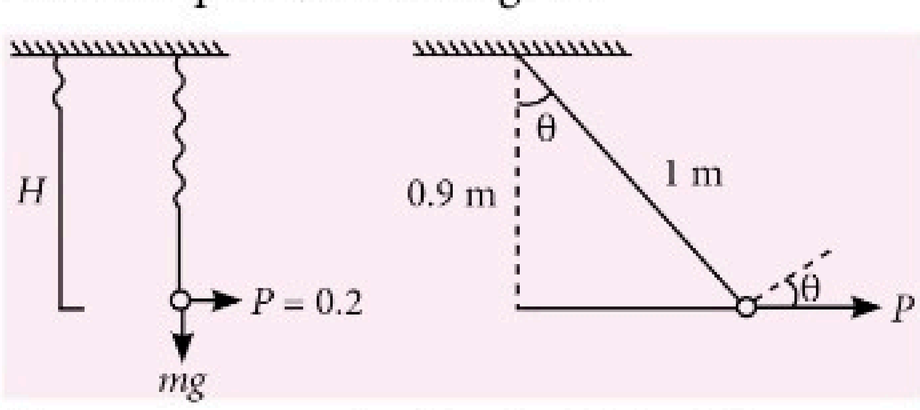
So, net magnetic field is zero somewhere between the loops.
(d) The net direction of magnetic field depends on the value of B_1 and B_2.
So, it is not sure.

7. (0.3)

8. (10) : Mass of tube, $m = 5$ gm.
Density of water, $\rho = 1$ gm cc^{-1}
Density of glass, = 2.5 gm cc^{-1}.
Atmospheric pressure, $P_0 = 10^5$ Pa
Volume of trapped air, $v_0 = 3.3$ cc
$\Delta V = X$ c.c, $\Delta P = Y \times 10^3$ Pa
When bottle is sinking :
Buoyant force, $F_B = mg$
$P(V_{glass} + V_{gas}) = m$
$1(2 + V_{gas}) = 5 \Rightarrow V_{gas} = 3$ cc.
So, $\Delta V = V_0 - V_{gas} = 3.3 - 3 = 0.3$ cc
So, $X = 0.3$
For isothermal process,
$P_1V_1 = P_2V_2$
$10^5 \times 3.3 = P_2 \times 3 \Rightarrow P_2 = 1.1 \times 10^5$
Now, $\Delta P = P_2 - P_1 = (1.1 - 1) \times 10^5 = 0.1 \times 10^5$
$\Delta P = 10 \times 10^3$ Pascal ; $Y = 10$

9. (0.18)

10. (0.16) : Mass of bob, $m = 0.1$ kg
Length of string, $L = 1$ m
Height of point of suspension, $H = 0.9$ m
Horizontal impulse, $P = 0.2$ kg m s^{-1}



Angular momentum, $J = L \times P = 0.9 \times 0.2$
$= 0.18$ kg m^2 s^{-1}

So, $J = 0.18$

When the string is tight, the velocity along the string is zero.
So, $mv_1 = P\cos\theta$

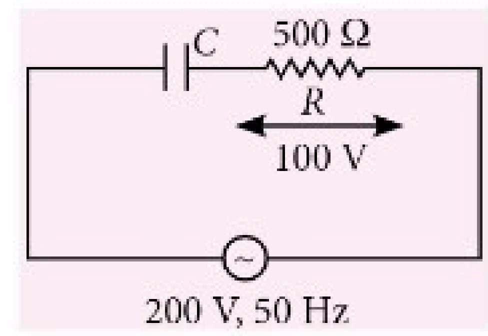
$$v_1 = \frac{P\cos\theta}{m} = 0.2 \times \frac{0.9}{1 \times 0.1} \Rightarrow v_1 = 1.8 \text{ m s}^{-1}$$

So, kinetic energy, $K = \frac{1}{2}mv_1^2$; $K = 0.162$ J

So, $K = 0.162$

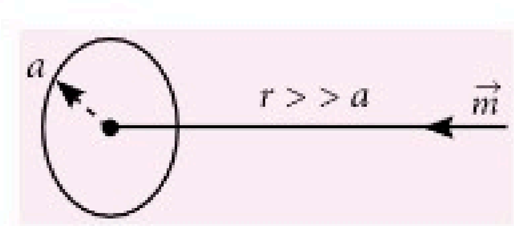
11. (100)

12. (60) : Capacitance, C μF;
Voltage, $V = 200$ V; Frequency = 50 Hz.
Power consumed, $P = 500$ W
Voltage drop = 100 V



$$E_{rms} = \sqrt{V_C^2 + V_R^2}$$
$$200^2 = V_C^2 + 100^2$$
$$V_C = 100\sqrt{3} \text{ V} \quad \text{...(i)}$$
$$\tan\phi = \frac{V_C}{V_R} = \frac{100\sqrt{3}}{100} = \sqrt{3} \Rightarrow \phi = 60^\circ$$
$$P = E_{rms}\, I_{rms} \cos\phi = \frac{E_{rms}^2}{Z} \times \frac{1}{2}$$
$$500 = \frac{200 \times 200}{Z} \times \frac{1}{2}$$
$$Z = \frac{200 \times 100}{500} = 40\ \Omega\ ;\ \cos\phi = \frac{R}{Z} = \frac{1}{2}$$
$$\frac{R}{40} = \frac{1}{2} \Rightarrow R = 20\ \Omega\ ;\ Z = \sqrt{R^2 + X_C^2}$$
$$40^2 = 20^2 + X_C^2 \Rightarrow X_C = 20\sqrt{3}$$
$$\frac{1}{\omega C} = X_C = 20\sqrt{3}$$
$$C = \frac{1}{20\sqrt{3} \times 2\pi \times 50} = \frac{1}{20 \times 2 \times 5 \times 50} = 10^{-4} \text{ F} = 100\ \mu\text{F}$$

13. (a) : The magnetic flux is $\phi = Li$
where L is self inductance.



$$i = \frac{\phi}{L} = \frac{B \cdot \text{Area}}{L} = \frac{\mu_0 m}{2\pi r^3 L} \times \pi a^2$$
$$i \propto \frac{m}{r^3}$$

14. (c) : Now $m' = i\pi a^2 = \dfrac{\mu_0 m \pi^2 a^4}{2\pi r^3 L}$

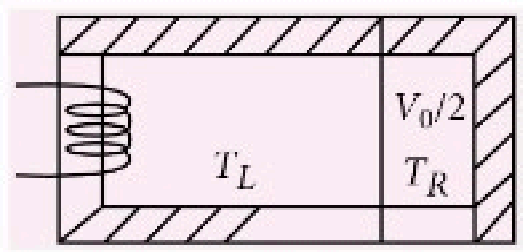
$$\text{Force, } F = \frac{k\,mm'}{r^4} = \frac{k \cdot m \times \mu_0\, m\pi^2 a^4}{2\pi r^7\, L}$$
$$F \propto \frac{m^2}{r^7} \Rightarrow \text{Work} = \int F \cdot dr \propto \int \frac{m^2}{r^7} \cdot dr$$
$$\text{Work} \propto \frac{m^2(-6)}{r^6} \Rightarrow \text{Work} \propto \frac{m^2}{r^6}$$

15. (a) : No. of moles = 1 ; $C_V = 2R$
Heat transfer = Q

Volume of right side = $\frac{V_0}{2}$

Initially the volume and temperature on both the chambers is V_0, T_0.

Finally, $V_L = \frac{3V_0}{2}, V_R = \frac{V_0}{2}$



$$C_V = \frac{R}{\gamma - 1} = 2R$$

$$\gamma - 1 = 0.5 \Rightarrow \gamma = \frac{3}{2}$$

As the process is adiabatic so, $T_0 V_0^{\gamma-1} = T_R V_R^{\gamma-1}$

$$\Rightarrow \quad T_0 V_0^{1/2} = T_R \cdot \left(\frac{V_0}{2}\right)^{1/2}$$

$$T_0 = \frac{T_R}{\sqrt{2}} \;;\; \frac{T_R}{T_0} = \sqrt{2}$$

16. (b) : Now, $P\left(\frac{V_0}{2}\right)^{\gamma} = P_0 V_0^{\gamma}$

$$P = P_0 2^{3/2} \qquad \text{...(i)}$$

$$\frac{PV}{T_L} = \frac{P_0 V_0}{T_0} \Rightarrow T_L = 2^{3/2} \times \frac{3}{2} T_0 = 3\sqrt{2}\, T_0$$

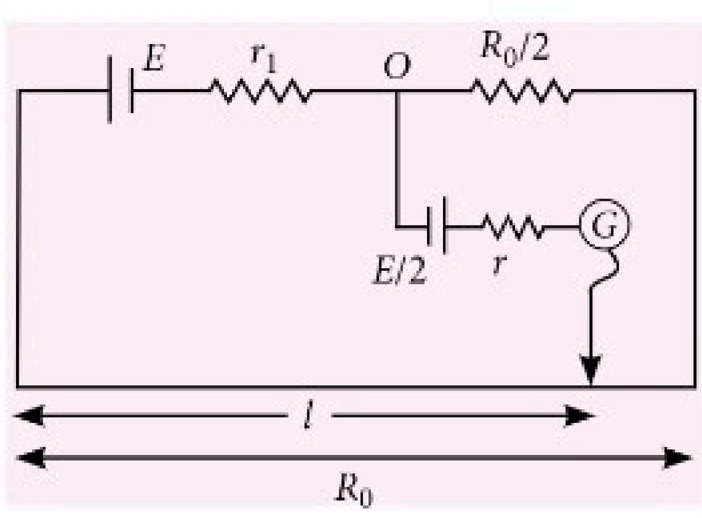
$$Q = nC_V \Delta T_1 + nC_V \Delta T_2$$

$$Q = 1 \times 2R(3\sqrt{2} - 1)T_0 + 1 \times 2R(\sqrt{2} - 1)T_0$$

$$\frac{Q}{RT_0} = 2 \times 3\sqrt{2} - 2 + 2\sqrt{2} - 2 = 8\sqrt{2} - 4 = 4(2\sqrt{2} - 1)$$

17. (3) : Resistance of remaining length = $0.28\, R_0$

So, by ohm's law $i\left(\frac{R_0}{2} + 0.28\, R_0\right) = \frac{E}{2}$



$$i \times 0.78\, R_0 = \frac{E}{2} \;;\; i = \frac{E}{2 \times 0.78\, R_0} = \frac{E}{r_1 + \frac{3}{2} R_0}$$

So, $r_1 + 1.5\, R_0 = 1.56\, R_0$
$r_1 = 0.06\, R_0 = 0.06 \times 50$; $r_1 = 3\ \Omega$

18. (9) : The time period of satellite is $T = 2\pi\sqrt{\frac{R^3}{Gm}}$
where, M is mass of planet and R is distance.

$$\text{For earth, } T = 2\pi\sqrt{\frac{R^3}{Gm_s}} \qquad \text{...(ii)}$$

$$\text{For binary for, } n \cdot T = 2\pi\sqrt{\frac{(9R)^3}{G(3M_S + 6M_S)}}$$

$$n \times 2\pi\sqrt{\frac{R^3}{GM_s}} = 2\pi \times 9^{3/2}\sqrt{\frac{R^3}{G \times 9\, M_s}} \;;\; n = \frac{9^{3/2}}{\sqrt{9}} = 9$$

19. (6) : $E_P = 2E_Q = 2E_R$
$\phi_P = 4$ eV ; $\phi_Q = 4.5$ eV ; $\phi_R = 5.5$ eV
For metal P and Q
$E_1 - 4 = E_P$
Let E_1 is incident energy, $E_1 - 4.5 = E_Q$
Here, $E_P = 2E_Q$
So, $E_1 - 4 = 2(E_1 - 4.5)$
$E_1 = 5$ eV ; $E_P = 1$ eV ; $E_Q = E_R = 0.5$ eV
For R : Let incident energy is E_2
$E_2 - 5.5 = 0.5 \Rightarrow E_2 = 6$ eV

SOLVED PAPER

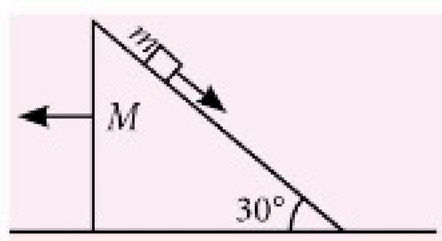
Held on
1st September, Evening Shift

JEE MAIN 2021

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. The temperature of an ideal gas in 3-dimensions is 300 K. The corresponding de-Broglie wavelength of the electron approximately at 300 K, is
[m_e = mass of electron = 9×10^{-31} kg,
h = Planck's constant = 6.6×10^{-34} J s,
k_B = Boltzmann constant = 1.38×10^{-23} J K^{-1}]
(a) 2.26 nm (b) 3.25 nm
(c) 8.46 nm (d) 6.26 nm

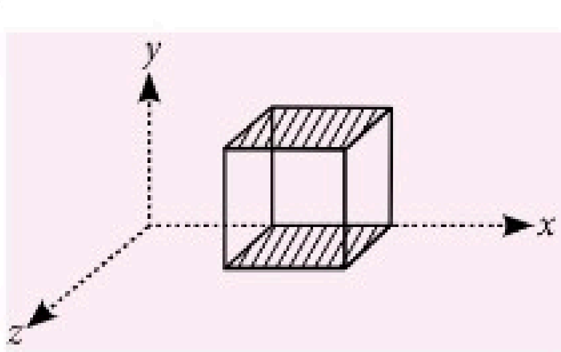
2. A block of mass m slides on the wooden wedge, which in turn slides backward on the horizontal surface. The acceleration of the block with respect to the wedge is Given m = 8 kg, M = 16 kg
Assume all the surfaces shown in the figure to be frictionless.



(a) $\frac{3}{5}g$ (b) $\frac{4}{3}g$ (c) $\frac{6}{5}g$ (d) $\frac{2}{3}g$

3. The ranges and heights for two projectiles projected with the same initial velocity at angles 42° and 48° with the horizontal are R_1, R_2 and H_1, H_2 respectively. Choose the correct option.
(a) $R_1 = R_2$ and $H_1 = H_2$ (b) $R_1 = R_2$ and $H_1 < H_2$
(c) $R_1 > R_2$ and $H_1 = H_2$ (d) $R_1 < R_2$ and $H_1 < H_2$

4. A cube is placed inside an electric field, $\vec{E} = 150y^2\hat{j}$. The side of the cube is 0.5 m and is placed in the field as shown in the given figure. The charge inside the cube is



(a) 8.3×10^{-11} C (b) 3.8×10^{-11} C
(c) 3.8×10^{-12} C (d) 8.3×10^{-12} C

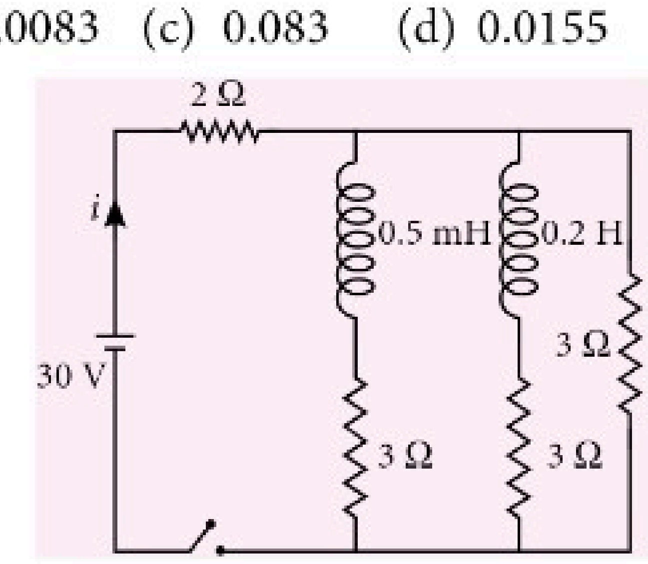
5. Due to cold weather a 1 m water pipe of cross-sectional area 1 cm^2 is filled with ice at –10 °C. Resistive heating is used to melt the ice. Current of 0.5 A is passed through 4 kΩ resistance. Assuming that all the heat produced is used for melting, what is the minimum time required ? (Given latent heat of fusion for water/ice = 3.33×10^5 J kg^{-1}, specific heat of ice = 2×10^3 J kg^{-1} and density of ice = 10^3 kg m^{-3})
(a) 3.53 s (b) 0.353 s (c) 35.3 s (d) 70.6 s

6. A student determined Young's Modulus of elasticity using the formula $Y = \frac{MgL^3}{4bd^3\delta}$. The value of g is taken to be 9.8 m s^{-2}, without any significant error, his observation are as following.

Physical Quantity	Least count of the Equipment used for measurement	Observed Value
Mass (M)	1 g	2 kg
Length of bar (L)	1 mm	1 m
Breadth of bar (b)	0.1 mm	4 cm
Thickness of bar (d)	0.01 mm	0.4 cm
Depression (δ)	0.01 mm	5 mm

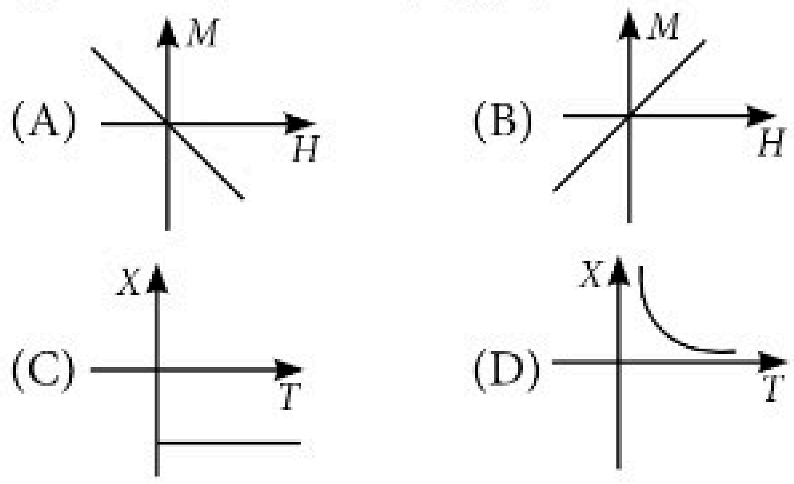
Then the fractional error in the measurement of Y is
(a) 0.155 (b) 0.0083 (c) 0.083 (d) 0.0155

7. For the given circuit the current i through the battery when the key is closed and the steady state has been reached is __________.



(a) 10 A (b) 6 A (c) 25 A (d) 0 A

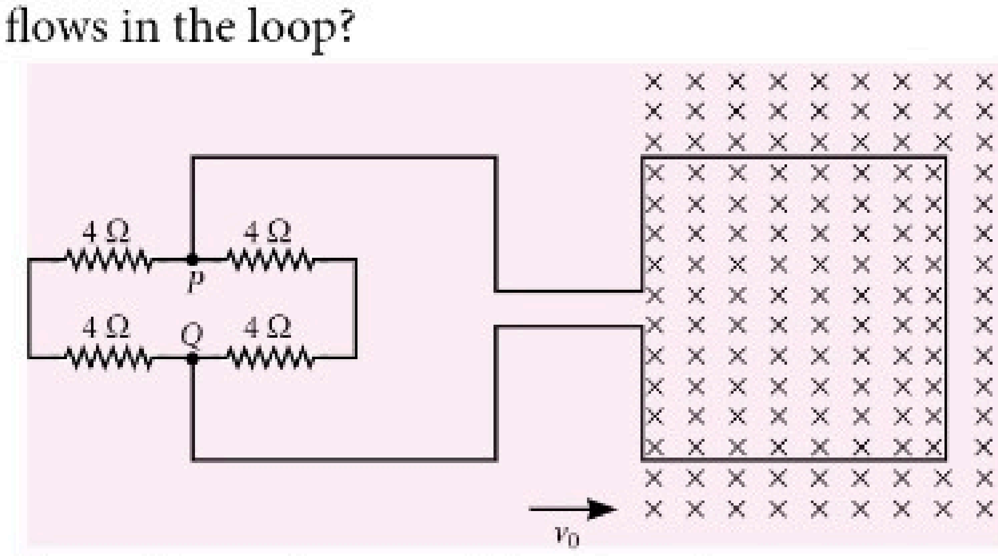
8. Following plots show Magnetization (M) *vs* Magnetising field (H) and magnetic susceptibility (χ) *vs* Temperature (T) graph.



Which of the following combination will be represented by a diamagnetic material?

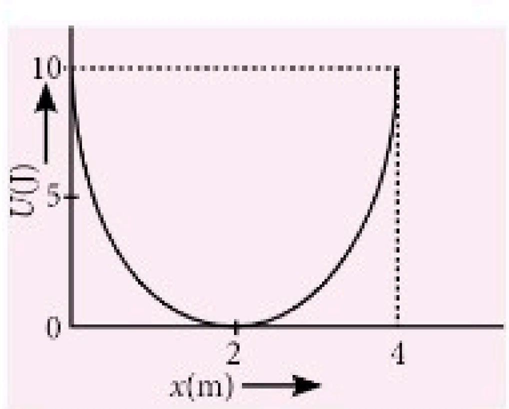
(a) (B), (C) (b) (B), (D)
(c) (A), (D) (d) (A), (C)

9. A square loop of side 20 cm and resistance 1 Ω is moved towards right with a constant speed v_0. The right arm of the loop is in a uniform magnetic field of 5 T. The field is perpendicular to the plane of the loop and is going into it. The loop is connected to a network of resistors each of value 4 Ω. What should be the value of v_0 so that a steady current of 2 mA flows in the loop?



(a) 10^{-2} cm s^{-1} (b) 1 cm s^{-1}
(c) 1 m s^{-1} (d) 10^2 m s^{-1}

10. A mass of 5 kg is connected to a spring. The potential energy curve of the simple harmonic motion executed by the system is shown in the figure.



A simple pendulum of length 4 m has the same period of oscillation as the spring system. What is the value of acceleration due to gravity on the planet where these experiments are performed?

(a) 4 m s^{-2} (b) 9.8 m s^{-2}
(c) 5 m s^{-2} (d) 10 m s^{-2}

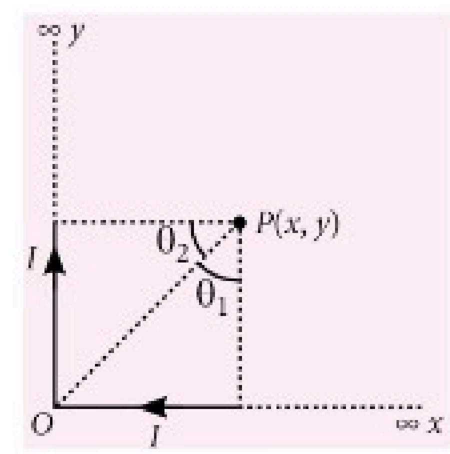
11. The half life period of a radioactive element x is same as the mean life time of another radioactive element y. Initially they have the same number of atoms. Then

(a) x and y decay at the same rate always
(b) x-will decay faster than y
(c) y-will decay faster than x
(d) x and y have same decay rate initially and later on different decay rate.

12. A body of mass 'm' dropped from a height 'h' reaches the ground with a speed of $0.8\sqrt{gh}$. The value of workdone by the air-friction is

(a) -0.68 mgh (b) mgh
(c) 0.64 mgh (d) 1.64 mgh

13. There are two infinitely long straight current carrying conductors and they are held at right angles to each other so that their common ends meet at the origin as shown in the figure given below. The ratio of current in both conductors is 1 : 1. The magnetic field at point P is ________.



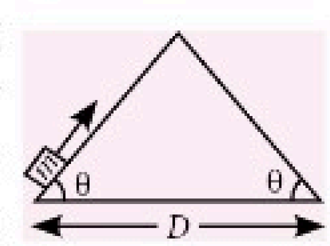
(a) $\dfrac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}-(x+y)\right]$

(b) $\dfrac{\mu_0 Ixy}{4\pi}\left[\sqrt{x^2+y^2}-(x+y)\right]$

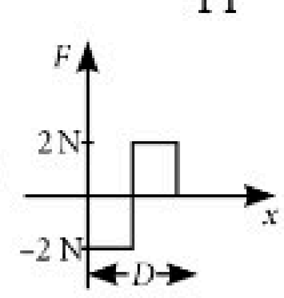
(c) $\dfrac{\mu_0 I}{4\pi xy}\left[\sqrt{x^2+y^2}+(x+y)\right]$

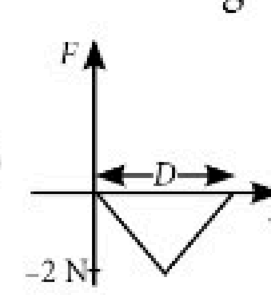
(d) $\dfrac{\mu_0 Ixy}{4\pi}\left[\sqrt{x^2+y^2}+(x+y)\right]$

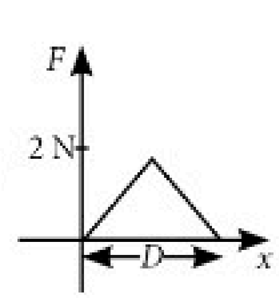
14. An object of mass 'm' is being moved with a constant velocity under the action of an applied force of 2 N along a frictionless surface with following surface profile.

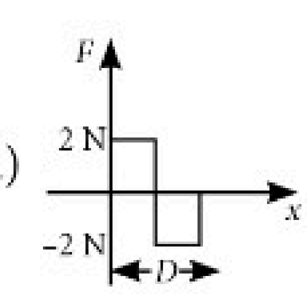


The correct applied force vs distance graph will be

(a) 

(b) 

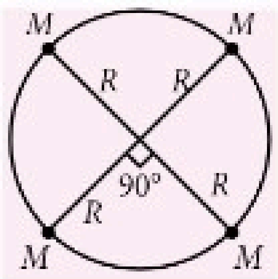
(c) 

(d) 

15. Electric field of a plane electromagnetic wave propagating through a non-magnetic medium is given by $E = 20\cos(2 \times 10^{10}\, t - 200x)$ V m^{-1}. The dielectric constant of the medium is equal to (Take $\mu_r = 1$)

(a) 2 (b) 1/3 (c) 9 (d) 3

16. Four particles each of mass M, move along a circle of radius R under the action of their mutual gravitational attraction as shown in figure. The speed of each particle is



(a) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}+1)}$ (b) $\frac{1}{2}\sqrt{\frac{GM}{R(2\sqrt{2}+1)}}$

(c) $\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2}-1)}$ (d) $\sqrt{\frac{GM}{R}}$

17. A glass tumbler having inner depth of 17.5 cm is kept on a table A student starts pouring water ($\mu = 4/3$) into it while looking at the surface of water from the above. When he feels that the tumbler is half filled, he stops pouring water. Up to what height, the tumbler is actually filled?

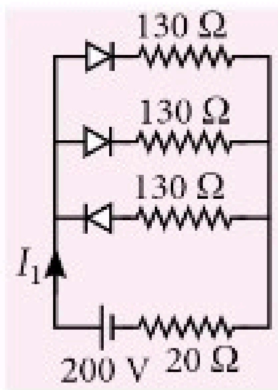
(a) 10 cm (b) 11.7 cm
(c) 7.5 cm (d) 8.75 cm

18. A capacitor is connected to a 20 V battery through a resistance of 10 Ω. It is found that the potential difference across the capacitor rises to 2 V in 1 μs. The capacitance of the capacitor is ________ μF.

Given $\ln\left(\frac{10}{9}\right) = 0.105$

(a) 0.95 (b) 9.52 (c) 1.85 (d) 0.105

19. In the given figure, each diode has a forward bias resistance of 30 Ω and infinite resistance in reverse bias. The current I_1 will be



(a) 2 A (b) 3.75 A
(c) 2.73 A (d) 2.35 A

20. Two resistors $R_1 = (4 \pm 0.8)$ Ω and $R_2 = (4 \pm 0.4)$ Ω are connected in parallel. The equivalent resistance of their parallel combination will be

(a) (4 ± 0.4) Ω (b) (2 ± 0.4) Ω
(c) (4 ± 0.3) Ω (d) (2 ± 0.3) Ω

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. A 2 kg steel rod of length 0.6 m is clamped on a table vertically at its lower end and is free to rotate in vertical plane. The upper end is pushed so that the rod falls under gravity. Ignoring the friction due to clamping at its lower end, the speed of the free end of rod when it passes through its lowest position is ________ m s^{-1}. (Take $g = 10$ m s^{-2})

22. The average translational kinetic energy of N_2 gas molecules at ________ °C becomes equal to the K.E. of an electron accelerated from rest through a potential difference of 0.1 volt. (Given $k_B = 1.38 \times 10^{-23}$ J K^{-1}) (Fill the nearest integer).

23. The width of one of the two slits in a Young's double slit experiment is three times the other slit. If the amplitude of the light coming from a slit is proportional to the slit-width, the ratio of minimum to maximum intensity in the interference pattern is $x : 4$ where x is ________.

24. Two satellites revolve around a planet in coplanar circular orbits in anticlockwise direction. Their period of revolutions are 1 hour and 8 hours respectively. The radius of the orbit of nearer satellite is 2×10^3 km. The angular speed of the farther satellite as observed from the nearer satellite at the instant when both the satellites are closest is $\frac{\pi}{x}$ rad h^{-1} where x is ________.

25. A steel rod with $y = 2.0 \times 10^{11}$ N m^{-2} and $\alpha = 10^{-5}$ °C^{-1} of length 4 m and area of cross-section 10 cm^2 is heated from 0°C to 400°C without being allowed to extend. The tension produced in the rod is $x \times 10^5$ N where the value of x is ________.

26. A uniform heating wire of resistance 36 Ω is connected across a potential difference of 240 V. The wire is then cut into half and a potential difference of 240 V is applied across each half separately. The ratio of power dissipation in first case to the total power dissipation in the second case would be $1 : x$ where x is ________.

27. A carrier wave with amplitude of 250 V is amplitude modulated by a sinusoidal base band signal of amplitude 150 V. The ratio of minimum amplitude to maximum amplitude for the amplitude modulated wave is $50 : x$, then value of x is ________.

28. The temperature of 3.00 mol of an ideal diatomic gas is increased by 40.0°C without changing the pressure of the gas. The molecules in the gas rotate but do not oscillate. If the ratio of change in internal energy of the gas to the amount of workdone by the gas is $x/10$. Then the value of x (round off to the nearest integer) is ________.

(Given $R = 8.31$ J mol^{-1} K^{-1})

29. When a body slides down from rest along a smooth inclined plane making an angle of 30° with the horizontal, it takes time T. When the same body slides down from the rest along a rough inclined plane making the same angle and through the same distance, it takes time αT, where α is a constant greater than 1. The co-efficient of friction between the body and the rough plane is $\frac{1}{\sqrt{x}}\left(\frac{\alpha^2-1}{\alpha^2}\right)$ where $x =$ ________.

30. An engine is attached to a wagon through a shock absorber of length 1.5 m. The system with a total mass of 40,000 kg is moving with a speed of 72 km h^{-1} when the brakes are applied to bring it to rest. In the process of the system being brought to rest, the spring of the shock absorber gets compressed by 1.0 m. If 90% of energy of the wagon is lost due to friction, the spring constant is ________ $\times 10^5$ N m^{-1}.

SOLUTIONS

1. (d): Temperature of gas = 300 K (Given)

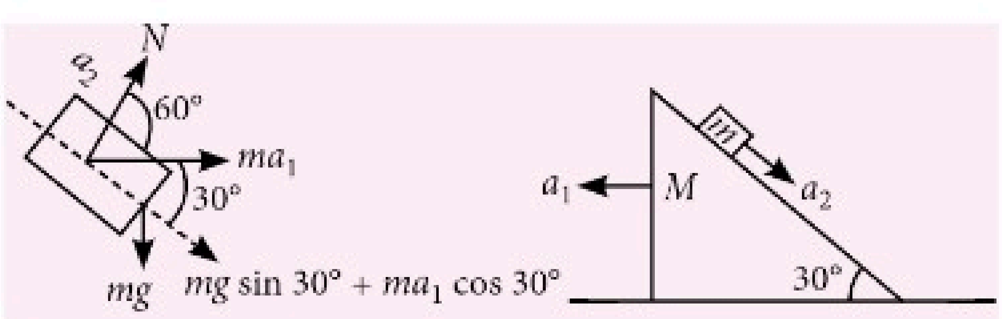
de-Broglie wavelength of electron is given by,

$$\lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.63\times10^{-34}}{\sqrt{3\times9.1\times10^{-31}\times1.38\times10^{-23}\times300}}$$

$= 0.0626 \times 10^{-7}$ m = 6.26 nm

2. (d): For wedge, $N\cos 60° = Ma_1 \Rightarrow N = 2Ma_1$, ...(i)

Free body diagram of block with respect to wedge is as shown



$N + ma_1 \sin 30° = mg\cos 30°$

From eqn (i), $2Ma_1 + \frac{ma_1}{2} = \frac{\sqrt{3}mg}{2}$

$a_1\left[2M + \frac{m}{2}\right] = \frac{\sqrt{3}}{2}mg$; $a_1\left[\frac{4M+m}{2}\right] = \frac{\sqrt{3}}{2}mg$

$a_1 = \frac{\sqrt{3}mg}{4M+m}$

Also, $mg\sin 30° + ma_1\cos 30° = ma_2$

$g\sin 30° + a_1\cos 30° = a_2$

$\frac{g}{2} + \frac{\sqrt{3}mg}{4M+m}\times\frac{\sqrt{3}}{2} = a_2$ ...(ii)

Put m = 8 kg and M = 16 kg in (ii), we get,

$a_2 = \frac{g}{2} + \frac{3\times8\times g}{2(72)}$ $\therefore a_2 = \frac{g}{2} + \frac{g}{6} = \frac{2g}{3}$

3. (b): For ranges θ and $90° - \theta$, range is same. So, for angle of projection 42° and 48°, range is same.

Maximum height $H = \frac{u^2\sin^2\theta}{2g}$

Since $\sin\theta$ is increasing function in $0° < \theta < 90°$, maximum height would be more for 48°.

Therefore, $R_1 = R_2$ and $H_1 < H_2$.

4. (a): Electric field, $\vec{E} = 150y^2\hat{j}$

Electric field is in sin y direction, so, electric flux is due to only top and bottom surfaces.

For bottom surface, $y = 0$, $\phi = 0$

Top surface, y = 0.5 m (Given)

$E = 150(0.5)^2 = \frac{150}{4}$

Now, flux, $\phi = EA = \frac{150}{4}\times\frac{1}{4} = \frac{150}{16}$

By Gauss's law, $\phi = \frac{q_{enclosed}}{\varepsilon_0}$

$q_{enclosed} = \phi\,\varepsilon_0 = \frac{150}{16}\times 8.85\times10^{-12} = 8.3\times10^{-11}$ C

5. (c) : Mass of ice = $\rho Al = 10^3 \times 10^{-4} \times 1 = 0.1$ kg

Energy required to melt the ice = $ms\Delta T + mL$

$= 0.1 \times 2 \times 10^3 (10) + 0.1 \times 3.33 \times 10^5$

$= 2 \times 10^3 + 3.33 \times 10^4 = 3.53 \times 10^4$ J ...(i)

Also, heat produced due to electric current

$H = i^2Rt = (0.5)^2 \times 4 \times 10^3 \times t$

$= 10^3 \times t$ ...(ii)

From equation, (i) and (ii)

$3.53 \times 10^4 = 10^3 t$

$\therefore t = 35.3$ s

6. (d): $Y = \frac{MgL^3}{4bd^3\delta}$

$g = 9.8$ m s^{-2} (constant)

$$\frac{\Delta Y}{Y} = \frac{\Delta M}{M} + \frac{3\Delta L}{L} + \frac{\Delta b}{b} + \frac{3\Delta d}{d} + \frac{\Delta\delta}{\delta}$$

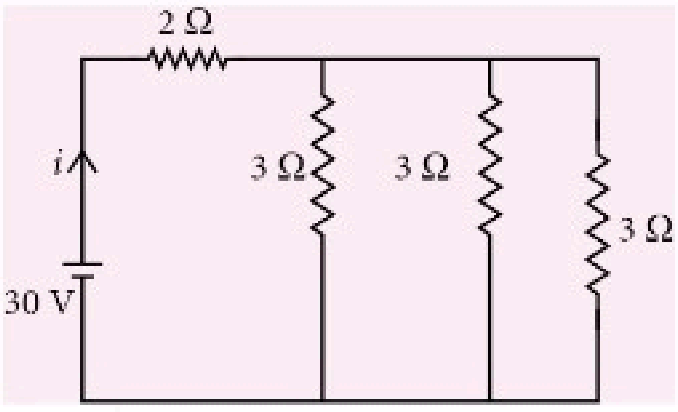
$$= \frac{1}{2000} + \frac{3}{1000} + \frac{0.1}{40} + \frac{0.03}{4} + \frac{0.01}{5}$$

$\therefore \frac{\Delta Y}{Y} = 0.0155$

So, fractional error is measurement of Y is 0.0155.

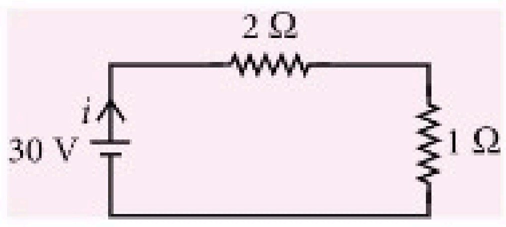
7. (a): In the steady state, inductor behaves as a conducting wire.

circuit diagram becomes,



Equivalent resistance of circuit is

$$\frac{1}{R_p} = \frac{1}{3}+\frac{1}{3}+\frac{1}{3} = 1$$



$\therefore \quad R_p = 1\ \Omega$

Now circuit becomes

Now, $R_s = 2\ \Omega + 1\ \Omega = 3\ \Omega$

$\therefore \quad R_{eq} = 3\ \Omega$

From Ohm's law, $i = \frac{V}{R} = \frac{30}{3} = 10\text{ A}$

8. (d): For a diamagnetic material, susceptibility (χ) is independent of temperature, so graph (C) is correct for diamagnetic material.

$\chi = -1$ (for diamagnetic material)

Also, $\chi = \frac{M}{H} \Rightarrow M = -H$

This is represented in graph (A).

9. (b): Resistance between P and $Q = 4\ \Omega + 1\ \Omega = 5\ \Omega$

Current, $I = 2$ mA

Emf, $\varepsilon = 5 \times 2 \times 10^{-3} = 10 \times 10^{-3}$ V

Also, induced emf, $\varepsilon = Blv_0$

$10 \times 10^{-3} = 5 \times 20 \times 10^{-2}\ v_0$

$\therefore \quad v_0 = \frac{1}{100}\text{ m s}^{-1} = 1\text{ cm s}^{-1}$

10. (a): Potential energy of spring, $U = \frac{1}{2}kx^2$

$10 = \frac{1}{2} \times k \times 2 \times 2$ (From given graph)

$\therefore \quad k = 5\text{ N m}^{-1}$

Time period of simple pendulum

= Time period of oscillation of spring system

$$2\pi\sqrt{l/g} = 2\pi\sqrt{k/m} \Rightarrow \frac{k}{m} = \frac{l}{g}$$

$\therefore \quad g = \frac{5}{5} \times 4 = 4\text{ m s}^{-2}$ [From given values]

11. (c): Half life of x = Mean life of y

$$t_{1/2_x} = t_y \Rightarrow \frac{0.693}{\lambda_x} = \frac{1}{\lambda_y}$$

$\therefore \quad \lambda_x = 0.693\ \lambda_y$

Since, decay rate = λN

Initially, number of atoms in x and y are equal. So, decay rate of y > decay rate of x.

12. (a): From work energy theorem,

work = gain in KE $= \frac{1}{2}m(0.8\sqrt{gh})^2 = 0.32\ mgh$

work done by gravity = mgh

work done by air friction = total work done – work done by gravity

$= 0.32\ mgh - mgh = -0.68\ mgh$

13. (c): Magnetic field due to wire along x axis at P is,

$$B_x = \frac{\mu_0}{4\pi}\frac{I}{y}[\sin\theta_1 + \sin 90°] = \frac{\mu_0}{4\pi}\frac{I}{y}\left[\frac{x}{\sqrt{x^2+y^2}}+1\right]$$

Magnetic field due to wire along y axis,

$$B_y = \frac{\mu_0}{4\pi}\frac{I}{x}\left[\frac{y}{\sqrt{x^2+y^2}}+1\right]$$

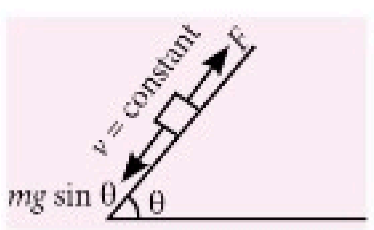
Total magnetic field, $\vec{B} = \vec{B}_1 + \vec{B}_2$

$$= \frac{\mu_0}{4\pi}I\left[\frac{x}{y\sqrt{x^2+y^2}} + \frac{y}{x\sqrt{x^2+y^2}} + \frac{1}{y} + \frac{1}{x}\right]$$

$$= \frac{\mu_0}{4\pi}I\left[\frac{x^2+y^2}{xy\sqrt{x^2+y^2}} + \frac{x+y}{xy}\right]$$

$$= \frac{\mu_0}{4\pi}\frac{I}{xy}\left[\sqrt{x^2+y^2} + x + y\right]$$

14. (d): During upward motion

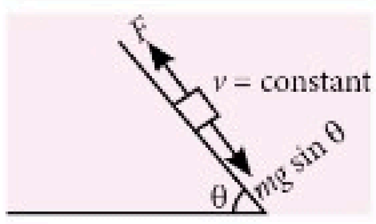


$F = 2$ N = constant (positive)

During downward motion

$F = 2$ N

= constant (negative)



So, the correct graph shown in option (d).

15. (c): Electric field,

$E = 20\cos(2 \times 10^{10}\ t - 200\ x)$ V m^{-1}

On comparing with, $E = E_0 \cos(\omega t - kx)$

we get, $k = 200$

$\omega = 2 \times 10^{10}$

$$v = \frac{\omega}{k} = \frac{2\times10^{10}}{200} = 10^8\text{ m s}^{-1}$$

as, $v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}}$

Given, $\mu_r = 1$

So, $v = \frac{1}{\sqrt{\mu_0\varepsilon_0\varepsilon_r}}$; $\varepsilon_r = \frac{1}{v^2\mu_0\varepsilon_0}$

$$= \frac{1}{10^{16}\times4\pi\times10^{-7}\times8.85\times10^{-12}} = \frac{1000}{4\pi\times8.85} = 9$$

So, $\varepsilon_r = 9$

16. (a) : Here, gravitational force will provide necessary centripetal force.

Force between two opposite masses $F = \frac{GM^2}{4R^2}$

Force between two adjacent masses $F' = \frac{GM^2}{2R^2}$

Net force on a particle of mass $M = F + \sqrt{2}F$

$$= \frac{GM^2}{R^2}\left[\frac{1}{4} + \frac{\sqrt{2}}{2}\right] = \frac{GM^2}{4R^2}\left[1 + 2\sqrt{2}\right]$$

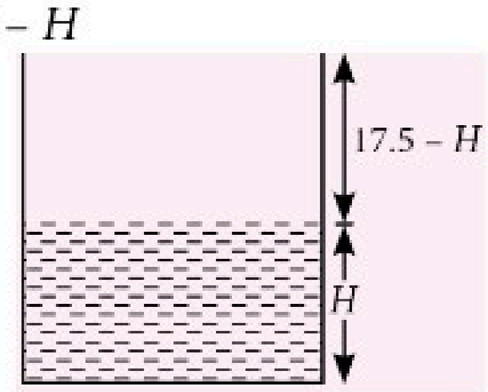
Also centripetal force $= \frac{mv^2}{R}$

So, $\frac{mv^2}{R} = \frac{GM^2}{4R^2}\left[1+2\sqrt{2}\right] \Rightarrow v = \frac{1}{2}\sqrt{\frac{GM}{R}\left(2\sqrt{2}+1\right)}$

17. (a) : From the figure,
Let real depth is H,

Apparent depth $= \frac{H}{\mu} = \frac{H}{4/3} = \frac{3H}{4}$

Also, apparent path = 17.5 – H



$\Rightarrow \quad \frac{3H}{4} = 17.5 - H$

$\Rightarrow \quad \frac{7H}{4} = 17.5$

$\therefore \quad H = 10$ cm

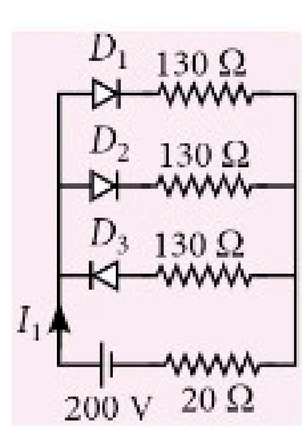
18. (a) : Potential across capacitor during charging

$V = V_0\,(1 - e^{-/RC})$ or $\frac{1}{10} = 1 - e^{-t/RC}$

$\Rightarrow \quad e^{-t/RC} = \frac{9}{10} \Rightarrow \frac{t}{RC} = \ln\left(\frac{10}{9}\right) = 0.105$

$\therefore \quad C = \frac{10^{-6}}{10 \times 0.105} = 0.95\ \mu\text{F}$

19. (a) : Since D_3 is in reverse bias, it will offer infinite resistance.
As D_1 and D_2 are connected in parallel, net resistance in circuit will be



$R_{eq} = R + \frac{D_1 D_2}{D_1 + D_2}$

From Ohm's law, $R_{eq} = 20 + \frac{160 \times 160}{160 + 160} = 100\ \Omega$

current, $I_1 = \frac{V}{R_{eq}} = \frac{200}{100} = 2$ A

20. (d): $R_1 = (4 \pm 0.8)\ \Omega$
$R_2 = (4 \pm 0.4)\ \Omega$

In parallel, $R = \frac{R_1 R_2}{R_1 + R_2} = \frac{4 \times 4}{4 + 4} = 2\ \Omega$

$$= \frac{\Delta R}{R^2} = \frac{\Delta R_1}{R_1^2} + \frac{\Delta R_2}{R_2^2}$$

$$\frac{\Delta R}{2 \times 2} = \frac{0.8}{4 \times 4} + \frac{0.4}{4 \times 4}$$

$\Delta R = 0.2 + 0.1 = 0.3$

So, equivalent resistance = $(2 \pm 0.3)\ \Omega$

21. (6): If l is the length of rod and ω is the angular velocity of rod at lowest position,
from energy conservation,

$$mgl = \frac{1}{2}I\omega^2 \Rightarrow mgl = \frac{1}{2}\left(\frac{1}{3}ml^2\right)\omega^2$$

$$\frac{6g}{l} = \omega^2 \quad \Rightarrow \quad \omega = \sqrt{\frac{6g}{l}}$$

$\Rightarrow \quad v = \omega l = \sqrt{6gl} = \sqrt{6 \times 10 \times 0.6} = \sqrt{36}$

$\therefore \quad v = 6\ \text{m s}^{-1}$

22. (500) : Given, translational kinetic energy of N_2

= kinetic energy of electron, $\frac{3}{2}k_B T = eV$

$\Rightarrow \quad \frac{3}{2} \times 1.38 \times 10^{-23} \times T = 1.6 \times 10^{-19} \times 0.1$

$\Rightarrow \quad T = \frac{2 \times 1.6 \times 10^{-20}}{3 \times 1.38 \times 10^{-23}}$

or $\quad T = 773$ K = (773 – 273) °C

$\therefore \quad T = 500$°C

23. (1): Intensity $\propto$ (amplitude)^2 $\propto$ (slit-width)^2

$$\frac{I_1}{I_2} = \left(\frac{3}{1}\right)^2 = 9$$

$$\frac{I_{min}}{I_{max}} = \frac{(\sqrt{I_1} - \sqrt{I_2})^2}{(\sqrt{I_1} + \sqrt{I_2})^2} = \left(\frac{\sqrt{\frac{I_1}{I_2}} - 1}{\sqrt{\frac{I_1}{I_2}} + 1}\right)^2 = \left(\frac{3-1}{3+1}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

So, $I_{min} : I_{max} = 1 : 4$

On comparing with x : 4, we get $x = 1$

24. (3): $T_1 = 1$ hour, $\omega_1 = 2\pi$ rad hr^{-1}

$T_2 = 8$ hour, $\omega_2 = \frac{\pi}{4}$ rad hr^{-1}

$R_1 = 2 \times 10^3$ km

From Kepler's third law

$T^2 \propto R^3$

or $\left(\frac{R_2}{R_1}\right)^3 = \left(\frac{T_2}{T_1}\right)^2 \Rightarrow \left(\frac{R_2}{2 \times 10^3}\right)^3 = (8)^2$

$\therefore \quad R_2 = 8 \times 10^3$ km

Relative angular velocity, $\omega = \dfrac{V_1 - V_2}{R_2 - R_1}$

$v_1 = R_1\omega_1 = 2 \times 10^3 \times 2\pi = 4\pi \times 10^3 \text{ km h}^{-1}$

$v_2 = R_2\omega_2 = 8 \times 10^3 \times \dfrac{\pi}{4} = 2\pi \times 10^3 \text{ km h}^{-1}$

So, $\omega = \dfrac{4\pi\times 10^3 - 2\pi\times 10^3}{8\times 10^3 - 2\times 10^3} = \dfrac{2\pi\times 10^3}{6\times 10^3} = \dfrac{\pi}{3}$ rad hr^{-1}

25. (8) : Given : $Y = 2 \times 10^{11} \text{ N m}^{-2}$, $\alpha = 10^{-5}\ {}^\circ\text{C}^{-1}$, $L = 4$ m, $A = 10 \times 10^{-4} \text{ m}^2 = 10^{-3} \text{ m}^2$, $\Delta T = 400\ ^\circ$C

Young's modulus, $Y = \dfrac{FL}{A\Delta L}$

So, $\Delta L = \dfrac{FL}{AY}$

Also, $\Delta L = L\,\alpha\,\Delta T$

So, $\dfrac{FL}{AY} = L\alpha\Delta T$

$\therefore \quad F = YA\,\alpha\,\Delta T$

$= 2 \times 10^{11} \times 10^{-3} \times 10^{-5} \times 400 = 8 \times 10^5$ N

26. (4) : Power, $P_1 = \dfrac{V^2}{R_1} = \dfrac{(240)^2}{36}$

When wire is cut into half, $R' = 18\ \Omega$

So, total power dissipation,

$P_2 = \dfrac{(240)^2}{18} + \dfrac{(240)^2}{18} = \dfrac{(240)^2}{9}$

So, $\dfrac{P_1}{P_2} = \dfrac{\frac{1}{36}}{\frac{1}{9}} = \dfrac{1}{9}$

$\therefore \quad P_1 : P_2 = 1 : 4$

On comparing, $P_1 : P_2 = 1 : x$

we get, $x = 4$

27. (200) : $A_{max} = A_c + A_m = 250 + 150 \text{ V} = 400$ V

$A_{min} = A_c - A_m = 250 - 150 = 100$ V

$\Rightarrow \quad \dfrac{A_{min}}{A_{max}} = \dfrac{100}{400} = \dfrac{50}{200}$

$\therefore \quad A_{min} : A_{max} = 50 : 200$

On comparing $A_{min} : A_{max} = 50 : x$

we get, $x = 200$

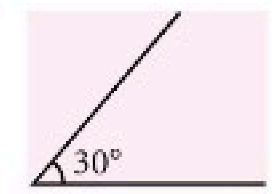
28. (25) : The process is isobaric as pressure remains constant so,

$$\Delta U = nC_V\Delta T = n \times \frac{5}{2}R\Delta T$$

and work done, $\Delta W = nR\Delta T$

So, $\dfrac{\Delta U}{\Delta W} = \dfrac{5}{2} = \dfrac{x}{10} \Rightarrow x = 25$

29. (3) : On smooth inclined plane



acceleration, $a = g \sin 30°$

displacement, $S = ut + \dfrac{1}{2}at^2$

$S = \dfrac{1}{2}(g\sin 30°)T^2$; $S = \dfrac{1}{4}gT^2$

On rough surface,

$$a = g\sin 30° - \mu\cos 30° = \left(1-\sqrt{3}\mu\right)\frac{g}{2}$$

displacement, $S = \dfrac{1}{2}\left(1-\sqrt{3}\mu\right)\dfrac{g}{2}\times(\alpha T)^2$

As displacement in both cases is same

$\dfrac{1}{4}gT^2 = \dfrac{1}{4}\left(1-\sqrt{3}\mu\right)g\alpha^2T^2 \Rightarrow \alpha^2 = \dfrac{1}{1-\sqrt{3}\mu}$

or $\quad \alpha^2 - \sqrt{3}\mu\alpha^2 = 1 = \dfrac{\alpha^2-1}{\alpha^2} = \sqrt{3}\mu$

$\therefore \quad \mu = \dfrac{1}{\sqrt{3}}\left(\dfrac{\alpha^2-1}{\alpha^2}\right)$

30. (16) : Length, $l = 1.5$ m

Total mass, $m = 40000$ kg

Speed, $v = 72 \text{ km h}^{-1} = 20 \text{ m s}^{-1}$

$\Delta x = 1$ m

90% of wagon energy is lost due to friction

From work - energy theorem.

$$\frac{-90}{100}\left(\frac{1}{2}mv^2\right) + W_{spring} = 0 - \frac{1}{2}mv^2$$

or $\quad W_{spring} = \dfrac{90}{100}\left(\dfrac{1}{2}mv^2\right) - \dfrac{1}{2}mv^2$

$\Rightarrow \quad \dfrac{-1}{2}k(x)^2 = \dfrac{-10}{100}\left(\dfrac{1}{2}mv^2\right)$

$\Rightarrow \quad k = \dfrac{1}{10}\times 40000\times(20)^2$

$\therefore \quad k = 16 \times 10^5 \text{ N m}^{-1}$

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(c)	**2.**	(d)	**3.**	(b)	**4.**	(a)	**5.**	(a)
6.	(d)	**7.**	(c)	**8.**	(b)	**9.**	(c)	**10.**	(b)
11.	(b)	**12.**	(d)	**13.**	(c)	**14.**	(b)	**15.**	(c)
16.	(c)	**17.**	(b)	**18.**	(c)	**19.**	(c)		
20.	(a,b,d)	**21.**	(a,b,c)	**22.**	(a,b,c)	**23.**	(b,c,d)		
24.	(b,c,d)	**25.**	(b,c,d)	**26.**	(28.8)	**27.**	(0.86)	**28.**	(50)
29.	(c)	**30.**	(b)						

CLASS-XI

BRUSH UP for NEET/JEE

Brush up your concepts to get high rank in NEET/JEE (Main and Advanced) by reading this column. This specially designed column is updated year after year by a panel of highly qualified teaching experts well-tuned to the requirements of these Entrance Tests.

Unit 4 System of Particles and Rotational Motion

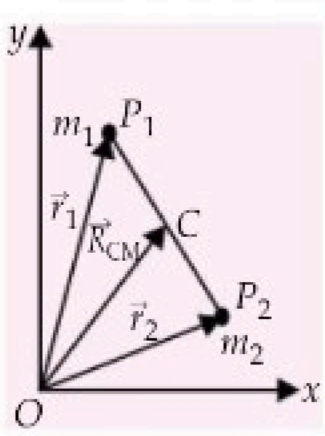
CENTRE OF MASS

The centre of mass of a system of particles is a point that moves as if

- All of the system mass is concentrated there and
- All external forces are applied there.

Centre of mass of a two particle system

- Consider a system of two particles P_1 and P_2 of masses m_1 and m_2 respectively. Let $\vec{r}_1$ and $\vec{r}_2$ be their position vectors with respect to the origin O, as shown in Fig. The position vector $\vec{R}_{\text{CM}}$ of the centre of mass C of the two particle system is given by



$$\vec{R}_{\text{CM}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

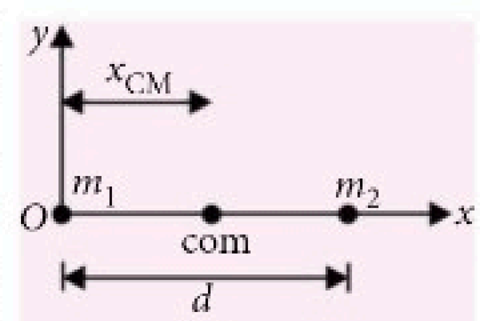
If $m_1 = m_2 = m$(say), then, $\vec{R}_{\text{CM}} = \dfrac{\vec{r}_1 + \vec{r}_2}{2}$

Thus the centre of mass of two equal masses lies exactly at the centre of the line joining the two masses.

If (x_1, y_1) and (x_2, y_2) are the coordinates of the locations of the two particles, then the coordinates of their centre of mass are given by

$$X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \text{ and } Y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$$

- In figure (a), two particles of masses m_1 and m_2 are separated by distance d. The position of the centre of mass (CM) of these two particles is given by



(a)

$$x_{\text{CM}} = \frac{m_2}{m_1 + m_2}d$$

Centre of mass of *n*-particles system

- For a system of n-particles of masses m_1, m_2, m_3, ..., m_n having the position vectors $\vec{r}_1, \vec{r}_2, ..., \vec{r}_n$ respectively with respect to coordinate system, the position of the centre of mass is given by a position vector

$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ... + m_n\vec{r}_n}{m_1 + m_2 + ... + m_n} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{M}$$

where, M is the total mass of the system.
The coordinates of centre of mass are given by

$$x_{\text{CM}} = \frac{\sum_{i=1}^{n} m_ix_i}{M} \; ; \; y_{\text{CM}} = \frac{\sum_{i=1}^{n} m_iy_i}{M} \; ; \; z_{\text{CM}} = \frac{\sum_{i=1}^{n} m_iz_i}{M}$$

Centre of mass of a rigid body (or continuous distribution of mass)

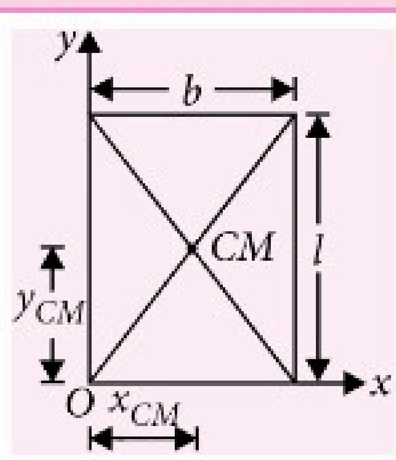
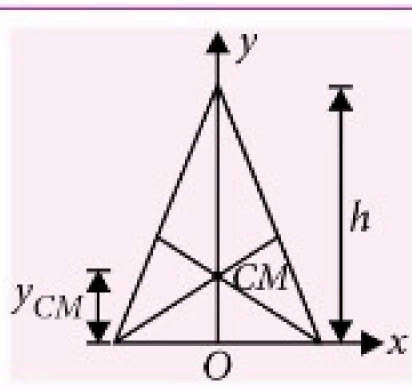
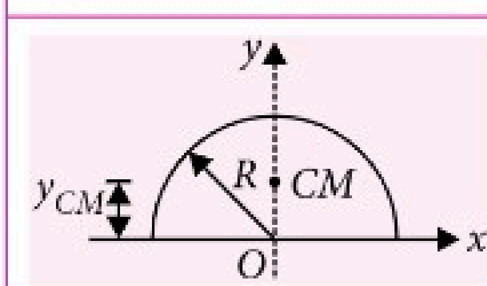
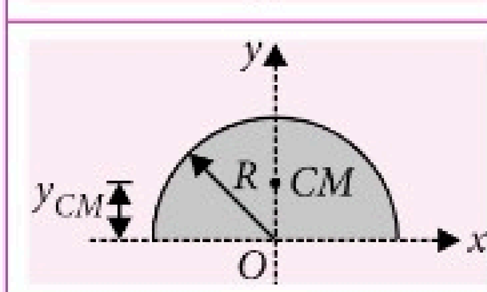
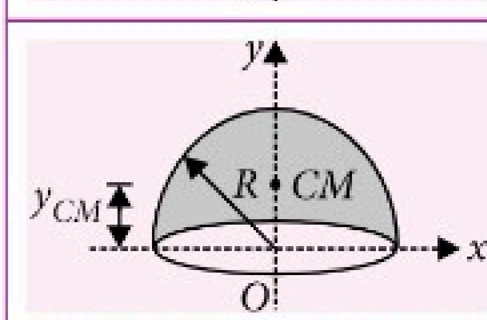
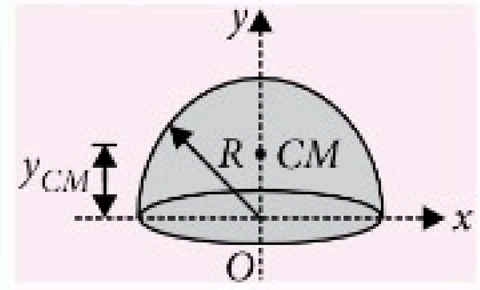
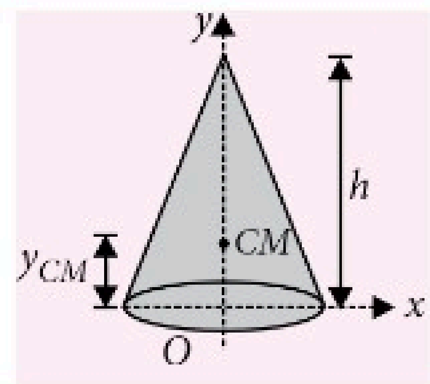
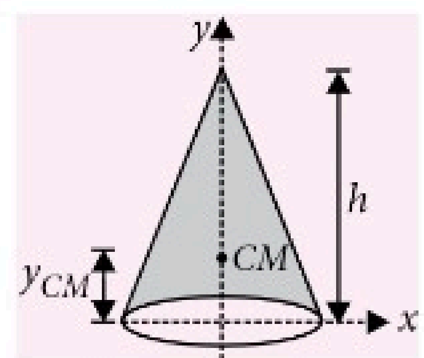
- The solid bodies contain so many particles (atoms) that we can treat the bodies as a continuous distribution of matter.

- The 'particles' then become differential mass elements dm, the sum become integrals, and the coordinates of the centre of mass are defined as

$$x_{CM} = \frac{1}{M}\int x\,dm\ ;\ y_{CM} = \frac{1}{M}\int y\,dm\ ;\ z_{CM} = \frac{1}{M}\int z\,dm$$

where M is the mass of the body.

- The position of the centre of mass of a system is independent of the choice of coordinate system.
- The position of the centre of mass depends on the shape of the body and the distribution of its mass. Hence it may lie within or outside the material of the body.
- In symmetrical bodies in which the distribution of mass is homogeneous, the centre of mass coincides with the centre of symmetry or geometrical centre.
- **Centre of mass of some well known symmetric rigid bodies :**

Position of Centre of Mass of different bodies	
	Rectangular plate $x_{CM} = \frac{b}{2}$, $y_{CM} = \frac{l}{2}$
	Triangular plate At the centroid, $y_{CM} = \frac{h}{3}$, $x_{CM} = 0$
	Semi-circular ring $y_{CM} = \frac{2R}{\pi}$, $x_{CM} = 0$
	Semi-circular disc $y_{CM} = \frac{4R}{3\pi}$, $x_{CM} = 0$
	Hemispherical shell $y_{CM} = \frac{R}{2}$, $x_{CM} = 0$
	Solid hemisphere $y_{CM} = \frac{3R}{8}$, $x_{CM} = 0$
	Circular cone (solid) $y_{CM} = \frac{h}{4}$, $x_{CM} = 0$
	Circular cone (hollow) $y_{CM} = \frac{h}{3}$, $x_{CM} = 0$

Motion of Centre of Mass

- For a system of particles, position of centre of mass is $\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...}$

So, Velocity of centre of mass

$$\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ...}{m_1 + m_2 + ...} \qquad \left(\because \frac{d\vec{r}}{dt} = \vec{v}\right)$$

- Acceleration of centre of mass

$$\vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...}{m_1 + m_2 + ...} \qquad \left(\because \vec{a} = \frac{d\vec{v}}{dt}\right)$$

- Linear momentum of a system of particles is equal to the product of mass of the system with the velocity of its centre of mass.
- If no external force acts on a system the velocity of its centre of mass remains constant, *i.e.*, velocity of centre of mass is unaffected by internal forces.

From Newton's second law $\vec{F}_{ext} = \frac{d(M\vec{v}_{CM})}{dt}$

If $\vec{F}_{ext} = 0$ then $\vec{v}_{CM}$ = constant.

Angular velocity and Angular acceleration

Angular Velocity

- It is defined as the time rate of change of angular displacement and is given by $\omega = \frac{d\theta}{dt}$.

- Angular velocity is directed along the axis of rotation. Angular velocity is a vector quantity. Its SI unit is rad s^{-1} and its dimensional formula is $[M^0L^0T^{-1}]$.

Relationship between Linear velocity and Angular Velocity

- The linear velocity of a particle of a rigid body rotating about a fixed axis is given by $\vec{v} = \vec{\omega} \times \vec{r}$ where $\vec{r}$ is the position vector of the particle with respect to an origin along the fixed axis.
- As in pure translation motion, all particles of the body have the same linear velocity at any instant of time, in pure rotational motion, all particles of the body have the same angular velocity at any instant of time.
- For rotation about a fixed axis, the direction of the angular velocity ω does not change with time. Its magnitude, however may change from instant to instant. In general rotation, both the magnitude and direction of ω may change from instant to instant.

Angular Acceleration

- It is defined as the time rate of change of angular velocity and is given by, $\alpha = \frac{d\omega}{dt}$
- Angular acceleration is a vector quantity. Its SI unit is rad s^{-2} and its dimensional formula is $[M^0L^0T^{-2}]$

Torque and Angular Momentum

Torque

- Torque is the turning effect of a force. If a force acting on an object has a tendency to rotate the body about an axis, the force is said to exert a torque on the body. It is a vector quantity.
- In vector form,

  Torque, $\tau = \vec{r} \times \vec{F}$

  In magnitude, $\tau = r\,F\sin\theta$.

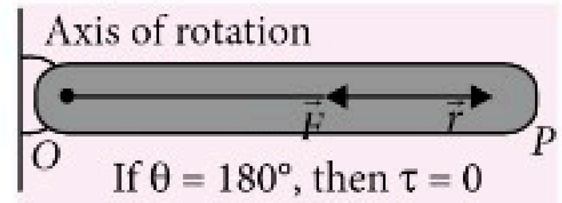
  Here θ is the angle between $\vec{r}$ and $\vec{F}$.
- Torque has the same dimensions as that of work *i.e.* $[ML^2T^{-2}]$. But work is a scalar quantity whereas torque is a vector quantity.
- By convention, anticlockwise moments are taken as positive and clockwise moments are taken as negative.

Special Cases

- 

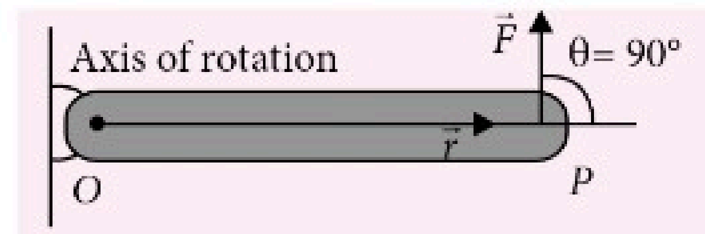
  If $\theta = 0°$, then $\tau = 0$

  (the rod will not rotate)

- 

  If $\theta = 180°$, then $\tau = 0$

  (the rod will not rotate)

- 

  If $\theta = 90°$, then $\tau = rF$ (maximum torque)

Work Done by a Torque and Power

- Work done, W = torque × angular displacement $= \tau \times \Delta\theta$
- Power, $P = \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega$

Angular Momentum

- Angular momentum of a particle about a given axis is the moment of linear momentum of the particle about that axis. It is denoted by symbol $\vec{L}$.
- Angular momentum $\vec{L} = \vec{r} \times \vec{p}$

  In magnitude, $L = rp\sin\theta$

  where θ is the angle between $\vec{r}$ and $\vec{p}$.
- Angular momentum is a vector quantity. Its SI unit is kg m^2 s^{-1}. Its dimensional formula is $[ML^2T^{-1}]$.

Relationship between Torque and Angular Momentum

Rate of change of angular momentum of a body is equal to the external torque acting upon the body.

Mathematically, $\vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$

Law of Conservation of Angular Momentum

If no external torque acts on a system then the total angular momentum is conserved.

i.e., $\vec{\tau}_{ext} = 0$ then $\frac{d\vec{L}}{dt} = 0$ or $\vec{L} = \text{constant}$

Thus, if no external torque acts on a system, total angular momentum of the system remains constant. This is a statement of law of conservation of angular momentum.

Applications of Conservation of Angular Momentum

- A planet revolves around the sun in an elliptical path. When it comes near the sun, the moment of inertia of the planet about the sun decreases.

In order to conserve the angular momentum, the angular velocity shall increase. Similarly, when the planet is away from the sun, there will be decrease in the angular velocity.

- An ice-skater or a ballet dancer can increase her angular velocity by folding her arms and bringing the stretched leg close to the other leg. When she stretches her hands and a leg outward, her moment of inertia increases and hence angular speed decreases to conserve angular momentum. When she folds her arms and brings the stretched leg close to the other leg, her moment of inertia decreases and hence angular speed increases.

EQUILIBRIUM OF A RIGID BODY

- A body is in rotational equilibrium, if the total external torque on it is zero, *i.e.* $\Sigma\tau_i = 0$.

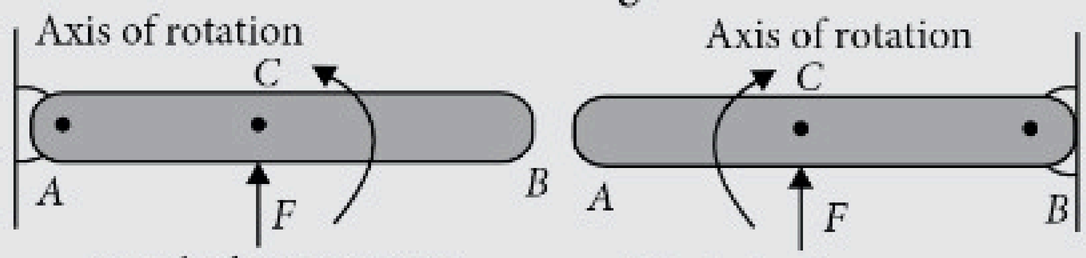
  $$\frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L} = \text{constant}$$

  i.e., the body rotates with a constant angular velocity.
- **Center of gravity :** A point of an extended body where the weight of the body acts and total gravitational torque on the body is zero, is known as the centre of gravity.

Key Point

- **Kinematics Equations of Rotational Motion**
- $\omega = \omega_0 + \alpha t$
- $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
- $\omega^2 = \omega_0^2 + 2\alpha\theta$

  where the symbols have their usual meanings.
- Same force acting at the same point can produce either anticlockwise or clockwise torque depending upon the location of the axis of rotation as shown in the figure.



Anticlockwise torque   Clockwise torque

MOMENT OF INERTIA

- The moment of inertia of a rigid body about a fixed axis is defined as the sum of the products of the masses of the particles constituting the body and the squares of their respective distances from the axis of rotation.
- The dimensional formula of moment of inertia is $[ML^2T^0]$. The SI unit of moment of inertia is kg m^2 and its CGS unit is g cm^2.
- Momentum of inertia is the rotational analogue of mass in linear motion.

  The moment of inertia of a body depends on
  - Mass of the body
  - Size and shape of the body
  - Distribution of mass about the axis of rotation.
  - Position and orientation of the axis of rotation with respect to the body.

Radius of Gyration

- The radius of gyration of a body about its axis of rotation may be defined as the distance from the axis of rotation at which, if the whole mass of the body were concentrated, its moment of inertia about the given axis would be the same as with the actual distribution of mass. It is denoted by symbol k.
- The relation between moment of inertia I and radius of gyration k is

  $$I = Mk^2 \text{ or } k = \sqrt{\frac{I}{M}}$$
- Radius of gyration of a body about an axis of rotation may also be defined as the root mean square distance of its particles from the axis of rotation.

  $$i.e., \ k = \sqrt{\frac{r_1^2 + r_2^2 + ... + r_n^2}{n}}$$

  The SI unit of radius of gyration is m.

Theorem of Perpendicular Axes

- It states that the moment of inertia of a plane lamina about an axis perpendicular to its plane is equal to the sum of the moments of inertia of the lamina about any two mutually perpendicular axes in its plane and intersecting each other at the point, where the perpendicular axes pass through the lamina. Mathematically,

  $$I_z = I_x + I_y$$

  where x-and y-axes lie in the plane of the lamina and z-axis is perpendicular to its plane and passes through the point of intersection of x and y-axes.

Theorem of Parallel Axes

- It states that the moment of inertia of a body about any axis is equal to the sum of the moment of inertia of the body about a parallel axis passing through its centre of mass and the product of its mass and the square of the distance between the two parallel axes. Mathematically, $I = I_{CM} + Md^2$

where I_{CM} is the moment of inertia of the body about an axis passing through its centre of mass and d is the perpendicular distance between the two parallel axes.

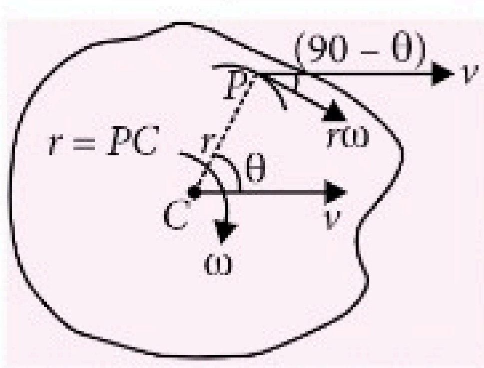
Moment of inertia and radius of gyration of some regular bodies about specific axis is given below :

S.No.	Body	Axis of rotation	Moment of inertia (I)	Radius of gyration (K)
1.	Uniform circular ring of mass M and radius R	(i) about an axis passing through the centre and perpendicular to its plane	MR^2	R
		(ii) about a diameter	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
		(iii) about a tangent in its own plane	$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$
		(iv) about a tangent perpendicular to its plane	$2MR^2$	$R\sqrt{2}$
2.	Uniform circular disc of mass M and radius R	(i) about an axis passing through its centre and perpendicular to its plane	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
		(ii) about a diameter	$\frac{1}{4}MR^2$	$R/2$
		(iii) about a tangent in its own plane	$\frac{5}{4}MR^2$	$\sqrt{5}\frac{R}{2}$
		(iv) about a tangent perpendicular to its own plane	$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$
3.	Solid sphere of radius R and mass M	(i) about its diameter	$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$
		(ii) about a tangential axis	$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$
4.	Hollow sphere of radius R and mass M	(i) about its diameter	$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$
		(ii) about a tangential axis	$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$
5.	Solid cylinder of length l, radius R and mass M	(i) about its own axis	$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
		(ii) about an axis passing through centre of mass and perpendicular to its own axis	$M\left[\frac{l^2}{12}+\frac{R^2}{4}\right]$	$\sqrt{\frac{l^2}{12}+\frac{R^2}{4}}$
		(iii) about the diameter of one of the faces of cylinder	$M\left[\frac{l^2}{3}+\frac{R^2}{4}\right]$	$\sqrt{\frac{l^2}{3}+\frac{R^2}{4}}$

6.	Hollow cylinder of mass M, length l and radius R	(i) about its own axis	MR^2	R
		(ii) about an axis passing through its centre of mass and perpendicular to its own axis	$M\left(\frac{R^2}{2}+\frac{l^2}{12}\right)$	$\sqrt{\frac{R^2}{2}+\frac{l^2}{12}}$
7.	Thin rod of length L and Mass M	(i) about an axis passing through the centre of mass and perpendicular to the rod	$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$
		(ii) about an axis passing through one end and perpendicular to rod	$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$
8.	Rectangular lamina of length l breadth b and mass M	about an axis passing through its centre of mass and perpendicular to plane	$M\left[\frac{l^2+b^2}{12}\right]$	$\sqrt{\frac{l^2+b^2}{12}}$
9.	Uniform solid cone of radius R, and mass M height h	about an axis through its centre of mass and joining its vertex to centre of base	$\frac{3}{10}MR^2$	$R\sqrt{\frac{3}{10}}$
10.	Parallelopiped of length l, breadth b and height h, mass M	about its central axis	$M\left(\frac{l^2+b^2}{12}\right)$	$\sqrt{\frac{l^2+b^2}{12}}$

Rolling

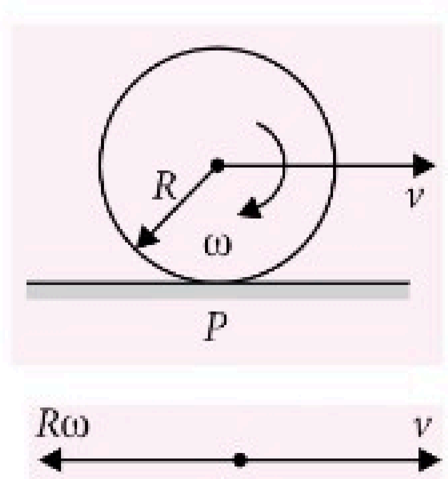
- Rolling is a special phenomenon and it can be understood from two perspectives. Firstly, it can be seen as a special combination of rotation and translation. Secondly, it can be seen as pure rotation about its point of contact.
- Here, velocity of point P is the vector sum of two terms v and $r\omega$. Here v is common for all points, while $r\omega$ is different for different points, as r is different.



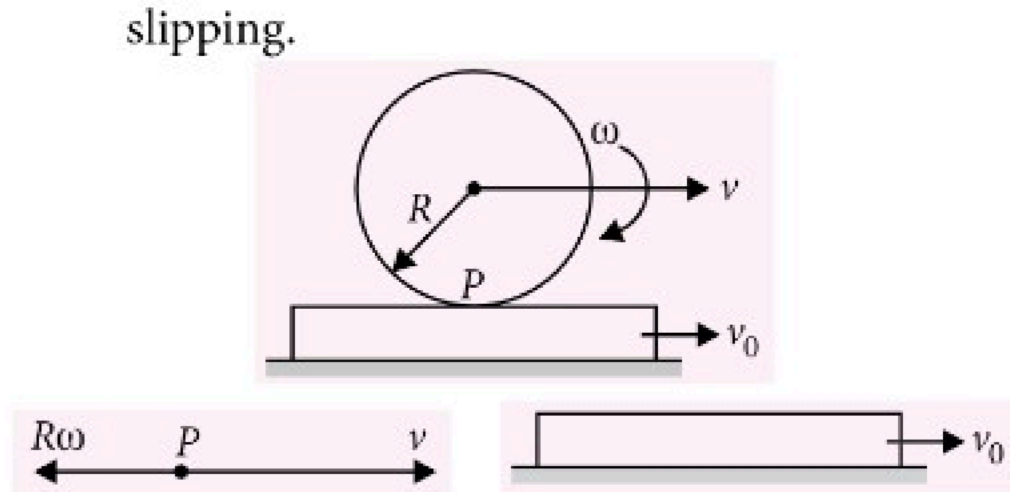
$$v_P = \sqrt{v^2 + (r\omega)^2 + 2(v)(r\omega)\cos(90° - \theta)}$$
$$= \sqrt{v^2 + r^2\omega^2 + 2vr\omega\sin\theta}$$

- Uniform pure rolling : In which v and ω remain constant.

Condition of pure rolling is $v = R\omega$. In this case bottommost point of the spherical body is at rest. It has no slipping with its contact point on ground. Because ground point is also at rest.



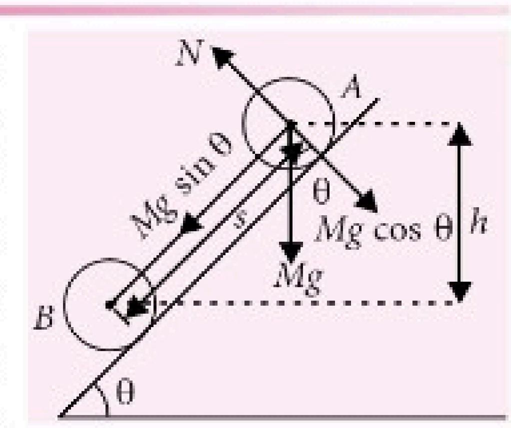
- If $v > R\omega$, then net velocity of point P is in the direction of v. This is called forward slipping.
- If $v < R\omega$, then net velocity of point P is in opposite direction of v. This is called backward slipping.



- If a spherical body is rolling over a plank, condition for no slipping between spherical body and plank is, $v - R\omega = v_0$

Motion of a Body Rolling without Slipping Down an Inclined Plane

- When a rigid body rolls down an inclined plane without slipping, it has a motion of translation as well as of rotation. As it rolls down, it suffers a vertical fall and therefore losses potential energy. At the same time it acquires linear and angular speeds and hence gains kinetic



energy of translation and that of rotation. If there is no loss of energy through friction etc., the loss in gravitational potential energy is equal to the gain in kinetic energy.

- Consider a plane inclined at an angle θ to the horizontal and M be the mass and R the radius of that circular section of the body which rolls on the plane.
- Let initially the body be at point A and at rest and after some time it reaches B (figure.).
- In moving from point A to B, the body travels a vertical distance h given by $h = s \sin\theta$, and hence the loss of gravitational potential energy of the body = $Mgh = Mgs \sin\theta$
- Because there is no slipping and it is assumed that there is no dissipation of energy in any other way, the loss in potential energy must be equal to the gain in kinetic energy of the rolling body. Thus,

$$Mgs\sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

Where I is moment of inertia of the body about an axis passing through its centre and perpendicular to its end faces. Now above equation can be written as

$$Mgs\sin\theta = \frac{1}{2}Mv^2 + \frac{1}{2}I\left(\frac{v^2}{R^2}\right) \quad \text{...(i)}$$

On solving,

$$v = \sqrt{\frac{2Mgh}{M + I/R^2}} = \sqrt{\frac{2gh}{1+\frac{k^2}{R^2}}}$$

- Differentiating equation (i), w.r.t. time, we get acceleration of rolling body,

$$a = \frac{Mg\sin\theta}{M + I/R^2} = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$$

Analogy between Translational Motion and Rotational Motion

The analogy between quantities that describe linear motion and the corresponding quantities that describe rotational motion is as shown in the table.

	Linear motion	Rotational motion about a fixed axis
1.	Displacement, x	Angular displacement, θ
2.	Velocity, $v = \frac{dx}{dt}$	Angular velocity, $\omega = \frac{d\theta}{dt}$
3.	Acceleration $a = \frac{dv}{dt}$	Angular acceleration $\alpha = \frac{d\omega}{dt}$
4.	Mass, M	Moment of inertia, I
5.	Force, $F = Ma$	Torque, $\tau = I\alpha$
6.	Work done, $W = Fs$	Work done, $W = \tau\theta$
7.	Translational kinetic energy, $K_T = \frac{Mv^2}{2}$	Rotational kinetic energy, $K_R = \frac{I\omega^2}{2}$
8.	Power, $P = Fv$	Power, $P = \tau\omega$
9.	Linear momentum, $p = Mv$	Angular momentum, $L = I\omega$
10.	Equations of translational motion $v = u + at$, $s = ut + \frac{1}{2}at^2$, $v^2 - u^2 = 2as$ where the symbols have their usual meaning.	Equations of rotational motion $\omega = \omega_0 + \alpha t$, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, $\omega^2 - \omega_0^2 = 2\alpha\theta$ where the symbols have their usual meaning.

Key Point

- Time of descent of a body rolling down an inclined plane, from equation $s = ut + \frac{1}{2}at^2$ will be,

$$t = \frac{1}{\sin\theta}\sqrt{\frac{2h}{gM}\left(M + \frac{I}{R^2}\right)} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{k^2}{R^2}\right)}$$

- Moment of Inertia of annular disc through an axis

Passing through the centre and perpendicular to the plane

$$I = \frac{M}{2}[R_1^2 + R_2^2]$$

Passing through its diameter.

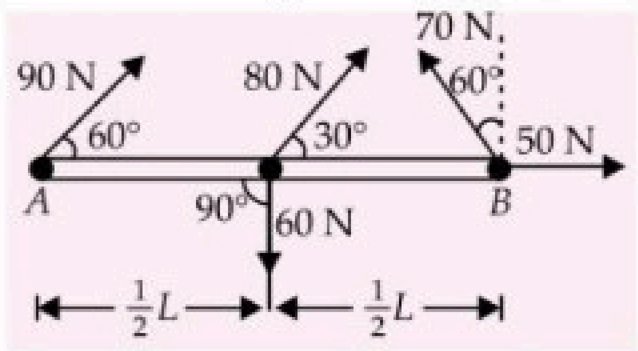
$I = \frac{M}{4}[R_1^2 + R_2^2]$, where R_1 and R_2 are the inner and outer radius respectively.

- Moment of inertia of an equilateral triangular lamina with side a passing through its centre of mass and perpendicular to the plane

$$I = \frac{Ma^2}{6}$$

WRAP it up!

1. The total torque about pivot A provided by the forces shown in the figure, for $L = 3.0$ m, is



(a) 210 N m (b) 140 N m
(c) 95 N m (d) 75 N m

2. The moment of inertia of a sphere of mass M and radius R about an axis passing through its centre is $\frac{2}{5}MR^2$. The radius of gyration of the sphere about a parallel axis to the above and tangent to the sphere is

(a) $\frac{7}{5}R$ (b) $\frac{3}{5}R$ (c) $\sqrt{\frac{7}{5}}R$ (d) $\sqrt{\frac{3}{5}}R$

3. A force $\vec{F} = \alpha\hat{i} + 3\hat{j} + 6\hat{k}$ is acting at a point $\vec{r} = 2\hat{i} - 6\hat{j} - 12\hat{k}$. The value of α for which angular momentum about origin is conserved is

(a) 1 (b) −1 (c) 2 (d) zero

4. A wire of length l and mass m is bent in the form of a rectangle $ABCD$ with $\frac{AB}{BC} = 2$. The moment of inertia of this wire frame about the side BC is

(a) $\frac{11}{252}ml^2$ (b) $\frac{8}{203}ml^2$
(c) $\frac{5}{136}ml^2$ (d) $\frac{7}{162}ml^2$

5. If the earth suddenly changes its radius x times the present value, the new period of rotation would be

(a) $6x^2$ hr (b) $12x^2$ hr
(c) $24x^2$ hr (d) $48x^2$ hr

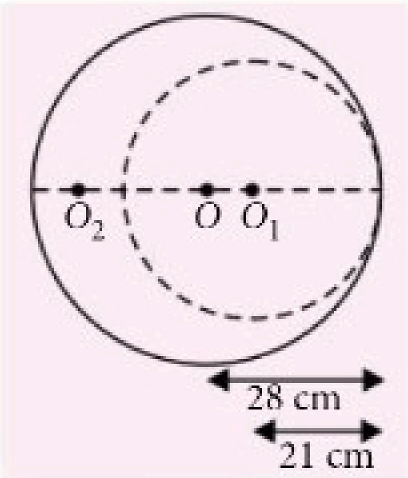
6. The ratio of angular momentum of earth for its orbital motion and its daily rotation is

(a) much greater than 1
(b) smaller than 1
(c) equal to 1
(d) slightly greater than 1

7. The moment of inertia of uniform circular disc of radius R and mass M about an axis passing from the edge of the disc and normal to the disc is

(a) $\frac{1}{2}MR^2$ (b) $\frac{7}{2}MR^2$ (c) $\frac{3}{2}MR^2$ (d) MR^2

8. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge as shown in figure. The centre of mass of the remaining portion from the centre of plate will be at



(a) 5 cm (b) 7 cm
(c) 9 cm (d) 11 cm

9. Which of the following statements is false for a particle moving in a circle with a constant angular speed?

(a) The velocity vector is tangent to the circle.
(b) The acceleration vector is tangent to the circle.
(c) The acceleration vector points to the centre of the circle.
(d) The velocity and acceleration vectors are perpendicular to each other.

10. Two solid spheres (A and B) are made of metals of different densities ρ_A and ρ_B respectively. If their masses are equal, the ratio of their moments of inertia (I_A/I_B) about their respective diameter is

(a) $\left(\frac{\rho_B}{\rho_A}\right)^{2/3}$ (b) $\left(\frac{\rho_A}{\rho_B}\right)^{2/3}$
(c) $\frac{\rho_A}{\rho_B}$ (d) $\frac{\rho_B}{\rho_A}$

11. Four particles each of mass m are lying symmetrically on the rim of a disc of mass M and radius R. Moment of inertia of this system about an axis passing through one of the particles and perpendicular to plane of disc is

(a) $16mR^2$ (b) $(3M+16m)\frac{R^2}{2}$
(c) $(3m+12M)\frac{R^2}{2}$ (d) zero

12. A rigid body rotates about a fixed axis with variable angular velocity equal to $(a - bt)$ at time t where a and b are constants. The angle through which it rotates before it comes to rest is

(a) $\frac{a^2}{b}$ (b) $\frac{a^2}{2b}$ (c) $\frac{a^2}{4b}$ (d) $\frac{a^2}{2b^2}$

13. An athlete throws a discus from rest to a final angular velocity of 15 rad s^{-1} in 0.270 s before releasing it. During motion, discus moves a circular arc of radius 0.810 m. Acceleration of discus before it is released is

(a) 45 m s^{-2} (b) 182 m s^{-2}
(c) 187 m s^{-2} (d) 192 m s^{-2}

14. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement is correct?

(a) Both cylinders P and Q reach the ground at the same time.
(b) Cylinder P has larger linear acceleration than cylinder Q.
(c) Both cylinders reach the ground with same translational kinetic energy.
(d) Cylinder Q reaches the ground with larger angular speed.

15. A uniform rod of length 1 m is bent at its midpoint to make 90° angle. The distance of the centre of mass from the centre of the rod is

(a) 36.1 cm (b) 25.2 cm
(c) 17.7 cm (d) zero

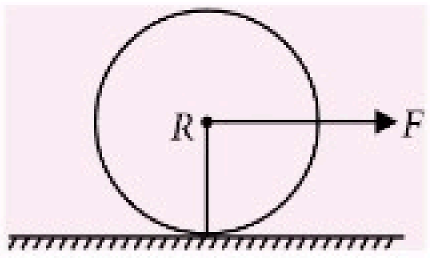
16. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is

(a) $I_Y = 32I_X$ (b) $I_Y = 16I_X$
(c) $I_Y = I_X$ (d) $I_Y = 64I_X$

17. The moment of inertia of a circular disc of mass M and radius R about an axis passing through the center of mass is I_0. The moment of inertia of another circular disc of same mass and thickness but half the density about the same axis is

(a) $\frac{I_0}{8}$ (b) $\frac{I_0}{4}$ (c) $8I_0$ (d) $2I_0$

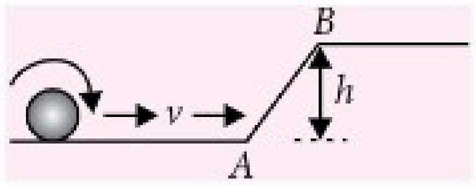
18. A uniform disc of mass M and radius R is resting on a table on its rim. The coefficient of friction between disc and table is μ.



Now the disc is pulled with a force F as shown in the figure. What is the maximum value of F for which the disc rolls without slipping?

(a) μMg (b) $2\mu Mg$ (c) $3\mu Mg$ (d) $4\mu Mg$

19. A solid sphere is rolling on a frictionless surface as shown in the figure,



with a translational velocity of v m s^{-1}. If it is to climb the inclined surface, then v should be

(a) $2gh$ (b) $\frac{10}{7}gh$

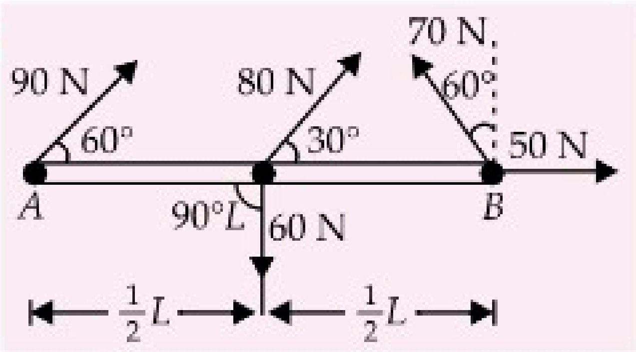
(c) $> \sqrt{2gh}$ (d) $\geq \sqrt{\frac{10}{7}gh}$

20. A solid cylinder rolls up a smooth inclined plane of angle of inclination 30°. At the bottom of the inclined plane the centre of mass of the cylinder has a speed of 5 m s^{-1}. How far will the cylinder go up the plane? (Take g = 10 m s^{-2})

(a) $\frac{10}{3}$ m (b) $\frac{3}{10}$ m (c) $\frac{4}{13}$ m (d) $\frac{15}{4}$ m

SOLUTIONS

1. (d):



Resolve the 90 N, 80 N and 70 N forces into x and y components. The line of action of 90 N, 50 N, and x-components of the 80 N and 70 N forces pass through the pivot point A, therefore they cause no rotation.

∴ The total torque about point A is

$$= (80\sin 30^\circ)\left(\frac{L}{2}\right) - (60)\left(\frac{L}{2}\right) + (70\cos 60^\circ)(L)$$

$$= (80)\left(\frac{1}{2}\right)\left(\frac{3}{2}\right) - (60)\left(\frac{3}{2}\right) + (70)\left(\frac{1}{2}\right)(3)$$

$$= 60 - 90 + 105 = 75 \text{ N m}$$

2. (c) : Given $I_c = \frac{2}{5}MR^2$

Using the theorem of parallel axes, moment of inertia of the sphere about a parallel axis tangential to the sphere is

$$I' = I_c + MR^2 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

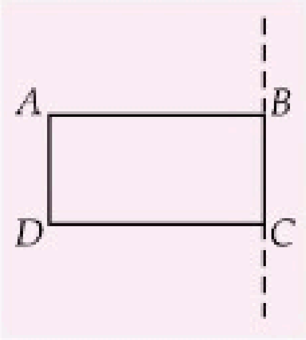
$$\therefore \quad I' = MK^2 = \frac{7}{5}MR^2 \text{ or } K = \sqrt{\frac{7}{5}}R$$

3. (b): Angular momentum, L = constant, when $\vec{\tau} = \vec{r} \times \vec{F} = 0$

i.e. $(2\hat{i} - 6\hat{j} - 12\hat{k}) \times (\alpha\hat{i} + 3\hat{j} + 6\hat{k}) = 0$

or $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & -12 \\ \alpha & 3 & 6 \end{vmatrix} = 0$, which gives $\alpha = -1$

4. (d): $\frac{AB}{BC} = 2 \quad \therefore \quad AB = DC = \frac{l}{3}$



and $BC = AD = \frac{l}{6}$

Similarly, $m_{AB} = m_{DC} = \frac{m}{3}$

and $m_{BC} = m_{AD} = \frac{m}{6}$

$\therefore$ Moment of inertia of the wire frame about the given axis is

$$I = I_{AB} + I_{AD} + I_{DC} + I_{BC}$$

$$= \frac{1}{3}\left(\frac{m}{3}\right)\left(\frac{l}{3}\right)^2 + \left(\frac{m}{6}\right)\left(\frac{l}{3}\right)^2 + \frac{1}{3}\left(\frac{m}{3}\right)\left(\frac{l}{3}\right)^2 + 0$$

$$= \frac{ml^2}{81} + \frac{ml^2}{54} + \frac{ml^2}{81} = \frac{7}{162}ml^2$$

5. (c) : As no torque is applied, angular momentum $L = I\omega$ = constant

or $\left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right)$ = constant

i.e., $\frac{R^2}{T}$ = constant or $\frac{R_1^2}{T_1} = \frac{R_2^2}{T_2}$

$$T_2 = \frac{R_2^2}{R_1^2}T_1 = \left(\frac{xR_1}{R_1}\right)^2 \times 24 \text{ hr} = 24x^2 \text{ hr}$$

6. (a) : For orbital motion of earth around the sun

$$L_1 = I_1\omega_1 = \text{Mass (radius of orbit)}^2 \times \frac{2\pi}{T_1}$$

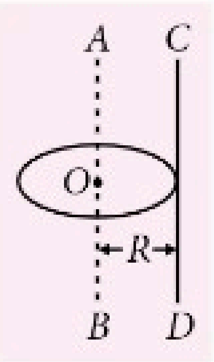
$$= 2.66 \times 10^{40} \text{ kg m}^2 \text{ s}^{-1}$$

For rotation around its axis, $L_2 = I_2\omega_2 = \frac{2}{5}MR^2 \times \frac{2\pi}{T_2}$

$$= 7.1 \times 10^{33} \text{ kg m}^2 \text{ s}^{-1}$$

$$\therefore \quad \frac{L_1}{L_2} >> 1$$

7. (c) : $I_{AB} = \frac{1}{2}MR^2$



I_{CD} = ?

Applying the theorem of parallel axes,

$$I_{CD} = I_{AB} + MR^2$$

$$= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

8. (c) : Let M be the mass of circular plate with centre O.

Mass per unit area $= \frac{M}{\pi(28)^2}$

$\therefore$ Mass of circular portion removed, with centre O_1

$$M_1 = \frac{M}{\pi(28)^2} \times \pi(21)^2 = \frac{9}{16}M$$

Mass left, $M_2 = M - M_1 = M - \frac{9}{16}M = \frac{7}{16}M$

Let O_2 be its centre of mass, where $OO_2 = x$.

Now, $M_1 \times OO_1 = M_2 \times OO_2$

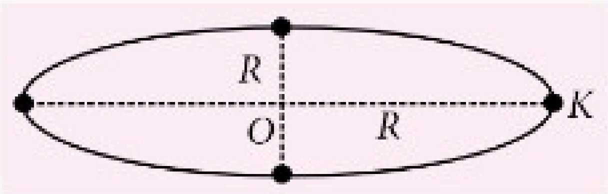
$$\frac{9}{16}M \times 7 = \frac{7}{16}M \times x \Rightarrow x = 9 \text{ cm}$$

9. (b): For a particle moving in a circle with constant angular speed, velocity vector is always tangent to the circle and the acceleration vector always points towards the centre of the circle or is always along radius of the circle. Since, tangential vector is perpendicular to radial vector therefore velocity vector will be perpendicular to the acceleration vector. But in no case acceleration vector is tangent to the circle.

10. (a)

11. (b): According to the theorem of parallel axes, moment of inertia of disc about an axis passing through K and perpendicular to plane of disc, is

$$= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$



Total moment of inertia of the system

$$= \frac{3}{2}MR^2 + m(2R)^2 + m(\sqrt{2}R)^2 + m(\sqrt{2}R)^2$$

$$= (3M + 16m)\frac{R^2}{2}$$

12. (b): $\omega = a - bt$

At time $t = 0$, $\omega = \omega_0 = a$

$$\alpha = \frac{d\omega}{dt} = -b$$

Using $\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow 0 = \omega_0^2 + 2\alpha\theta$ or $\theta = -\frac{\omega_0^2}{2\alpha} = \frac{a^2}{2b}$

13. (a)

14. (d): Acceleration of a body rolling down a fixed inclined plane of inclination θ is, $a=\dfrac{g\sin\theta}{1+\dfrac{k^2}{R^2}}$ ...(i)

As cylinder P has most of its mass concentrated near its surface, while cylinder Q has most of its mass concentrated near the axis, therefore, radius of gyration,

$k_P > k_Q$

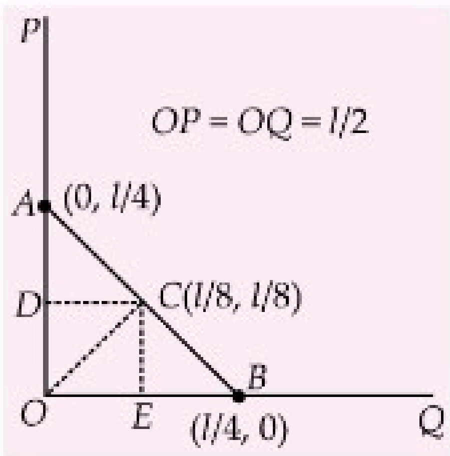
From (i), $a_P < a_Q$

Therefore, at bottom of incline, $v_P < v_Q$

or $(\omega_P R) < (\omega_Q R)$ or $\omega_P < \omega_Q$ or $\omega_Q > \omega_P$

i.e., cylinder Q reaches the ground with larger angular speed.

15. (c) : Rod POQ of length l = 1 m = 100 cm is bent at its midpoint O so that $\angle POQ$ = 90° (see figure). The mass of part PO of length $l/2$ can be taken to be concentrated at its midpoint A whose coordinates are (0, $l/4$) and of part OQ of length $l/2$ at its midpoint B whose coordinates are ($l/4$, 0). The centre of mass of these two equal masses is at midpoint C between A and B. The coordinates of C are ($l/8$, $l/8$).



$$\therefore\ OC=\sqrt{(OE)^2+(CE)^2}=\sqrt{\left(\frac{l}{8}\right)^2+\left(\frac{l}{8}\right)^2}$$

$$=\frac{l}{\sqrt{32}}=\frac{100\text{ cm}}{\sqrt{32}}=17.7\text{ cm}$$

16. (d): Mass of disc (X), $m_X=\pi R^2 t\rho$

where ρ = density of material of disc

$$\therefore\ I_X=\frac{1}{2}m_XR^2=\frac{1}{2}\pi R^2 t\rho R^2 \text{ or } I_X=\frac{1}{2}\pi\rho t R^4$$

Mass of disc (Y)

$$m_Y=\pi(4R)^2\frac{t}{4}\rho=4\pi R^2 t\rho$$

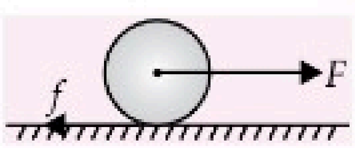
$$I_Y=\frac{1}{2}m_Y(4R)^2=\frac{1}{2}4\pi R^2 t\rho\times 16R^2$$

or $I_Y=32\pi t\rho R^4$ $\therefore\ \dfrac{I_Y}{I_X}=\dfrac{32\pi t\rho R^4}{\frac{1}{2}\pi\rho t R^4}=64$

or $I_Y=64I_X$

17. (d)

18. (c) : Let a be the acceleration of the centre of mass of disc. Then



$Ma=F-f$ ...(i)

If there is no slipping, angular acceleration of the disc,

$\alpha=\dfrac{a}{R}$ ... (ii)

Now torque of the disc, $\tau=I\alpha=Rf$

or $\tau=\left(\frac{1}{2}MR^2\right)\alpha=Rf$ $\left[\because \text{ For a disc, } I=\frac{1}{2}MR^2\right]$

$\therefore\ \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right)=Rf \Rightarrow Ma=2f$ (using (ii))

Substituting this in eq. (i), we get

$2f=F-f$ or $f=\dfrac{F}{3}$

Since there is no slipping,

$\therefore\ f\le\mu Mg \Rightarrow F\le 3\mu Mg$

$\therefore\ F_{max}=3\mu Mg$

19. (d): When the solid sphere rolls, its rolling K.E. at A

$$K_A=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2=\frac{1}{2}mv^2+\frac{1}{2}\cdot\frac{2}{5}mR^2\cdot\frac{v^2}{R^2}=\frac{7}{10}mv^2$$

Its P.E. at $B=mgh$

Thus to reach B, $\dfrac{7}{10}mv^2=mgh$

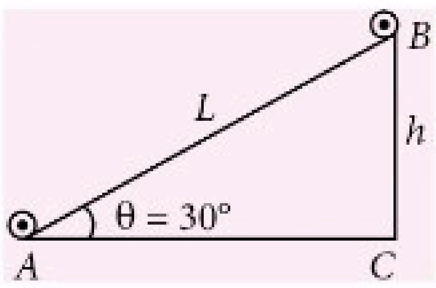
$$\therefore\ v^2=\frac{10gh}{7} \Rightarrow v=\sqrt{\frac{10gh}{7}}$$

This is the minimum velocity.

$\therefore$ v should be $\ge\sqrt{\dfrac{10}{7}gh}$

20. (d) : From figure, $\sin\theta=\dfrac{h}{L}$; $L=\dfrac{h}{\sin\theta}=\dfrac{h}{1/2}=2h$

Suppose that the solid cylinder of radius R goes from A to B, where $AB=L$ and height $h=BC$.



At A, the rolling energy of the cylinder $=\dfrac{1}{2}mv^2+\dfrac{1}{2}I\omega^2$

For the solid cylinder, $I=\dfrac{mR^2}{2}$ and $\omega=\dfrac{v}{R}$

$$\therefore\ E_{Rolling}=\frac{1}{2}mv^2+\frac{1}{2}\cdot\frac{mR^2}{2}\times\frac{v^2}{R^2}=\frac{3}{4}mv^2$$

It stops at B and have the P.E. = mgh

By the principle of conservation of energy,

$$\therefore\ mgh=\frac{3}{4}mv^2 \Rightarrow h=\frac{3v^2}{4g}$$

$$\therefore\ L=2h=\frac{3v^2}{2g}=\frac{3\times5\times5}{2\times10}=\frac{15}{4}\text{ m}$$

CBSE warm-up!

CLASS-XI

TERM-I OBJECTIVE TYPE QUESTIONS

Practice Paper 2021

Time allowed : 90 minutes
Maximum marks : 35

GENERAL INSTRUCTIONS

1. The Question Paper contains three sections.
2. Section A has 25 questions. Attempt any 20 questions.
3. Section B has 24 questions. Attempt any 20 questions.
4. Section C has 6 questions. Attempt any 5 questions.
5. All questions carry equal marks.
6. There is no negative marking.

SECTION - A

This section consists of 25 multiple choice questions with overall choice to attempt any 20 questions. In case more than desirable number of questions are attempted, ONLY first 20 will be considered for evaluation.

1. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as

(a) t^3 (b) t^2 (c) t (d) $t^{1/2}$

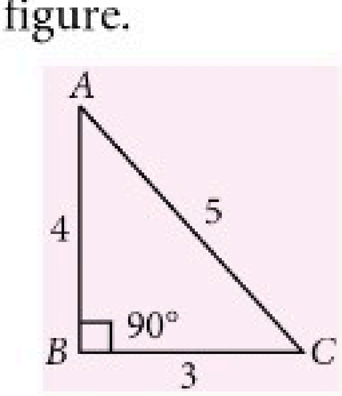
2. When a speeding bus stops suddenly, passengers are thrown forward from their seats because

(a) the back of seat suddenly pushes the passengers forward.
(b) inertia of rest stops the bus and takes the body forward.
(c) upper part of the body continues to be in the state of motion whereas the lower part of the body in contact with seat remains at rest.
(d) upper part of the body come to rest whereas the lower part of the body in contact with seat begins to move.

3. A 30 m deep well is having water upto 15 m. An engine evacuates it in one hour. The power of the engine, if the diameter of the well is 4 m is

(a) 11.55 kW (b) 1155 kW
(c) 23.10 kW (d) 2310 kW

4. ABC is a triangular plate of uniform thickness. Its sides are in the ratio shown in the figure. I_{AB}, I_{BC} and I_{CA} are the moments of inertia of the plate about AB, BC and CA as axes respectively. Which one of the following relations is correct?



(a) $I_{AB} + I_{BC} = I_{CA}$
(b) I_{CA} is maximum
(c) $I_{AB} > I_{BC}$ (d) $I_{BC} > I_{AC}$

5. The SI unit of electron mobility is

(a) $m^2\ V^{-1}\ s^{-1}$ (b) $m\ V^{-1}\ s$
(c) $m\ V\ s^{-1}$ (d) $m^2\ V^{-2}\ s^{-2}$

6. The acceleration a in m s^{-2} of a particle is given by $a = 3t^2 + 2t + 2$, where t is the time. If the particle starts out with a velocity $u = 2$ m s^{-1} at $t = 0$, then the velocity at the end of 2 seconds is

(a) 12 m s^{-1} (b) 18 m s^{-1} (c) 27 m s^{-1} (d) 36 m s^{-1}

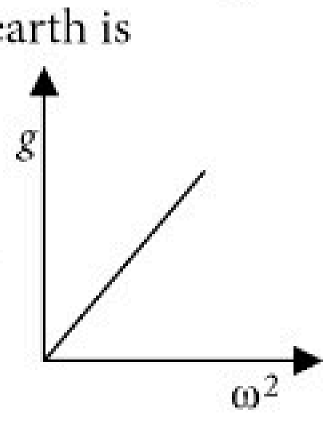
7. An astronaut accidentally gets separated out of his small spaceship accelerating in interstellar space at a constant rate of 100 m s^{-2}. What is the acceleration of the astronaut the instant after he is outside the spaceship?
(Assume that there are no nearby stars to exert gravitational force on him)
(a) zero (b) 10 m s^{-2}
(c) 50 m s^{-2} (d) 100 m s^{-2}

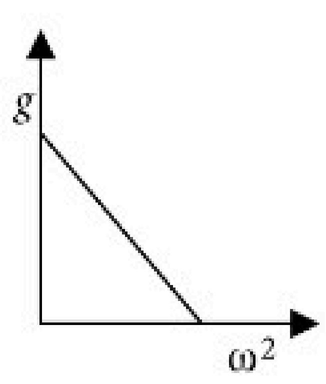
8. Two springs of spring constants 1000 N m^{-1} and 2000 N m^{-1} are stretched with same force. They will have potential energy in the ratio of
(a) 2 : 1 (b) $2^2 : 1$ (c) 1 : 2 (d) $1 : 2^2$

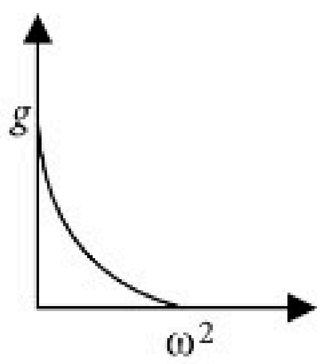
9. In the rotational motion of a rigid body, all particles move with
(a) the same linear velocity and angular velocity
(b) the same linear velocity and different angular velocities
(c) different linear velocities and same angular velocity
(d) different linear velocities and different angular velocities.

10. A metre stick made of half wood and half steel is pivoted at the wooden end and a force is applied perpendicular to its length at the steel end. Then the stick is pivoted at the steel end and the same force is applied perpendicular to its length at the wooden end. The angular acceleration produced is
(a) smaller in the first case
(b) smaller in the second case
(c) equal in both the cases
(d) larger in the first case.

11. The graph that represents variation of g at the equator with square of angular velocity of rotation of earth is

(a)


(b)


(c)


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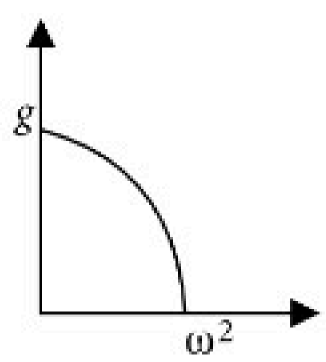


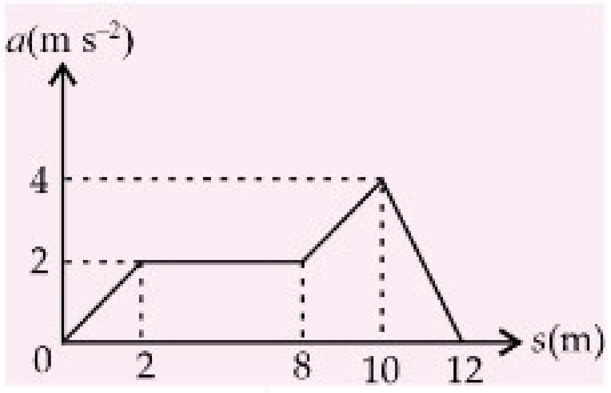
12. A child is standing with folded hands at the centre of a platform rotating about its central axis. The kinetic energy of the system is K. The child now stretches his arms so that the moment of inertia of the system becomes doubled. The kinetic energy of the system now is
(a) $2K$ (b) $\frac{K}{2}$ (c) $\frac{K}{4}$ (d) $4K$

13. Match the following.

p.	Capacitance	i.	volt $(\text{ampere})^{-1}$
q.	Magnetic induction	ii.	volt sec $(\text{ampere})^{-1}$
r.	Inductance	iii.	newton $(\text{ampere})^{-1}$ $(\text{metre})^{-1}$
s.	Resistance	iv.	coulomb^2 $(\text{joule})^{-1}$

(a) p – ii, q – iii, r – iv, s – i
(b) p – iv, q – iii, r – ii, s – i
(c) p – iii, q – iv, r – i, s – ii
(d) p – iv, q – i, r – ii, s – iii

14. The acceleration-displacement graph of a particle moving in a straight line is as shown in the figure. Initial velocity of particle is zero. Velocity of the particle when displacement of the particle is s = 10 m is



(a) $\sqrt{10}$ m s^{-1}
(b) $2\sqrt{10}$ m s^{-1}
(c) $3\sqrt{10}$ m s^{-1}
(d) $10\sqrt{3}$ m s^{-1}

15. A constant retarding force of 50 N is applied to a body of mass 10 kg moving initially with a speed of 10 m s^{-1}. The body comes to rest after
(a) 2 s (b) 4 s (c) 6 s (d) 8 s

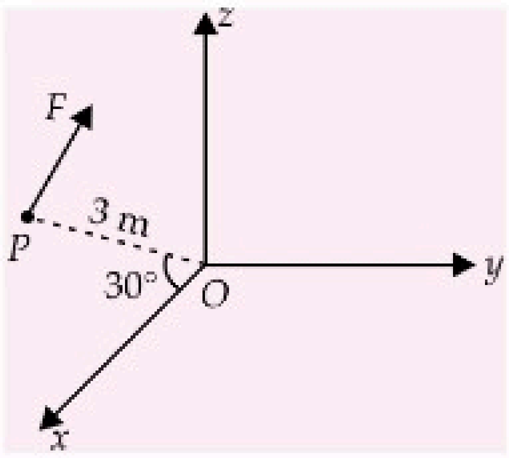
16. Consider a one-dimensional motion of a particle with total energy E. There are four regions A, B, C and D in which the relation between potential energy V, kinetic energy K and total energy E is as given below:
Region $A : V > E$ Region $B : V < E$
Region $C : K > E$ Region $D : V > K$
Which of the following regions the particle cannot be found?
(a) Region A (b) Region B
(c) Region C (d) Region D

17. Which of the following statements is correct regarding a geostationary satellite?
(a) A geostationary satellite goes around the earth in east-west direction.
(b) A geostationary satellite goes around the earth in west-east direction.

(c) The time-period of a geostationary satellite is 48 hours.

(d) The angle between the equatorial plane and the orbital plane of geostationary satellite is 90°.

18. A force $F = 2.0$ N acts on a particle P in the xz-plane. The force F is parallel to x-axis. The particle P (as shown in the figure) is at a distance 3 m and the line joining P with the origin makes angle 30° with the x-axis. The magnitude of torque on P with respect to origin O (in N m) is



(a) 2 (b) 3 (c) 4 (d) 5

19. A force F is applied onto a square plate of side L. If the percentage error in determining L is 2% and that in F is 4%, the permissible percentage error in determining the pressure is

(a) 2% (b) 4% (c) 6% (d) 8%

20. A ball is dropped from the top of a building 100 m high. At the same instant another ball is thrown upwards with a velocity of 40 m s^{-1} from the bottom of the building. The two balls will meet after

(a) 3 s (b) 2 s (c) 2.5 s (d) 5 s

21. Match Column I with Column II.

	Column I		Column II
(A)	Definition of force	(p)	Newton's third law
(B)	Measure of force	(q)	Impulse
(C)	Effect of force	(r)	Newton's second law
(D)	Recoiling of gun	(s)	Newton's first law

(a) A – q, B – p, C – r, D – s

(b) A – p, B – q, C – r, D – s

(c) A – s, B – r, C – q, D – p

(d) A – s, B – q, C – r, D – p

22. A ball bounces to 80% of its original height. What fraction of its potential energy is lost in each bounce?

(a) 0.20 (b) 0.60 (c) 0.40 (d) 1

23. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, work required to pull the hanging part on to the table is

(a) MgL (b) $\frac{MgL}{3}$ (c) $\frac{MgL}{9}$ (d) $\frac{MgL}{18}$

24. The ratio of the radii of gyration of a circular disc and a circular ring of the same radius and same mass about a tangential axis perpendicular to plane of disc or ring is

(a) $\sqrt{3}:2$ (b) $2:3$

(c) $1:2$ (d) $\sqrt{5}:\sqrt{6}$

25. A rope of negligible mass is wound around a hollow cylinder of mass 4 kg and radius 40 cm. What is the angular acceleration of the cylinder, if the rope is pulled with a force of 4 N? Assume that there is no slipping.

(a) 2 rad s^{-2} (b) 1.5 rad s^{-2}

(c) 2.5 rad s^{-2} (d) 3 rad s^{-2}

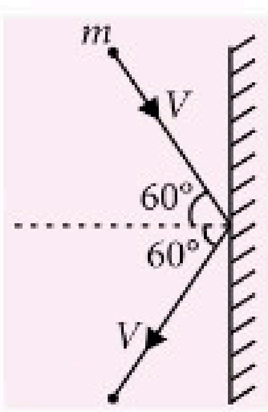
SECTION - B

This section consists of 24 multiple choice questions with overall choice to attempt any 20 questions. In case more than desirable number of questions are attempted, ONLY first 20 will be considered for evaluation.

26. Two trains are moving with equal speed in opposite directions along two parallel railway tracks. If the wind is blowing with speed u along the track so that the relative velocities of the trains with respect to the wind are in the ratio 1 : 2, then the speed of each train must be

(a) $3u$ (b) $2u$ (c) u (d) $4u$

27. A rigid ball of mass m strikes a rigid wall at 60° and gets reflected without loss of speed as shown in the figure. The value of impulse imparted by the wall on the ball will be



(a) mV (b) $2\,mV$

(c) $\frac{mV}{2}$ (d) $\frac{mV}{3}$

28. The correct order in which the dimensions of time decreases in the following physical quantities is

1. Stefan's constant
2. Coefficient of volume expansion
3. Work done
4. Velocity gradient

(a) 2, 4, 3, 1 (b) 1, 2, 3, 4

(c) 4, 3, 2, 1 (d) 1, 2, 4, 3

29. When unit vector $\hat{n} = a\hat{i} + b\hat{j}$ is perpendicular to $(\hat{i} + \hat{j})$, then a and b are

(a) 1, 0 (b) –2, 0

(c) 3, 0 (d) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

MAP

Every piece of music you hear, from Hindustani classical to film songs, depends on performers producing waves and your detection of those waves.

Wave Motion

A means of transferring momentum and energy from one point to another without any actual transportation of matter

Progressive Wave Parameters

- Displacement, $y = A\sin(\omega t + kx)$

$$y = A\sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right) = A\sin\frac{2\pi}{\lambda}(vt+x)$$

- Phase, $\phi = 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)+\phi_0$

  where ϕ_0 is the initial phase.
- Phase change with time,

$$\Delta\phi = \frac{2\pi}{T}\Delta t.$$

- Phase change with position,

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x.$$

- Instantaneous particle velocity,

$$u = \frac{dy}{dt} = \frac{2\pi A}{T}\cos 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)$$

- Velocity amplitude,

$$u_0 = \frac{2\pi A}{T} = \omega A$$

- Instantaneous particle acceleration,

$$a = \frac{du}{dt} = -\frac{4\pi^2}{T^2}A\sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)$$
$$= -\omega^2 y$$

- Acceleration amplitude,

$$a_0 = \frac{4\pi^2}{T^2}A = \omega^2 A$$

Wave Travelling Along a String

- Speed, $v = \sqrt{\frac{T}{m}}$,

  where, T = tension in the string,
  m = mass per unit length.
- Average rate at which kinetic energy or potential energy transported $= \frac{1}{4}\frac{\omega^2 A^2 T}{v}$
- Average power transmitted along the string by a sine wave

$$P_{av} = \frac{1}{2}\frac{\omega^2 A^2 T}{v} = 2\pi^2 m v A^2 \upsilon^2$$

Principle of Superposition of Waves

- According to the principle of superposition of waves, when any number of waves interact at a point in a medium, the net displacement of the point at a given time is the algebraic sum of the displacements due to each wave at that instant of time.

Stationary Waves

- The stationary wave formed by the superposition of incident wave and reflected wave is given by

$$y = 2A\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi t}{T}$$

  Nodes are formed at the positions

Transverse and Longitudinal Waves

- A transverse wave is one in which the disturbance occurs perpendicular to the direction of travel of the wave.
- A longitudinal wave is one in which the disturbance occurs parallel to the line of travel of the wave.

Velocity of Longitudinal Waves

- Velocity of longitudinal waves in a solid of bulk modulus κ, modulus of rigidity η and density ρ is

$$v = \sqrt{\frac{\kappa + \frac{4}{3}\eta}{\rho}}$$

- Velocity of longitudinal waves in a long solid rod of Young's modulus Y and density ρ is given by

$$v = \sqrt{\frac{Y}{\rho}}$$

- Velocity of longitudinal waves in a fluid of bulk modulus κ and density ρ is

$$v = \sqrt{\frac{\kappa}{\rho}}$$

- Newton's formula for the velocity of sound in a gas is

$$v = \sqrt{\frac{\kappa_{iso}}{\rho}} = \sqrt{\frac{P}{\rho}}.$$

  here, P = pressure of the gas
- Laplace formula for the velocity of sound in a gas is

$$v = \sqrt{\frac{\kappa_{adia}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}},$$

- Intensity of sound waves

$$I = \frac{1}{2}\frac{\omega^2 A^2 \kappa}{v} = \frac{2\pi^2\kappa}{v}A^2\upsilon^2 = \frac{P_0^2 v}{2\kappa} = \frac{P_0^2}{2\rho v}$$

Factors Affecting Velocity of Sound through Gases

- Effect of density, $v \propto \frac{1}{\sqrt{\rho}}$

  i.e., $\frac{v_2}{v_1} = \sqrt{\frac{\rho_1}{\rho_2}}$
- Effect of temperature, $v \propto \sqrt{T}$

$$\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+t}{273}}$$

- No change in velocity of sound with change in pressure provided temperature is kept constant.

Doppler's Effect

- If v, v_o, v_s and v_m are the velocities of sound, observer, source and medium respectively, then the apparent frequency,

$$\upsilon' = \frac{v \pm v_m \pm v_o}{v \pm v_m \mp v_s}\times\upsilon$$

- If the medium is at rest, ($v_m = 0$) then

Organ Pipes

- Open organ pipe
  Fundamental mode,

$$\upsilon_1 = \frac{v}{2L} = \upsilon \text{ (First harmonic)}$$

  Second mode, $\upsilon_2 = 2\upsilon$
  (Second harmonic or first overtone)

  n^{th} mode, $\upsilon_n = \frac{nv}{2L}$
  (n^{th} harmonic or $(n-1)^{th}$ overtone)
- Closed organ pipe
  Fundamental mode,

$$\upsilon_1 = \frac{v}{4L} = \upsilon \quad \text{(First harmonic)}$$

  Second mode, $\upsilon_2 = 3\upsilon$
  (Third harmonic or first overtone)
  Third mode, $\upsilon_3 = 5\upsilon$
  (Fifth harmonic or second overtone)
  n^{th} mode, $\upsilon_n = (2n-1)\upsilon$
  [$(2n-1)^{th}$ harmonic or $(n-1)^{th}$ overtone]
- Laplace correction $e = 0.6r$ (in closed pipe) and $2e = 1.2r$ (in open pipe)

$$\upsilon = n\left[\frac{v}{2(l+1.2r)}\right] \text{ (in open pipe)}$$

$$\upsilon = n\left[\frac{v}{4(l+0.6r)}\right] \text{ (in closed pipe)}$$

Modes of Vibration of Strings

- String fixed at both ends
  Frequency of vibration

$$\upsilon = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

  where L = length of string
  n = mode of vibration
  Fundamental frequency

$$\upsilon_0 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{m}},$$

  Second harmonic or 1st overtone, $\upsilon_2 = 2\upsilon_0$
  Third harmonic or 2nd overtone, $\upsilon_3 = 3\upsilon_0$ and so on.
- String fixed at one end
  Frequency of vibration

$$\upsilon = \left(n+\frac{1}{2}\right)\frac{v}{2L} = \frac{\left(n+\frac{1}{2}\right)}{2L}\sqrt{\frac{T}{m}}$$

  Fundamental frequency,

$$\upsilon_0 = \frac{v}{4L} = \frac{1}{4L}\sqrt{\frac{T}{m}}$$

- Law of length
  υL = constant
  or $\upsilon_1 L_1 = \upsilon_2 L_2$

Beats

- Beat frequency = Number of beats/sec
  = Difference in frequencies of two sources.
  $\upsilon_{beat} = (\upsilon_1 - \upsilon_2)$ or $(\upsilon_2 - \upsilon_1)$
  $\therefore \quad \upsilon_2 = \upsilon_1 \pm \upsilon_{beat}$

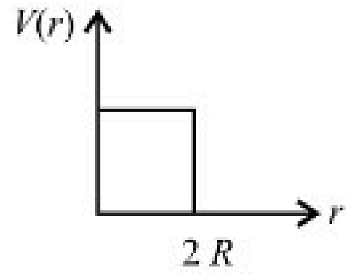
30. If the force acting on a body is inversely proportional to its speed, then its kinetic energy is
(a) linearly related to time
(b) inversely proportional to time
(c) inversely proportional to the square of time
(d) a constant.

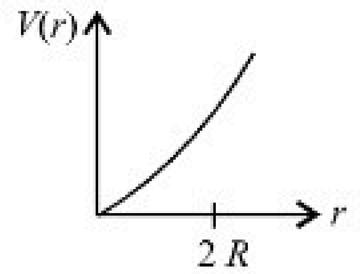
31. The ratio of the radii of gyration of a circular disc to that of a circular ring, each of the same mass and radius, around their respective axis is
(a) $\sqrt{3}:\sqrt{2}$ (b) $1:\sqrt{2}$
(c) $\sqrt{2}:1$ (d) $\sqrt{2}:\sqrt{3}$

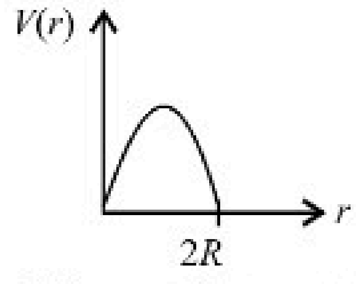
32. A person travels towards north by 4 m and then turns to west and travels by 3 m. The distance and displacement are
(a) 7 m and 5 m (b) 7 m and 7 m
(c) 7 m and 1 m (d) 7 m and 3.5 m

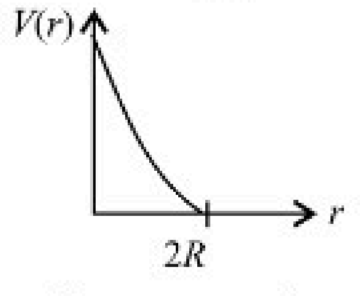
33. Three concurrent co-planar forces 1 N, 2 N and 3 N acting along different directions on a body
(a) can keep the body in equilibrium if 2 N and 3 N act at right angle.
(b) can keep the body in equilibrium if 1 N and 2 N act at right angle.
(c) can not keep the body in equilibrium.
(d) can keep the body in equilibrium if 1 N and 3 N act at an acute angle.

34. Which of the following potential energy curves possibly describe the elastic collision of two billiard balls each of radius R ?
Here r is the distance between centres of the balls.

(a)


(b)


(c)


(d)


35. An adult weighing 600 N raises the centre of gravity of his body by 0.25 m while taking each step of 1 m length in jogging. If he jogs for 6 km, the energy utilised by him in jogging is
(a) 9×10^6 J (b) 9×10^5 J
(c) 6×10^6 J (d) 6×10^5 J

36. Torque of equal magnitude are applied to a thin hollow cylinder and a solid sphere, both having the same mass and radius. Both of them are free to rotate about their axis of symmetry. If α_c and α_s are the angular accelerations of the cylinder and the sphere respectively, then the ratio α_c/α_s will be
(a) 5/2 (b) 2/5 (c) 4/3 (d) 3/4

37. In the relation $P = \frac{\alpha}{\beta} e^{-\frac{\alpha z}{k\theta}}$, P is pressure, z is distance, k is Boltzmann constant and θ is the temperature. The dimensional formula of β will be
(a) $[M^0L^2T^0]$ (b) $[ML^2T]$
(c) $[ML^0T^{-1}]$ (d) $[M^0L^2T^{-1}]$

38. Three projectiles A, B and C are projected at an angle of 30°, 45°, 60° respectively. If R_A, R_B and R_C are ranges of A, B and C respectively then (velocity of projection is same for A, B and C)
(a) $R_A = R_B = R_C$ (b) $R_A = R_C > R_B$
(c) $R_A < R_B < R_C$ (d) $R_A = R_C < R_B$

39. Which of the following statements is correct about friction?
(a) The coefficient of friction between a given pair of substances is largely independent of the area of contact between them.
(b) The frictional force can never exceed the reaction force on the body from the support surface.
(c) Rolling friction is only slightly smaller than sliding friction.
(d) The main source of friction is the irregularity of the surfaces in contact.

40. The escape velocity from a spherical satellite is V_e. The escape velocity from another satellite of double the radius and half the mean density will be
(a) $V_e/2$ (b) $2V_e$ (c) $\sqrt{2}V_e$ (d) $V_e/3$

41. The period of revolution of an earth's satellite close to surface of earth is 90 min. The time period of another satellite in an orbit at a distance of four times the radius of earth from its surface will be
(a) $90\sqrt{9}$ min (b) 270 min
(c) 720 min (d) 360 min

42. Two satellites are in the parking orbits around the earth. Mass of one is 5 times that of the other. The ratio of their periods of revolution is
(a) 25 (b) $\sqrt{5}$ (c) 1 (d) $\sqrt[3]{5}$

43. A stone tied at the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 25 revolutions in 14 s, what is the magnitude of acceleration of the stone ?
(a) 90 m s^{-2} (b) 100 m s^{-2}
(c) 110 m s^{-2} (d) 120 m s^{-2}

44. The mass of a bicycle rider along with the bicycle is 100 kg. He wants to cross over a circular turn of radius 100 m with a speed of 10 m s^{-1}. If the coefficient of friction between the tyres and the road is 0.6, the frictional force required by the rider to cross the turn, is

(a) 300 N (b) 600 N (c) 1200 N (d) 150 N

45. **Given below are two statements labelled as Assertion (A) and Reason (R)**

Assertion (A) : Mass is not conserved separately, but mass and energy are conserved as a single entity called mass - energy.

Reason (R) : Mass and energy are interconvertible in accordance with Einstein's relation, $E = mc^2$.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

46. **Given below are two statements labelled as Assertion (A) and Reason (R)**

Assertion (A) : A player lowers his hands while catching a cricket ball.

Reason (R) : The time of catch increases when cricketer lowers its hand while catching a ball.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

47. **Given below are two statements labelled as Assertion (A) and Reason (R)**

Assertion (A) : The maximum horizontal range of projectile is proportional to square of velocity.

Reason (R) : The maximum horizontal range of projectile is equal to maximum height attained by projectile.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

48. **Given below are two statements labelled as Assertion (A) and Reason (R)**

Assertion : The average and instantaneous velocities have same value in a uniform motion.

Reason : In uniform motion, the velocity of an object increases uniformly.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

49. **Given below are two statements labelled as Assertion (A) and Reason (R)**

Assertion : It is harder to open and shut the door if we apply force near the hinge.

Reason : Torque is maximum at hinge of the door.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

SECTION - C

This section consists of 6 multiple choice questions with an overall choice to attempt any 5. In case more than desirable number of questions are attempted, ONLY first 5 will be considered for evaluation.

50. The coefficient of friction between the ground and the wheels of a car moving on a horizontal road is 0.5. If the car starts from rest, what is the minimum distance in which it can acquire a speed of 72 km h^{-1} without slipping? (Take $g = 10$ m s^{-2}).

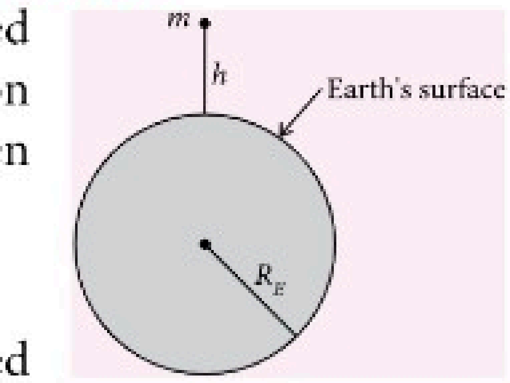
(a) 10 m (b) 20 m (c) 30 m (d) 40 m

51. A body of mass m is raised to a height h from the surface of the earth where the acceleration due to gravity is g. If R is the radius of the earth and $h << R$, then the loss in weight due to variation in g is approximately

(a) $\frac{2mgh}{R}$ (b) $\frac{2mgR}{h}$ (c) $\frac{mgR}{h}$ (d) $\frac{mgh}{R}$

Case Study : Read the following paragraph and answer the questions :

Acceleration Due to Gravity Below and Above the Surface of the Earth



Effect of altitude : If $F(h)$ denoted the magnitude of the force on the point mass m, then

$$F(h) = \frac{GM_E m}{(R_E + h)^2}$$

The acceleration experienced

by the point mass is $F(h)/m = g(h)$ and we get

$$g(h) = \frac{F(h)}{m} = \frac{GM_E}{(R_E+h)^2}$$

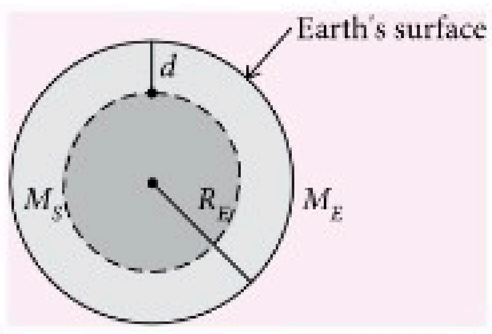
$$g(h) = \frac{GM_E}{R_E^2(1+h/R_E)^2} = g(1+h/R_E)^{-2}$$

For $\frac{h}{R_E} << 1$, using binomial expression,

$$g(h) \cong g\left(1 - \frac{2h}{R_E}\right).$$

This equation tells us that for small heights h above the Earth's surface the value of g decreases by a factor $(1 - 2h/R_E)$.

Effect of depth : A point mass is situated at the depth d below the surface of the Earth is shown in the figure.



If M_s is the mass of the smaller sphere, then

$M_s/M_E = (R_E - d)^3 / R_E^3$ ...(i)

In this case only the smaller sphere of radius (R_E-d) contributes to g.

Thus the force on the point mass is

$$F(d) = \frac{GM_S m}{(R_E-d)^2} = \frac{GM_E m(R_E-d)}{R_E^3}$$

Acceleration due to gravity at a depth d,

$$g(d) = \frac{F(d)}{m} = \frac{GM_E}{R_E^3}(R_E-d) = g\frac{R_E-d}{R_E} = g(1-d/R_E)$$

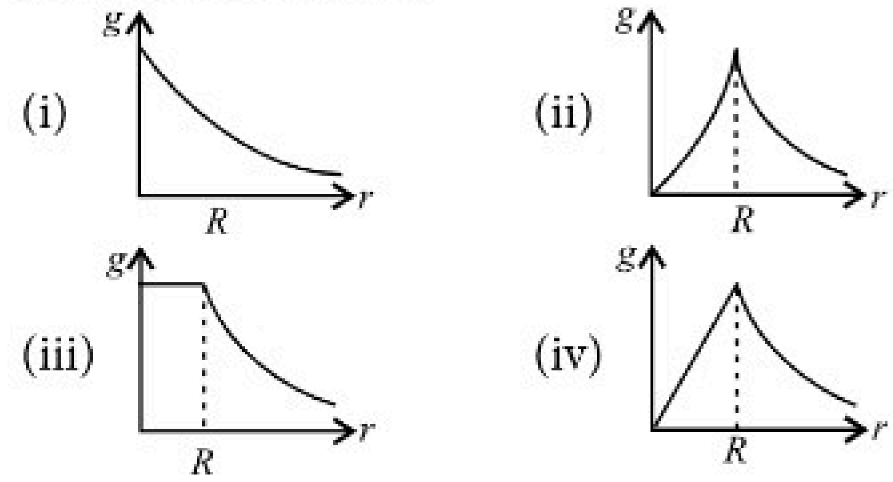
Thus, as we go down below Earth's surface, the acceleration due to gravity changes by a factor of $(1 - d/R_E)$.

52. Value of g is
(a) maximum at poles (b) maximum at equator
(c) same everywhere (d) minimum at poles.

53. Earth is flattened at the poles and bulges at the equator. This is due to the fact that
(a) the Earth revolves around the sun in an elliptical orbit
(b) the angular velocity of spinning about its axis is more at the equator
(c) the centrifugal force is more at the equator than at poles
(d) none of these.

54. The acceleration due to gravity at the poles and the equator is g_p and g_e respectively. If the Earth is a sphere of radius R_E and rotating about its axis with angular speed ω, then $g_p - g_e$ is given by
(a) $\frac{\omega^2}{R_E}$ (b) $\frac{\omega^2}{R_E^2}$ (c) $\omega^2 R_E^2$ (d) $\omega^2 R_E$

55. The dependence of acceleration due to gravity g on the distance r from the centre of the Earth assumed to be a sphere of radius R of uniform density is as shown in the figure.



The correct figure is
(a) (i) (b) (ii) (c) (iii) (d) (iv)

SOLUTIONS

1. (b) : $v = \alpha\sqrt{x}$ or $\frac{dx}{dt} = \alpha\sqrt{x}$ or $\frac{dx}{\sqrt{x}} = \alpha dt$

or $\int \frac{dx}{\sqrt{x}} = \alpha \int dt$ or $2x^{1/2} = \alpha t + C_1$

where C_1 is the constant of integration.
Given : $x = 0$ at $t = 0$, $\therefore C_1 = 0$

$\therefore$ $2x^{1/2} = \alpha t$ or $x = \left(\frac{\alpha}{2}\right)^2 t^2$ or $x \propto t^2$

2. (c) : When the speeding bus stops suddenly, the lower part of the passengers body in contact with the seat remains at rest whereas the upper part of the body of the passengers continues to be in state of motion due to inertia. Hence, the passengers are thrown forward.

3. (a) : Mass of water, m = volume × density = $\pi r^2 l \times \rho$
$= \pi(2)^2 \times 15 \times 1000$ kg

$$h = \frac{30+15}{2} = 22.5 \text{ m}$$

$$\therefore \quad P = \frac{W}{t} = \frac{mgh}{t} = \frac{\pi(2)^2 \times 15 \times 1000 \times 9.8 \times 22.5}{3600}$$
$= 11.55$ kW

4. (d) : In the given triangular plate, BC is the shortest side. The M.I. of the triangular plate is maximum about BC as the axis, because the particles of the plate are distributed at maximum distances from BC.
From the figure, we find that
$I_{BC} > I_{AB} > I_{AC}$ or $I_{BC} > I_{AC}$

5. (a) : Mobility $(\mu_e) = \frac{\text{Drift velocity } (v_d)}{\text{Electric field } (E)}$

$$= \frac{\text{m s}^{-1}}{\text{V m}^{-1}} = \text{m}^2\,\text{V}^{-1}\,\text{s}^{-1}$$

The SI unit of electron mobility is $\text{m}^2\,\text{V}^{-1}\,\text{s}^{-1}$.

6. (b) : Given, $a = \frac{dv}{dt} = 3t^2 + 2t + 2$

or $dv = (3t^2 + 2t + 2)dt$

Integrating both sides, we get

$$\int_u^v dv = \int_0^t (3t^2 + 2t + 2)dt \quad \text{or} \quad v - u = \left(\frac{3t^3}{3} + \frac{2t^2}{2} + 2t\right)_0^t$$

or $v = u + t^3 + t^2 + 2t$

At $t = 2$ s, $v = 2 + (2)^3 + (2)^2 + 2 \times 2 = 18$ m s^{-1}

7. (a)

8. (a) : Potential energy

$$U = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{F^2}{2k}$$

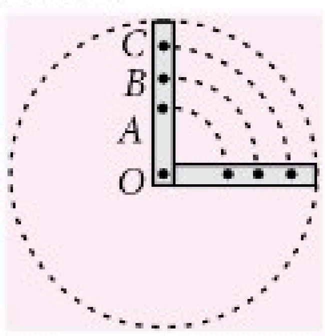
$$U_1 \times k_1 = U_2 \times k_2 \quad \text{or} \quad \frac{U_1}{U_2} = \frac{k_2}{k_1} = \frac{2000}{1000} = \frac{2}{1}$$

9. (c) : As the body is rigid therefore angular velocity of all particles will be same *i.e.*, ω = constant.

From $v = r\omega$, $v \propto r$

(if ω = constant).

It means linear velocity of that particle will be more, whose distance from the centre is more.



i.e., $v_A < v_B < v_C$ but $\omega_A = \omega_B = \omega_C$.

10. (a) : In the first case, steel is at the outer end. The M.I. depends upon the masses of the particles and their distribution from the axis of rotation.

As density of steel > density of wood, the M.I. is more in the first case or $I_1 > I_2$ and $\because \alpha = \frac{\tau}{I}, \alpha_1 < \alpha_2$

11. (b) : $g_\phi = g - R\omega^2$; $Y = -mx + c$.

It is a straight line with negative slope.

12. (b) : According to the principle of conservation of angular momentum, $I\omega$ = constant As I is doubled, ω becomes half. Now KE of rotation, $K = \frac{1}{2}I\omega^2$

Since I is doubled and ω is halved, KE will become half *i.e.* $K/2$.

13. (b)

14. (b) : Acceleration, $a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = v\frac{dv}{ds}$

or $vdv = ads$

$$\therefore \frac{v^2}{2} = \int ads \qquad \text{...(i)}$$

But $\int ads$ = area under a-s graph

From equation (i), $v = \sqrt{2(\text{area under } a\text{-}s \text{ graph})}$

Area under a-s graph

$$= \frac{1}{2}(2)(2) + 6 \times 2 + \frac{1}{2}(2)(2) + 2 \times 2 = 2 + 12 + 2 + 4 = 20$$

$\therefore v = \sqrt{2 \times 20} = 2\sqrt{10}$ m s^{-1}

15. (a) : Here, $F = -50$ N (– ve sign for retardation)

$m = 10$ kg, $u = 10$ m s^{-1}, $v = 0$

As $F = ma \quad \therefore \quad a = \frac{F}{m} = \frac{-50 \text{ N}}{10 \text{ kg}} = -5 \text{ m s}^{-2}$

Using $v = u + at \quad \therefore \quad t = \frac{v - u}{a} = \frac{0 - 10 \text{ m s}^{-1}}{-5 \text{ m s}^{-2}} = 2$ s

16. (a) : Total energy of the particle

= Kinetic energy + Potential energy

$E = K + V$ or $K = E - V$

Since kinetic energy can never be negative. The particle cannot be found in the region where its kinetic energy would become negative. In region A, $V > E$

$\therefore$ K becomes negative in this region. Hence, the particle cannot be found in region A.

17. (b) : A geostationary satellite goes around the earth in west-east direction.

The time period of a geostationary satellite is 24 hours. The angle between the equatorial plane and the orbital plane of geostationary satellite is 0°.

18. (b) **19. (d)** **20. (c)**

21. (c) : Newton's first law defines force. Newton's second law gives us a measure of force. Impulse gives us the effect of force. Recoiling of gun is accounted for by Newton's 3rd law.

22. (a) : Let the ball is dropped from height h.

Initial potential energy = mgh.

Potential energy after first bounce = $mg \times 80$ % of h

= 0.80 mgh.

Potential energy lost in each bounce = 0.20 mgh

Fraction of potential energy lost in each bounce

$$= \frac{0.20\, mgh}{mgh} = 0.20$$

23. (d) : The weight of hanging part $\left(\frac{L}{3}\right)$ of chain is $\left(\frac{1}{3}Mg\right)$. This weight acts at centre of gravity of the hanging part, which is at a distance of $\left(\frac{L}{6}\right)$ from the table.

As work done = force × distance

$$\therefore W = \frac{Mg}{3} \times \frac{L}{6} = \frac{MgL}{18}$$

24. (a)

25. (c) : The M.I. of the hollow cylinder about its geometric axis is given by $I = MR^2 = 4 \times (0.4)^2$
$= 0.64 \text{ kg m}^2$

When the rope is pulled with a force $F = 4$ N, a torque, $\tau = F \times R = 4 \times 0.4 = 1.6$ N m is produced and it produces an angular acceleration, α.

$$\therefore \alpha = \frac{\tau}{I} = \frac{1.6}{0.64} = 2.5 \text{ rad s}^{-2} \qquad (\because \tau = I\alpha)$$

26. (a) : Let the speed of each train be x.
According to the given question

$$\frac{x-u}{x+u} = \frac{1}{2} \text{ or } 2x - 2u = x + u \text{ or } x = 3u$$

27. (a) **28. (a)**

29. (d) : $1 = \sqrt{a^2 + b^2}$; or $a^2 + b^2 = 1$ ...(i)

And $(a\hat{i} + b\hat{j}) \cdot (\hat{i} + \hat{j}) = 0$ or $a + b = 0$; or $b = -a$

From (i), $a^2 + (-a)^2 = 1$

or $a = \frac{1}{\sqrt{2}}$ and $b = -\frac{1}{\sqrt{2}}$

30. (a) : $F \propto \frac{1}{v}, F = \frac{C}{v}$

Where, C is a constant of proportionality.

$$\Rightarrow ma = \frac{C}{v} \text{ or } m\frac{dv}{dt} = \frac{C}{v} \text{ ; } vdv = \frac{Cdt}{m}$$

Integrating both sides, we get

$$\frac{v^2}{2} = \frac{Ct}{m} \text{ or } \frac{1}{2}mv^2 = Ct$$

or Kinetic energy, $K = \frac{1}{2}mv^2 = Ct$ or $K \propto t$

31. (b) : Radius of gyration is given by, $K = \sqrt{\frac{I}{M}}$

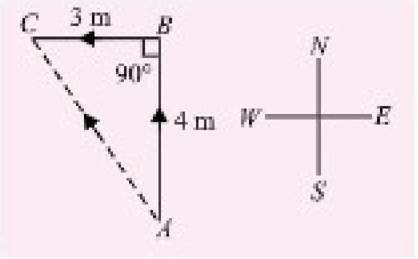
For given problem, $\frac{K_{\text{disc}}}{K_{\text{ring}}} = \sqrt{\frac{I_{\text{disc}}}{I_{\text{ring}}}}$ ...(i)

But $I_{\text{disc}} = \frac{1}{2}MR^2$ and $I_{\text{ring}} = MR^2$ (about its axis)

where, R is the radius of both bodies.
Therefore, eqn (i) becomes

$$\frac{K_{\text{disc}}}{K_{\text{ring}}} = \sqrt{\frac{(1/2)MR^2}{MR^2}} = 1 : \sqrt{2}$$

32. (a): Displacement = CA



$CA^2 = CB^2 + BA^2 = (3)^2 + (4)^2 = 25$

$\therefore$ $CA = 5$ m

Distance = 3 + 4 = 7 m

Displacement = 5 m

33. (c) : If 1 N and 2 N act in the same direction and 3 N acts in opposite direction, then equilibrium is possible.

34. (d) : Here the potential energy of a system of two masses exists only during collision. This potential energy is due to deformation.

35. (b) : Here, $mg = 600$ N, $h = 0.25$ m
Length of each step = 1 m

$\therefore$ No of steps in 6 km is $N = \frac{6 \times 10^3 \text{ m}}{1 \text{ m}} = 6000$

$\therefore$ Energy utilised in jogging is
$E = Nmgh = 6000 \times 600 \text{ N} \times 0.25 \text{ m} = 9 \times 10^5$ J

36. (b) : $\tau = I\alpha$

Torque of equal magnitude are applied to the hollow cylinder and the solid sphere.

$$\therefore I_c\alpha_c = I_s\alpha_s \Rightarrow \frac{\alpha_c}{\alpha_s} = \frac{I_s}{I_c} = \frac{\frac{2}{5}MR^2}{MR^2} = \frac{2}{5}$$

37. (a) : In the given equation, $\frac{\alpha z}{k\theta}$ should be dimensionless.

$$\alpha = \frac{k\theta}{z} \text{ or } [\alpha] = \frac{[\text{ML}^2\text{T}^{-2}\text{K}^{-1}][\text{K}]}{[\text{L}]} = [\text{MLT}^{-2}]$$

$$P = \frac{\alpha}{\beta} \text{ or } [\beta] = \left[\frac{\alpha}{P}\right] = \frac{[\text{MLT}^{-2}]}{[\text{ML}^{-1}\text{T}^{-2}]} = [\text{M}^0\text{L}^2\text{T}^0]$$

38. (d) : Range, $R = \frac{u^2 \sin 2\theta}{g}$

Here g is constant and u is same for the projectiles A, B and C.

$\therefore$ $R \propto \sin 2\theta$

$\Rightarrow R_A : R_B : R_C = \sin 60° : \sin 90° : \sin 120°$

$$R_A : R_B : R_C = \frac{\sqrt{3}}{2} : 1 : \frac{\sqrt{3}}{2} = \sqrt{3} : 2 : \sqrt{3}$$

Hence, $R_A = R_C < R_B$.

39. (a) : The coefficient of friction between a given pair of substances is independent of the area of contact between them. $\mu > 1$ for copper on cast iron.
Rolling friction is much smaller than sliding friction.
Irregularity of the surfaces in contact is not a main contributor of friction.

40. (c): $V_{e_1} = \sqrt{\frac{2GM_1}{R_1}} = \sqrt{\frac{2G \times \frac{4}{3}\pi R_1^3 \times \rho_1}{R_1}}$

$$\therefore V_{e_1} = \frac{2\sqrt{2}R_1}{\sqrt{3}}\sqrt{\pi G\rho_1} \text{ ; } V_{e_2} = \frac{2\sqrt{2} \times 2R_1}{\sqrt{3}} \times \sqrt{\pi G\frac{\rho_1}{2}}$$

$$\therefore \frac{V_{e_2}}{V_{e_1}} = 2\sqrt{\frac{\rho_1}{2\rho_1}} = \frac{2}{\sqrt{2}} = \sqrt{2} \therefore V_{e_2} = \sqrt{2}V_{e_1}$$

41. (c) : From Kepler's law, $T^2 \propto R^3 \Rightarrow T \propto R^{3/2}$

$$\Rightarrow \frac{T'}{T} = \left(\frac{R'}{R}\right)^{3/2} \Rightarrow \frac{T'}{T} = \left(\frac{4R}{R}\right)^{3/2} = 8$$

$\therefore\ T' = 8T = 8 \times 90 = 720$ min

42. (c) : For a satellite, the period $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

where h is the height of the satellite from the surface of the earth. Thus T depends upon the mass, radius of the earth and h. It is independent of the mass of the satellite.

As both are at the same height, $T_1 = T_2$ or $\frac{T_1}{T_2} = 1$

43. (b) : Length of the string, $l = 80$ cm $= 0.8$ m

Number of revolutions = 25, Time taken = 14 s

$\therefore\ \upsilon = \frac{25}{14} = 1.78\ \text{s}^{-1}$

$\therefore\ \omega = 2\pi\upsilon = 2\pi \times 1.78 = 11.18\ \text{rad s}^{-1}$

$a = \omega^2 l = (11.18)^2 \times (0.8) = 99.99\ \text{m s}^{-2} \approx 100\ \text{m s}^{-2}$

44. (b) : Centripetal force

$$= \frac{mv^2}{r} = \frac{100 \times 10 \times 10}{100} = 100\ \text{N}$$

Required frictional force to cross the turn,

$= \mu mg = 0.6 \times 100 \times 10 = 600$

As the frictional force is greater than the centripetal force, so the rider will be able to cross the turn.

45. (a)

46. (a) : According to law of conservation of linear momentum, the momentum of the ball remains constant. As we know that, Impulse = $F \times t$ = change in momentum of the ball = constant, therefore , when the time of catch (t) increases, F decreases *i.e.* hands of the player are not hurt much.

47. (c) : Horizontal range of projectile, $R = \frac{u^2 \sin 2\theta}{g}$

For maximum horizontal range, $\theta = 45°$

$\therefore\ R_{max} = u^2/g$

Maximum height attained by projectile, $H = \frac{u^2 \sin^2\theta}{2g}$

$\therefore$ For $\theta = 45°$, $H = u^2/4g$

48. (c) : An object is said to be in uniform motion if it undergoes equal displacement in equal intervals of time.

$$\therefore\ v_{av} = \frac{s_1 + s_2 + s_3 + ...}{t_1 + t_2 + t_3 + ...} = \frac{s + s + s + ...}{t + t + t + ...} = \frac{ns}{nt}$$

and $v_{ins} = \frac{s}{t}$.

Thus, in uniform motion average and instantaneous velocities have same value.

49. (c) : Torque=Force×perpendiculardistanceoflineof force from the axis of rotation. Hence for a given applied force, torque or true tendency of rotation will be high for large value of d. If distance d is smaller, then greater force is required to cause the same torque, hence it is harder to open or shut down the door by applying a force near the hinge.

50. (d) : Here $\mu = 0.5$, $u = 0$

$$v = 72\ \text{km h}^{-1} = 72 \times \frac{5}{18} = 20\ \text{m s}^{-1}$$

As $\mu = \frac{f}{R} = \frac{ma}{mg} = \frac{a}{g}$ $\quad (\because f = ma$, without slipping)

$\therefore\ a = \mu g = 0.5 \times 10 = 5\ \text{m s}^{-2}$

As $v^2 - u^2 = 2as$

$\therefore\ (20)^2 - 0^2 = 2 \times 5 \times s$ or $s = \frac{400}{10} = 40$ m

51. (a) : For $h << R$, $g_h = g\left(1 - \frac{2h}{R}\right) = g - \frac{2gh}{R}$

or $g - g_h = \frac{2gh}{R}$

$\therefore$ Loss in weight due to variation in g

$$= mg - mg_h = m(g - g_h) = \frac{m \times 2gh}{R} = \frac{2mgh}{R}.$$

52. (a) : Value of g is maximum at poles.

53. (c)

54. (d) : Acceleration due to gravity at a place of latitude λ due to the rotation of Earth is

$g' = g - R_E\omega^2\cos^2\lambda$

At equator, $\lambda = 0°$, $\cos 0° = 1$

$\therefore\ g' = g_e = g - R_E\omega^2$

At poles, $\lambda = 90°$, $\cos 90° = 0$

$\therefore\ g' = g_p = g\ \therefore\ g_p - g_e = g - (g - R_E\omega^2) = R_E\omega^2$

55. (d) : The acceleration due to gravity at a depth d below the surface of Earth is

$$g' = g\left(1 - \frac{d}{R}\right) = g\left(\frac{R-d}{R}\right) = g\frac{r}{R} \quad \text{... (i)}$$

where $R - d = r$ = distance of location from the centre of the Earth. When $r = 0$, $g' = 0$

From (i), $g \propto r$ till $R = r$, for which $g' = g$

For $r > R$, $g' = \frac{gR^2}{(R+h)^2} = \frac{gR^2}{r^2}$ or $g' \propto \frac{1}{r^2}$

Here, $R + h = r$

Therefore, the variation of g with distance r from centre of Earth will be as shown in figure (iv). Thus, option (d) is correct.

MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

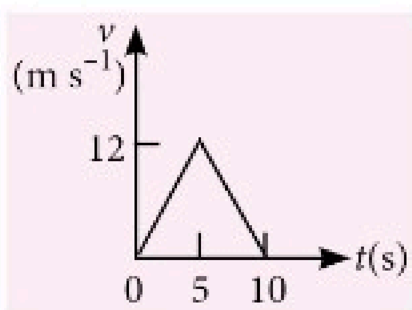
Time Taken : 60 Min. | Total Marks : 120

Kinematics

NEET

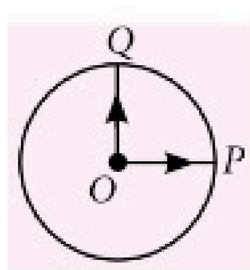
Only One Option Correct Type

1. The speed-time graph of a particle moving along a fixed direction as shown in the figure. The distance traversed by the particle between $t = 0$ s to 10 s is



(a) 20 m (b) 40 m
(c) 60 m (d) 80 m

2. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference and returns to the centre along QO as shown in the figure. If the round trip takes ten minutes, the net displacement and average speed of the cyclist (in metre and kilometre per hour) is



(a) 0, 1 (b) $\frac{\pi+4}{2}, 0$
(c) $21.4, \frac{\pi+4}{2}$ (d) 0, 21.4

3. The component of a vector $\vec{r}$ along x-axis will have maximum value if

(a) $\vec{r}$ is along positive y-axis
(b) $\vec{r}$ is along positive x-axis
(c) $\vec{r}$ makes an angle of 45° with the x-axis
(d) $\vec{r}$ is along negative y-axis.

4. It is a common observation that rain clouds can be at about 1 km altitude above the ground. If a rain drop falls from such a height freely under gravity. What will be its speed in km h^{-1}?

(Take $g = 10$ m s^{-2})

(a) 508 (b) 608 (c) 708 (d) 908

5. A player throws a ball upwards with an initial speed of 29.4 m s^{-1}. What is the velocity of the ball at the highest point of its motion?

(a) 0 (b) 29.4 m s^{-1}
(c) 2.94 m s^{-1} (d) 5.88 m s^{-1}

6. For a particle performing uniform circular motion, choose the incorrect statement from the following.

(a) Magnitude of particle velocity (speed)remains constant.
(b) Particle velocity remains directed perpendicular to radius vector.
(c) Direction of acceleration keeps changing as particle moves.
(d) Magnitude of acceleration does not remain constant.

7. A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 m s^{-1}. Neglect air resistance. With what speed the stone hits the ground?

(a) 15 m s^{-1} (b) 90 m s^{-1}
(c) 99 m s^{-1} (d) 49 m s^{-1}

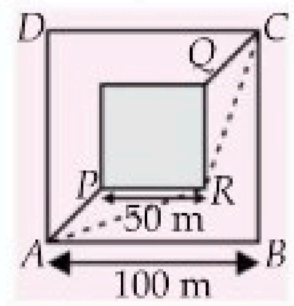
8. In one dimensional motion, instantaneous speed v satisfies $0 \le v < v_0$, Then,

(a) the displacement in time T must always take non-negative values
(b) the displacement x in time T satisfies $-v_0T < x < v_0T$
(c) the acceleration is always a non-negative number
(d) the motion has no turning points.

9. The position of an object moving along x-axis is given by $x = a + bt^2$ where $a = 8.5$ m and $b = 2.5$ ms^{-2} and t is measured in seconds. Its

velocity at $t = 2$ s and the average velocity between $t = 2$ s and $t = 4$ s is
(a) $2 \text{ m s}^{-1}, 5 \text{ m s}^{-1}$ (b) $15 \text{ m s}^{-1}, 10 \text{ m s}^{-1}$
(c) $10 \text{ m s}^{-1}, 15 \text{ m s}^{-1}$ (d) $10 \text{ m s}^{-1}, 20 \text{ m s}^{-1}$

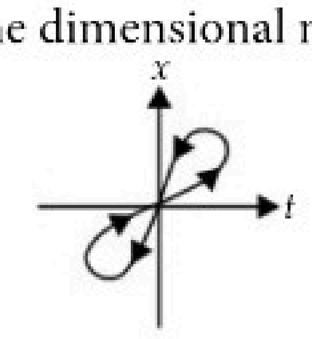
10. A man wants to reach from *A* to the opposite corner of the square *C* (Figure). The sides of the square are 100 m. A central square of 50 m × 50 m is filled with sand. Outside this square, he can walk at a speed 1 m s^{-1}.



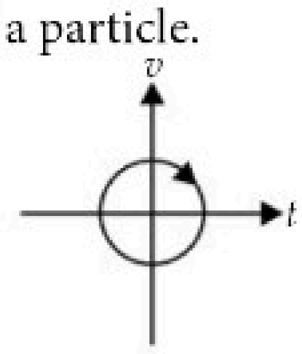
In the central square, he can walk only at a speed of $v \text{ m s}^{-1}$ ($v < 1$). What is smallest value of v for which he can reach faster via a straight path through the sand than any path in the square outside the sand?
(a) 0.18 m s^{-1} (b) 0.81 m s^{-1}
(c) 0.5 m s^{-1} (d) 0.95 m s^{-1}

11. A football is kicked into the air vertically upwards with velocity *u*. What is its velocity at the highest point?
(a) *u* (b) zero (c) 2*u* (d) 4*u*

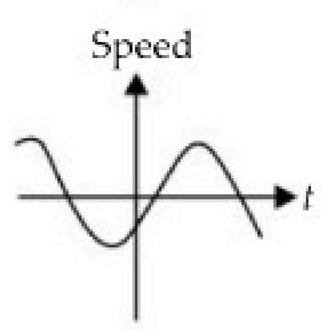
12. Look at the graphs (i) to (iv) carefully and choose, which of these cannot possibly represent one dimensional motion of a particle.



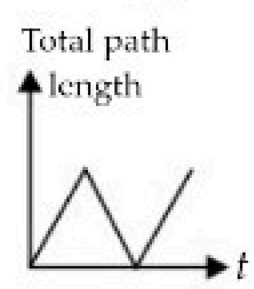
(i)



(ii)



(iii)



(iv)

(a) (i) and (ii) (b) (ii) and (iii)
(c) (i), (ii) and (iii) (d) (i), (ii), (iii) and (iv)

13. A body starts from rest and travels a distance *S* with uniform acceleration, then moves uniformly a distance 2*S*, and finally comes to rest after moving further 5*S* distance under uniform retardation. The ratio of the average velocity to maximum velocity is
(a) 2/5 (b) 3/5 (c) 4/7 (d) 5/7

14. A plane flying horizontally at 100 m s^{-1} releases an object which reaches the ground in 10 s. At what angle with horizontal it hits the ground?
(a) 55° (b) 45° (c) 60° (d) 75°

15. A projectile has initially the same horizontal velocity as it would acquire if it had moved from rest with uniform acceleration of 3 m s^{-2} for 0.5 min. If the maximum height reached by it is 80 m, then the angle of projection is ($g = 10 \text{ m s}^{-2}$)
(a) $\tan^{-1}3$ (b) $\tan^{-1}(3/2)$
(c) $\tan^{-1}(4/9)$ (d) $\sin^{-1}(4/9)$

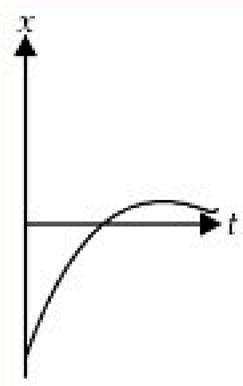
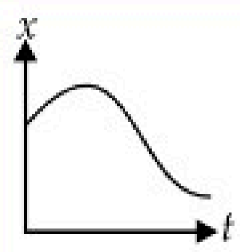
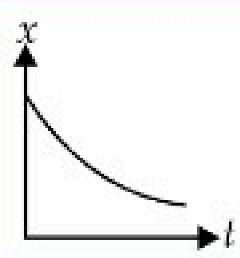
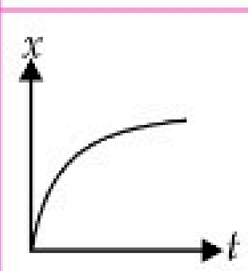
JEE MAIN / JEE ADVANCED

Only One Option Correct Type

16. A man holds an umbrella at 30° with the vertical to keep himself dry. He, then, runs at a speed of 10 m s^{-1} and finds the rain drops to be hitting vertically. Speed of the rain drops with respect to the running man and with respect to earth are respectively are
(a) 20 m s^{-1} and 10 m s^{-1}
(b) 10 m s^{-1} and $20\sqrt{3} \text{ m s}^{-1}$
(c) $10\sqrt{3} \text{ m s}^{-1}$ and 20 m s^{-1}
(d) 20 m s^{-1} and $10\sqrt{3} \text{ m s}^{-1}$

17. Two towns *A* and *B* are connected by a regular bus service with a bus leaving in either direction every *T* minutes. A man cycling with a speed of 20 km h^{-1} in the direction *A* to *B* notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. The period *T* of the bus service is
(a) 4.5 min (b) 9 min
(c) 12 min (d) 24 min

18. Match the column I with column II.

	Column I (Graph)		Column II (Characteristic)
A.		p.	Has $v > 0$ and $a < 0$ throughout.
B.		q.	Has $x > 0$ throughout and has a point with $v = 0$ and a point with $a = 0$.
C.		r.	Has a point with zero displacement for $t > 0$.
D.		s.	Has $v < 0$ and $a > 0$.

(a) A - p, B - q, C - r, D - s
(b) A - q, B - p, C - s, D - r
(c) A - r, B - q, C - s, D - p
(d) A - s, B - p, C - r, D - q

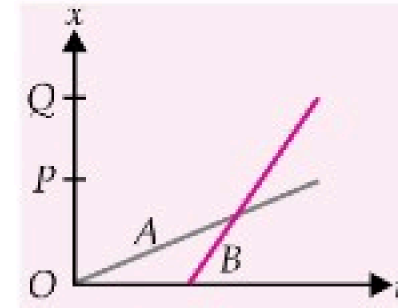
19. If $|\vec{A}| = 2$ and $|\vec{B}| = 4$, then match the relations in column I with the angle θ between $\vec{A}$ and $\vec{B}$ in column II.

	Column I		Column II
(A)	$\lvert\vec{A}\times\vec{B}\rvert = 0$	(p)	$\theta = 30°$
(B)	$\lvert\vec{A}\times\vec{B}\rvert = 8$	(q)	$\theta = 45°$
(C)	$\lvert\vec{A}\times\vec{B}\rvert = 4$	(r)	$\theta = 90°$
(D)	$\lvert\vec{A}\times\vec{B}\rvert = 4\sqrt{2}$	(s)	$\theta = 0°$

(a) A → p, B → r, C → q, D → s
(b) A → r, B → q, C → s, D → p
(c) A → s, B → r, C → p, D → q
(d) A → q, B → p, C → r, D → s

More than One Options Correct Type

20. The position-time (x-t)graphs for two children A and B returning from their school O to their homes P and Q respectively are as shown in the figure. Choose the correct statements regarding these graphs.



(a) A lives closer to the school than B.
(b) A starts from the school earlier than B.
(c) A walks faster than B.
(d) A and B reach home at same time.

21. Which of the following statements is/are incorrect about the average velocity of a particle?

(a) $\vec{v}_{\text{average}} = \dfrac{\vec{v}(t_2) - \vec{v}(t_1)}{2}$

(b) $\vec{v}_{\text{average}} = \vec{a}_{\text{average}}(t_2 - t_1)$

(c) $\vec{v}_{\text{average}} = \dfrac{\vec{v}(t_2) + \vec{v}(t_1)}{2}$

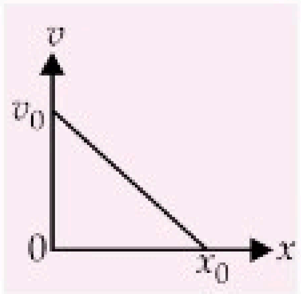
(d) $\vec{v}_{\text{average}} = \dfrac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1}$

(The average stands for average of the quantity over the time interval t_1 to t_2)

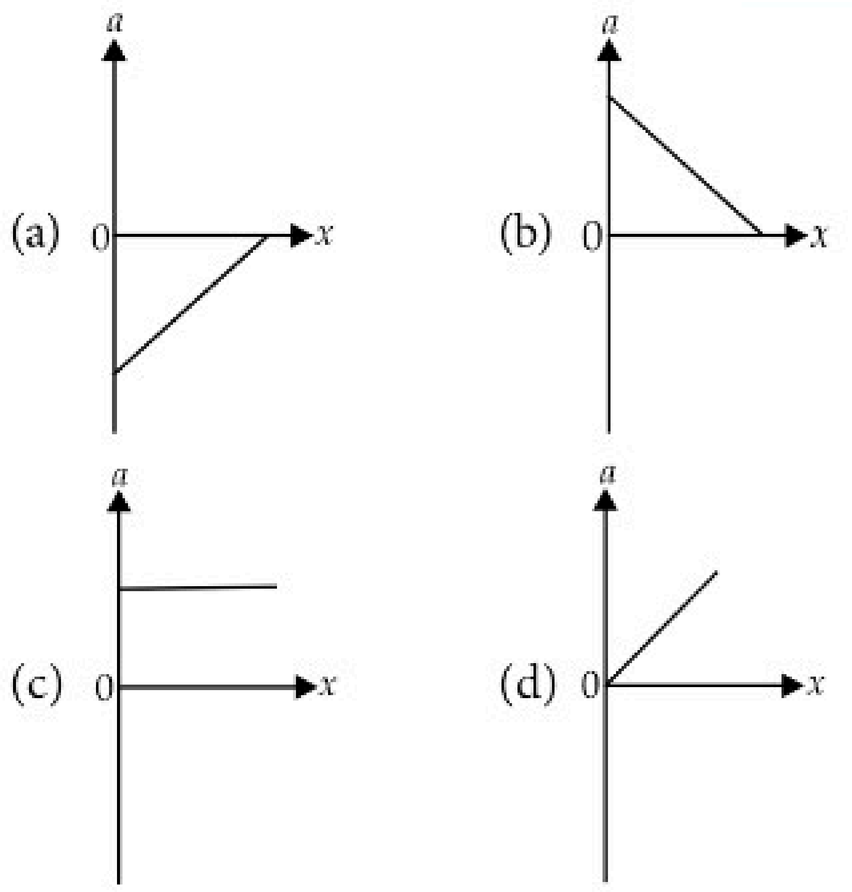
22. Centripetal acceleration is
(a) a constant vector
(b) a constant scalar
(c) a magnitude changing vector
(d) not a constant vector.

23. Two particles are projected in air with speed u at angles θ_1 and θ_2 (both acute) to the horizontal, respectively. If the height reached by the first particle is greater than that of the second, then which one of the following is/are incorrect?
(a) $\theta_1 > \theta_2$ (b) $\theta_1 = \theta_2$
(c) $T_1 < T_2$ (d) $T_1 = T_2$
where T_1 and T_2 are the time of flight

24. The velocity-displacement graph of a particle is as shown in the figure.



Which of the following graphs are incorrectly represents the variation of acceleration with displacement ?



25. Let the resultant of two vectors $\vec{A}$ and $\vec{B}$ be $\vec{R}$. Let the angle between $\vec{A}$ and $\vec{R}$ be α and the angle between $\vec{B}$ and $\vec{R}$ be β. Let the magnitudes of $\vec{A}$, $\vec{B}$ and $\vec{R}$ be represented by A, B and R respectively. Which of the following statements is/are correct?

(a) $\dfrac{R}{\sin(\alpha+\beta)} = \dfrac{A}{\sin\alpha} = \dfrac{B}{\sin\beta}$
(b) $R\sin\alpha = B\sin(\alpha + \beta)$
(c) $A\sin\alpha = B\sin\beta$
(d) $R\sin\beta = A\sin(\alpha + \beta)$

Numerical Value Type

26. A bird is tossing (flying to and fro) between two cars moving towards each other on a straight road. One car has speed of 27 km h^{-1} while the other has the speed of 18 km h^{-1}. The bird starts moving from first car towards the other and is moving with the speed of 36 km h^{-1} when the two cars were separated by 36 km. What is the total distance (in km) covered by the bird?

27. A cyclist is riding with a speed of 27 km h^{-1}. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at the constant rate of 0.50 m s^{-1} every second. What is the magnitude of the net acceleration (in m s^{-2}) of the cyclist on the circular turn?

28. A cricketer can throw a ball to a maximum horizontal distance of 100 m. With the same speed how much high (in m) above the ground can the cricketer throw the same ball?

Comprehension Type

Two particles A and B are initially 40 m apart, A is behind B. Particle A is moving with uniform velocity of 10 m s^{-1} towards B. Particle B starts moving away from A with constant acceleration of 2 m s^{-2}.

29. The time at which there is a minimum distance between the two is
(a) 2 s (b) 4 s (c) 5 s (d) 6 s

30. The minimum distance between the two is
(a) 20 m (b) 15 m (c) 25 m (d) 30 m

◈◈

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SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
< 60%	NOT SATISFACTORY!	Revise thoroughly and strengthen your concepts.

CLASS-XII

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Unit 4

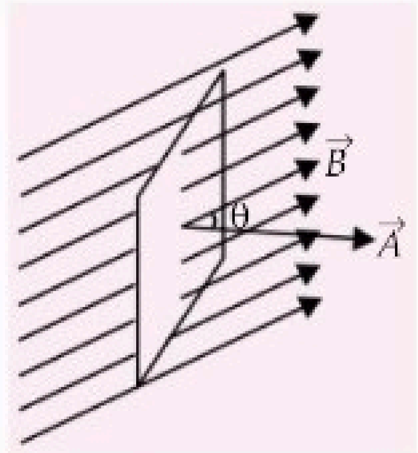
Electromagnetic Induction | Alternating Current

ELECTROMAGNETIC INDUCTION

- Electromagnetic induction is the phenomenon of production of emf in a coil, when the magnetic flux linked with the coil is changed. The emf so produced is called induced emf and the current so produced is called induced current.

Magnetic Flux (ϕ)

- The total number of magnetic field lines crossing normally through a surface placed in a magnetic field is called magnetic flux (ϕ) linked with the surface.



$\phi = \vec{B}.\vec{A} = BA\cos\theta$,
where B is the magnetic field, A is the area of the surface and θ is the angle between the direction of the magnetic field and normal to the surface.

- The SI unit of magnetic flux is weber (Wb).
- Magnetic flux can be changed by
  - changing the intensity of the magnetic field.
  - changing the orientation of coil with respect to the magnetic field.
  - changing the area of the closed circuit.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

- **First law:** Whenever magnetic flux linked with a circuit (a loop of wire or a coil or an electric circuit in general) changes, induced emf is produced. The induced emf lasts as long as the change in the magnetic flux continues.
- **Second law:** The magnitude of the induced emf is directly proportional to the rate of change of the magnetic flux.

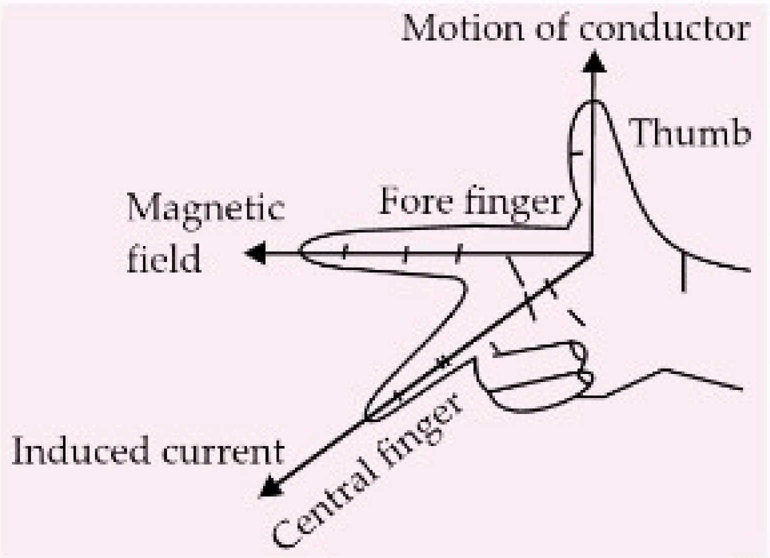
  Induced emf, $\varepsilon = -\frac{d\phi}{dt} = -\frac{\phi_2 - \phi_1}{t}$

LENZ'S LAW

- It states that the induced current produced in a circuit always flows in such a direction that it opposes the change or the cause that produces it.
- Lenz's law can be used to find the direction of induced current. Lenz's law is in accordance with the law of conservation of energy.

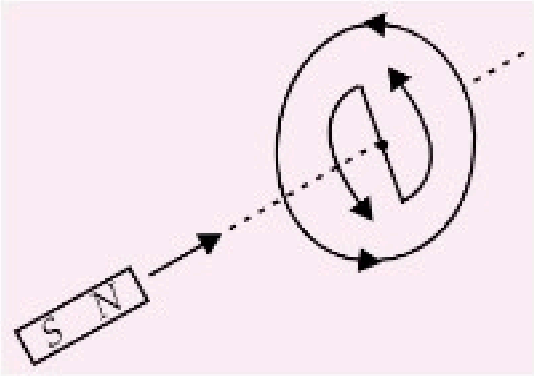
Fleming's Right Hand Rule

- Fleming's right hand rule gives us the direction of induced emf or current, in a conductor moving in a magnetic field.
- According to this rule, if we stretch the fore finger, central finger and thumb of our right hand in mutually perpendicular directions such that fore finger points along the direction of the field and thumb is along the direction of motion of the conductor, then the central finger would give us the direction of induced current or emf.

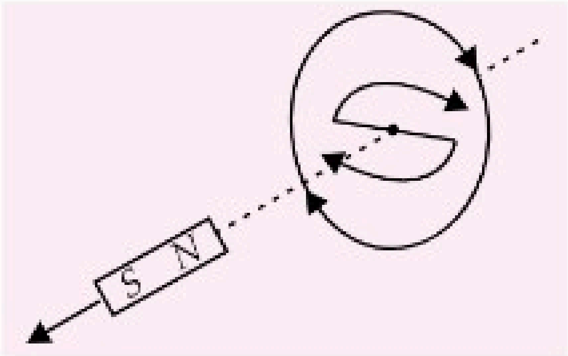


Applications of Lenz's law

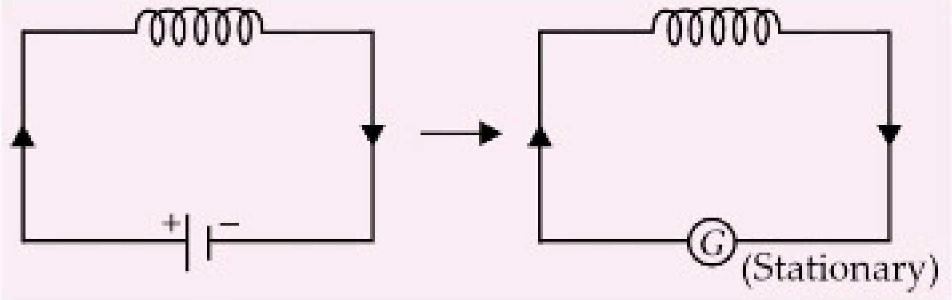
- When the North pole of a bar magnet is moved towards a coil as shown in the figure, the current induced in the coil will be in anticlockwise direction.



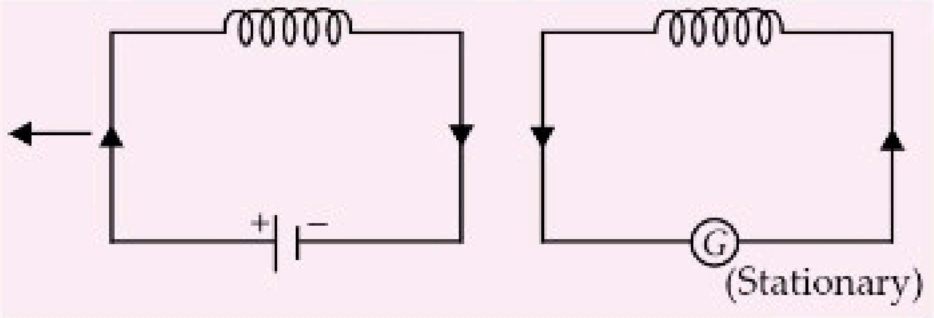
- When the North pole of a bar magnet is moved away from the coil as shown in the figure, the current induced in the coil will be in clockwise direction.



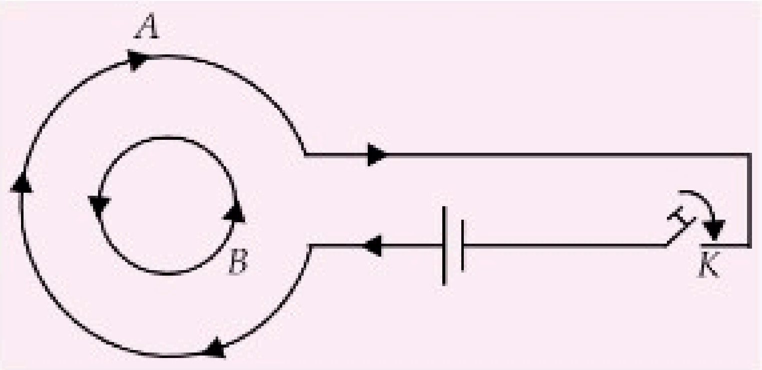
- When a current carrying coil is moved towards a stationary coil, the direction of current induced in stationary coil is as shown in figure below.



- When a current carrying coil is moved away from a stationary coil, the direction of current induced in stationary coil is as shown in figure below.



- When two coils *A* and *B* are arranged as shown in figure, then on pressing *K*, current in *A* increases in clockwise direction. Therefore, induced current in *B* will be in anticlockwise direction.



- However, when key *K* is released, current in *A* decreases in clockwise direction. Therefore, induced current in *B* will be in clockwise direction.

Motional Electromotive Force

- If a conductor is moving with velocity $\vec{v}$ in a magnetic field, electrons inside it experience a force $\vec{F} = q(\vec{v} \times \vec{B})$ and accumulate at the end of the conductor. Very soon, an electric field is established. Eventually component of magnetic force along the conductor length is balanced by the electric field force and the drifting of electrons stops and an emf is established.
- Now, $\varepsilon = -\oint \vec{E}\cdot d\vec{l} = \int(\vec{v}\times\vec{B})\cdot d\vec{l} \Rightarrow \varepsilon = \int(\vec{v}\times\vec{B})\cdot d\vec{l}$
- This is general expression for induced emf in a conducting wire. If $\vec{v}, \vec{B}$ and l are mutually perpendicular to each other, then $\varepsilon = Bvl$.
- If conducting rod moves on two parallel conducting rails, then phenomenon of induced emf can be understood by the concept of changing area (lvt).

  Hence induced emf $|\varepsilon| = \frac{d\phi}{dt} = Bvl$

  Induced Current : $I = \frac{\varepsilon}{R} = \frac{Bvl}{R}$

  Magnetic Force : $F_m = BIl = B\left(\frac{Bvl}{R}\right)l = \frac{B^2vl^2}{R}$

  Power dissipated in moving the conductor :

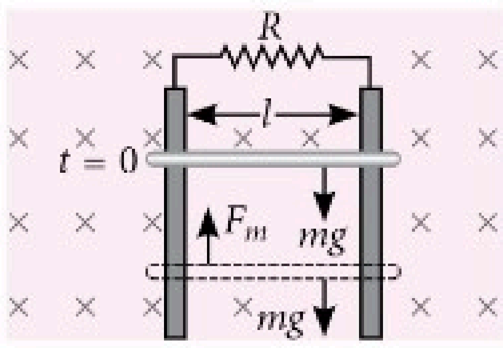
  $$P_{mech} = P_{ext} = \frac{dW}{dt} = F_{ext}\cdot v = \frac{B^2vl^2}{R}\times v = \frac{B^2v^2l^2}{R}$$

- **Electrical Power :** $P_{thermal} = \frac{H}{t} = I^2R = \left(\frac{Bvl}{R}\right)^2\cdot R$

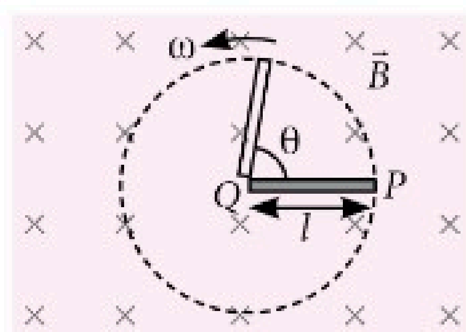
  $\therefore\ P_{thermal} = \frac{B^2v^2l^2}{R}$

- **Motion of conducting rod in a vertical plane**
  - Rod will achieve a constant maximum (terminal) velocity v_T if $F_m = mg$

So, $\frac{B^2 v_T l^2}{R} = mg; v_T = \frac{mgR}{B^2 l^2}$



- Motional emf due to rotational motion
  - emf induced across the ends of the rod is,



$$\varepsilon = \frac{1}{2} Bl^2 \omega = Bl^2 \pi \upsilon$$

$$= \frac{Bl^2 \pi}{T}$$

Eddy Currents

- The currents induced in the body of a conductor, when the magnetic flux linked with the conductor changes are called eddy currents.
- The direction of the eddy currents in the conductor can be found by applying Lenz's law or Fleming's right hand rule.
- **Applications of eddy currents**
  - Induction furnace is based on the heating effect of eddy currents.
  - Speedometer is a device used to measure the instantaneous speed of a vehicle.
  - Concept of eddy current is used in energy meter to record the consumption of electricity.

Inductance

- Inductance the analogous to inertia in mechanics, because inductance of an electrical circuit opposes any change of current in the circuit.
- Inductance is a scalar quantity. It has dimensions of $[ML^2T^{-2}A^{-2}]$. SI unit of inductance is henry.
- **Self Inductance**
  - When the current in a coil changes, it induces a back emf in the same coil. The self-induced emf is given by, $\varepsilon = -L\frac{dI}{dt}$,

    where L is the self-inductance of the coil.

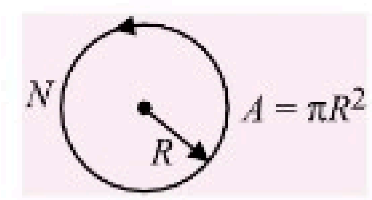
    $$L = \frac{N\phi}{I} = \frac{NBA}{I} = \frac{\phi_{total}}{I}$$

    It is a measure of the inertia of the coil against the change of current through it.
  - The self inductance of a long solenoid, the core of which consists of a magnetic material of permeability μ_r is

    $$L = \frac{\mu_0 \mu_r N^2 A}{l} = \mu_r \mu_0 n^2 Al = \mu_0 \mu_r n^2 V$$

    Here V = volume of solenoid = Al
- Self inductance of a planar coil of radius R

$$L = \frac{\mu_0 N^2 \pi R}{2}$$



- **Mutual inductance**
  - When an emf is produced in a coil because of change in current in a coupled coil, the effect is called mutual inductance.

    $$M_{12} = \frac{N_1 \phi_1}{I_2} \text{ and } M_{21} = \frac{N_2 \phi_2}{I_1}$$
  - For same length and different number of turns per unit length of two solenoids mutual inductance is given by

    $M_{12} = M_{21} = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l = M$

    r_1 = radius of inner coil
- Mutual inductance of two concentric and coplanar coils $M_{C_1C_2} = \frac{N_2 B_1 A_2}{I_1} = \frac{\mu_0 N_1 N_2 \pi r_2^2}{2r_1}$
- Magnetic energy per unit volume, $u_B = \frac{U_B}{V} = \frac{B^2}{2\mu_0}$
- **Combination of inductance**
  - **Series combination**

    $L_S = L_1 + L_2$ (take $M = 0$). If $M \neq 0$ then $L_S = L_1 + L_2 \pm 2M$.

    The plus sign occurs if windings in the two coils are in the same sense, while minus sign occurs if windings are in opposite sense.
  - **Parallel combination,** $\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2}$

    $\therefore \quad L_p = \frac{L_1 L_2}{L_1 + L_2}$ (take $M = 0$)

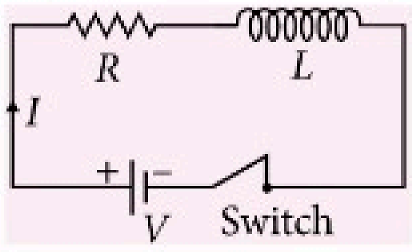
    When $M \neq 0$ *i.e.*, they situated close to each other, $L_p = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm M}$

Energy Stored in an Inductor

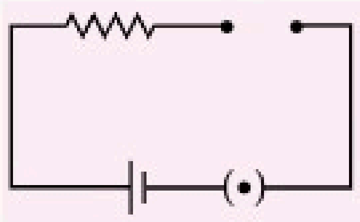
- When a current I flows through an inductor, the energy stored in it, is given by $U_B = \frac{1}{2} LI^2$
- The energy stored in an inductor is in the form of magnetic energy.

Current Growth in an *LR* Circuit

- Emf equation :
$$V = IR + L\frac{dI}{dt}$$

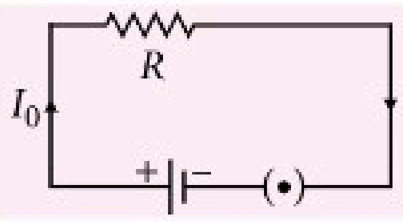


- Current at any instant : When key is closed the current in circuit increases exponentially with respect to time. The current in circuit at any instant t is given by $I = I_0(1 - e^{-t/\tau})$
- Just after the closing of key, inductance behaves like open circuit and current in circuit is zero.



(Open circuit, $t = 0, I = 0$)
(Inductor provide infinite resistance)

- Some time after closing of the key inductance behaves like short circuit and current in circuit is constant.



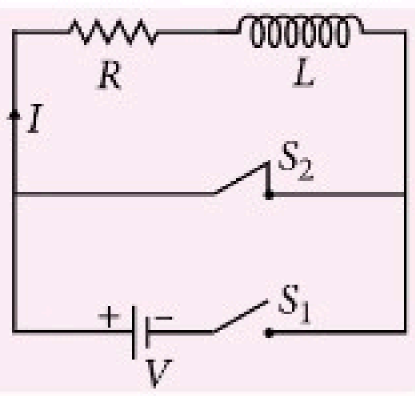
(Short circuit, $t \to \infty, I \to I_0$)
(Inductor provide zero resistance)

  - $I_0 = \frac{V}{R}$ (maximum or peak value of current)
  - Peak value of current in circuit does not depend on self inductance of coil.

Current Decay in an *LR* Circuit

- Emf equation : $IR + L\frac{dI}{dt} = 0$

Initially, S_1 : ON
S_2 : OFF
Finally, S_1 : OFF
S_2 : ON



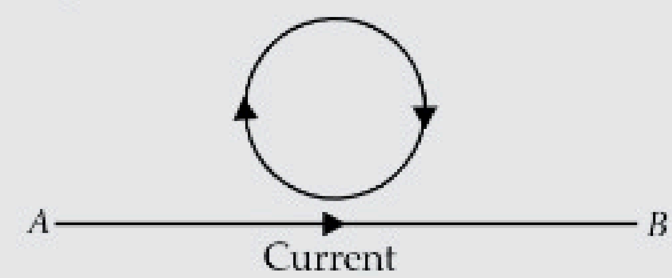
- Current at any instant : Once current acquires its final maximum steady value, if suddenly required switching positions (S_1 and S_2) are interchanged then current starts decreasing exponentially with respect to time. The current in the circuit at any instant t is given by $I = I_0\, e^{-t/\tau}$
  - Just after opening of key ($t = 0$) $\Rightarrow$ $I = I_0 = \frac{V}{R}$
  - Some time after opening of key $(t \to \infty) \Rightarrow I_0 \to 0$

AC Generator

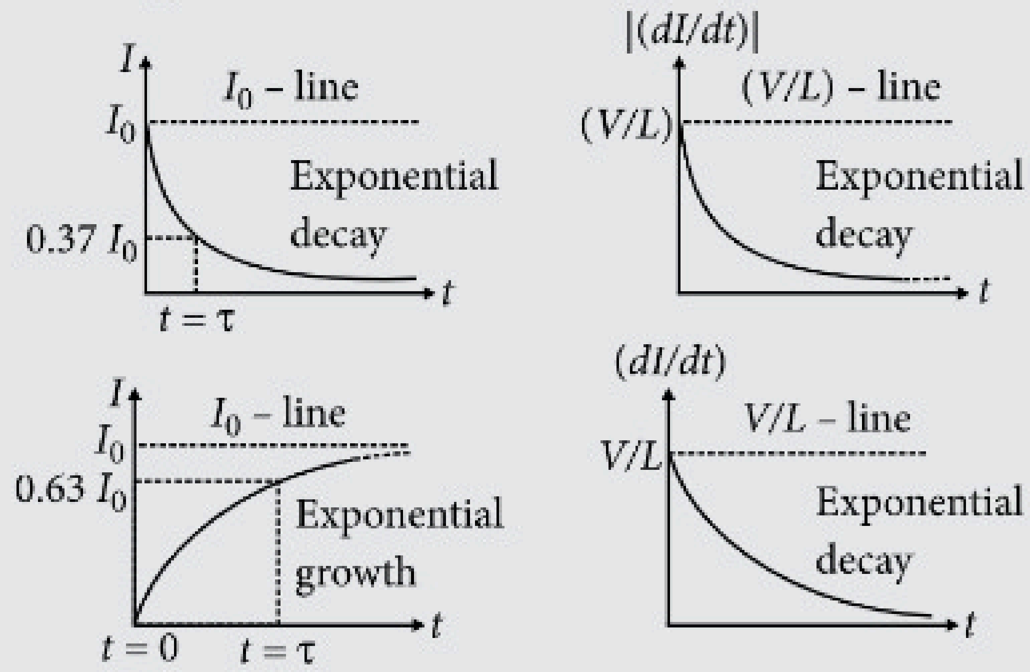
- It is a device used to obtain a supply of alternating emf by converting rotational mechanical energy into electrical energy. It is based on the phenomenon of electromagnetic induction. *i.e.* when a coil is rotated in uniform magnetic field, an induced emf is produced in it.
- The instantaneous value of the emf produced is given by $\varepsilon = NBA\omega\sin\omega t$, where N is number of turns of the coil, A is the area of coil and ω is angular frequency of rotation of the coil in a magnetic field strength B.

Key Point

- When current in a straight conductor AB is increased, induced current in loop will be in clockwise direction as shown in the figure. If current in AB is decreasing, the induced current in the loop will be in anticlockwise direction.



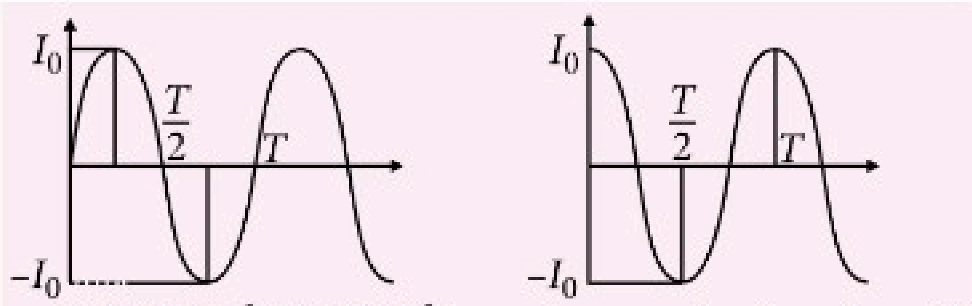
- Time constant (τ) : It is a time in which current decreases up to 37% (0.37 times) or increases up to 63% (0.67 times) of peak current value and given as $\tau = L/R$.



Alternating Current

- It is the current which varies continuously in magnitude and periodically in direction. It can be represented by $I = I_0 \sin\omega t$ or $I = I_0\cos\omega t$.
- where I_0 is peak value of current and is known as amplitude of ac and I is the instantaneous value of alternating current.

- Angular frequency, $\omega = \frac{2\pi}{T} = 2\pi\upsilon$ where T is period of ac and υ is frequency of ac.



I as a sine function of t | I as a cosine function of t

Mean or Average Value of AC

- Average value of the alternating current over a half cycle is the value of direct current which will sends the same amount of charge in a circuit in a time of half cycle as is sent by the given ac in the same circuit in the same time. For the complete cycle,

$$I_m \text{ or } \overline{I} \text{ or } I_{av} = \frac{\int_0^T I_0 \sin\omega t\, dt}{\int_0^T dt} = 0$$

$$V_m \text{ or } \overline{V} \text{ or } V_{av} = \frac{\int_0^T V_0 \sin\omega t\, dt}{\int_0^T dt} = 0$$

- Average value of alternating current for first half cycle is

$$I_{av} = \frac{\int_0^{T/2} I_0 \sin\omega t\, dt}{\int_0^{T/2} dt} = \frac{2I_0}{\pi} = 0.637 I_0$$

- Similarly, for alternating voltage, the average value over first half cycle is

$$V_{av} = \frac{\int_0^{T/2} V_0 \sin\omega t dt}{\int_0^{T/2} dt} = \frac{2V_0}{\pi} = 0.637 V_0$$

- Average value of alternating current for second cycle is

$$I_{av} = \frac{\int_{T/2}^{T} I_0 \sin\omega t dt}{\int_{T/2}^{T} dt} = -\frac{2I_0}{\pi} = -0.637\, I_0$$

- Similarly, for alternating voltage, the average value over second half cycle is

$$V_{av} = \frac{\int_{T/2}^{T} V_0 \sin\omega t dt}{\int_{T/2}^{T} dt} = -\frac{2V_0}{\pi} = -0.637\, V_0$$

Root Mean Square Value of Alternating Current

- The value of that direct current which produces heat at the same rate as the alternating current in a given resistor is known as the rms value of alternating current. *rms* value is also known as virtual value or effective value. All ac instruments measure virtual value.

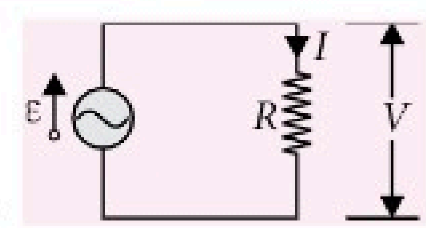
$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0 \text{ or } V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = 0.707\, V_0$$

$$\text{Form factor} = \frac{I_{\text{rms}}}{I_{\text{av}}} = \frac{0.707 I_0}{0.637 I_0} = 1.11$$

AC Voltage Applied to a Resistor

The varying potential difference

$V = V_0 \sin\omega t$



then $I = \frac{V}{R} = \frac{V_0 \sin\omega t}{R} = I_0 \sin\omega t$

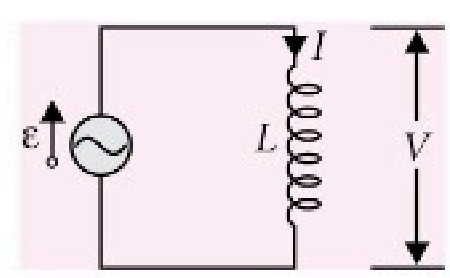
- Here the alternating voltage is in phase with current, when ac flows through the resistor.

AC Voltage Applied to an Inductor

- Potential difference across the inductor,

$V = V_0 \sin\omega t$

or $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$



where $I_0 = \frac{V_0}{\omega L}$

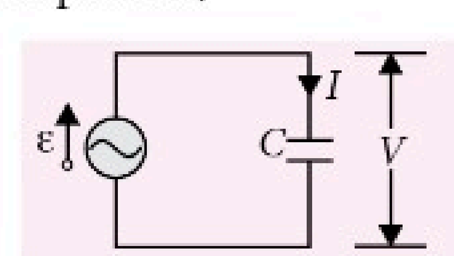
- The alternating current lags behind the alternating voltage by a phase angle of $\pi/2$ when ac flows through an inductor.
- **Inductive reactance :** It is the opposition offered by the inductor to the flow of alternating current through it. $X_L = \omega L = 2\pi\upsilon L$

AC Voltage Applied to a Capacitor

- Potential difference across capacitor,

$V = V_0 \sin\omega t$

or $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$

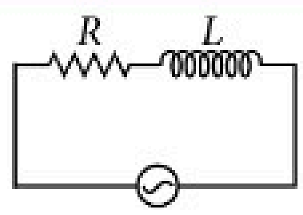
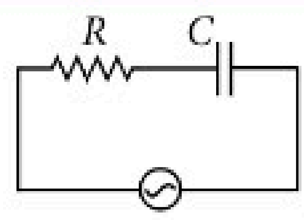
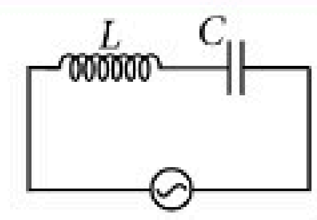
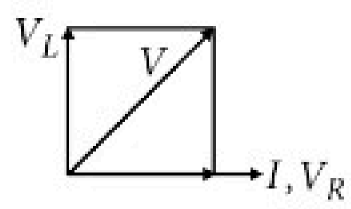
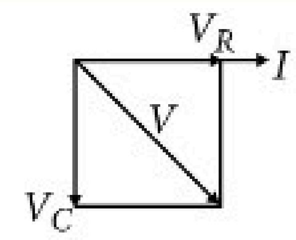
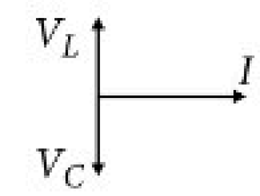


where $I_0 = (\omega C) V_0$

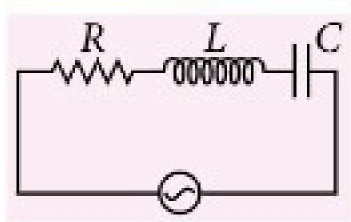
- The alternating current leads the voltage by a phase angle of $\pi/2$, when ac flows through a capacitor.
- **Capacitive reactance :** It is the opposition offered by the capacitor to the flow of alternating current through it.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \upsilon C}$$

Combination of *R – L*, *R – C* and *L – C* in an AC Circuit

Term	*R – L*	*R – C*	*L – C*
Circuit	 I is same in R and L	 I is same in R and C	 I is same in L and C
Phasor diagram	 $V^2 = V_R^2 + V_L^2$	 $V^2 = V_R^2 + V_C^2$	 $V = V_L - V_C \quad (V_L > V_C)$ $V = V_C - V_L \quad (V_C > V_L)$
Phase difference between V and I	V leads I $\left(0 \text{ to } \frac{\pi}{2}\right)$	V lags I $\left(-\frac{\pi}{2} \text{ to } 0\right)$	V lags I $\left(-\frac{\pi}{2}, \text{if } X_C > X_L\right)$ V leads I $\left(+\frac{\pi}{2}, \text{if } X_L > X_C\right)$
Impedance	$Z = \sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + X_C^2}$	$Z = \lvert X_L - X_C \rvert$
At very low υ	$Z \simeq R \; (X_L \to 0)$	$Z \simeq X_C$	$Z \simeq X_C$
At very high υ	$Z \simeq X_L$	$Z \simeq R \quad (X_C \to 0)$	$Z \simeq X_L$

Series *LCR*-Circuit



- Let $V = V_0 \sin\omega t$
  Then, $I = I_0 \sin(\omega t - \phi)$
  where $I_0 = \frac{V_0}{Z}$
  Here Z is the impedance of the series LCR circuit.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

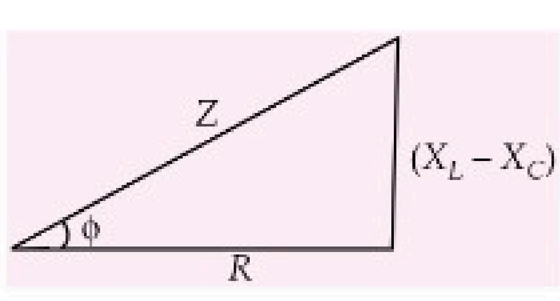
- The alternating current lags behind the voltage by a phase angle ϕ.

$$\tan\phi = \frac{X_L - X_C}{R}$$

- When $X_L > X_C$, $\tan\phi$ is positive. Therefore, ϕ is positive. Hence current lags behind the voltage by a phase angle ϕ. The ac circuit is inductance dominated circuit.
- When $X_L < X_C$, $\tan\phi$ is negative. Therefore, ϕ is negative. Hence current leads the voltage by a phase angle ϕ. The ac circuit is capacitance dominated circuit.

Impedance triangle

- It is a right angled triangle, whose base represents ohmic resistance (R), perpendicular represents reactance ($X_L - X_C$) and hypotenuse represents impedance (Z) of the series LCR circuit as shown in figure.



Impedance of circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Admittance

The reciprocal of the impedance of an ac circuit is known as admittance. It is represented by Y.

$\therefore$ Admittance $= \dfrac{1}{\text{Impedance}}$ or $Y = \dfrac{1}{Z}$

- The unit of admittance is $(\text{ohm})^{-1}$ or siemen.

Susceptance

- The reciprocal of the reactance of an ac circuit is known as susceptance. It is represented by S.

$\therefore$ Susceptance $= \dfrac{1}{\text{Reactance}}$

The unit of susceptance is $(\text{ohm})^{-1}$ or siemen.

- Inductive susceptance $= \dfrac{1}{\text{Inductive reactance}}$

or $S_L = \dfrac{1}{X_L} = \dfrac{1}{\omega L}$

- Capacitive susceptance $= \dfrac{1}{\text{Capacitive reactance}}$

or $S_C = \dfrac{1}{X_C} = \dfrac{1}{1/\omega C} = \omega C$

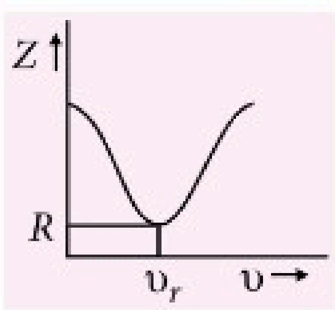
Resonance in Series *LCR* Circuit

- A circuit is said to be resonant when the natural frequency of the circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.
- At resonance,
  - $X_L = X_C$, $V_L = V_C$
  - $\phi = 0$ (V and I are in same phase)
  - $Z_{min} = R$, $I_{max} = \dfrac{V}{R}$
- Resonant frequency :

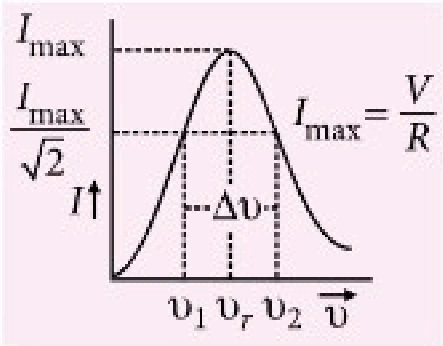
$\because X_L = X_C \Rightarrow \omega L = \dfrac{1}{\omega C}$

$\omega = \dfrac{1}{\sqrt{LC}}$ or, $\upsilon_r = \dfrac{1}{2\pi\sqrt{LC}}$

- Variation of Z with υ
  - If $\upsilon < \upsilon_r$ then $X_L < X_C$, circuit is capacitive, (ϕ negative).
  - At $\upsilon = \upsilon_r$, $X_L = X_C$, circuit is resistive, ϕ = zero.
  - If $\upsilon > \upsilon_r$ then $X_L > X_C$ circuit is inductive, (ϕ positive).
- As υ increases, Z first decreases then increases.



- Variation of I with υ



as υ increases, I first increases then decreases.

- **Bandwidth** $\Delta\upsilon = \upsilon_2 - \upsilon_1$
- **Quality factor (Q) :** Q-factor of ac circuit basically gives an idea about stored energy and lost energy.

$Q = 2\pi \dfrac{\text{Maximum energy stored per cycle}}{\text{Maximum energy lost per cycle}}$

  - It represents the sharpness of resonance.
  - It is unitless and dimensionless quantity.
  - $Q = \dfrac{(X_L)_r}{R} = \dfrac{(X_C)_r}{R} = \dfrac{2\pi\upsilon_r L}{R}$

    $= \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{\upsilon_r}{\Delta\upsilon} = \dfrac{\upsilon_r}{\text{bandwidth}}$
  - Sharpness $\propto$ Quality factor

    R decrease $\Rightarrow Q$ increases $\Rightarrow$ sharpness increases

Power in AC Circuit

- Let $V = V_0 \sin \omega t$ and $I = I_0 \sin(\omega t - \phi)$

Instantaneous power $P = V_0 \sin \omega t \cdot I_0 \sin(\omega t - \phi)$

$= V_0 I_0 \sin \omega t (\sin \omega t \cos\phi - \sin\phi \cos \omega t)$

- Average power <P>

$= \dfrac{1}{T}\int_0^T (V_0 I_0 \sin^2 \omega t \cos\phi - V_0 I_0 \sin \omega t \cos \omega t \sin\phi)dt$

$\langle P \rangle = \dfrac{V_0 I_0 \cos\phi}{2} \Rightarrow \langle P \rangle = V_{rms} I_{rms} \cos\phi$

- rms power, $P_{rms} = V_{rms} I_{rms}$

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1. (a)	2. (b)	3. (b)	4. (a)	5. (b)
6. (c)	7. (a)	8. (d)	9. (c)	10. (d)
11. (c)	12. (b)	13. (a)	14. (d)	15. (a)
16. (d)	17. (c)	18. (b)	19. (c)	
20. (a,,b,c,d)	21. (a, b, c)		22. (b, d)	
23. (a,b,d)	24. (b, c, d)	25. (a, c)	26. (4)	27. (2)
28. (6)	29. (d)	30. (d)		

- Power factor $(\cos\phi) = \dfrac{\text{Average power}}{\text{rms power}}$

  $\cos\phi = \dfrac{R}{Z}$
- Power dissipation is maximum in resistive circuit or at resonance in a LCR series circuit.
- No power is dissipated in purely inductive or capacitive circuit even a current is flowing in the circuit $\left(\because \phi = \dfrac{\pi}{2}\right)$. This current is referred as wattless current.
- Power is dissipated only in resistor even circuit has RL, RC or LCR combination.

Choke Coil

- Circuit with a choke coil is a series $L - R$ circuit. If resistance of choke coil = r (very small). Current in the circuit, $I = \dfrac{V}{Z}$ with $Z = \sqrt{(R+r)^2 + (\omega L)^2}$
- It has a high inductance and negligible resistance coil.
- It is used to control current in ac circuit at negligible power loss.

  $\therefore \quad \cos\phi = \dfrac{V}{Z} = \dfrac{r}{\sqrt{r^2 + \omega^2 L^2}} \approx \dfrac{r}{\omega L} \to 0$
- Resistance of an ideal coil is zero.

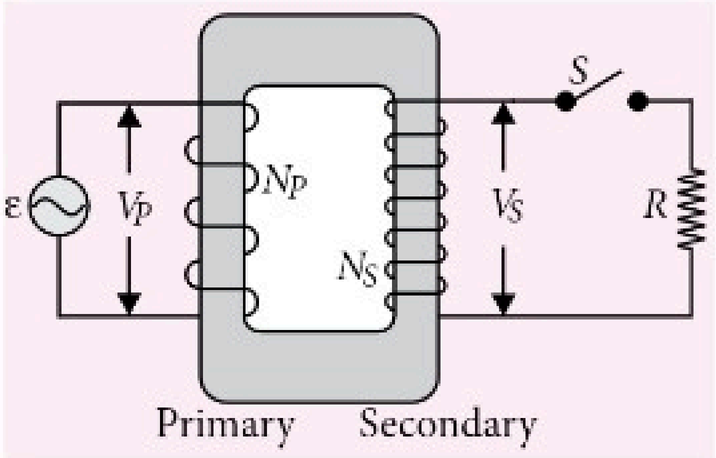
LC Oscillation

- The oscillation of energy between capacitor (electric field energy) and inductor (magnetic field energy) is called LC-oscillation.
- Frequency of oscillation $\upsilon = \dfrac{1}{2\pi\sqrt{LC}}$
- If charge varies sinusoidally with time t as $q = q_0 \cos \omega t$, then current varies periodically with time t as $I = \dfrac{dq}{dt} = q_0 \omega \cos\left(\omega t + \dfrac{\pi}{2}\right)$
- If initial charge on the capacitor is q_0 then electrical energy stored in capacitor is $U_E = \dfrac{1}{2}\dfrac{q_0^2}{C}$
- If the capacitor is fully discharged, then total electrical energy is stored in the inductor in the form of magnetic energy.

  $U_B = \dfrac{1}{2} L I_0^2$, where I_0 = Maximum current

Transformer

- A transformer is an electrical device which is used for changing alternating voltages. It is based on the phenomenon of mutual induction.



- If it is assumed that there is no loss of energy in the transformer, then the power input = the power output, and since $P = I \times V$ then $I_P V_P = I_S V_S$
- Although some energy is always lost, this is a good approximation, since a well designed transformer may have an efficiency of more than 95%.

  $\dfrac{I_P}{I_S} = \dfrac{V_S}{V_P} = \dfrac{N_S}{N_P}$
- A transformer affects the voltage and current. We have

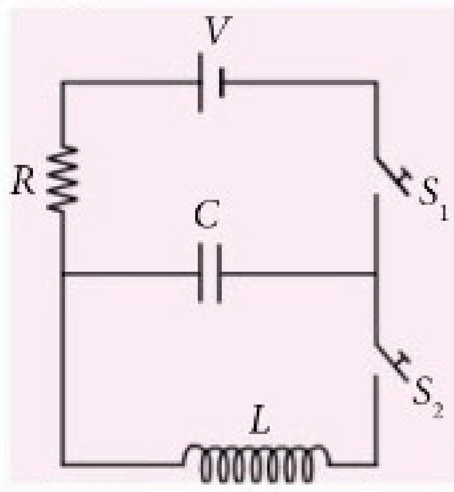
  $V_S = \left(\dfrac{N_S}{N_P}\right) V_P$ and $I_S = \left(\dfrac{N_P}{N_S}\right) I_P$
- Now, if $N_S > N_P$, the voltage is stepped up $(V_S > V_P)$. This type of arrangement is called a step-up transformer.
- If $N_S < N_P$, we have a step-down transformer. In this case $V_S < V_P$ and $I_S > I_P$. The voltage is stepped down and the current is increased.

Key Point

- At resonance impedance of the series resonant circuit is minimum so it is called acceptor circuit as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.
- The co-efficient of coupling k between two coils of inductance L_1 and L_2 having mutual inductance M is given by, $k = \dfrac{M}{\sqrt{L_1 L_2}}$
- The value of k varies from 0 to 1.

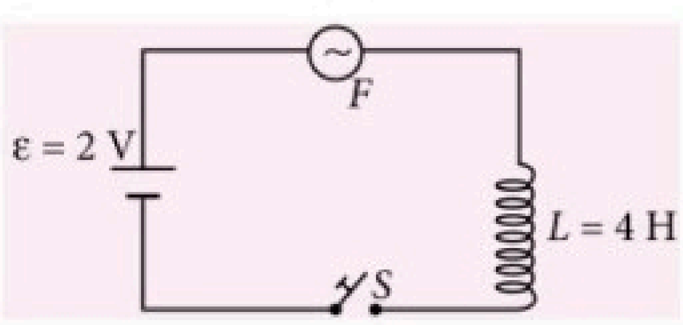
WRAP it up!

1. In an LCR circuit as shown in figure, both switches are open initially. Now, switch S_1 is closed, S_2 is kept open. If q is charge on capacitor and $\tau = RC$ is capacitative time constant, then which of the following statements is correct?



(a) Work done by the battery is half of the energy dissipated in the resistor.
(b) At $t = \tau$, $q = CV/2$.
(c) At $t = 2\tau$, $q = CV(1 - e^{-2})$.
(d) At $t = \frac{\tau}{2}$, $q = CV(1 - e^{-1})$.

2. In the circuit shown in figure, the cell is ideal. The coil has an inductance of 4 H and zero resistance. F is fuse of zero resistance and will blow when the current through it reaches 5 A. The switch is closed at $t = 0$. The fuse will blow



(a) almost at once (b) after 2 s
(c) after 5 s (d) after 10 s

3. A current I_0 is flowing through an L-R circuit of time constant τ. The source of current is switched off at time $t = 0$. Let r be the value of $(-dI/dt)$ at time $t = 0$. Assuming this ratio to be constant, the current will reduce to zero in a time interval of

(a) $\frac{\tau}{e}$ (b) τ
(c) $e\tau$ (d) $\left(1 - \frac{1}{e}\right)\tau$

4. A conducting circular loop of radius a and resistance R is kept on a horizontal plane. A vertical time varying magnetic field $B = 2t$ is switched on at time $t = 0$. Then which of the following statements is correct?

(a) Power generated in the coil at any time t is constant.
(b) Flow of charge per unit time from any section of the coil is constant.
(c) Total charge passed through any section between time $t = 0$ to $t = 2$ s is $\left(\frac{4\pi a^2}{R}\right)$.
(d) All of these.

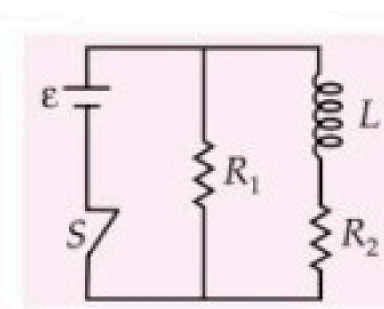
5. Two different coils have self-inductances, $L_1 = 8$ mH and $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At this time, the current, the induced voltage and the energy stored in the first coil are I_1, V_1 and U_1 respectively. Corresponding values for the second coil at the same instant are I_2, V_2 and U_2 respectively. Then which of the following is incorrect?

(a) $\frac{U_2}{U_1} = 4$ (b) $\frac{V_2}{V_1} = \frac{1}{4}$
(c) $\frac{I_1}{I_2} = \frac{1}{4}$ (d) None of these

6. A 3 H inductor is placed in series with a 10 Ω resistor. An emf of 30 V is being applied suddenly to the combination. At 0.3 s (which is one inductive time constant) after start, find at which rate energy is stored in the inductor.

(a) 20.93 W (b) 2.083 W
(c) 209.3 W (d) 31.83 W

7. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 2\,\Omega$ and $R_2 = 2\,\Omega$ are connected to a battery of emf 12 V as shown in figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is



(a) $6e^{-5t}$ V
(b) $\frac{12}{t}e^{-3t}$ V
(c) $6\,(1 - e^{-t/0.2})$ V
(d) $12\,e^{-5t}$ V

8. An inductance of $\frac{200}{\pi}$ mH, a capacitance of $\frac{10^{-3}}{\pi}$ F and a resistance of 10 Ω are connected in series with an ac source of 220 V, 50 Hz. The phase angle of the circuit is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

9. A coil has 1000 turns and 500 cm^2 as its area. The plane of the coil is placed perpendicular to a uniform magnetic field of 2×10^{-5} T. The coil is rotated through 180° in 0.2 s. The average emf induced in the coil, in mV is

(a) 5 (b) 10 (c) 15 (d) 20

10. What is the value of inductance L for which the current is maximum in a series LCR circuit with $C = 10\ \mu F$ and $\omega = 1000\ s^{-1}$?

(a) 100 mH
(b) 1 mH
(c) 10 mH
(d) can not be calculated unless R is known

11. A 150 Ω resistor and an inductance L are connected in series to a 200 V, 50 Hz source of negligible impedance. The current comes to 1 A. When the source is changed to 400 V, 100 Hz, the current will be

(a) less than 1 A
(b) 1 A
(c) between 1 A and 2 A
(d) between 2 A and 4 A

12. The flux linked with a coil is given by $\phi_B = 4t^2 - 2t + 5$. The ratio of the average emf between 0 to 3 s and instantaneous emf at 3 s is

(a) 1 (b) 5/11 (c) 11/5 (d) 11

13. A coil has an inductance of 0.7 H and is joined in series with a resistance of 220 Ω. When an alternative emf. of 220 V at 50 cps is applied to it, then the wattles component of the current in the circuit is

(a) 5 A (b) 0.5 A (c) 0.7 A (d) 7 A

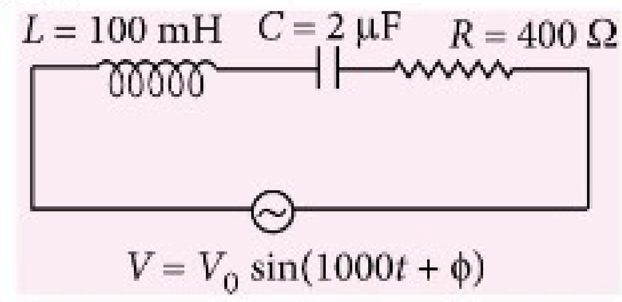
14. In a series LCR circuit, the voltage across the resistance, capacitance and inductance is 10 V each. If the capacitance is short circuited, the voltage across the inductance will be

(a) 10 V (b) $10\sqrt{2}$ V (c) $\frac{10}{\sqrt{2}}$ V (d) 20 V

15. A capacitor of capacitance 1 μF is charged to a potential of 1 V. It is connected in parallel to an inductor of inductance 10^{-3} H. The maximum current that will flow in the circuit has the value

(a) $\sqrt{1000}$ m A (b) 1 m A
(c) 10 m A (d) 1000 m A

16. Find the value of the phase difference between the current and the voltage in the series LCR circuit shown below.

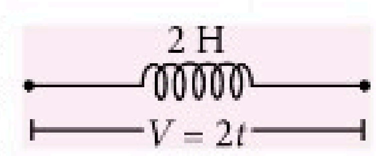


(a) 30° (b) 45° (c) 60° (d) 50°

17. A pure resistive circuit element X when connected to an ac supply of peak voltage 200 V gives a peak current of 5 A which is in phase with the voltage. A second circuit element Y, when connected to the same ac supply also gives the same value of peak current but the current lags behind by 90°. If the series combination of X and Y is connected to the same supply, what will be the rms value of current?

(a) $\frac{10}{\sqrt{2}}$ A (b) $\frac{5}{\sqrt{2}}$ A
(c) (5/2) A (d) 5 A

18. A variable voltage $V = 2t$ is applied across an inductor of inductance $L = 2$ H as shown in figure. Then choose wrong option.



(a) Current versus time graph is a parabola.
(b) Energy stored in magnetic field at $t = 2$ s is 4 J.
(c) Potential energy at time $t = 1$ s in magnetic field is increasing at a rate of 1 J s^{-1}.
(d) Energy stored in magnetic field is zero all the time.

19. A capacitor of 50 μF is connected to a power source $V = 220 \sin 50t$ (V in volt, t in second). The value of rms current (in ampere) is

(a) $\frac{\sqrt{2}}{0.55}$ A (b) 0.55 A
(c) $\sqrt{2}$ A (d) $\frac{0.55}{\sqrt{2}}$ A

20. An inductance L, a capacitance C and a resistance R may be connected to an ac source of angular frequency ω, in three different combinations of RC, RL and LC in series. Assume that $\omega L = \frac{1}{\omega C}$. The

powers consumed by the three combinations are P_1, P_2 and P_3 respectively. Then,

(a) $P_1 = P_2 < P_3$ (b) $P_1 > P_2 > P_3$
(c) $P_1 = P_2 = P_3$ (d) $P_1 = P_2 > P_3$

SOLUTIONS

1. (c) : In the figure, when switch S_1 is closed and switch S_2 is kept open, the condenser gets charged through resistance. At any time t, the charge acquired by the condenser is

$q = q_0(1 - e^{-t/RC})$, where $q_0 = CV$

At $t = 2\tau$, $q = q_0(1 - e^{-2\tau/\tau}) = CV(1 - e^{-2})$

2. (d): As $|\varepsilon| = L\frac{dI}{dt}$ $\therefore$ $dI = \frac{\varepsilon}{L}dt = \frac{2}{4}dt = \frac{dt}{2}$

As fuse will blow, when $I = 5$ A,

$\therefore$ $\frac{dt}{2} = dI = 5$ or $dt = 10\,\text{s}$

3. (b): In LR circuit, decaying current at any time t is given by $I = I_0e^{-t/\tau}$,
where τ_0 is time constant.

$$\frac{dI}{dt} = I_0e^{-t/\tau}\left(-\frac{1}{\tau}\right) \text{ or } -\frac{dI}{dt} = \frac{I_0}{\tau}e^{-t/\tau}$$

At $t = 0$, $-\frac{dI}{dt} = \frac{I_0}{\tau}e^0$, *i.e,* $r = \frac{I_0}{\tau}\times 1$

As r is constant, therefore, current will reduce to zero in time τ.

4. (d)

5. (d): Here, $L_1 = 8$ mH, $L_2 = 2$ mH

As induced voltage, $V = L\frac{dI}{dt}$ and

$\frac{dI}{dt}$ is constant for the two coils,

$\therefore$ $V \propto L$ or $\frac{V_2}{V_1} = \frac{L_2}{L_1} = \frac{2}{8} = \frac{1}{4}$

As power $= VI =$ constant

$\therefore$ $I \propto \frac{1}{V}$; $\frac{I_1}{I_2} = \frac{V_2}{V_1} = \frac{1}{4}$

As $U = \frac{1}{2}LI^2$

$$\therefore \frac{U_2}{U_1} = \left(\frac{L_2}{L_1}\right)\left(\frac{I_2}{I_1}\right)^2 = \left(\frac{2}{8}\right)\left(\frac{4}{1}\right)^2 = \frac{4}{1}$$

6. (a) : Current after one inductive time constant

$$I = I_0(1 - e^{-R/L\,t}) = I_0\left(1 - e^{-\frac{10}{3}\times 0.3}\right)$$

$= I_0(1 - e^{-1}) = 3(1 - 0.368)$ A $= 1.896$ A

$$\left[\because I_0 = \frac{\varepsilon}{R} = \frac{30}{10} = 3\,\text{A}\right]$$

As $\varepsilon - L\frac{dI}{dt} = RI, \Rightarrow \frac{dI}{dt} = \frac{\varepsilon}{L} - \frac{R}{L}I$

or $\frac{dI}{dt} = \frac{30}{3} - \frac{10}{3}\times 1.896 = 3.68\,\text{A s}^{-1}$

Rate at which energy is stored $= \frac{d}{dt}\left(\frac{1}{2}LI^2\right)$

$= LI\frac{dI}{dt} = (3\times 1.896\times 3.68)\,\text{W} = 20.93\,\text{W}$

7. (d)

8. (b): Here, $L = \frac{200}{\pi}$ mH $= \frac{200\times 10^{-3}}{\pi}$ H $= \frac{0.2}{\pi}$ H

$V_{rms} = 220$ V, $\upsilon = 50$ Hz

The inductive reactance is

$$X_L = \omega L = 2\pi\upsilon L = 2\pi\times 50\times\frac{0.2}{\pi} = 20\ \Omega$$

The capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\upsilon C} = \frac{\pi}{2\pi\times 50\times 10^{-3}} = 10\ \Omega$$

As $\tan\phi = \frac{(X_L - X_C)}{R} = \frac{20-10}{10} = 1 \Rightarrow \phi = \frac{\pi}{4}$

9. (b): When the plane of coil is perpendicular to the field, the flux linked with the coil is

$\phi_1 = NBA\cos 0° = NBA$

When the coil is turned through 180°, the flux linked with the coil is

$\phi_2 = NBA\cos 180° = -NBA$ $(\because \cos 180° = -1)$

$\therefore$ Change in flux, $\Delta\phi = \phi_2 - \phi_1$

$= -NBA - NBA = -2NBA$

$|\Delta\phi| = 2NBA$

Magnitude of average induced is

$$|\varepsilon_{av}| = \frac{|\Delta\phi|}{\Delta t} = \frac{2NBA}{\Delta t}$$

$$= \frac{2\times 1000\times 2\times 10^{-5}\times 500\times 10^{-4}}{0.2}$$

$= 10\times 10^{-3}$ V $= 10$ mV

10. (a) **11. (c)**

12. (b) : As ϕ_B (at $t = 0$) $= 5$, ϕ_B (at $t = 3$ s) $= 35$,

$$|\varepsilon_{average}| = \frac{35-5}{3-0} = 10\text{ unit}$$

Now, $|\varepsilon_{inst.}| = \frac{d\phi_B}{dt} = 8t - 2$

$\varepsilon_{inst.}$ (at $t = 3$ s) $= 8\times 3 - 2 = 22$ unit

$$\frac{|\varepsilon_{average}|}{|\varepsilon_{inst.}|} = \frac{10}{22} = \frac{5}{11}$$

13. (b): $\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{2\pi \times 50 \times 0.7}{220} = 1$

$\Rightarrow \quad \phi = 45^\circ$ and $Z = \sqrt{R^2 + X_L^2} = \sqrt{220^2 + 220^2}$

$= 220\sqrt{2}\ \Omega$

Wattles component of current = $I_v \sin\phi$

$= \frac{E_v}{Z}\sin 45^\circ = \frac{220}{220\sqrt{2}} \times \frac{1}{\sqrt{2}} = 0.5$ A

14. (c) : As $V_R = V_L = V_C = 10$ V

$\therefore \quad R = X_L = X_C$

$Z = R$

$V = IR = 10$ V

When capacitor is short circuited,

Then $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + R^2} = R\sqrt{2}$

New current $I' = \frac{V}{Z} = \frac{V}{R\sqrt{2}} = \frac{10}{\sqrt{2}R}$

$\therefore$ Potential drop across inductance = $I'X_L$

$= I'R = \frac{10 \times R}{\sqrt{2}R} = \frac{10}{\sqrt{2}}$ V

15. (a) : Charge on the emf capacitor,

$q_0 = CV = 1 \times 10^{-6} \times 1 = 10^{-6}$ C = 1 μC

Here, $q = q_0 \sin \omega t$

$\Rightarrow \quad \frac{dq}{dt} = q_0 \omega \cos \omega t$

Maximum current $I_0 = \omega q_0$

Now, $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 10^{-6}}} = \frac{1}{\sqrt{10^{-9}}} = (10^9)^{1/2}$

$\therefore \quad I_0 = (10^9)^{1/2} \times (1 \times 10^{-6}) = \sqrt{1000}$ mA

16. (b)

17. (c) : As current is in phase with the applied voltage, X must be R.

$R = \frac{V_0}{I_0} = \frac{200\text{ V}}{5\text{ A}} = 40\ \Omega$

As current lags behind the voltage by 90°, Y must be an inductor.

$X_L = \frac{V_0}{I_0} = \frac{200\text{ V}}{5\text{ A}} = 40\ \Omega$

In the series combination of X and Y,

$Z = \sqrt{R^2 + X_L^2} = \sqrt{40^2 + 40^2} = 40\sqrt{2}\ \Omega$

$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_0}{\sqrt{2}Z} = \frac{200}{\sqrt{2}(40\sqrt{2})} = \frac{5}{2}$ A

18. (d): $V = 2t$

$\therefore \quad L \cdot \frac{di}{dt} = 2t$ or $2 \cdot \frac{di}{dt} = 2t$

or $\quad \frac{di}{dt} = t$ or $(di) = t(dt)$

Integrating, we get, $i = \frac{t^2}{2}$

i.e., $i - t$ graph is a parabola.

At $t = 2$ s, $i = 2$ A

$\therefore \quad U = \frac{1}{2}Li^2 = \frac{1}{2} \times 2 \times 4 = 4$ J

$\frac{dU}{dt} = Li\left(\frac{di}{dt}\right) = (2)\left(\frac{t^2}{2}\right)(t) = t^3$

At $t = 1$ second, $\frac{dU}{dt} = 1$ J s^{-1}

19. (b): Given, $C = 50$ μF $= 50 \times 10^{-6}$ F

$V = 220 \sin (50\,t)$ ...(i)

But we know that,

$V = V_0 \sin \omega t$ ...(ii)

On comparing both equations we get

$V_0 = 220$ V, $\omega = 50$ rad s^{-1}

The capacitive reactance of the circuit is given by

$X_C = \frac{1}{\omega C} = \frac{1}{50 \times 50 \times 10^{-6}} = 400\ \Omega$

the rms values of current in the circuit are given as,

$i_0 = \frac{V_0}{X_C} = \frac{220}{400} = \frac{11}{20} = 0.55$ A

20. (d): When L and C are joined in series, there is no power loss.

$\therefore \quad P_3 = 0$

There is no loss of power across L or C.

When R and L are joined in series, the impedance

$Z_1 = \sqrt{R^2 + \omega^2 L^2}$

and when R and C are joined in series,

$Z_2 = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$

But it is given that $\omega L = \frac{1}{\omega C}$

$\therefore \quad Z_1 = Z_2 = R$

$\therefore$ Powers consumed by the combinations RC and RL are equal *i.e.*, $P_1 = P_2$ and $P_3 = 0$

$\therefore \quad P_1 = P_2 > P_3$

CBSE warm-up!

Exam on 10^{th} December 2021

CLASS-XII

TERM-I OBJECTIVE TYPE QUESTIONS

Practice Paper 2021

Time allowed : 90 minutes
Maximum marks : 35

GENERAL INSTRUCTIONS

1. The Question Paper contains three sections.
2. Section A has 25 questions. Attempt any 20 questions.
3. Section B has 24 questions. Attempt any 20 questions.
4. Section C has 6 questions. Attempt any 5 questions.
5. All questions carry equal marks.
6. There is no negative marking.

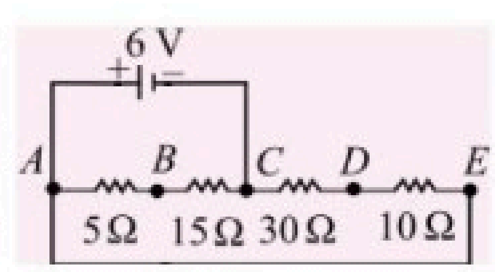
SECTION - A

This section consists of 25 multiple choice questions with overall choice to attempt any 20 questions. In case more than desirable number of questions are attempted, ONLY first 20 will be considered for evaluation.

1. In an atom electrons revolves around the nucleus along a path of radius 0.72 Å making 9.4×10^{18} revolution per second. The equivalent current is ($e = 1.6 \times 10^{-19}$ C)

(a) 1.2 A (b) 1.5 A (c) 1.4 A (d) 1.8 A

2. Four resistors are connected as shown in the figure. A 6 V battery of negligible resistance is connected across terminals A and C. The potential difference across terminals B and D will be



(a) Zero (b) 1.5 V (c) 2 V (d) 3 V

3. The magnetic flux of 3×10^{-4} Wb are passing through a coil of 100 turns. If the e.m.f. induced in the coil is 1.5 V, the time interval will be

(a) 1 sec (b) 0.1 sec
(c) 0.02 sec (d) 0.4 sec

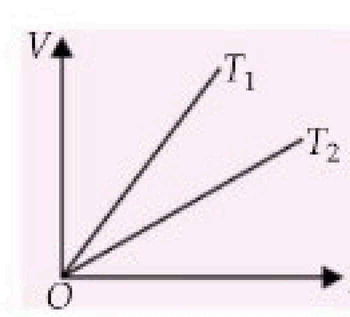
4. Force between two identical charges placed at a distance r in vacuum is F. Now a slab of dielectric constant $K = 4$ is inserted between these two charges. The thickness of the slab is $r/2$. The force between the charges will now become

(a) $F/4$ (b) $F/2$ (c) $3F/5$ (d) $4F/9$

5. The electrical resistance of a conductor depends upon

(a) size of the conductor
(b) temperature of the conductor
(c) geometry of the conductor
(d) all of these.

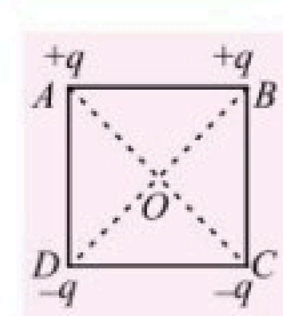
6. The voltage V and current I graphs for a conductor at two different temperatures T_1 and T_2 are shown in the figure. The relation between T_1 and T_2 is



(a) $T_1 > T_2$ (b) $T_1 < T_2$

(c) $T_1 = T_2$ (d) $T_1 = \frac{1}{T_2}$

7. Four charges are arranged at the corners of a square $ABCD$ as shown in the figure. The force on the charge kept at the centre O is



(a) along the diagonal BD

(b) along the diagonal AC
(c) zero
(d) perpendicular to side AB

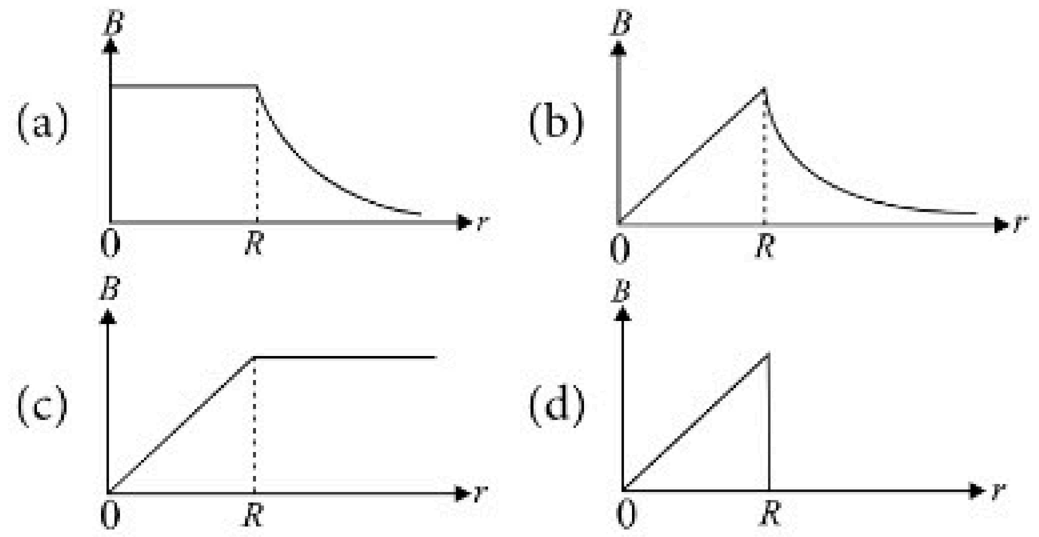
8. A sphere has surface charge density s. It is surrounded by a spherical shell. The surface charge density on the spherical shell is

(a) σ (b) $-\sigma$ (c) zero (d) $\frac{\sigma}{2}$

9. An LC circuit contains a 40 mH inductor and a 25 μF capacitor. The resistance of the circuit is negligible. The time is measured from the instant the circuit is closed. The energy stored in the circuit is completely magnetic at times (in milliseconds)

(a) 0, 3.14, 6.28 (b) 0, 1.57, 4.71
(c) 1.57, 4.71, 7.85 (d) 1.57, 3.14, 4.71

10. The correct plot of the magnitude of magnetic field $\vec{B}$ vs distance r from centre of the wire is, if the radius of wire is R



11. A circular coil of radius 10 cm having 100 turns carries a current fo 3.2 A. The coil is placed in a vertical plane and is free to rotate about a horizontal axis which coincides with its diameter. A uniform magnetic field of 5 T in the horizontal direction exists such that initially the axis of the coil is in direction of the field. The coil rotates through an angle of 60° under the influence of magnetic field. The magnitude of torque on the coil in the final position is

(a) 25 N m (b) $25\sqrt{3}$ N m
(c) 40 N m (d) $40\sqrt{3}$ N m

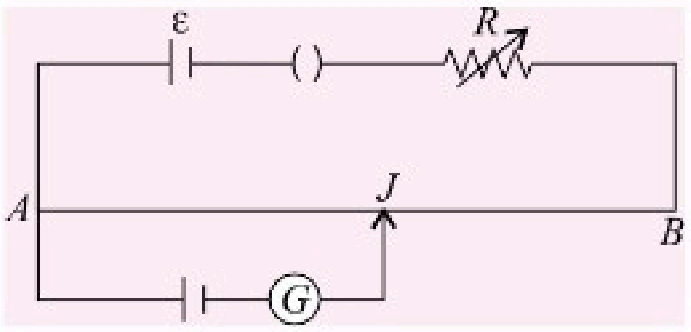
12. A parallel plate capacitor is connected to a 5 V battery and charged. The battery is then disconnected and a glass slab is introduced between the plates. Then the quantities that decrease are

(a) charge and potential difference
(b) charge and capacitance
(c) energy stored and potential difference
(d) energy stored and capacitance

13. When a current of 2 A flows in a battery from negative to positive terminal, the potential difference across it is 12 V. If a current of 3 A flowing in the opposite direction produces a potential difference of 15 V, the emf of the battery is

(a) 12.6 V (b) 13.2 V (c) 13.5 V (d) 14.0 V

14. AB is a wire of potentiometer with the increase in the value of resistance R, the shift in the balance point J will be



(a) towards B (b) towards A
(c) remains constant
(d) first towards B then back towards A.

15. The unit of inductance is equivalent to

(a) $\frac{\text{volt} \times \text{ampere}}{\text{second}}$ (b) $\frac{\text{ampere}}{\text{volt} \times \text{second}}$
(c) $\frac{\text{volt}}{\text{ampere} \times \text{second}}$ (d) $\frac{\text{volt} \times \text{second}}{\text{ampere}}$

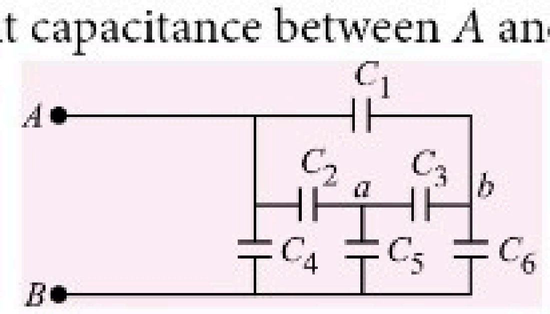
16. If the number of turns per unit length of a coil of solenoid is doubled, the self-inductance of the solenoid will

(a) remain unchanged (b) be halved
(c) be doubled (d) become four times.

17. If an electron is moving with velocity $\vec{v}$ produces a magnetic field $\vec{B}$, then

(a) the direction of field $\vec{B}$ will be same as the direction of velocity $\vec{v}$.
(b) the direction of field $\vec{B}$ will be opposite to the direction of velocity $\vec{v}$.
(c) the direction of field $\vec{B}$ will be perpendicular to the direction of velocity $\vec{v}$.
(d) the direction of field $\vec{B}$ does not depend upon the direction of velocity $\vec{v}$.

18. Six equal capacitors each of capacitance C are connected as shown in the figure. Then the equivalent capacitance between A and B is



(a) $6C$ (b) C (c) $2C$ (d) $C/2$

19. A conducting wire is dropped along east west direction, then
(a) induced current flows from east to west
(b) induced current flows from west to east
(c) no current is induced
(d) no e.m.f. is induced.

20. When a current carrying conductor is placed in a magnetic field, it experiences a force, the direction of which is given by
(a) Fleming's left hand rule
(b) Maxwell's rule
(c) Fleming's right hand rule
(d) Faraday's rule

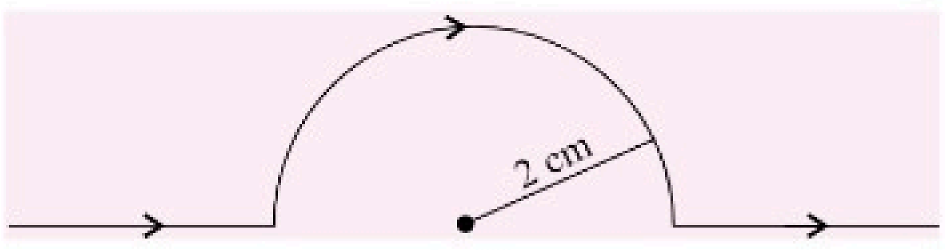
21. The selectivity of a series *LCR* a.c. current is large, when
(a) *L* is large and *R* is large
(b) L is small and *R* is small
(c) *L* is large and *R* is small
(d) *LR*

22. Two spherical conductors *A* and *B* of radii 1 mm and 2 mm are separated by a distance of 5 mm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium conditions, the ratio of the magnitude of the electric fields at the surfaces of spheres of *A* and *B* is
(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

23. For which of the following dependences of drift velocity v_d on electric field *E*, is Ohm's law obeyed?
(a) $v_d \propto E$ (b) $v_d \propto E^2$
(c) $v_d \propto \sqrt{E}$ (d) $v_d \propto \frac{1}{E}$

24. In parallel combination of *n* cells, we obtain
(a) more voltage (b) more current
(c) less voltage (d) less current.

25. A straight wire carrying a current of 13 A is bent into a semi-circular arc of radius 2 cm as shown in figure. The magnetic field is 1.5×10^{-4} T at the centre of arc, then the magnetic field due to straight segment is

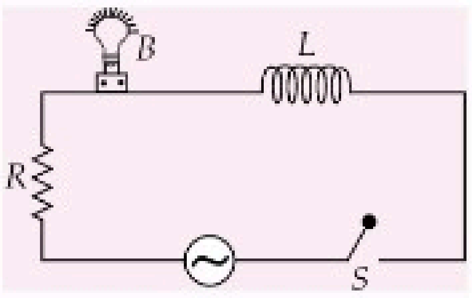


(a) 1.5×10^{-4} T (b) 2.5×10^{-4} T
(c) zero (d) 3×10^{-4} T

SECTION - B

This section consists of 24 multiple choice questions with overall choice to attempt any 20 questions. In case more than desirable number of questions are attempted, ONLY first 20 will be considered for evaluation.

26. Switch *S* is closed at $t = 0$. After sufficiently long time, an iron rod is inserted into the inductor *L*. Then, the light bulb



(a) glows more brightly
(b) gets dimmer
(c) glows with the same brightness
(d) gets momentarily dimmer and then glows more brightly

27. The magnetic induction associated with current flowing in a hollow copper tube will be
(a) only inside
(b) only outside
(c) both inside and outside
(d) neither inside nor outside

28. A moving coil galvanometer has a coil of 10 turns each of length 12 cm and breadth 8 cm. The coil of M.C.G. carries a current of 125 μA. The coil is kept perpendicular to a uniform magnetic field of induction 10^{-2} T. The twist constant of phosphor bronze fibre is 12×10^{-9} N m per degree. Find the deflection produced.
(a) 10° (b) 20° (c) 30° (d) 40°

29. Coulomb's law for the force between electric charges most closely resembles with
(a) law of conservation of energy
(b) Newton's law of gravitation
(c) Newton's 2nd law of motion
(d) law of conservation of charge

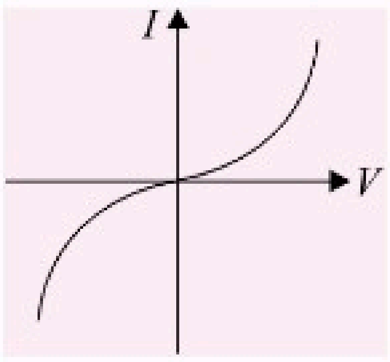
30. If only 1% of the total current is to be passed through a galvanometer of resistance *G*, then the resistance of the shunt will be
(a) $\frac{G}{25}\,\Omega$ (b) $\frac{G}{35}\,\Omega$ (c) $\frac{G}{49}\,\Omega$ (d) $\frac{G}{99}\,\Omega$

31. A negatively charged oil drop is prevented from falling under gravity by applying a vertical electric field 100 V m^{-1}. If the mass of the drop is

1.6×10^{-3} g, the number of electrons carried by the drop is ($g = 10$ m s^{-2})

(a) 10^{18} (b) 10^{15} (c) 10^{12} (d) 10^{9}

32. The I-V characteristics shown in figure represents



(a) ohmic conductors
(b) non-ohmic conductors
(c) insulators
(d) superconductors.

33. A rod lies along the x-axis with one end at the origin and the other at $x \to \infty$. It carries a uniform charge λ C m^{-1}. The electric field at the point $x = -a$ on the axis will be

(a) $\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 a}(-\hat{i})$ (b) $\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 a}(\hat{i})$

(c) $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 a}(-\hat{i})$ (d) $E = \frac{\lambda}{2\pi\varepsilon_0 a}(\hat{i})$

34. An electric dipole is placed in an uniform electric field with the dipole axis making an angle θ with the direction of the electric field. The orientation of the dipole for stable equilibrium is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) 0 (d) $\frac{\pi}{2}$

35. Two copper wires of length l and $2l$ have radii, r and $2r$ respectively. What is the ratio of their specific resistances?

(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) 1 : 3

36. When the rate of change of current is unity, the induced e.m.f. is equal to

(a) thickness of coil
(b) number of turns in coil
(c) coefficient of self inductance
(d) total flux linked with coil.

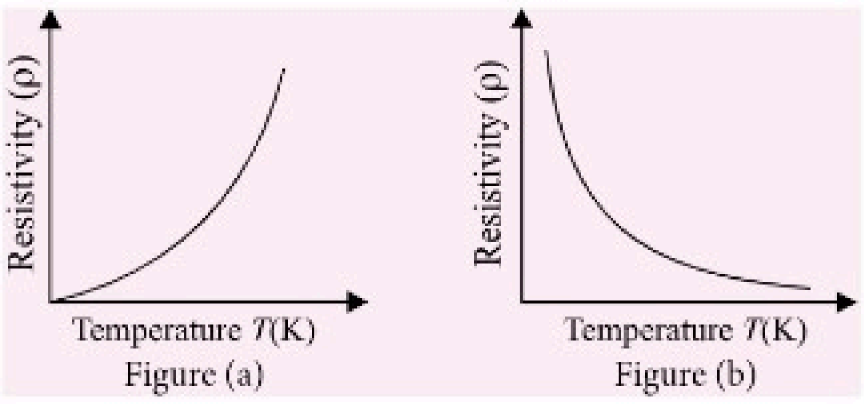
37. In an ideal transformer, the primary and the secondary voltages always have

(a) equal magnitude
(b) the same phase
(c) a phase difference of 90°
(d) a phase difference of 180°.

38. A galvanometer may be converted into an ammeter or a voltmeter. In which of the following cases is the resistance of the device so obtained least?

(a) Ammeter of range 1 A
(b) Ammeter of range 10 A
(c) Voltmeter of range 1 V
(d) Voltmeter of range 10 V

39. Figure (a) and figure (b) both are showing the variation of resistivity (ρ) with temperature (T) for some materials. Identify the type of these materials.



(a) Conductor and semiconductor
(b) Conductor and Insulator
(c) Insulator and semiconductor
(d) Both are conductors

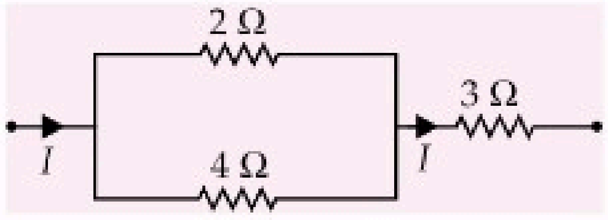
40. If n identical drops of mercury are combined to form a bigger drop, then the capacity of bigger drop, if capacity of each drop of mercury is C, is

(a) $n^{1/3}C$ (b) $n^{2/3}C$ (c) $n^{1/4}C$ (d) nC

41. A light bulb is rated at 100 W for a 220 V ac supply. The resistance of the bulb is

(a) 284 Ω (b) 384 Ω (c) 484 Ω (d) 584 Ω

42. In the circuit shown in figure heat developed across 2 Ω, 4 Ω and 3 Ω resistances are in the ratio of



(a) 2 : 4 : 3 (b) 8 : 4 : 12
(c) 4 : 8 : 27 (d) 8 : 4 : 27

43. Eight dipoles of charges of magnitude e are placed inside a cube. The total electric flux coming out of the cube will be

(a) $\frac{8e}{\varepsilon_0}$ (b) $\frac{16e}{\varepsilon_0}$ (c) $\frac{e}{\varepsilon_0}$ (d) zero

44. The total resistance in the parallel combination of three resistances 9 Ω, 7 Ω and 5 Ω is

(a) 1.22 Ω (b) 2.29 Ω (c) 4.22 Ω (d) 2.20 Ω

45. Given below are two statements labelled as Assertion (A) and Reason (R)

Assertion (A) : An ammeter is connected in series in the circuit.

Reason (R) : An ammeter is a high resistance galvanometer.

Select the most appropriate answer from the options given below:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

46. Given below are two statements labelled as Assertion (A) and Reason (R)

Assertion (A) : In order to have maximum current, a series combinations of cells is used where their internal resistance is much smaller than the external resistance.

Reason (R) : Current flowing through the circuit is inversely proportional to total resistance.

Select the most appropriate answer from the options given below:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

47. Given below are two statements labelled as Assertion (A) and Reason (R)

Assertion (A) : A point charge is lying at the centre of a cube of each side . The electric flux emanating from each surface of the cube is $\frac{1}{6}^{\text{th}}$ of total flux.

Reason (R) : According to Gauss theorem, total electric flux through a closed surface enclosing a charge is equal to $1/\varepsilon_0$ times the magnitude of the charge enclosed.

Select the most appropriate answer from the options given below:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

48. Given below are two statements labelled as Assertion (A) and Reason (R)

Assertion (A) : A copper sheet is placed in a magnetic field. If we pull it out of the field or push it into the field, we experience an opposing force.

Reason (R) : According to Lenz's law eddy current produced in sheet opposes the motion of the sheet.

Select the most appropriate answer from the options given below:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

49. Given below are two statements labelled as Assertion (A) and Reason (R)

Assertion (A) : A dynamo converts mechanical energy into electrical energy.

Reason (R) : The dynamo is based on the principle of electromagnetic induction.

Select the most appropriate answer from the options given below:

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

SECTION - C

This section consists of 6 multiple choice questions with an overall choice to attempt any 5. In case more than desirable number of questions are attempted, ONLY first 5 will be considered for evaluation.

50. A proton, a deutron and an α-particle with the same kinetic energy enter a region of uniform magnetic field, moving at right angle to B. What is the ratio of the radius of their circular paths?

(a) $1:\sqrt{2}:1$ (b) $1:\sqrt{2}:\sqrt{2}$
(c) $\sqrt{2}:1:1$ (d) $\sqrt{2}:\sqrt{2}:1$

51. An inductance L, a capacitance C and a resistance R may be connected to an ac source of angular frequency ω, in three different combinations of RC, RL and LC in series. Assume that $\omega L = \frac{1}{\omega C}$. The power drawn by the three combinations are P_1, P_2, P_3 respectively. Then,

(a) $P_1 > P_2 > P_3$ (b) $P_1 = P_2 < P_3$
(c) $P_1 = P_2 > P_3$ (d) $P_1 = P_2 = P_3$

Case Study : Read the following paragraph and answer the questions :

Self Inductance and Mutual Inductance

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with it will also change. As a result of this, an emf is induced in the coil or the circuit which opposes the change that causes it. This phenomenon is known as self induction and the emf induced is known as self induced emf or back emf. When a current I flows through a coil and ϕ is the magnetic flux linked with the coil, then $\phi \propto I$ or $\phi = LI$ where L is coefficient of self induction or self inductance of the coil.

The self induced emf is $\varepsilon = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$

Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighbouring coil or circuit will also change. Hence an emf will be induced in the neighbouring coil or circuit. This phenomenon is known as mutual induction.

Let I_P be the current flowing through primary coil at any instant. If ϕ_S is the flux linked with secondary coil then $\phi_S \propto I_P$ or $\phi_S = MI_P$ where M is coefficient of mutual inductance of the two coils.

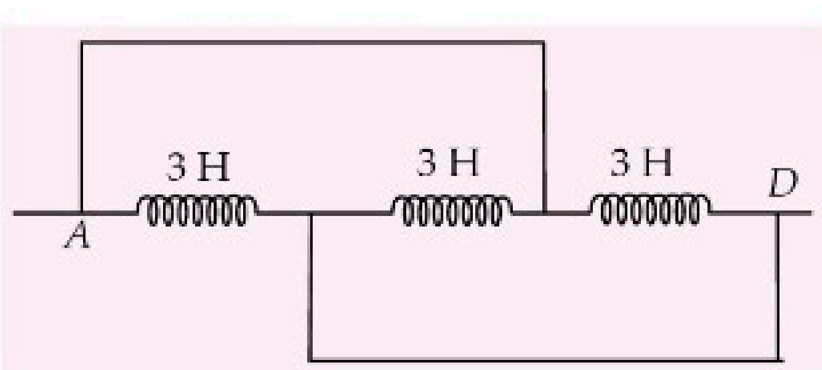
The emf induced in the secondary coil is given by $\varepsilon_S = -M\frac{dI_P}{dt}$

The SI unit of M and L is henry (H) and its dimensional formula is $[ML^2T^{-2}A^{-2}]$.

52. If N is the number of turns in a coil, the value of self-inductance varies as
(a) N^0 (b) N (c) N^2 (d) N^{-2}

53. A short solenoid of radius a, number of turns per unit length n_1, and length L is kept coaxially inside a very long solenoid of radius b, number of turns per unit length n_2. What is the mutual inductance of the system?
(a) $\mu_0\pi b^2 n_1 n_2 L$ (b) $\mu_0\pi a^2 n_1 n_2 L^2$
(c) $\mu_0\pi a^2 n_1 n_2 L$ (d) $\mu_0\pi b^2 n_1 n_2 L^2$

54. The inductance between A and D as shown in figure is



(a) 1 H (b) 9 H (c) 0.66 H (d) 0.99 H

55. The self inductance of a long solenoid cannot be increased by
(a) increasing its area of cross section
(b) increasing its length
(c) increasing the current through it
(d) increasing the number of turns in it.

SOLUTIONS

1. (b) : Radius of electron orbit, $r = 0.72$ Å $= 0.72 \times 10^{-10}$ m

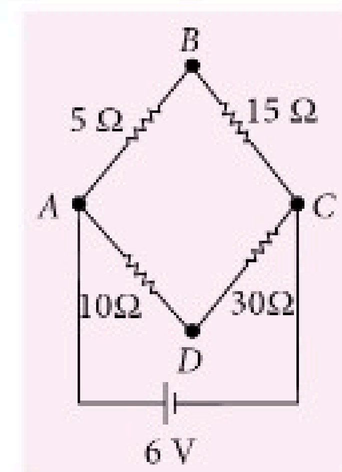
Frequency of revolution of electron in orbit of a given atom $\upsilon = 9.4 \times 10^{18}$ rev s^{-1}

(where T is time period of revolution of electron in orbit)

$\therefore$ Then equivalent current is

$I = \frac{e}{T} = e\upsilon = 1.6 \times 10^{-19} \times 9.4 \times 10^{18} = 1.504$ A

2. (a) : The given figure is a circuit of balanced Wheatstone bridge as shown in the figure. Points B and D would be at the same potential,
i.e., $V_B - V_D = 0$ volt



3. (c) : $e = \frac{Nd\phi}{dt}$

$\therefore \quad dt = \frac{Nd\phi}{e} = \frac{100 \times 3 \times 10^{-4}}{1.5} = 2 \times 10^{-2}\ \text{s} = 0.02$ s

4. (d) : According to Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \quad \text{...(i)}$$

Suppose force between the charges is same when charges are r' distance apart in dielectric.

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{Kr'^2} \quad \text{...(ii)}$$

From (i) and (ii), $Kr'^2 = r^2$ or $r = \sqrt{K}r'$

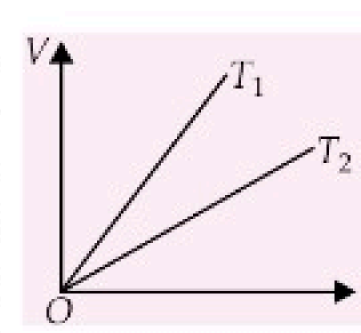
Thus distance r' of dielectric is equivalent to $r/\sqrt{K}$ distance of air,

In the given situation, force between the charges would be

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\left(\frac{r}{2} + \sqrt{4}\frac{r}{2}\right)^2} = \frac{4}{9}\frac{q^2}{4\pi\varepsilon_0 r^2} = \frac{4}{9}F.$$

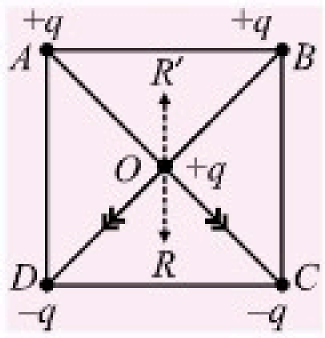
5. (d) : The electrical resistance of a conductor is depend upon all factors *i.e.*, size, temperature and geometry of conductor.

6. (a) : The slope of V-I graph gives the resistance of a conductor at a given temperature. From the graph, it follows that resistance of a conductor at temperature T_1 is greater than at temperature T_2. As the resistance of a conductor is more at higher temperature and less at lower temperature, hence $T_1 > T_2$.



7. (d) : Since $ABCD$ is a square, the centre O is equispaced from all the corners of the square.

Let the charge at O be $+q$.



Therefore force on the charge at O due to charges at A and C will be along $\overrightarrow{OC}$ as one force is repulsive and the other is attractive. So the resultant of these two forces will be along $\overrightarrow{OC}$.

Similarly the resultant force due to charges at B and D will be along $\overrightarrow{OD}$. Hence the resultant of all the forces at O will be along $\overrightarrow{OR}$. In case the charge at O be $-q$, then the resultant will be along $\overrightarrow{OR'}$. Since both $\overrightarrow{OR}$ and $\overrightarrow{OR'}$ are perpendicular to the side AB, so the resultant force on any charge placed at O due to the given configuration of charges will be perpendicular to the side AB.

8. (c) : The surface charge density on the spherical shell is zero.

9. (c) : Here, $L = 40$ mH $= 40 \times 10^{-3}$ H, $C = 25\ \mu$F $= 25 \times 10^{-6}$ F

$$\upsilon = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{40\times10^{-3}\times25\times10^{-6}}} = \frac{10^3}{2\pi}\text{ Hz}$$

$$\therefore\ T = \frac{1}{\upsilon} = \frac{2\pi}{10^3}\text{ s} = 2\pi\times10^{-3}\text{ s} = 2\pi\text{ ms}$$

Energy stored is completely electrical at times

$$t = 0, \frac{T}{2}, T, \frac{3}{2}T, \ldots\ldots\ldots$$

Energy stored is completely magnetic at times

$$t = \frac{T}{4}, \frac{3}{4}T, \frac{5}{4}T, \ldots\ldots\ldots\ldots$$

Hence, $t = \frac{\pi}{2}\text{ ms}, \frac{3\pi}{2}\text{ ms}, \frac{5\pi}{2}\text{ ms}$

$= 1.57$ ms, 4.71 ms, 7.85 ms

10. (b) : The magnetic field from the centre of wire of radius R is given by

$$B = \left(\frac{\mu_o I}{2R^2}\right)r \qquad (r < R) \Rightarrow B \propto r$$

and $$B = \frac{\mu_o I}{2\pi r} \qquad (r > R) \Rightarrow B \propto \frac{1}{r}$$

From this description, we can say that the graph (b) is a correct representation.

11. (b) : Torque $|\vec{\tau}| = |\vec{m}\times\vec{B}| = mB\sin\theta$

Here, $m = 10$ A m^2, $B = 5$ T

Now initially $\theta = 0°$

Thus, initial torque, $\tau_i = 0$

In final position $\theta = 60°$

$$\therefore\ \tau_f = mB\sin 60° = 10\times5\times\frac{\sqrt{3}}{2} = 25\sqrt{3}\text{ N m}$$

12. (c) : When a charging battery is disconnected and a glass slab is introduced between the plates, then

(i) Charge remains the same.

(ii) Capacitance increases.

(iii) Potential difference across the plates decreases.

(iv) Energy stored decreases.

13. (b) : Let ε be emf and r be internal resistance of the battery.

In first case, $12 = \varepsilon - 2r$ … (i)

In second case, $15 = \varepsilon + 3r$ … (ii)

Substract (ii) from (i), we get, $r = \frac{3}{5}\ \Omega$

Putting this value of r in eqn. (i), we get

$$\varepsilon = 12 + \frac{2\times3}{5} = \frac{60+6}{5} = \frac{66}{5} = 13.2\text{ V}$$

14. (a) : Due to increase in resistance R the current through the wire will decrease and hence the potential gradient also decreases, which results in increase in balancing length. So J will shift towards B.

15. (d) : As $\varepsilon = L\frac{dI}{dt},\ L = \varepsilon\frac{dt}{dI}$

$$\Rightarrow\ L = \frac{\text{volt}\times\text{second}}{\text{ampere}}$$

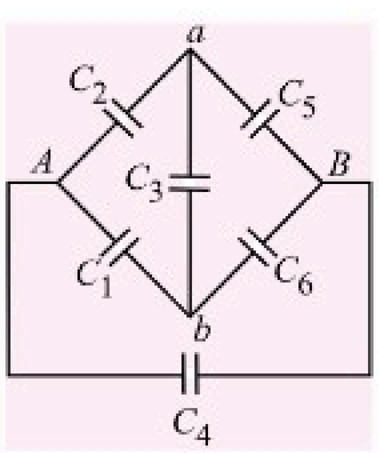
16. (d) : Self inductance of a solenoid $= \mu_0 n^2 Al$, where n is the number of turns per unit length.

$\therefore$ Self inductance is directly proportional to n^2, so self inductance becomes 4 times when n is doubled.

17. (c) : According to Biot-Savart law, the magnetic field $\vec{B} = \frac{\mu_o}{4\pi}\cdot\frac{q(\vec{v}\times\vec{r})}{r^3}$

The direction of $\vec{B}$ will be along $\vec{v}\times\vec{r}$ *i.e.*, perpendicular to the plane containing $\vec{v}$ and $\vec{r}$.

18. (c) : The given network of capacitors can be redrawn as follows C_4 is in parallel to a balanced Wheatstone bridge made from the rest five capacitors as shown in the figure. Therefore, equivalent capacitance $= C + C = 2C$.



19. (b) : East-West direction is perpendicular to north-south direction. Hence induced current flows from West to East by Fleming's left hand rule.

20. (a) : The direction is given by Fleming's left hand rule.

21. (c) : Selectivity depends on the quality factor, $Q = \frac{\omega_0 L}{R}$.

22. (b) : Here, $r_1 = 1$ mm, $r_2 = 2$ mm, $d = 5$ mm
When spheres are connected by a conducting wire, charge flows from the sphere at higher potential to the sphere at lower potential, till their potentials become equal.

Now $\frac{C_1}{C_2} = \frac{r_1}{r_2} = \frac{1}{2}$

As potential V is same, therefore $\frac{q_1}{q_2} = \frac{C_1 V}{C_2 V} = \frac{C_1}{C_2} = \frac{1}{2}$

$$\text{Now } \frac{E_1}{E_2} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}}{\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}} = \frac{q_1}{q_2} \times \left(\frac{r_2}{r_1}\right)^2 = \frac{1}{2}\left(\frac{2}{1}\right)^2 = 2$$

23. (a) : $v_d = \frac{I}{nAe} = \frac{J}{ne} = \left(\frac{\sigma}{ne}\right)E$

24. (b) : In parallel combination of cells the voltage across the terminals is same and resistance is minimum. Therefore from $V = IR$, the current drawn from cell combination will be more.

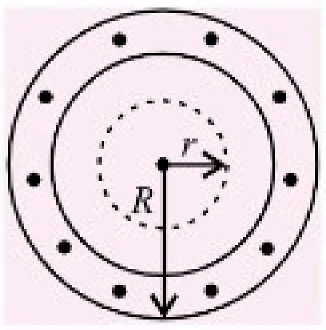
25. (c) : Since dl and r for each element of the straight segments are parallel. Therefore

$\vec{dl} \times \vec{r} = 0$

Hence, B is also zero.

26. (b) : As the rod is inserted, inductance increases and hence the voltage across inductor increases. This causes a drop in the voltage across the bulb and hence it gets dimmer.

27. b) : Cross-section of a hollow tube with current coming out.

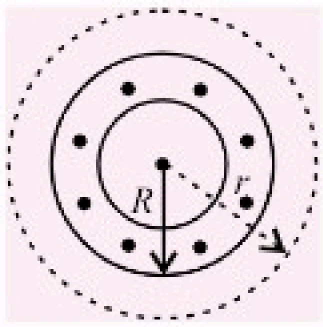


For $B(r < R)$, Using Ampere's law,

$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_p$

$B(2\pi r) = 0 \Rightarrow B = 0$

B For $(r > R)$,



$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_p$

$B(2\pi r) = \mu_0 I$

$B = \frac{\mu_0 I}{2\pi r}$

$\Rightarrow B$ exists outside only.

28. (a) : Current flowing through coil of M.C.G.

$I = \left(\frac{c}{nBA}\right)\theta$

$\theta = \frac{nBA}{c} I$

$$\theta = \frac{10\times10^{-2}\times12\times10^{-2}\times8\times10^{-2}\times125\times10^{-6}}{12\times10^{-9}}$$

$\theta = 10°$

29. (b) : Coulomb's law for the force between electric charges most closely resembles with Newton's law of gravitation.

30. (d) : $\frac{I_g}{I} = \frac{1}{100} \Rightarrow n = \frac{I}{I_g} = 100$

$\therefore \quad S = \frac{G}{n-1} = \frac{G}{99}\ \Omega$

31. (c) : For the drop to be stationary,
Force on the drop due to electric field = Weight of the drop $qE = mg$

$\therefore \ q = \frac{1.6\times10^{-6}\times10}{100} = 1.6 \times 10^{-7}$ C

Number of electrons carried by the drop is

$N = \frac{q}{e} = \frac{1.6\times10^{-7}\ \text{C}}{1.6\times10^{-19}\ \text{C}} = 10^{12}$

32. (b) : The given figure shows I-V characteristics of non-ohmic or non-linear conductors.

33. (a) : The electric field at the point $x = -a$ on the axis will be $\vec{E} = \frac{\lambda}{4\pi\varepsilon_0 a}(-\hat{i})$

34. (c) : For stable equilibrium, $\theta = 0$ *i.e.* electric dipole is aligned along the electric field.

35. (c) : The specific resistance is independent of the dimensions of the wire but depends upon the nature of the material of the wire.

36. (c) : As, $e = L\frac{dI}{dt}$

When $\frac{dI}{dt} = 1$, $\quad \therefore \quad e = L$

37. (b) : In an ideal transformer, the primary and the secondary voltages always have the same phase.

38. (b) : A voltmeter always has high resistance as R is in series.
To increases the range of ammeter *i.e.* to increase I, its resistance must decrease.

$\therefore$ High range $\Rightarrow$ low resistance.

39. (a) : In conductors due to increase in temperature the resistivity increases and in semiconductors it decreases exponentially.

40. (a) : Let radius of small drop = r

Volume of n drops $= n\frac{4}{3}\pi r^3$

Let R be the radius of bigger drop.
Volume will remain same.

$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi n r^3$ or $R = n^{1/3} r$ $\therefore$ $C = 4\pi\varepsilon_0 r; C' = 4\pi\varepsilon_0 R$

Hence, $C' = Cn^{1/3}$

41. (c) : Here, $P = 100$ W, $V_{rms} = 220$ V
Resistance of the bulb is

$$R = \frac{V^2_{rms}}{P} = \frac{(220)^2}{100} = 484\ \Omega$$

42. (d) : Current through 2 Ω resistor $I_1 = \frac{2I}{3}$
Heat produced per second,

$$H_1 = I_1^2 \times 2 = \left(\frac{2I}{3}\right)^2 \times 2 = \frac{8I^2}{9}$$

Current through 4 Ω resistor $I_2 = \frac{I}{3}$

Heat produced per second

$$H_2 = I_2^2 \times 4 = \left(\frac{I}{3}\right)^2 \times 4 = \frac{4I^2}{9}$$

Current through 3 Ω resistor = I

Heat produced $H_3 = I^2 \times 3 = 3I^2 = \frac{27I^2}{9}$

$\therefore$ $H_1 : H_2 : H_3 = 8 : 4 : 27.$

43. (d) : Since net charge inside the cube is zero, $\therefore \phi_E = 0$

44. (d) : In the parallel combination of three resistances, the equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

or $\frac{1}{R_{eq}} = \frac{1}{9} + \frac{1}{7} + \frac{1}{5} = \frac{143}{315}$; $R_{eq} = \frac{315}{143} = 2.20\ \Omega$

45. (c) : An ammeter is a low resistance galvanometer. It is used to measure the current in amperes. To measure the current of a circuit, the ammeter is connected in series in the circuit so that the current to be measured must pass through it. Since, the resistance of ammeter is low, so its inclusion in series in the circuit does not change the resistance and hence the main current in the circuit.

46. (a) : The current flowing through the circuit is given by $I = \frac{nE}{R + nr}$, so when $R << nr$, $I = E/r$ = current due to single cell, when $R >> nr$, $I = nE/R = n$ times the current due to single cell. Thus when cell are connected in series, we get maximum current where total internal resistance of the cells is negligible.

47. (b) : The electric flux through the cube, $\phi = q/\varepsilon_0$.
A cube has six faces of equal area. Therefore, electric flux through each face $= \frac{1}{6}\phi = (q/\frac{1}{6})$.

Therefore equal to $\frac{1}{6}^{\text{th}}$ of the total flux.

48. (a) : When we pull a copper plate out of the magnetic field or push it into the magnetic field, magnetic flux linked with the plate changes. As a result of this eddy currents are produced in the plate which oppose its motion (according to Lenz's law).

49. (a)

50. (a) : If a magnetic field is perpendicular to velocity of particle,

$$\frac{mv^2}{r} = Bqv \text{ or } r = \frac{mv}{Bq} \text{ and } E_k = \frac{1}{2}mv^2$$

$$mv = \sqrt{2m\ E_k}\text{ . So, } r = \frac{\sqrt{2mE_k}}{Bq} \text{ or } r \propto \frac{\sqrt{m}}{q}$$

For same value of E_k and B

$$r_p : r_d : r_\alpha = \frac{\sqrt{m}}{e} : \frac{\sqrt{2m}}{e} : \frac{\sqrt{4m}}{2e} \Rightarrow 1 : \sqrt{2} : 1$$

51. (c) : The LC circuit draws no power.

When $\omega L = \frac{1}{\omega C}$ the impedance of the RC and LR circuits are equal, and hence they draw the same power.

$\therefore$ $P_1 = P_2 > P_3$

52. (c) : Self inductance $= \frac{\mu_0 N^2 A}{l}$

53. (c) : The mutual inductance of the system is
$M = \mu_0 n_1 n_2 \pi a^2 L$

54. (a) : The three coils are in parallel.

$$\therefore \frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1 \text{ or } L_p = 1\text{ H}$$

55. (c) : The self inductance of a long solenoid is given by $L = \mu_r \mu_0 n^2 A l$
where, n = Number of turns per unit length
l = Length
A = Cross-sectional area
μ_r = Relative permeability of the medium
From the given expression it is clear that the self inductance of a long solenoid does not depend upon the current flowing through it.

◈◈

MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Time Taken : 60 Min.

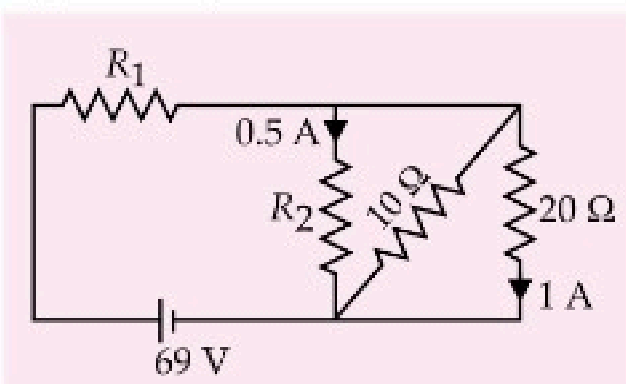
Current Electricity

Total Marks : 120

NEET

Only One Option Correct Type

1. In the circuit shown below, the resistances R_1 and R_2 are respectively

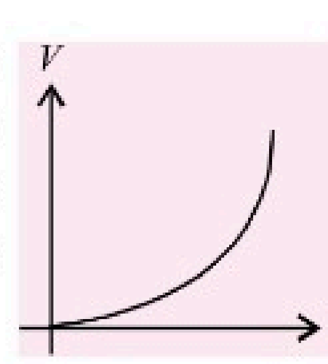


(a) 14 Ω and 40 Ω (b) 40 Ω and 14 Ω
(c) 40 Ω and 30 Ω (d) 14 Ω and 30 Ω

2. A current carrying wire in the shape of a circle as the current progresses along the wire the direction of current density changes in an exact manner while the current I remains unaffected. The responsible factor for it is

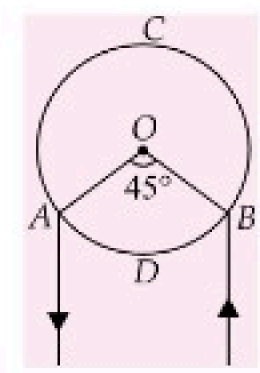
(a) the charges ahead.
(b) electric field produced by charges accumulated on the surface of wire.
(c) the charges just behind a given segment of wire which push them, right away by repulsion.
(d) none of these.

3. The voltage current characteristic curve of a non-ohmic device is $V = 4i^2$ as shown in figure. Then, what is the resistance of device when current is 1 A?



(a) 4 Ω (b) 8 Ω
(c) 2 Ω (d) 6 Ω

4. A and B are two points on a uniform ring of resistance 15 Ω. The $\angle AOB = 45°$. The equivalent resistance between A and B is



(a) 1.64 Ω (b) 2.84 Ω
(c) 4.57 Ω (d) 2.64 Ω

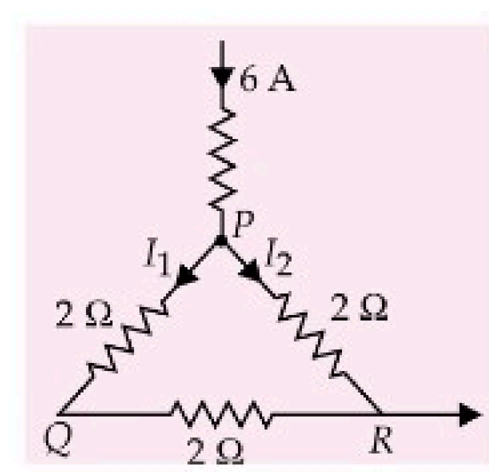
5. When a current of 2 A flows in a battery from negative to positive terminal, the potential difference across it is 12 V. If a current of 3 A flowing in the opposite direction produces a potential difference of 15 V, the emf of the battery is

(a) 12.6 V (b) 13.2 V (c) 15.5 V (d) 14 V

6. The mean free path of electrons in a metal is 4×10^{-8} m. The electric field which can give on an average 2 eV energy to an electron in the metal will be in units V m^{-1}

(a) 5×10^{-11} (b) 8×10^{-11}
(c) 5×10^{7} (d) 8×10^{7}

7. A current of 6 A enters one corner P of an equilateral triangle PQR having 3 wires of resistances 2 Ω each and leaves by the corner R. Then the currents I_1 and I_2 are

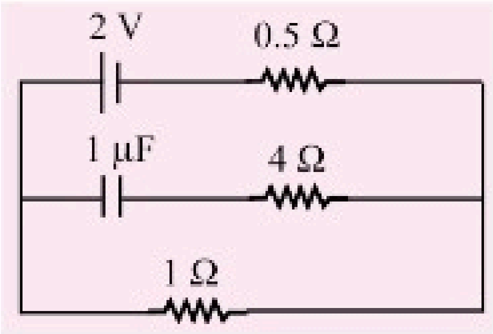


(a) 2 A, 4 A (b) 4 A, 2 A
(c) 1 A, 2 A (d) 2 A, 3 A

8. Three resistances 2 Ω, 4 Ω, 5 Ω are combined in series and this combination is connected to a battery of 12 V emf and negligible internal resistance. The potential drop across these resistances are respectively

(a) (5.45, 4.36, 2.18) V (b) (2.18, 5.45, 4.36) V
(c) (4.36, 2.18, 5.45) V (d) (2.18, 4.36, 5.45) V

9. What is the charge stored by 1 μF as shown in the figure?



(a) 2.33 μC (b) 3.33 μC
(c) 1.33 μC (d) 4.33 μC

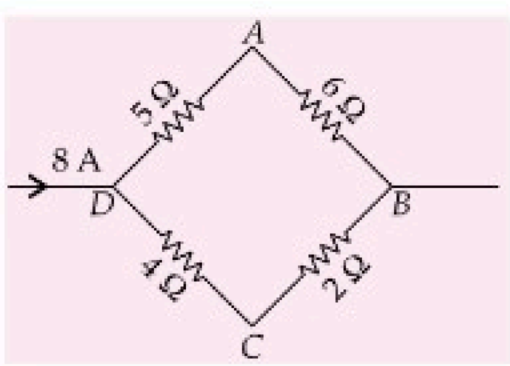
10. A battery of e.m.f. E and internal resistance r is connected to an external resistance R. The condition for maximum power transfer is

(a) $r < R$ (b) $r > R$ (c) $r = 1/R$ (d) $r = R$

11. If voltage across a bulb rated 220 V, 100 W drops by 2.5% of its rated value, the percentage of the rated value by which the power would decrease is

(a) 20% (b) 2.5% (c) 5% (d) 10%

12. A current of 8 A flows in a system of resistors as shown in figure. The potential difference $V_C - V_A$ will be



(a) $\frac{45}{23}$ V
(b) 6.6 V
(c) 3.3 V
(d) 9.9 V

13. In a potentiometer a cell of emf 1.5 V gives a balanced point at 32 cm length of the wire. If the cell is replaced by another cell then the balance point shifts to 65 cm then the emf of second cell is

(a) 3.05 V (b) 2.05 V (c) 4.05 V (d) 6.05 V

14. In a potentiometer the balancing with a cell is at length of 220 cm. On shunting the cell with a resistance of 3 Ω balance length becomes 130 cm. What is the internal resistance of this cell ?

(a) 4.5 Ω (b) 7.8 Ω (c) 6.3 Ω (d) 2.08 Ω

15. Three resistors of resistances 3 Ω, 4 Ω and 5 Ω are combined in parallel. This combination is connected to a battery of emf 12 V and negligible internal resistance, the current through each resistor in ampere are respectively

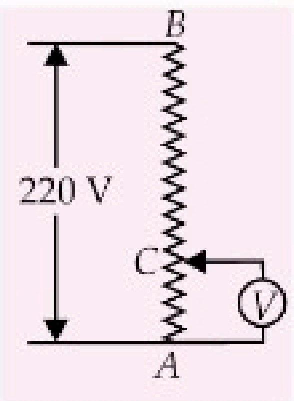
(a) 4, 3, 2.4 (b) 8, 7, 3.4
(c) 2, 5, 1.8 (d) 5, 5, 8.2

Only One Option Correct Type

16. A wire of mass m, length l, density d and area of cross-section A is stretched in such a way that its length increases by 10% of its original value. Percentage change in resistance is

(a) 18% (b) 19% (c) 20% (d) 21%

17. A potential difference of 220 V is maintained across a 12000 ohm rheostat AB as shown in figure. The voltmeter V has a resistance of 6000 ohm and point C is at one fourth of the distance from A to B. What is the reading in the voltmeter ?

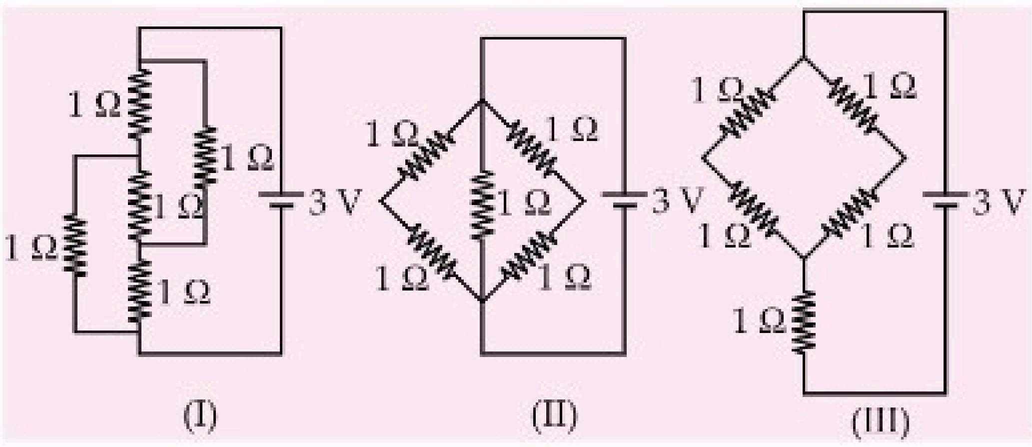


(a) 32 V (b) 36 V (c) 40 V (d) 42 V

18. The charge supplied by source varies with time t as $Q = at - bt^2$. The total heat produced in resistor $2R$ is

(a) $\frac{a^3R}{6b}$ (b) $\frac{a^3R}{27b}$ (c) $\frac{a^3R}{3b}$ (d) $\frac{a^2R}{27b}$

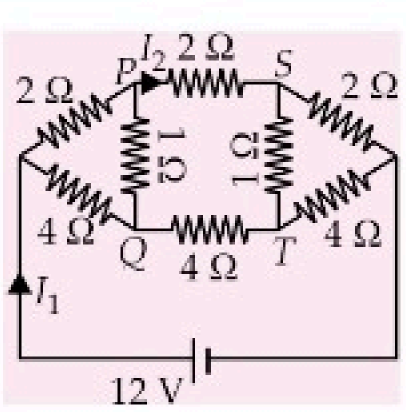
19. Given figure shows three resistor configurations I, II and III connected to 3 V battery. If the power dissipated by the configuration I, II and III is P_I, P_{II} and P_{III} respectively, then



(a) $P_I > P_{II} > P_{III}$ (b) $P_I > P_{III} > P_{II}$
(c) $P_{II} > P_I > P_{III}$ (d) $P_{III} > P_{II} > P_I$

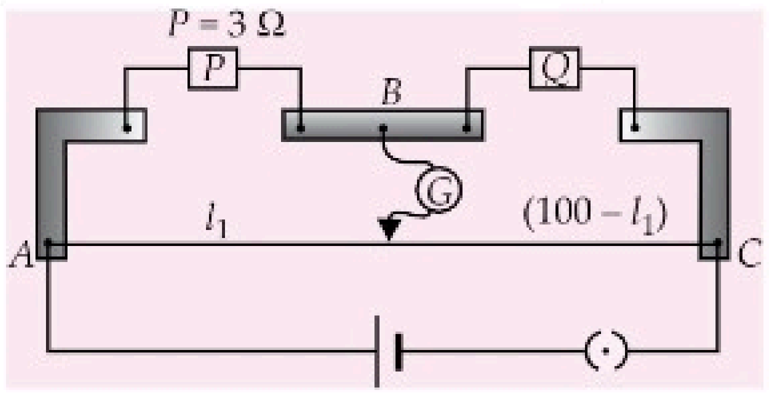
More than One Options Correct Type

20. For the resistance network shown in the figure, choose the correct option(s).



(a) The current through PQ is zero
(b) $I_1 = 3$ A
(c) The potential at S is less than that at Q
(d) $I_2 = 2$ A

21. In a metre bridge experiment, resistances are connected as shown in the figure. The balancing length l_1 is 55 cm. Now an unknown resistance x is connected in series with P and the new balancing length is found to be 75 cm. Then,



(a) $\frac{P}{Q} = \frac{55}{45}$ (b) $Q = \frac{27}{11}\,\Omega$

(c) Value of x is $\frac{48}{11}\,\Omega$. (d) Value of x is $\frac{11}{48}\,\Omega$.

22. A voltmeter and an ammeter are joined in series to an ideal cell, giving readings V and A respectively. If a resistance equal to the resistance of ammeter is now joined in parallel to the ammeter

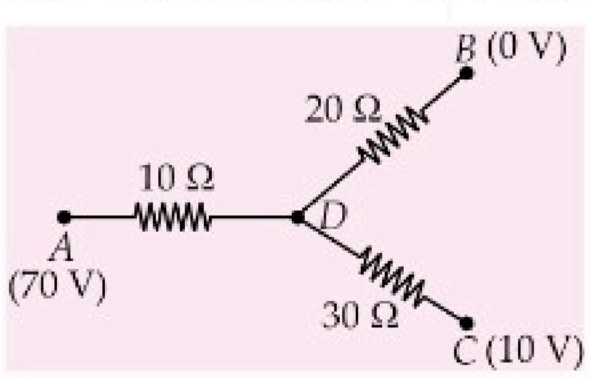
(a) V will not change

(b) V will increase slightly

(c) A will become exactly half of its initial value

(d) A will become slightly more than half of its initial value.

23. In the network shown, points A, B and C are at potentials of 70 V, zero and 10 V respectively. Then



(a) the potential of point D is 40 V

(b) the currents in the sections AD, DB, DC are in the ratio 3 : 2 : 1

(c) the currents in the sections AD, DB, DC are in the ratio of 1 : 2 : 3

(d) the network draws a total power of 200 W.

24. A boy has two spare light bulbs in his drawer. One is marked 240 V and 100 W and the other is marked 240 V and 60 W. Which of the following statements are incorrect about the light bulbs?

(a) The 60 W light bulb has more resistance and therefore burns less brightly.

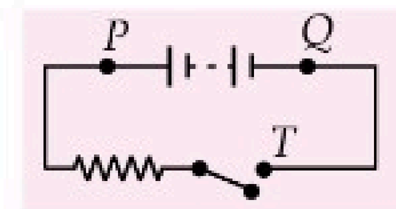
(b) The 60 W light bulb has less resistance and therefore burns less brightly.

(c) The 100 W bulb has more resistance and therefore burns more brightly.

(d) The 100 W bulb has less resistance and therefore burns less brightly.

25. A battery, an open switch and a resistor are connected in series as shown below.

Consider the following three statements concerning the circuit. A voltmeter will read zero if it is connected across points



(a) P and T (b) P and Q

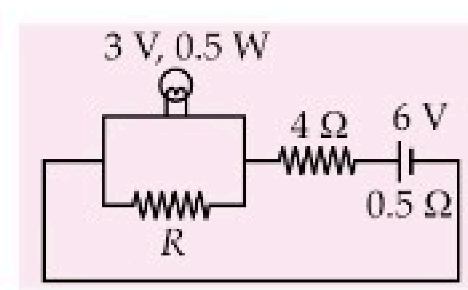
(c) Q and T (d) All of the above

Numerical Value Type

26. When two identical batteries of internal resistance 1 Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\,J_2$, then find the value of R in ohm.

27. 32 cells each of emf 3 V are connected in series and kept in a box. Externally, the combination shows an emf of 84 V. How many number of cells are connected reversely?

28. In the circuit shown below what would be the value of resistance R (in ohm) so that the electric bulb consumes the rated power?



Comprehension Type

Consider a block of conducting material of resistivity ρ shown in the figure. Current I enters at A and leaves from D. We apply superposition principle to find voltage ΔV developed between B and C. The calculation is done in the following steps:

(i) Take current I entering from A and assume it to spread over a hemispherical surface in the block.

(ii) Calculate field $E(r)$ at distance r from A by using Ohm's law $E = \rho j$, where j is the current per unit area at r.

(iii) From the r dependence of $E(r)$, obtain the potential $V(r)$ at r.

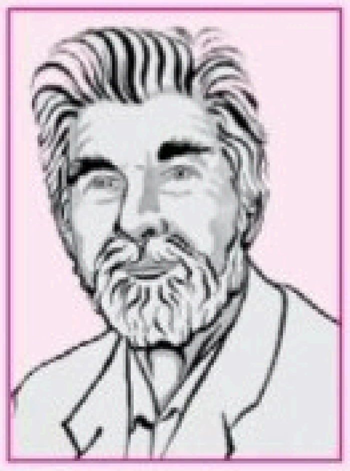
(iv) Repeat (i), (ii) and (iii) for current I leaving D and superpose results for A and D.

The Nobel Prize in Physics 2021

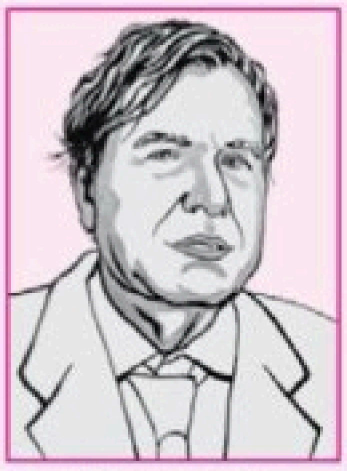
The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics 2021"for ground breaking contributions to our understanding of complex physical systems" with one half jointly to Syukuro Manabe Princeton University, USA, Klaus Hasselmann Max Planck Institute for Meteorology, Hamburg, Germany "for the physical modelling of Earth's climate, quantifying variability and reliably predicting global warming" and the other half to Giorgio Parisi

Sapienza University of Rome, Italy "for the discovery of the interplay of disorder and fluctuations in physical systems from atomic to planetary scales"

Syukuro Manabe
Prize share : 1/4



Klaus Hasselmann
Prize share : 1/4



Giorgio Parisi
Prized share : 1/2

Physics for climate and other complex phenomena

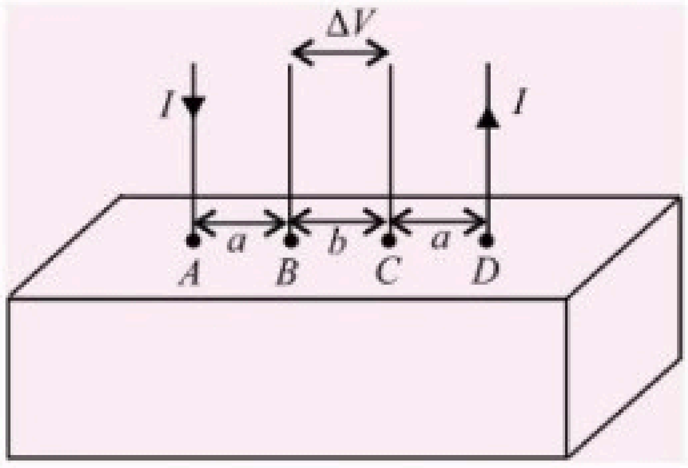
Three Laureates share this year's Nobel Prize in Physics for their studies of chaotic and apparently random phenomena. Syukuro Manabe and Klaus Hasselmann laid the foundation of our knowledge of the Earth's climate and how humanity influences it. Giorgio Parisi is rewarded for his revolutionary contributions to the theory of disordered materials and random processes. Complex systems are characterised by randomness and disorder and are difficult to understand. This year's Prize recognises new methods for describing them and predicting their long-term behaviour.

One complex system of vital importance to humankind is Earth's climate. Syukuro Manabe demonstrated how increased levels of carbon dioxide in the atmosphere lead to increased temperatures at the surface of the Earth. In the 1960s, he led the development of physical models of the Earth's climate and was the first person to explore the interaction between radiation balance and the vertical transport of air masses. His work laid the foundation for the development of current climate models.

About ten years later, Klaus Hasselmann created a model that links together weather and climate, thus answering the question of why climate models can be reliable despite weather being changeable and chaotic. He also developed methods for identifying specific signals, fingerprints, that both natural phenomena and human activities imprint in the climate. His methods have been used to prove that the increased temperature in the atmosphere is due to human emissions of carbon dioxide.

Around 1980, Giorgio Parisi discovered hidden patterns in disordered complex materials. His discoveries are among the most important contributions to the theory of complex systems. They make it possible to understand and describe many different and apparently entirely random materials and phenomena, not only in physics but also in other, very different areas, such as mathematics, biology, neuroscience and machine learning. "The discoveries being recognised this year demonstrate that our knowledge about the climate rests on a solid scientific foundation, based on a rigorous analysis of observations. This year's Laureates have all contributed to us gaining deeper insight into the properties and evolution of complex physical systems," says Thors Hans Hansson, chair of the Nobel Committee for Physics.

For more details visit www.nobelprize.org



29. ΔV measured between B and C is

(a) $\dfrac{\rho I}{2\pi(a-b)}$ (b) $\dfrac{\rho I}{\pi a}-\dfrac{\rho I}{\pi(a+b)}$

(c) $\dfrac{\rho I}{a}-\dfrac{\rho I}{(a+b)}$ (d) $\dfrac{\rho I}{2\pi a}-\dfrac{\rho I}{2\pi(a+b)}$

30. For current entering at A, the electric field at a distance r from A is

(a) $\dfrac{\rho I}{4\pi r^2}$ (b) $\dfrac{\rho I}{8\pi r^2}$

(c) $\dfrac{\rho I}{r^2}$ (d) $\dfrac{\rho I}{2\pi r^2}$

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