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# THE ESSENTIAL GUIDE TO THE ACT!

Features  
"ZERO HOUR THREAT",  
the secret weapon  
interactive SAT and ACT  
prep game designed  
by I.D.E.A.S. located  
at Disney/MGM Studios.

THE *TEST*  
IS SOON AND I  
KNOW I'M  
NOT READY!

WE NEED *SERIOUS*  
HELP BUILDING  
TEST *SKILLS*... AND  
IT CAN'T BE  
BORING!

YOU WANT TO *BOOST* THOSE SCORES?  
YOU WANT *PROVEN* RESULTS? WE DON'T CALL  
THIS THE *ESSENTIAL GUIDE* FOR NOTHING!!!



# **The Essential Guide to the ACT**

**2012**

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**2016**

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The material in this Guide reflects changes to the ACT that began with the September 2015 testing cycle. These changes include the following:

1. Within the Reading subsection, the student will see a set of paired passages. This means that instead of four long passages (one each of Prose Fiction, Social Science, Humanities, and Natural Science), one of the types will have two shorter passages which the student will compare in terms of structure and content.
2. Also within the Reading subsection, a new question type will appear in which the student will indicate which of four sentences in the passage the answer to the previous question is based.
3. The Essay (still optional) prompt now includes a paragraph stating a problem or issue with three perspectives on the issue given. The student must then write a persuasive essay analyzing the three perspectives and support his own position on the topic. The student will receive a raw score of up to 48 based on 4 different domain areas which will then be converted to a scaled score.

For additional resources regarding the ACT please go to [www.actstudent.org](http://www.actstudent.org).

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(Answer sheets for taking the Practice Test can be found at the back of the *Guide*.)



## Introduction to The Essential Guide

Welcome to the world of *ACT* test prep! Like thousands of other high school students, you are probably both excited and anxious about all of the steps that will lead you to a college that is right for you. Among those steps is taking college entrance exams, and perhaps you are wondering what you can do to make sure that you achieve your very best on these tests. Well, not to worry. Help is on the way!

### Overview and Goals of the *ACT* Prep Program

Our materials are designed for students who want to increase their scoring potential or improve on previous scores on the *ACT* and for those who are seeking a refresher course in particular English, reading, math, science, and/or writing skills. In this *Guide*, you will find those things, and more.

Our program is also designed to boost your confidence in taking college entrance exams by providing:

- vital information about the nature and construction of the *ACT* and about test registration;
- tips for mastering the art of taking standardized tests in general and the *ACT* specifically;
- formal instruction and tips for tackling test prep and taking the exam;
- help for increasing your reading vocabulary through our use of college level writing and the inclusion of unfamiliar words throughout the content of the *Guide*;
- the student perspective on issues related to college entrance testing;
- a fun way—in video game format—to review material and practice test-taking skills; and
- a brief list of other resources to aid you in preparing for the *ACT*.

In fact, in this *Guide* you will find a unique approach to mastering the *ACT*. Our approach is tailored to students who are self-motivated—those who can and will work on their own in preparing for the *ACT* and are not interested in using the typical “big, fat book” of 300+ pages to do so. Instead, included with this guide is a fun, easy-to-use practice test in the form of an *ACT* test prep game called *Zero Hour Threat*. Created by I.D.E.A.S. at Disney-MGM Studios, it is an interactive action game designed to increase standardized test scores as well as enhance general mathematics, science, reading, and vocabulary skills. With each correct answer, students move one step closer to decoding a virus that international criminals have set in place to infect the United States’ banking systems. You will be able to work on enhancing your *ACT* scores while having fun through playing a state-of-the-art video game. The *Zero Hour Threat* CD is not a stand-alone study program. It is designed for use in conjunction with the review materials, questions, and other tools provided in *The Essential Guide to the ACT*.

In addition to the CD game, you will find skill-building exercises, along with one practice test (in the basic format and time frame of the real test) in each content category of the *ACT* to provide you with simulated experience in taking the actual *ACT*. For each of these, we will provide the opportunity for you to score your test by providing the correct answers and the rationale behind each of those answers.

## Frequently Asked Questions and Student Concerns

Below is a list of some frequently asked questions and concerns raised by students regarding the *ACT*.

- Why should I take the *ACT*?
- What is the difference between the *ACT* and the *SAT Reasoning Test*?
- How do I register for the *ACT*?
- What subjects are covered on the *ACT*?
- How is the *ACT* scored?
- What tools may I or should I bring to the *ACT* test site on the day of the exam?
- What is the format of the exam?
- How can I prepare for the exam?
- Which method of preparation for the *ACT* is best?
- How long should I spend on each question?
- If I don't know the answer to some questions, should I guess?

Each of these questions will be addressed specifically in the appropriate chapters of the *Guide*. You should read each chapter carefully and thoroughly in order to make the most of the material provided. You should also jot down any other questions that come to mind as you read and engage in the activities found in each chapter. If you find that you have additional questions or concerns that are not answered as you move through the *Guide*, you should consult the guidance or college counselor at your school or visit the *ACT* web site at [www.act.org/index.html](http://www.act.org/index.html) for further information about the test program.

### Optimizing Your Use of *The Essential Guide to the ACT*

To get the most out of the *Guide*, we suggest that you follow the directions to the letter. You should plan to review one chapter at a time, setting aside time each day for studying the content and completing the exercises. Review the vocabulary charts in Chapter Three daily, and use the suggested strategies for increasing your reading comprehension skill. You can, of course, play *Zero Hour Threat* as frequently as time permits. (Remember that getting into college depends largely on your academic performance and that success at school is a result of hard work in the classroom and completion of your homework and other assignments.)

We hope that you will find our approach exciting and rewarding. After utilizing this unique test preparation method that combines work and play, you should be on your way to increasing your *ACT* scoring power.

Using *The Essential Guide to the ACT* will...

- 1) Boost your confidence
- 2) Increase your potential for high scores
- 3) Increase your vocabulary
- 4) Help you master the art of test-taking

## Chapter One: Essential Facts

### What is *the ACT*?

The *American College Testing (ACT)* examination measures skill in four areas: English, reading, science, and mathematics. In addition, there is an optional writing test offered in a version of the exam called *ACT Plus Writing*. The *ACT* is designed and developed by *American College Testing* in keeping with standards and codes set by the *National Council on Measurement in Education*, *The American Psychological Association*, *The American Educational Research Association*, and *The Joint Committee on Testing Practices*. Students are allocated 2 hours and 55 minutes to answer the two hundred and fifteen (215) multiple-choice questions on the exam. The optional writing section requires an additional 40 minutes of testing time. Since some colleges require the writing test, you should check with the colleges to which you are applying to ascertain whether this portion of the *ACT* is a requirement.

Each of the four test areas of the *ACT* (English, mathematics, reading, and science) is scored 1 to 36, with 36 being the highest possible score. These four test scores are then averaged and rounded to the nearest whole number to obtain a composite score from 1 to 36. If you take the *ACT Plus Writing*, your essay will be evaluated by two professional readers who will assess the overall quality of your written expression and award your essay with a subscore. (For details regarding the writing score, visit <http://www.actstudent.org/writing/scores/index.html>.)

Below is the format of the *ACT* exam:

<b>English</b>	(45 minutes/60 questions) (1-36 points) Measures skill in rhetoric and standard written English
<b>Math</b>	(60 minutes/60 questions) (1-36 points) Measures skill in mathematics normally gained from instruction through 11th grade (numbers, operations, algebra/functions, geometry, trigonometry)
<b>Reading</b>	(35 minutes/40 questions) (1-36 points) Measures skill in reading comprehension
<b>Science</b>	(35 minutes/40 questions) (1-36 points) Measures skill in reasoning, analysis, interpretation, evaluation, and problem-solving related to scientific material and evidence
<b>Writing</b>	(Optional—40 minutes, in response to one writing prompt) Measures skill in writing normally developed through high school instruction and expected for success in completing early college writing assignments

## **Who Should Take the *ACT*?**

As a college entrance examination, the *ACT* is taken by high school juniors and seniors. Along with a record of your academic performance (the high school transcript), evidence of involvement in extra-curricular activities, recommendations, essays, and other supportive documents — the *ACT* is accepted by the majority of the colleges and universities in the United States as a part of the application profile. The *ACT* is administered five times per year between October and June; however, some states also offer the exam in late September as well.

Since the exam measures mathematics skills through the third year of college preparatory study, we recommend that you take your first *ACT* no earlier than the spring of your junior year in high school unless you are highly advanced in your academic work. If you wish to try to improve your initial score, you should feel free to take the exam again in June or in the first semester of your senior year, prior to your college application deadlines.

## ***How do I register for the *ACT* and what fees apply?***

*ACT* test registration materials are available in your high school guidance or college counseling office. You will find test dates and registration deadlines posted in both of these locations as well.

You can also register online at [www.actstudent.org/regist/elecereg](http://www.actstudent.org/regist/elecereg), where you will find the testing dates for the current year. At this website, you will also find high school, college, and test center codes and information about eligibility for fee waivers and special circumstances related to *ACT* testing. Whether you register by mail or online, it is wise to read the registration material carefully, since other fees may apply under various conditions.

The basic fees for taking the *ACT* are outlined in the registration materials for the exam. These charges cover the administration of the test and your score report and allow you to send your scores to as many as four colleges.

Fee waiver information is also available through your guidance or college counselor, and fee waiver cards can only be procured from the counselor, even for online registration. Home-schooled students must provide the local high school proof of eligibility for fee waivers. To use the card, you must register for the *ACT* exams according to posted regular deadlines

## Chapter Two: Test-Taking Tips

In this and the next three chapters, we will focus on reviewing skills related to the areas tested on the *ACT*. Along with skill review, you will find tips and strategies for tackling specific types of questions. This chapter will center on general tips for taking standardized college entrance tests. While we cannot guarantee that these tips will work for every student, you should consider them as good advice for any test-taker.

Nora, a student who recently took the ACT, offers the following advice:

- **Dress in layers and bring a sweater or jacket.**  
Test sites use the cafeteria, auditorium, or other spaces that are large enough to hold all of the test takers, but schools often turn off the heat or air conditioning on the weekends. It is difficult to concentrate when you are too hot or too cold.
- **Bring a watch.**  
Basic time management is up to you! The proctors tell you how much time you have for each section, when to start, when to stop, and when you have 5 minutes left. You should pay attention to how long you are taking on the questions and pace yourself during the test.
- **Skip hard questions or use your best guess.**  
You can usually narrow your choices down to two possible correct answers. That will give you a 50% chance of answering the questions correctly. (We will cover guessing in more detail below.)
- **Replace, in your mind, long hard story character names like Jedidiah, Beaugregard, Shaneequa with Bill or Jane or just use the first letter of the name for each character.**  
You can get lost in the names and it can draw your attention away from the question.
- **Read carefully!**  
Make sure you understand the main idea of the story. Ask yourself these two questions:
  - 1) What is the story about -- the subject?
  - 2) How does the subject of the story relate to the questions?
- **Review the basic concepts** — arithmetic operations and their order in solving problems, algebraic formulas, etc.
- **Prepare yourself by studying a few days of basic math formulas** a week before the test (Example:  $A=bh^2$ ).
- **Know a bit about word problems** and how to apply basic formulas to them.
- **Know how to solve 3-D and 2-D shape problems** involving circumference, perimeter, and area.
- **Be prepared for lots of questions using algebra.** (Example:  $2x = 4$  or  $xy = 0$ ).

- **Be aware that a few questions may include seldom-used units of measurement,** such as “stones.” The key is to use “proportion.” Do not worry about the unit. (Example: If 100 bags of sand weigh 300 stones, how much would 170 bags weigh? Simply cross multiply to find “x”.)
- **Bring a graphing calculator.** It will help in converting decimals to fractions and vice versa, with square root, etc., and will save a lot of time.
- **Sharpen your science skills** in reading and understanding graphs, charts, tables, and other schematic forms. Review the scientific terms you have learned in your high school science courses.
- **Familiarize yourself** with reading and understanding **scientific research summaries.** Be able to recognize and understand related and conflicting viewpoints when reading scientific data.
- **Write on the test!** Underline important material and make notes that might help you answer the questions correctly.

### To Guess or Not to Guess?

As mentioned above, the question of guessing on the responses to the *ACT* is a common one. While some would say that guessing is cheating, it is important to understand that there is no penalty for wrong answers on the *ACT*. Therefore, you should always guess if you do not know the answer to an *ACT* question. Before you guess, however, you should try to **eliminate at least one wrong answer before choosing a response.** (Eliminating two is even better!) Guessing wisely rather than randomly will increase the likelihood that your guess will yield the right answer.

### Some Final Thoughts

Other general test-taking advice includes the following:

- **Get a good night’s sleep before the test.**
- **Eat a light breakfast and take a snack and something to drink for break times.**
- **Arrive at the test site early or on time.**  
(You will not be allowed to take the test if you are late!)
- **Along with a watch and calculator, bring several sharpened #2 pencils and erasers.**
- **Try to relax; then concentrate on following directions and answering the questions to the best of your ability.**

By utilizing these tips and strategies along with those offered in Chapters 3, 4, and 5, you will increase your chances for success in taking the *ACT*. So jump right in, review the comments provided in this guide, play the CD game, and have fun preparing for the *ACT*.

## Chapter Three: English and Reading Skills Review

The English section of the ACT consists mainly of multiple-choice questions that test your knowledge of grammar, conventions, and sentence structure. Only the most grammar-savvy students should face this section of the test without some grammar review. Below are some of the most common types of grammatical issues you'll encounter on this part of the test.

### Subject/Verb Agreement

- This is by far the test writers' favorite type of question.
- It is important that your subjects agree with your verbs. In other words, you must write *he jumps* instead of *he jump* because a singular subject (*he*) takes a singular verb (*jumps*).
- Subject/verb agreement gets tricky when a prepositional phrase comes between the subject and the verb. However, you should ignore the prepositional phrase and make the verb agree with the subject. (One of the girls likes the movie.)
- Note that the following pronouns are singular: *each, either, neither, someone, somebody, everyone, everybody, anyone, anybody, no one, nobody*. (Neither of the movies was very good.)
- When two or more subjects are joined by *and*, you must use a plural verb. (Hiking and camping are fun.)
- When two or more subjects are joined by *or* or *nor*, the verb agrees with the subject nearest the verb. (Neither the players nor the coach is late for practice.)
- Amounts are usually singular, as are titles. (Two hours is a long time to wait. Great Expectations is a great book.)

### Try this practice question:

Help for improving test-taking skills and reading skills are also available.

- A. NO CHANGE
- B. for reading skills are
- C. reading skills is
- D. for reading skills is

The **correct answer is C**. The subject (*Help*) is singular and therefore requires a singular verb (*is*, not *are*). The prepositional phrase (*for improving test-taking skills and reading skills*) is meant to trick you because it's plural. It's also long so that by the time you reach the verb, you've lost track of the subject, so be careful!

### Pronoun/Antecedent Agreement

- This concept is very similar to subject/verb agreement and is another favorite of the test writers.
- A noun that a pronoun refers to is called its antecedent. For example, in the sentence, *The boy rode his bike*, *boy* is the antecedent of the pronoun *his*. Just as a subject has to agree with its verb, a pronoun has to agree with its antecedent.

- If the antecedent is singular, the pronoun must be singular; if the antecedent is plural, the pronoun must be plural.
- Pronoun/antecedent agreement also gets tricky when the antecedent is followed by a prepositional phrase. However, you should ignore the prepositional phrase and make the pronoun agree with the antecedent. (One of the girls forgot her pencil.)
- Note that the following pronouns are singular: *each, either, neither, someone, somebody, everyone, everybody, anyone, anybody, no one, nobody*. (Neither of the players remembered to tie his shoes.)
- When two or more antecedents are joined by *and*, you must use a plural pronoun. (Katianne and Ralston took off their shoes.)
- When two or more antecedents are joined by *or* or *nor*, the pronoun agrees with the antecedent closest to it. (Neither the players nor the coach brought his stopwatch.)

**Try this sample question:**

If a student wishes to be involved in the class, they must get parental consent first.

- A. NO CHANGE
- B. he or she must
- C. they have to
- D. he or she have to

The **correct answer is B**. The antecedent of the pronoun is *student*, which is singular. Therefore, the pronoun must be singular as well (*he or she* rather than *they*). Choice D is incorrect because when two subjects are joined by *or* (*he or she*), the verb agrees with the one it's closer to (*she*).

**Parallel Structure**

- When you join two or more ideas—in the form of words, phrases, or even clauses—in a sentence (usually with the help of a conjunction), the ideas you join must be parallel. In other words, the grammatical structure of each one must be the same. For example, *Michelle is kind, helpful, and likes puppies* is not parallel because we're joining two adjectives (*kind and helpful*) and one verb (*likes*). To correct the sentence, you could write *Michelle is kind, is helpful, and likes puppies* (three verbs) or *Michelle is kind, helpful, and loving* (three adjectives).

**Try this sample question:**

Lucie is not only a loving character, but she is a strong character as well.

- A. NO CHANGE
- B. character, in addition to that she is also very strong
- C. character, but she is also a strong character
- D. character but also a strong character

The **correct answer is D**. In the original sentence, an adjective and noun follow the *not only* part of the correlating conjunction, but a subject and verb follow the *but* part of the correlating conjunction. Instead, we need an adjective and noun to follow the *but*. Note that the correct answer is also the shortest, as is often the case.

### Passive Voice

- Although passive voice is not grammatically incorrect, it is often considered to be stylistically unpleasant.
- Passive voice consists of a *to be* verb (*is, be, am, are, was, were, been, being*) and a past participle (such as *stolen, pushed, or enjoyed*).
- The subject of the passive voice verb is receiving the action rather than doing the action. – for example, *The play was enjoyed by the audience*. Although *play* is the subject, the object of the preposition (*audience*) is actually doing the enjoying. The sentence would sound much better if it were active: *The audience enjoyed the play*.
- Not all past participles indicate passive voice, however. For example, in the sentence *I have listened to that CD all day long, have listened* is not passive voice.
- Although there are some cases in which passive voice is acceptable, use active voice whenever possible.

### Try this sample question:

The letter is written by Sir Andrew and Sir Toby as a trick to make a fool of Malvolio.

- A. NO CHANGE
- B. Sir Andrew and Sir Toby write the letter
- C. The letter, which is written by Sir Andrew and Sir Toby,
- D. Sir Andrew and Sir Toby write the letter, and they do it

The **correct answer is B** because the subjects (*Sir Andrew and Sir Toby*) do the action (*write*). In other words, the sentence is in active voice rather than passive voice. While D is also active voice, it's wordy. Remember, on the *ACT* there may be more than one **correct** answer, but you're looking for the **best** one. Stay away from wordiness.

### Use of Modifiers

- A modifier is a word that describes another word.
- Modifiers come in **three degrees of comparison: positive** (tall, carefully), **comparative** (taller, more carefully), and **superlative** (tallest, most carefully).
  - Use the **positive** form when you're just describing one person/thing. (Joe is tall.)
  - Use the **comparative** form when you're comparing two persons/ things. (Joe is taller than John.)
  - Use the **superlative** form when you're comparing three or more persons/things. (Joe is the tallest person in his class.)
- Avoid using an adjective to modify a verb. (*He ran quickly*, not *He ran quick*.)

- When you're comparing something with a group it belongs to, you must include the word *other* or the word *else*. (This class is more fun than any other class. Mrs. Miller is nicer than anyone else I know.)
- Note that *less*, *amount*, and *much* refer to collective nouns (uncountable things like traffic and rain); fewer, number, and many refer to nouns you can count (like cars and rain drops).
- A modifier is said to be “dangling” when it refers to a word not directly stated in the sentence.
- A misplaced modifier is one that is in the wrong place in sentence.
- Always put modifiers as close to the words they modify as possible, otherwise, your sentences are unclear.

### Try these practice questions:

1) When you captivate the attention of your audience, you have less conduct problems.

- A. NO CHANGE
- B. one has less
- C. one has fewer
- D. you have fewer

The **correct answer is D**. Since you can count conduct problems, you must use *fewer* instead of *less*. Don't be tricked into choosing C. Changing *you* to *one* would result in a shift from second person in the first clause (*you* captivate) to third person (*one* has) in the second clause.

2) The sirens cause passing sailors to go completely insane by singing beautiful songs.

- A. NO CHANGE
- B. Singing beautiful songs, the sirens cause passing sailors to go completely insane.
- C. The sirens, which are always singing beautiful songs, cause sailors to go completely insane.
- D. The sirens, who cause passing sailors to go completely insane, are always singing beautiful songs.

The **correct answer is B** because *singing beautiful songs* modifies the sirens. Choice A suggests that the sailors go insane by singing beautiful songs themselves! Choices C and D are wordy and awkward.

### Pronoun Usage

We already talked about pronoun/antecedent agreement, but you must also know when to use which pronoun case (such as *I* versus *me*).

- You must use nominative pronouns (such as *I*, *he*, and *they*) for subjects and predicate nominatives (for example, *Jacob and I are studying for our math test*).
- You must use objective pronouns (such as *me*, *her*, and *them*) for direct objects, indirect objects, and objects of prepositions. (*Will you eat lunch with Rudy and me?*)
- Most students have trouble deciding when to use *who* and when to use *whom*.
  - o ***Who* (nominative case) is for subjects and predicate nominatives.**

- o **Whom (objective case) is for direct objects, indirect objects, and objects of prepositions.**

Avoid the use of reflexive pronouns (myself, yourself, himself, etc.)  
Also, remember that *hissself*, *theirselves*, and *themselves* are not words!

**Try this practice question:**

Since Mr. Key wasn't sure who was responsible, he gave both Lauren and I detention.

- A. NO CHANGE
- B. gave both Lauren and myself
- C. both gave Lauren and I
- D. gave both Lauren and me

The **correct answer is D**. You must use the objective pronoun *me* rather than the nominative pronoun *I* because *Lauren and me* is an indirect object. Note that the use of *who* in this sentence is correct because it acts as the subject of the clause *who was* responsible.

**Verb Tense**

- As a general rule, you should use past tense (*walked*) to write about something that happened in the past, present tense (*walks*) to write about something that is happening right now, and future tense (*will walk*) to write about something that hasn't yet happened.
- In a verb phrase, the tense is determined by the helping verb. For example, *has walked* is present perfect tense, but *had walked* is past perfect tense.
- Avoid switching verb tense within your writing. For example, if you're telling a story and using past tense, don't randomly switch to present tense.

**Try this sample question:**

When Kyle comes over to my house, we started playing video games immediately.

- A. NO CHANGE
- B. start playing
- C. will start playing
- D. started to play

The **correct answer is B**. *Started* is a past tense verb, whereas *comes* is present tense. Choices C and D are both wrong because C offers a future tense verb and D offers a past tense verb.

## Verb Usage

Sometimes you might use the right verb tense and correct subject/verb agreement, but you use the wrong verb or verb form. Here are a few tips regarding verbs that people sometimes misuse:

- *Lie, sit, and rise* are all intransitive verbs and are therefore not followed by a direct object. (I like to *lie* under the tree.)
- *Lay, set, and raise* are transitive and therefore are followed by a direct object. (*Set* the books on the table.)
- *Shall* goes with *I* and *we* (*I shall* go to the movie.) *Will* goes with everyone else. (She *will* go to the movie.)
- Use *may* when you're referring to permission. (*May* I have that pencil?)
- Use *can* when you're referring to ability. (*Can* you run twelve miles?)
- Avoid using the wrong form of a verb. For example, a past tense verb (*spoke*) does not need a helping verb, but a past participle (*has spoken*) does.

### Try this practice question:

On weekends I like to lay around and watch movies.

- A. NO CHANGE
- B. to lie around
- C. laying around
- D. to keeping lying around

The **correct answer is B**. Here we have to use the intransitive verb *lie* rather than the transitive verb *lay* because we don't have a direct object. Although *around* may look like a direct object, it tells "where" rather than "what." Choice D does use *lying* rather than *laying*, but it is unnecessarily wordy.

## Run-on Sentences

- When you have two complete whole sentences stuck together with only a comma in between them, you have a type of run-on called a comma splice (such as *I'm starving, I hope we can eat soon*).
- A comma is not strong enough to hold two sentences together.
- You must add a coordinating conjunction (such as *I'm starving, so I hope we can eat soon*), change the comma to a semicolon (such as *I'm starving; I hope we can eat soon*), create two separate sentences (such as *I'm starving. I hope we can eat soon*), or reword the whole thing (such as *I hope we can eat soon because I'm starving*).
- When you have two whole sentences (or even more than two) stuck together with nothing at all in between them, you have a run-on sentence (such as *I'm starving I hope we can eat soon*).
- You can correct the run-on sentence the same way you correct the comma splice.

**Try this practice question:**

We have two hours to get there, that's plenty of time.

- A. NO CHANGE
- B. there that's plenty of time.
- C. there, which is plenty of time.
- D. there, which certainly should be plenty of time.

The **correct answer is C**. Choices A and B are both run-on sentences. Although D is technically not wrong, it is wordy, making C a better choice.

**Sentence Fragments**

- A sentence fragment is a sentence that isn't complete.
- In order to be complete, a sentence must be a complete thought with a subject and a verb.
- Beware of sentences that start with subordinating conjunctions (like *if*, *because*, *since*, and *although*).
- A clause that starts with a subordinating conjunction is dependent and cannot stand alone as a sentence, so be sure that you have another clause—an independent one—following the dependent one.

**Try this sample question:**

Although now I can say I'm special and mean it.

- A. NO CHANGE
- B. Although, now I can say I'm special and mean it.
- C. Although I called myself special before, now I really mean it.
- D. Now I call myself special, however I didn't mean it before.

The **correct answer is C**. It consists of a dependent clause (*Although I called myself special before*) and an independent clause (*now I really mean it*). Choice A consists of a dependent clause only. So does choice B. The comma after *although* doesn't change that fact. Choice D is a comma splice.

**Wordiness**

- Clear, effective writing says more with fewer words.
- Inexperienced writers sometimes think that they need to add extra words to their papers to make them sound better.
- Your goal should be to add more content (information, examples, description, explanation, etc.) instead of to add empty words.
- When in doubt, choose the shorter answer. Although you won't be correct every time, more often than not you will be.

Try this sample question:

This is an excellent book that is extremely fun and helpful to read.

- A. NO CHANGE
- B. This is an excellent book which is extremely fun and helpful.
- C. This excellent book, which is extremely fun and helpful.
- D. This helpful book is extremely fun to read.

The **correct answer is D** because it is short and sweet. A and B contain unnecessary words. Choice C makes the sentence into a fragment.

### Word Choice (Diction)

Sometimes you will have to identify words that are misused rather than grammatically incorrect. For example, a sentence may use the word *imply* where it should use *infer*. Below is a brief list of commonly confused words. (Many more are available in grammar textbooks and online.) You may want to review the definitions of these words in order to ensure that you are using them properly.

**accept** (verb): to agree or take something offered

**except** (preposition): excluding

**disinterested** (adjective): impartial

**uninterested** (adjective): not interested

**imply** (verb): to indirectly suggest meaning

**infer** (verb): to draw a conclusion

**disperse** (verb): to scatter

**disburse** (verb): to pay out

**than** (conjunction): used for comparison

**then** (adverb): next

**farther** (adjective): more distant; more advanced

**further** (adjective): in addition; extending beyond a certain point

**affect** (verb): to produce an effect or change

**effect** (noun): a result

**who** (pronoun): referring to a person or to people

**which** (pronoun): referring to a singular or plural thing

**that** (pronoun): referring to things or a group of people

**conscience** (noun): sense of right and wrong

**conscious** (adjective): awake; aware

**Try this practice question:**

Their were too many players on the field, so the referee called a foul.

- A. NO CHANGE
- B. Their was
- C. There were
- D. There was

The **correct answer is C**. In this sentence, we need *there* rather than *their*. Don't be fooled by choice D; we can't use the verb *was* with the subject *players*.

**Double Negatives**

- In math, two negatives make a positive. The same is true in writing.
- If we use two negatives, they cancel each other out. For example, consider the sentence "I don't have no marbles." If I don't have **no** marbles, then I must have **some** marbles!
- When you want to make a negative statement, you should use only one negative word.
- Common negative words include *nothing, none, never, not, neither, scarcely, barely, hardly, and without*.
- Be careful—negative prefixes such as 'un-' and 'dis-' also count as negatives.

**Try this question:**

I couldn't never have finished that difficult job by myself.

- A. NO CHANGE
- B. could never have
- C. couldn't hardly of
- D. couldn't ever of

The **correct answer is B**. Since the sentence already utilizes a negative word (*couldn't*), you don't need the second one (*never*). Don't be fooled by choice D. Although *couldn't ever* would work in this sentence, we must say *have finished* rather than *of finished*.

**Idioms**

- An idiom is a word or expression that cannot be taken literally. For example, "He *pulled the wool over my eyes*."
- There are no rules to learn about idioms. They either sound right, or they don't.
- The most common idiom errors you will encounter are preposition errors. For example, it would be incorrect to say, "He pulled the wool *across* my eyes."

**Try this question:**

If you keep practicing the oboe, you'll eventually get the hang for it.

- A. NO CHANGE
- B. get a hang for it
- C. get a hang of it
- D. get the hang of it

The **correct answer is D**. The correct way to write the idiom is “the hang *of* it.”

**Paragraphs**

Some of the multiple-choice questions you'll see on the English section of the *ACT* will deal with a paragraph as a whole. In those instances, you'll be asked to identify sentences that are out of order, sentences that should be rewritten or combined with other sentences, and sentences that should be removed from the paragraph altogether. You'll also be asked to select appropriate introductory and concluding sentences, transitional words, and supporting examples. You may also be asked to put the paragraphs of a passage in the correct sequential order.

**Try this sample question:**

**We visited a local animal shelter on Saturday. We wanted to pick out a new dog. As we walked through the corridor, each dog had its own way of sharing something about its personality. Some dogs barked, others jumped up and down, and still others covered in the corners of their cages. We visited with several different dogs, but the one who stole our hearts was Bogart. This big, black dog with droopy ears and a wagging tail is now the newest member of our family.**

What is the best way to combine the first two sentences of this paragraph?

- A. We visited a local animal shelter on Saturday, we wanted to pick out a new dog.
- B. We visited a local animal shelter on Saturday because we were hoping we would be able to pick out a new dog.
- C. Visiting a local animal shelter on Saturday, we were wanting to pick out a new dog.
- D. We visited a local animal shelter on Saturday to pick out a new dog.

The **correct answer is D** because it combines the two sentences without being wordy. Choice A, although not wordy, is a comma splice.

**The Reading Section**

Most students will tell you that the reading section is the most difficult part of the *ACT*. Why? Because the lengthy passages require you to stay focused! In order to remain alert and to concentrate during the critical reading section of the *ACT*, you must remember **Rule # 1: Stay engaged with the text**. In other words, interact with the passages in front of you.

Here's how:

- First, **write on the passage**. Underline information that you think may be important. Underline transitional or signal words such as *however*, *therefore*, *since*, *nevertheless*, and *above all*. Circle words that are unfamiliar to you. Put stars next to examples the author provides. Make brief notes in the margins about the author's purpose, point, or attitude. Don't worry about marking details like dates and percentages, though, because questions that ask about specific details will refer you to the right line number. Writing on the passage serves three important purposes. It helps you to make more sense of what you're reading; it helps you to remember what you've read; and (here's the clincher) it helps you to stay tuned in while you're reading!
- The second way to stay engaged with the text is to **keep a conversation going in your head while you're reading**. Go ahead. No one else will know. Talk back to the author of the passage. Ask him questions like "What point are you trying to make?" or "Why did you describe the situation that way?" Make accusations like "Wow, you obviously don't like this character very much" or "Well, I can tell you think global warming is nothing but a scam." Get inside his head by saying "You're trying to be sarcastic, aren't you?" or "Oh, I see where you're going with this example." These conversations may feel awkward at first, but good readers have them all the time. They help you to think like the author (which means you'll have an easier time answering the questions), and they help you to (once again) stay tuned in to the reading!

Although interacting with the passages is important, don't get too carried away. Keep in mind that you have a time limit, and try to follow these guidelines:

- You may want to skim the passage first to identify the topic.
- Get familiar with the questions. Find out what you are looking for.
- Go through the passage one time, marking it and talking to it as you go.
- Try to answer questions as you read.
- Read through each question and answer the ones you can immediately.
- Go back to the ones about which you are unsure.
- When you go back to the passage to figure these out, remember that, generally, the questions follow the order of the passage. In other words, you should be able to find the answer to the first question near the beginning of the passage
- Answer questions that pertain to the whole passage last. By this time you have worked with the whole passage and the details in the passage which should lead to a sense of what the whole passage is about.
- If reading the whole passage confuses you, read a portion (a couple of paragraphs or a column) and answer any questions that pertain.

You'll have to answer questions about the point of the passage and how the author uses his words to make a point. Specifically, you need to practice figuring out a passage's main idea, the author's attitude or tone toward the subject matter, and what the passage implies (says indirectly or between the lines).

## **Paired Passages**

Starting in September of 2015, the Reading Section may include a set of paired passages in one of the four passage types. Instead of one long passage there will be two passages addressing a particular subject. In this case work with one passage at a time. Read and answer the questions relating to the first passage. Then, read and the answer the questions related to the second passage. Finally work on the questions (usually 3-4) that direct you to compare/contrast the two passages. You may be asked to compare the two authors' thinking and/or point of view.

## **Vocabulary in Context**

Sometimes you will be asked to figure out the meaning of a word in context. In this case, the question will give you the line location of the word. Perhaps you will have already identified the word as one that is unfamiliar to you. Either way, you'll need to read the sentence (**and** the ones immediately before and after it) to look for context clues—words in the sentence that give away the meaning of the target word.

You should also review your knowledge of root word, prefixes, and suffixes. Below are charts that will help you to do this. While you are not expected to memorize the information on the charts, familiarizing yourself with these word parts will help you to understand the difficult words you may encounter in the Reading Section of the *ACT*.

## A Root is Worth a Thousand Words

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### Important Prefixes and Suffixes

ROOT	MEANING
acerb	bitter
ag, act	drive, urge, act
agon	contest, struggle
alt	high
amic	friend
anim, anima	1. life 2. mind, soul, spirit
arch	rule, govern
aug, auct	increase
autrui	other people
ban	to command
bel, bell	1. fair, fine 2. war
bene	well
brev	short
cad, cas	fall
cand	glow, burn
cap, cip, cep	to take
ced, cess	go, yield
cens	give opinion, appraise
chron	time
ciarl	prattle, babble
cit, citat	arouse, summon, call out
clam, claim	call out
clement	lenient, mild
concili	bring together
copi	abundance
cor, cord	heart
corp	body
corrig	to correct
cred	believe, make trust
crit	a judge
cur	care, attention
curr, curs	run
cycl	circle, wheel
deleter	destroyer
derm	skin
didact	to teach
dol	grieve, worry
don	gift, give
dyn	power
enigm	riddle
equ	1. equal 2. horse
erro, erron	wanderer

ROOT	MEANING
exter, extra	beyond, outside
fac, fact, fec	make, form, act, do
fall, fals	err, beguile, deceive
fam	1. hunger 2. report
fastidi	loathing
ferv	boil
firm	steadfast, firm, strong
flagr	burn
flex, flect	bend
fund, fus	pour, melt
garr	chatter
gen, gener	origin, race, species
geo	earth
ger	bear, carry, rule
graph, gram	to write
grav	heavy
greg	flock, herd
gyro	turn
helio	sun
hemera	day
hibit	have, hold
host, hosp	host, guest
humil	humble
hydr	water
inan	void, empty
it, itiner	to go, journey
judic	judge
jur	1. swear 2. law, right
labor	to work
langu	be weak
leg, legis	1. appoint, send 2. law
leth	oblivion
lev	1. light 2. lift, raise, rise
line	line
listen	desire
liter	letter
log	speech, word, reason
loqu, locut	speak, talk
luc	1. shine 2. light
lud, lus	sport, play, laugh, mock
luna	moon
magn	great
mal	bad, ill
malle	hammer
man	hand
mar, mer	sea

<b>ROOT</b>	<b>MEANING</b>
mater, matr	maternal
metr, meter	measure
metus	fear
minn, minut	diminish, lessen
miser	miserable
mitt, miss	to send, let go
mor, mos	will
mord, mors	bite
morph	form, shape
mut	change
nat	1. born 2. swim
nav, naus	ship
noc	hurt, harm
nounce	bring tidings, tell
nov	new
omin	omen
oper	to work
ox	poison
pac	peace
pais	peace
par	1. equal 2. get ready, set
pars, part, partit	divide, share, separate, part
part	father
pat, pass	suffer, feel, endure
path	feel, suffer
pauc	little
ped	1. boy, child 2. foot
pell, puls	1. drive, urge 2. skin, fur
pet, petit	to seek
phan	to show
phem	speech
phone	voice, sound
pi	1. appease 2. devout
plac	please
plaud, plaus	clap hands
ple, plet	fill
plic	fold, bend, embrace
potent	powerful
povre	poor
prehend, prehens	take, seize, grasp
prob, prov	test
prodigi	token, omen
prol	offspring, increase
pud	feel shame, blush
put	1. think 2. cleanse, lop off
quiesc	rest, become quiet

ROOT	MEANING
quir, quer	to ask, seek
rat	think, calculate, settle
rect	ruled, right, straight
rept	creep
riv	stream
rog, rogat	ask, demand
rupt	to break
sanct	holy
scind	to cut
scrib, script	write
scrut	search into carefully
sent, sens	perceive, feel, think
sequi	to follow
serv	serve, keep
solemn	religious
solv, solut	loosen
son	sound
spec, spect	look, see, appear
spond, spons	promise, answer
stat	stand, standing
stru, struct	to build
suav	sweet
sume, sump	take, use, waste
surrect, surreg	rise
syc	fig
tac	be silent
tedi	irksomeness, irritating
ten, tend, tent	hold, stretch, strain
termin	boundary, end
the	1. a god 2. place, put
trepid	trembling, agitated
tric	hindrances, wiles, snares
trit	rub
trud, trus	thrust
turb	disturb, drive
un	one
ven, vent	to come
ver	1. spring 2. true, truth
verb	word
vert, vers	turn
vi	way, road
vir	1. man 2. poison
viv	life, to live
voc	1. voice 2. call
vol	1. wish, will 2. fly

## Important Prefixes and Suffixes

PREFIX	SUFFIX	MEANING
a		without, not; to, toward, into, at; from; out; off; on
ab, abs		from, away
ad, ac, af, ag, al, am, an, ap, ar, as, at		to, toward, before, near
ana		up, back, again
ante		before
anti, contra, counter		against
apo		from, off
arch, archi		chief
auto, aut		self, same, automatic
be		to cause
bi		two, both, double
bio		life
cata		down, thoroughly
con, com, col, co, cor		together, with
de		down, from
deca, dec, deka, dek		ten
di		double
dia		through, between, across
dis		apart, away, un
epi		upon, to, besides
eu		good
ex, e, ec, ef, exo		out, out of
for		intensely, utterly
hyper		over
in, ir, el, em, en, il, im		in, into
inter		between
mal		bad
meta		among, with, after, over
mis		wrongly, ill
mono		single, sole
multi		many
ob, oc, of, op		to, toward, before
octa		eight
over		above; superior; excessively
pan, omni		all
par, per		through
para		beside
peri		around, about
poly		many

port, pros		toward
post		after, behind
preter		beyond
pro, pur, pre		before, forward
proto		first
quadri, quadr, quadra, quadru		four, square
re, red, ana		back, again
se		aside, apart
se, sed, de, des, dis		away, apart
semi		half
sub, suc, suf, sum, sup, sur, sus		under, after
super, supra		above, over, beyond
syl, sym, syn		together
tra, trans		beyond, across
tri		three
un		reverse
un, non, ir, in, il, im		not
uni		one
	able, ible	able, capable (makes words into adjectives)
	al	referring (makes words into adjectives)
	ary, ory	place where (turns words into nouns)
	cle, cule	small
	ed	makes a regular verb past tense
	er	makes the comparative degree
	er, or	one who performs a specific action
	est	makes the superlative degree
	fy, ate	to make (turns words into verbs)
	ing	makes present participle form of verbs
	ist	one who, that which
	ive	inclined to (makes words into adjectives)
	ly	like, resembling
	or, ant, ar, ist, an, ian, ent	one who (turns words into nouns)
	ous	full of (makes words into adjectives)
	phobia	excessive fear
	s, es	makes a word plural
	sion, tion, ancy, ment, ency, ty, ance, ence, ity	the state of, the act of (makes words into nouns)
	y, ey	like, full of (makes words into adjectives)

## Practice Questions

Try interacting with this short reading passage and answering the questions that follow it.

*This passage is excerpted from a 1992 issue of National Geographic magazine.*

**The United States and Canadian governments are mounting a defense against a Russian invasion. No, the clock has not been turned back. These invaders are Asian gypsy moths, foliage-chomping insects that can cause billions of dollars in damage. Experts believe they entered North America from Russia last year in egg masses attached to grain vessels. The larger Asian gypsy moth is a more voracious feeder than the common North American strain and can feed on Pacific Northwest tree species. Unlike the flightless North American female, an Asian female can fly 20 miles between mating and egg-laying. The United States Health Inspection Service is barring from West Coast ports ships found carrying egg masses. The Tacoma and Portland areas, as well as Vancouver, British Columbia, have been sprayed with a biopesticide.**

1. The author of this passage refers to the increase in Asian gypsy moths as a “Russian invasion” in order to
  - A. make a political statement.
  - B. stress the severity of the problem.
  - C. warn readers about underhanded activities of the Russian government.
  - D. make light of a serious situation.

The **correct answer** to this question is **B**. By comparing the moth problem with a well-known historical situation, the author emphasizes the severity of the increase in the Asian gypsy moth population. Choice A might trick some test-takers because the comparison is political in nature. However, the rest of the passage doesn’t deal with political matters at all. Choice C is incorrect for the same reason. Given the information the author provides in the passage, we can be certain that he isn’t making light of the situation (D).

2. Based on its use in the passage, the word *voracious* most likely means
  - A. speedy.
  - B. able to go days without eating.
  - C. picky.
  - D. consuming large amounts of food.

The **correct answer** to this question is **D**. The Asian gypsy moth is a threat because it eats so much. The word *larger* is a good context clue because it suggests that these moths eat more than do common North American moths.

3. Which group of words from the passage best reflects the writer's tone?

- A. mounting, barring, sprayed
- B. egg masses, mating, egg-laying
- C. invaders, foliage-chomping, voracious
- D. Canadian, Russian, Asian, North American

The **correct answer is C**. We're looking for subjective words here--words that the author chooses to express his attitude. Because the writer uses words like *invaders*, *foliage-chomping*, and *voracious*, we can tell that he is clearly concerned about this problem. Choices B and D offer purely objective words. The words *mounting* and *barring* in choice A seem tempting, but *sprayed* doesn't fit the bill.

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**Here are some additional practice questions on which to test your skills.** (*Answers are on page 34.*)

I. *This passage is excerpted from a 1986 article about the use of social studies textbooks in elementary schools.*

**The inherent difficulty of social studies content stems mainly from the heavy technical concept load of social studies textbook passages. Technical concepts are one- or two-word "ideas" which have specialized meaning in social studies (for example: government, delta, immigrants, interdependence, economy, constitution, federal, cotton belt, division of labor, and political party). These words may have little or no meaning for students unless specific vocabulary or concept development lessons precede their first encounter with such terms. Yet basal social studies textbooks are notorious for heavy technical concept load and "thin" discussion of topics, making even the most careful independent reading low in potential benefit. Hard-to-pronounce names of cities, faraway countries, and foreign language names contribute to the complexity of textbook content. Many adult readers are stopped by these words, yet social studies is neither complete nor accurate without them.**

**Add to the above problems frequent references to long periods of time or huge distances, and it becomes even more apparent why children have trouble learning from their social studies textbooks. What must a child of 9 or 10 think when the book says, "Our country was founded over 200 years ago"--or perhaps worse, "long, long ago"? What do expressions such as "far to the north" or "over a thousand miles to the east" mean to students who are not sure which direction is which and who have never traveled further than across the state or out of town?**

1. The word "inherent" (line 1) means

- A. naturally occurring.
- B. worst.
- C. least important.
- D. intentional.

2. According to the author of this passage, which of the following does not contribute to the ineffectiveness of social studies textbooks?

- A. difficult terminology
- B. references to long periods of time
- C. uninteresting topics
- D. skimpy explanations of topics

3. The author's main argument is that

- A. social studies texts have no place in the classroom.
- B. social studies texts should be rewritten so that they are easier to understand.
- C. children today are not as intelligent as children in the past.
- D. the difficulties of social studies texts are necessary hurdles that must be overcome.

4. In the last paragraph of the passage, the author drives his point home with

- A. rhetorical questions.
- B. imagery.
- C. hyperboles.
- D. an oxymoron.

II. *This passage is excerpted from a 2003 article about the impact of electronic communication on writing.*

**Word processing and e-publishing have brought about interesting developments in the way writers write. In general, the malleable nature of electronic text has made the physical process of composing more “elastic” in that writers are quicker to commit thought to writing and to reorganize content because it is simple to make changes on the electronic screen. Even young children find it easy to insert and manipulate images and video or audio clips in their texts. In addition, writers who publish on the Web perceive it as a new rhetorical space that provides options for using non-linear, alternative structures, making it necessary for them to anticipate how audiences might physically navigate through their hypertext compositions. This consciousness creates complex perspectives and a heightened awareness of traditional rhetorical elements in a way that text alone never could.**

5. The word “malleable” (line 2) means

- A. inflexible.
- B. simplistic.
- C. pliable.
- D. insincere.

6. The author of this passage suggests that word processing and e-publishing have made the writing and reading of text
- A. more complex than ever before.
  - B. quicker than ever before.
  - C. more simple than ever before.
  - D. more decorative than ever before.

III. *This passage is excerpted from a 2002 article about ecosystems.*

**When someone asks us where we are from or what we do, most of us mention the town or city where we live, our occupation, where we attended school, or our family heritage. We respond in terms of human communities, cultures, and geopolitical boundaries. We seldom, if ever, describe ourselves in terms of our ecological status in the natural world. We humans have so completely ordered, designed, and defined our physical environs and social milieu that our ecological connections have slipped from consciousness. Perhaps this is why we seem so unaware of our impact on nature and our rapid destruction of natural systems. We simply do not perceive ourselves as being part of the natural order of beings.**

**All of us live within ecological systems, or “ecosystems,” and through our commerce, food distribution, and use of natural resources we each indirectly participate in the custodianship of many ecosystems worldwide. Ironically, we are simultaneously the most potent forces within most ecosystems, and yet nearly oblivious to the ecological effects of our daily lifestyles. There has never been a time when a deep understanding of ecosystems and our roles within them has been more critical. Indeed, the world’s freshwater ecosystems are so degraded that their ability to support plant and animal life, including humans, is viewed by many as being in peril. Learning about ecosystems is more than an expected focus in biology classes; it has become a study in survival.**

**Ecosystems are functional units of interacting abiotic, biotic, and cultural (anthropogenic) components. All natural ecosystems are open systems where energy and matter are transferred in and out through the complex interactions of energy, water, carbon, oxygen, nitrogen, phosphorus, sulfur, and other cycles. Unfortunately, many scientists contend, we humans have disrupted the balance of transfers across ecosystem boundaries. In addition to learning our place within ecosystems, we must learn to become better stewards and managers of ecosystems.**

7. The author’s tone in this passage can best be described as
- A. apologetic.
  - B. scolding.
  - C. warning.
  - D. encouraging.

8. The first two sentences of this passage are primarily intended to
- A. capture the reader's attention with an anecdote.
  - B. demonstrate our unawareness of our place in the natural world.
  - C. cause the reader to think about his or her own role in society.
  - D. emphasize the importance of a person's background.
9. The author's attitude in this passage is represented by all of the following phrases except
- A. "must learn to become better stewards and managers of ecosystems."
  - B. "oblivious to the ecological effects of our daily lifestyles."
  - C. "rapid destruction of natural systems."
  - D. "indirectly participate in the custodianship of many ecosystems worldwide."

*IV. This passage is excerpted from a 1922 novel about a Midwestern American's journey to the front during World War I.*

**Claude backed the little Ford car out of its shed, ran it up to the horse-tank, and began to throw water on the mud-crusts wheels and windshield. While he was at work the two hired men, Dan and Jerry, came shambling down the hill to feed the stock. Jerry was grumbling and swearing about something, but Claude wrung out his wet rags and, beyond a nod, paid no attention to them. Somehow his father always managed to have the roughest and dirtiest hired men in the country working for him. Claude had a grievance against Jerry just now, because of his treatment of one of the horses.**

**Molly was a faithful old mare, the mother of many colts; Claude and his younger brother had learned to ride on her. This man Jerry, taking her out to work one morning, let her step on a board with a nail sticking up in it. He pulled the nail out of her foot, said nothing to anybody, and drove her to the cultivator all day. Now she had been standing in her stall for weeks, patiently suffering, her body wretchedly thin, and her leg swollen until it looked like an elephant's. She would have to stand there, the veterinary said, until her hoof came off and she grew a new one, and she would always be stiff. Jerry had not been discharged, and he exhibited the poor animal as if she were a credit to him.**

**Mahailey came out on the hilltop and rang the breakfast bell. After the hired men went up to the house, Claude slipped into the barn to see that Molly had got her share of oats. She was eating quietly, her head hanging, and her scaly, dead-looking foot lifted just a little from the ground. When he stroked her neck and talked to her she stopped grinding and gazed at him mournfully. She knew him, and wrinkled her nose and drew her upper lip back from her worn teeth, to show that she liked being petted. She let him touch her foot and examine her leg.**

**When Claude reached the kitchen, his mother was sitting at one end of the breakfast table, pouring weak coffee, his brother and Dan and Jerry were in their chairs, and Mahailey was**

baking griddle cakes at the stove. A moment later Mr. Wheeler came down the enclosed stairway and walked the length of the table to his own place. He was a very large man, taller and broader than any of his neighbours. He seldom wore a coat in summer, and his rumpled shirt bulged out carelessly over the belt of his trousers. His florid face was clean shaven, likely to be a trifle tobacco-stained about the mouth, and it was conspicuous both for good-nature and coarse humour, and for an imperturbable physical composure. Nobody in the county had ever seen Nat Wheeler flustered about anything, and nobody had ever heard him speak with complete seriousness. He kept up his easy-going, jocular affability even with his own family.

As soon as he was seated, Mr. Wheeler reached for the two-pint sugar bowl and began to pour sugar into his coffee. Ralph asked him if he were going to the circus. Mr. Wheeler winked. "I shouldn't wonder if I happened in town sometime before the elephants get away." He spoke very deliberately, with a State-of-Maine drawl, and his voice was smooth and agreeable. "You boys better start in early, though. You can take the wagon and the mules, and load in the cowhides. The butcher has agreed to take them."

Claude put down his knife. "Can't we have the car? I've washed it on purpose." "And what about Dan and Jerry? They want to see the circus just as much as you do, and I want the hides should go in; they're bringing a good price now. I don't mind about your washing the car; mud preserves the paint, they say, but it'll be all right this time, Claude."

10. Claude's attitude toward Jerry is one of

- A. respect.
- B. resentment.
- C. jealousy.
- D. camaraderie.

11. The expression "imperturbable physical composure" suggests that Nat Wheeler

- A. has a hard time keeping his face clean.
- B. is terribly overweight.
- C. cannot be irritated.
- D. always appears calm by his facial expressions.

12. Which statement by Mr. Wheeler represents an example of verbal irony?

- A. "You can take the wagon and the mules, and load in the cowhides."
- B. "I shouldn't wonder if I happened in town sometime before the elephants getaway."
- C. "They want to see the circus just as much as you do, and I want the hides should go in; they're bringing a good price now."
- D. "I don't mind about your washing the car; mud preserves the paint, they say, but it'll be all right this time, Claude."

## Answers to Reading practice questions

1. A. Because of the highly technical language of social studies, the difficult terminology discussed in this paragraph is a naturally occurring problem.
2. C. The passage specifically mentions all of the other problems, but at no time does the author suggest that social studies is not interesting.
3. D. The author states that the difficulties of the texts are inherent (or natural) and that social studies texts would be “neither complete nor accurate without” difficult words. Rather than passing judgment on the texts or on their readers, the author identifies necessary hurdles that must be overcome.
4. A. The author uses two rhetorical questions, or questions meant to make a point rather than to be answered.
5. C. The word “elastic” provides a good context clue, as does the statement that “it is simple to make changes on the electronic screen.” The word “malleable” suggests that the words on the screen are pliable, or can be easily altered.
6. A. This question is a little tricky because the author utilizes many of the words that are in the wrong answer choices (such as quicker and simple). None of these terms, however, represents his point. The use of alternative structures, images, and other nontraditional rhetorical elements makes electronic communication more complex than traditional texts.
7. C. The author is certainly not apologetic or encouraging. Although some of his words seem scolding, his overall purpose is more to warn than to scold. Therefore, warning is a better choice than scolding.
8. B. Although the opening sentences of the passage may do any of the things listed in the answer choices, their intended purpose is to show that we are oblivious to our place in nature. The focus of the rest of the first paragraph reinforces this purpose.
9. D. All of the other answer choices contain negative words and phrases that warn of threats to ecosystems.
10. B. Claude resents Jerry because Jerry injured a horse that is special to Claude. Evidence of Claude’s resentment can be seen in his description of Jerry’s carelessness and of the horse’s resulting injuries.
11. D. The sentence following the expression sums up its meaning: no one had ever seen Mr. Wheeler flustered. Although C is also a tempting choice, the author doesn’t say that Mr. Wheeler *can’t* be irritated, just that he never looks irritated.
12. D. Verbal irony involves saying the opposite of what you really mean. Mr. Wheeler teases Claude by suggesting that he would rather have a dirty car.

## The English Test

The English Test is composed of five passages presented in an essay format. Two types of questions are presented: those involving usage and mechanics and those involving rhetorical devices such as passage organization and author's point of view. The English Test relies heavily on the conventions of standard English which is the more formal English you find in material such as text books.

**Try the following questions to test your skill in identifying errors in a written passage** (*answers are on page 38*):

My friend Ansley and me volunteer at a local animal shelter every weekend. We arrive at  
1  
the shelter at 9:30 a.m. and start walking the dogs. Most of them are real excited about  
2  
getting out of their cages. It's usually difficult to walk the puppies because they don't  
have no idea what they're suppose to do while they're outside. They usually just want to  
3 4  
play and roll around in the grass. Which is fine because we want them to have fun. While  
5 6  
we walk the dogs, the cages are cleaned by another volunteer. Then we return the dogs to  
7  
their cages and start our other chores including doing laundry, washing dog dishes, and  
8  
we mop the floors. After all of the chores are finished, we enjoy some playtime with the  
9  
animals. Each of the dogs need socialization, that way they can all find good homes  
10  
someday.

11 12

1.
  - A. NO CHANGE
  - B. my friend Ansley and I
  - C. me and my friend Ansley
  - D. myself and my friend Ansley
  
2.
  - A. NO CHANGE
  - B. get real excited about
  - C. are really excited about
  - D. are really excited over

- 3.
- A. NO CHANGE
  - B. they got no idea
  - C. they have no idea
  - D. they don't got any idea
- 4.
- A. NO CHANGE
  - B. supposed to do while
  - C. suppose to be doing while
  - D. supposed to do any time that
- 5.
- A. NO CHANGE
  - B. grass. This is fine
  - C. grass and that is fine
  - D. grass, which is fine
- 6.
- A. NO CHANGE
  - B. another volunteer is who cleans the cages
  - C. another volunteer cleans the cages
  - D. the cages are being cleaned by another volunteer
- 7.
- A. NO CHANGE
  - B. doing laundry, and we wash dog dishes and also mop the floors
  - C. doing laundry, washing dog dishes, and mopping the floors
  - D. laundry, dog dishes, and mopping the floors
- 8.
- A. NO CHANGE
  - B. all of the chores is finished
  - C. all the chores had been finished
  - D. the finishing of all of the chores
- 9.
- A. NO CHANGE
  - B. All of the dogs need
  - C. Each of the dogs is in great need of
  - D. Each of the dogs are in need of

- 10.
- A. NO CHANGE
  - B. socialization, this will help them
  - C. socialization, which will help them
  - D. socialization seeing as how that way they can
11. The organization of this paragraph can be best described as
- A. chronological.
  - B. spatial.
  - C. order of importance.
  - D. cause and effect.
12. Which of the following sentences would make an effective conclusion for this paragraph?
- A. All animals deserve a chance to find good homes.
  - B. In conclusion, everyone should volunteer at an animal shelter on weekends.
  - C. We volunteer at this same shelter every weekend.
  - D. We're exhausted when we finally go home, but we feel good about helping these homeless animals.

## Answers to practice questions (English)

1. B. friend and I make up the compound subject of the sentence. Therefore, we must use the nominative case pronoun (I) rather than the objective (me).
2. C. *Real* is an adjective. We need really (an adverb) in this sentence to modify excited (a participle).
3. C. Don't have no is a double negative. Although choices B and D eliminate the double negative as well, they use the incorrect verb form (got). Got must be used with a helping verb (such as have).
4. B. Don't leave off the d in phrases like supposed to and used to. Although choice D uses supposed, it is wordier than choice B and therefore not the better choice.
5. D. As it is, which begins a sentence fragment. All of the answer choices correct this problem, but the choices are not all equal. Choice B presents a pronoun reference error (What does this refer to?). Choice C also presents a pronoun reference error (What does that refer to?) and is missing a comma before the conjunction that joins two independent clauses.
6. C. The original wording is passive voice, as is choice D. Choices B and C are active voice, but C is more concise and therefore a better choice.
7. C. The original wording is not parallel; rather, it consists of two gerund phrases and one independent clause. Choice C is the only one that uses parallel structure (three gerund phrases).
8. A. This sentence is correct as it is. Choice, B and C use wrong subject/verb agreement and wrong verb tense. Choice D is unnecessarily wordy.
9. B. The original wording is incorrect because the singular pronoun each does not agree with the verb need. Choice B corrects the subject/verb agreement problem and corrects a pronoun/antecedent agreement problem that would otherwise exist with the next part of the sentence (each/they).
10. C. The original wording contains a comma splice, as does choice B. Choice C corrects the problem, while choice D is terribly wordy.
11. A. The paragraph starts at the beginning of the volunteer experience and moves chronologically (in order of time) through to the end.
12. D. Choice A could logically follow the sentence that is currently last, but it doesn't relate to the paragraph as a whole. Because the paragraph focuses on the experiences of the author and her friend, choice B would be ineffective. Choice C merely repeats the topic sentence. Choice D provides closure for the whole paragraph.

## The Essay

Teaching students to write an essay is not the purpose of this guide. Instead, we will devote the remainder of this chapter to providing you with *ACT*'s criteria for evaluating the essay you will complete on the writing test. Information about scoring the essay, combined with the skill review provided above, should go a long way in helping you do well on the writing section of the *ACT*. All the skills covered in the English skills section are applicable to writing your essay.

Beginning in September 2015, the format of the essay prompt has been changed. It now includes:

- Introductory paragraphs explaining an issue to be discussed,
- Three perspectives detailing a particular way of thinking about the issue.

Your task is to:

- Analyze and evaluate the perspectives given;
- State and develop your own perspective (which may be one of the three);
- Explain the relationship between your perspective and those given.

You will have 40 minutes to complete the essay. It is very important that you take some time at the beginning of the session to develop an outline. This will be very helpful to you when you actually start writing.

The essay will be scored by two people who will assign a score of 1-6 in four areas. These areas include Ideas and Analysis, Development and Support, Organization, and Language Use and Conventions. The scores will be combined and converted to a scale score.

You can find more detailed information on how the essay is evaluated at the *ACT* website—[www.act.org](http://www.act.org)—*The ACT Test*. (Click on “The Writing Test Option.”)

## Chapter Four: Math Skills Review

So many students approach this section of the test with an unhealthy amount of fear or anxiety. Familiarizing yourself with the format of the *ACT* math questions and becoming comfortable working with the equations you will need to answer the questions will help you overcome this. Approaching this part of the test with a positive attitude and a plan of attack will help you achieve your desired score.

The mathematical reasoning sections are designed to test your problem-solving abilities in topics that are generally covered in the math classes taken by most high school students. These topics include pre-algebra, algebra, and geometry. Although you will be tested on your knowledge of these subjects, it is your ability to **apply** what you have learned that will determine your success on the test.

You will need to familiarize yourself with basic mathematical formulas. These are not provided for you on the test. You are being tested on your ability to apply these formulas when needed to arrive at the correct answers. Therefore, memorizing the formulas and their corresponding equations is not enough. Your comfort in applying your knowledge of what they mean is vital to success in solving mathematical problems on the test.

### Thy Days Are Numbered: Properties of Numbers

A finite number of days remain before you are going to take the *ACT*, and it is always a good idea to begin preparing well in advance. Remember, you have been preparing for this day for several years now. This is where all of those hours spent in math classes finally pay off.

For the majority of the mathematical reasoning questions, obtaining the correct answer will depend on your ability to manipulate numerical values. Approximately eighty percent (80%) of your answers will be a numerical value. In this section, you will review the types of numbers that may be encountered and some of their basic properties.

### Classes and Types of Numbers

Yes, numbers do come in different types! We will begin by covering some of the basic terminology used to describe the different classes or types of numbers.

- **Counting Numbers**

The **counting numbers**, also referred to as the **natural numbers**, are probably the first numbers you ever encountered in life. Beginning with one, they continue infinitely in the positive direction.

Examples: 1, 2, 3, 4, 5, ...

- **Integer**

The integers, also referred to as **whole numbers**, are the counting numbers together with their negatives (and zero). They continue infinitely in both the negative and positive directions.

Examples: ..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...

- **Decimal**

The decimal system allows us to write numbers that are arbitrarily small. We can represent **numbers or parts of a number that are less than 1 by using a decimal**. The place values to the left of the decimal represent tenths, hundredths, thousandths, and so on.

Examples: 1.2, 1.003, 0.00234, and 1.235

- **Rational Numbers**

**Rational numbers** can be written as a **ratio or fraction involving two integers**. Any number that can be written as a terminating or a repeating decimal is a rational number!

Examples:  $\frac{1}{2}$ , 0.033, and 2

- **Irrational Numbers**

Irrational numbers consist of any real numbers that are not rational numbers, that is, they cannot be written as a ratio of two integers.

Examples:  $\sqrt{2}$ ,  $\sqrt{3}$ , and  $\pi$

- **Real Numbers**

The real numbers consist of **all rational and irrational numbers**.

- **Negative, Positive, or Not**

This is included here because you will often encounter this terminology when describing classes of numbers.

- o **A negative number is any number that is less than zero.**
  - -1,  $-\sqrt{2}$ , and  $-\frac{1}{33}$  are negative numbers.
- o A positive number is any number greater than zero.
  - 2,  $\sqrt{3}$ , and  $\frac{34}{53}$  are positive numbers.
- o **Zero is neither negative nor positive.**
- o **We cannot classify imaginary or complex numbers as positive or negative, although we can classify the real or the imaginary component.**
  - $3-2i$  is neither positive nor negative, although the real component (3) is positive and the imaginary component (-2) is negative.
- o **Non-negative numbers include zero and all positive numbers.**
  - 0, 3, and  $\sqrt{5}$  are examples.
- o **Non-positive numbers include zero and all negative numbers.**
  - 0,  $-\pi$ , and  $-\frac{1}{28}$  are examples.

## Multiples, Factors, and Primes

A multiple of a number is any product of that number and an integer. So, 4, 8, 12, 16, and 20 are multiples of 4 because  $4 \times 1 = 4$ ,  $4 \times 2 = 8$ ,  $4 \times 3 = 12$ ,  $4 \times 4 = 16$ , and  $4 \times 5 = 20$ . The test writers are typically going to consider the positive multiples of a number. They may even refer to the positive multiples as simply the multiples. However, multiples of a number can be negative as well. Note that  $-4$ ,  $-8$ ,  $-12$ ,  $-16$ , and  $-20$  are also multiples of 4. Multiples of a number need not be positive or negative. Although the chances it will come up on the test may be small, it certainly doesn't hurt to know that 0 is a multiple of every number, since 0 is an integer and any number  $x \times 0 = 0$ . The lowest (positive) multiple shared by two numbers is called the **least common multiple**. The multiples of 3 and 8 are

**Multiples of 3:** 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, ...

**Multiples of 8:** 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, ...

As you can see, the first multiple to appear in both lists is 24. Therefore, 24 is the least common multiple of 3 and 8.

**A factor of a number is any (positive) integer that evenly divides (remainder of 0) into the number.** The number 18 has factors of 2, 3, 6, and 9 because  $18/9 = 2$ ,  $18/6 = 3$ ,  $18/3 = 6$ , and  $18/2 = 9$ . Any given number is a multiple of all its factors. The term **greatest common factor** is given to the largest factor shared by two numbers. The greatest common factor of 21 and 28 is 7, since 7 is the largest number that is a factor of both numbers. Although it is unlikely that you will simply be asked to factor a number for the test, being able to factor numbers will be important to answering many of the questions.

**A prime number is any integer greater than 1 whose only factors (positive divisors) are 1 and itself.** Three (3) is a prime number because the only factors of 3 are 1 and 3. Six (6) is not a prime number because the factors of 6 are 1, 2, 3, and 6. Although we won't get into why here, note that 1 is not considered a prime number. We can write every positive integer as a unique product of prime numbers. This is referred to as the *prime factorization* of a number. The number 4620 has a prime factorization of  $2 \cdot 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11$ . The prime factorization of a number is unique; that is to say, there is no other combination of prime factors that will give the same product. This leads us to another concept that you may encounter, although probably not more than once on a given test. Two numbers are said to be *relatively prime* if they have no factors (other than 1) in common. The numbers  $198 = 2 \cdot 3 \cdot 3 \cdot 11$  and  $455 = 5 \cdot 7 \cdot 13$  are relatively prime because they share no common factors in their prime factorization.

When completely factoring a number, begin by removing any obvious factors such as 2, 5, and 10. This will make it easier to examine any of the less obvious factors. In factoring a number, the following divisibility rules will be helpful.

1. One (1) is a factor of any whole number.
2. Any even number (ends in 0, 2, 4, 6, or 8) will be divisible by 2.
  - 312, 256 are divisible by 2, while 221 is not.

3. Add up the digits in the number. If the sum of the digits is divisible by 3, then the number is divisible by 3 as well. Note that you can do this recursively for large numbers.
  - How would you determine if the number 937, 689, 865, 863 is divisible by 3? Add up the digits:  $9 + 3 + 7 + 6 + 8 + 9 + 8 + 6 + 5 + 8 + 6 + 3 = 78$ . Is 78 divisible by 3?  $7 + 8 = 15$  and  $15 \div 3 = 5$ , so the original number is divisible by 3.
4. If the last two digits are divisible by 4, then the number is divisible by 4.
  - 432, 235, 916 is divisible by 4 because 16 is divisible by 4.
5. Any number ending in 0 or 5 is divisible by 5.
6. Any even number that is divisible by 3 will be divisible by 6. (This is just the combination of rules 2 and 3 above)
  - 14 is divisible by 2 (because it ends in 4) but not by 3 ( $1 + 4 = 5$ , which is not divisible by 3), so 14 is not divisible by 6.
  - 15 is divisible by 3 ( $1 + 5 = 6$ , which is divisible by 3) but not by 2 (because it ends in 5), so 15 is not divisible by 6.
  - 18 is divisible by 2 (because it ends in 8) and by 3 (because  $1 + 8 = 9$ , which is divisible by 3), so 18 is divisible by 6.
7. Double the last digit and subtract from the remaining digits. If this number is divisible by 7, then the original number is also divisible by 7. This can be done recursively as well.
  - To determine if 19,565 is divisible by 7, do the following.
 
$$\begin{array}{r} 1956 \text{ (} 5 \times 2 = 10 \text{)} \\ - 10 \\ \hline 194 \text{ (} 6 \times 2 = 12 \text{)} \\ - 12 \\ \hline 18 \text{ (} 2 \times 2 = 4 \text{)} \\ - 4 \\ \hline 14 / 7 = 2 \end{array}$$
8. If the last three digits are divisible by 8, then the entire number is divisible by 8. This isn't as helpful as some of the others. You can also just look for successive factors of 2 (remember that  $2 \cdot 2 \cdot 2 = 8$ ).
  - The number 257,392,104 is divisible by 8 because 104 is divisible by 8. Also, note that  $257,392,104/2 = 128,696,052$ ,  $128,696,052/2 = 64,348,026$ , and  $64,348,026/2 = 32,174,013$ .
9. This rule is similar to rule 3 above. If the sum of the digits is divisible by 9, then the number is divisible by 9.
  - 423,421,264,413 is divisible by 9 because the sum of the digits is  $36 = (9 \times 4)$ .
10. Any number ending in 0 is divisible by 10.

## Properties of Numbers Practice Problems

1. True or false: All natural numbers are non-negative.
  - A) True
  - B) False
2. Which of the following are positive integers?
  - I. 2
  - II.  $1/3$
  - III. 0
  - IV.  $-3$
  - V.  $\pi$
  - A) I, II, III, and IV
  - B) I, II, and III
  - C) I and III
  - D) I only
  - E) II only
3. True or false: All non-negative integers are natural numbers.
  - A) True
  - B) False
4. Which of the following are non-positive rational numbers?
  - I.  $-1$
  - II. 0
  - III.  $-\sqrt{2}$
  - IV.  $-\sqrt{4}$
  - V.  $1/3$
  - A) I, II, III, and IV
  - B) I, II, and IV
  - C) I, II, and V
  - D) I, III, and IV
  - E) I, and V
5. What is  $1/500$  written as a decimal?
  - A) 0.01
  - B) 0.02
  - C) 0.05
  - D) 0.002
  - E) 0.005

6. Which of the following are multiples of 8?

- I. 24
- II. 36
- III. 4
- IV. 1,624
- V. 3,001

- A) I, II, and III
- B) I, II, and IV
- C) I and IV
- D) II and IV
- E) I and V

7. The least common multiple of 10 and 18 is...

- A) 2.
- B) 80.
- C) 90.
- D) 180.
- E) 360.

8. Which of the following is not a factor of 20,790?

- A) 2
- B) 3
- C) 6
- D) 11
- E) 20

9. The greatest common factor of 874 and 1,995 is...

- A) 3.
- B) 7.
- C) 15.
- D) 19.
- E) 23.

10. The correct prime factorization of 22,770 is...

- A)  $6 \cdot 5 \cdot 11 \cdot 23$ .
- B)  $2 \cdot 3 \cdot 5 \cdot 11 \cdot 23$ .
- C)  $2 \cdot 3 \cdot 3 \cdot 5 \cdot 11 \cdot 23$ .
- D)  $2 \cdot 3 \cdot 3 \cdot 3 \cdot 5 \cdot 7 \cdot 11$ .
- E)  $3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 \cdot 7 \cdot 7$ .

11. Of the following pairs of numbers, which pair is mutually prime?
- A) 43,010 and 150,423
  - B) 3,705 and 7,116
  - C) 99,009 and 144,177,345
  - D) 68 and 13,247,352,132
  - E) 22 and 319,209,649

Note: Answers may be found on Page 105.

## Frustrations with Fractions

If fractions get you frustrated, you are not alone. Many students are uncomfortable working with fractions. **A fraction is used to describe a part of a whole.** The **numerator (top) describes the part, and the denominator (bottom) describes the whole.** In the fraction  $\frac{2}{3}$ , the numerator is 2 and the denominator is 3. We'll discuss operations such as addition and multiplication in a later section. For now, we just want you to familiarize yourself with the concepts of equivalent fractions, reducing fractions, and common denominators.

### Equivalent Fractions

Stated simply, **two fractions that describe the same part of the whole are considered equivalent fractions.** If two friends cut a pie in half, they can each take  $\frac{1}{2}$  of the whole. What if they both cut their piece of the pie into two pieces? They now each have two pieces of the pie, which is now in 4 pieces. So, they each have  $\frac{2}{4}$  of the pie.  $\frac{2}{4}$  and  $\frac{1}{2}$  are equivalent fractions. In fact,  $\frac{362}{724}$  and  $\frac{1}{2}$  are equivalent fractions. Two equivalent fractions will be the same when completely reduced (written in simplest form). Both  $\frac{2}{4}$  and  $\frac{362}{724}$  simplify to  $\frac{1}{2}$ . When one or both of the fractions contain large numbers it may not be in your best interest to try to reduce or simplify them, although you could certainly do this. We can compare two fractions by simply cross multiplying. For example, if we are asked to compare  $\frac{1,728}{4,032}$  and  $\frac{3,972}{9,268}$  we probably don't want to simplify these fractions. You will want to cross multiply.

$$(1,728 \times 9,268 = 16,015,104) \quad \frac{1,728}{4,032} \times \frac{3,972}{9,268} \quad (4,032 \times 3,972 = 16,015,104)$$

Because the cross products are the same, the fractions are equivalent. Had the product on the left been larger, the fraction to the left would be the larger of the two. Similarly, had the product on the right been larger, the fraction to the right would be the larger of the two. For example,  $\frac{74}{100} < \frac{3}{4} < \frac{76}{100}$ .

$$(74 \times 4 = 296) \quad \frac{74}{100} \times \frac{3}{4} \quad (100 \times 3 = 300)$$

$$(3 \times 100 = 300) \quad \frac{3}{4} \times \frac{76}{100} \quad (4 \times 76 = 304)$$

## Reducing Fractions

Reducing and simplifying fractions will require you to be able to identify the common factors between the numerator and the denominator. This is where the divisibility rules will come into play. You will be looking for factors that can divide both the numerator and the denominator. You can do this either by writing out the prime factorization of both the numerator and the denominator or just by examination. Consider the problem of simplifying the fraction  $\frac{420}{630}$ . You could simply write out the prime factorization of 420 and 630, canceling any common factors. After some work you would arrive at the correct answer.

$$\frac{420}{630} = \frac{\cancel{2} \times 2 \times \cancel{3} \times \cancel{5} \times 7}{\cancel{2} \times 2 \times \cancel{3} \times \cancel{5} \times 7} = \frac{2}{3}$$

A second, and often quicker, approach will be simply to look for common factors by inspection. We can easily see that both 420 and 630 are divisible by 10. This leaves us with  $\frac{42}{63}$ . Both 42 and 63 are divisible by 3 (the sum of the digits are 6 and 9, respectively), so we are left with  $\frac{14}{21}$ . Removing the final common factor of 7 will leave the correct answer of  $\frac{2}{3}$ .

You will probably encounter one or more *improper fractions* on the test. An improper fraction is one that represents a whole number plus 1 remainder. Answers for the multiple-choice questions will typically be written as *mixed numbers*. Mixed numbers represent an improper fraction as a whole number and a fractional component. For example,  $\frac{7}{3}$  is an improper fraction. Three (3) divides into 7 twice with a remainder of 1. We could thus write  $\frac{7}{3}$  as the mixed number  $2\frac{1}{3}$ . You will also encounter the situation where you will have to convert a mixed number to an improper fraction, especially when multiplying or dividing with fractions. To do this, simply multiply the whole number by the denominator and add the result to the numerator. Using our previous example, writing  $2\frac{1}{3}$  as an improper fraction would look something like

$$2\frac{1}{3} = \frac{(2 \times 3) + 1}{3} = \frac{6 + 1}{3} = \frac{7}{3}$$

## Finding Common Denominators

The most common problem students encounter with fractions involves finding a common denominator, or more importantly, the *least common denominator*. Before you can add or subtract fractions, you will need to find a common denominator. Probably the simplest way to find a common denominator will be to take the product of the two denominators. For example,  $\frac{2}{3}$  and  $\frac{1}{4}$  can be written with a common denominator of 12.

$$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12} \qquad \frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$$

Although this can be done quickly, it may result in answers requiring a significant amount of simplification. This is because the common denominator that results is not necessarily the *least*

*common denominator.* To find the least common denominator will require a couple of steps. Begin by factoring both of the denominators. With this factorization in hand, we will take as the least common denominator the union of these two sets. We thus simply multiply the numerator and the denominator of each fraction by the missing factors to arrive at equivalent fractions with the least common denominator as the denominator. Consider the fractions  $\frac{91}{660}$  and  $\frac{37}{42}$ . Begin by factoring both denominators:

$$\frac{91}{\frac{660}{2 \cdot 2 \cdot 3 \cdot 5 \cdot 11}} \quad \frac{53}{\frac{210}{2 \cdot 3 \cdot 5 \cdot 7}}$$

The least common denominator will thus be  $2 \cdot 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 = 4,620$ . All we need to do now is multiply the numerator and denominator of each fraction by the missing factors.

$$\frac{91}{\frac{660}{2 \cdot 2 \cdot 3 \cdot 5 \cdot 11}} \times \frac{7}{7} = \frac{637}{4,620} \quad \frac{53}{\frac{210}{2 \cdot 3 \cdot 5 \cdot 7}} \times \frac{2 \cdot 11}{2 \cdot 11} = \frac{1,166}{4,620}$$

It may not be obvious since we still have fractions with rather large numbers, but this really did make things simpler. Had we simply multiplied the two denominators together, we would have gotten a common denominator of 138,600.

## Fractions Practice Problems

1. Which of the following fractions are equivalent to  $\frac{4}{7}$ ?

I.  $\frac{49,428}{86,500}$       IV.  $\frac{47,325}{34,321}$

II.  $\frac{9,460}{16,555}$       V.  $\frac{36}{63}$

III.  $\frac{12,357}{31,367}$

- A) I, II, III, and V.
- B) I, II, and V.
- C) I and II.
- D) I and V.
- E) II and V.

2. Which of the following are greater than  $\frac{5}{11}$ ?

I.  $\frac{1}{2}$       IV.  $\frac{34}{77}$

II.  $\frac{65}{132}$       V.  $\frac{40}{87}$

III.  $\frac{17}{33}$

- A) I, II, III, and V.
- B) I, II, and III.
- C) I, II, and V.
- D) I and II.
- E) I and V.

3. The fraction  $\frac{720}{780}$  is equivalent to which of the following?

- A)  $\frac{5}{6}$
- B)  $\frac{3}{4}$
- C)  $\frac{6}{7}$
- D)  $\frac{12}{13}$
- E)  $\frac{13}{17}$

4. Write  $\frac{9,702}{6,930}$  as a mixed number in simplest form.

- A)  $\frac{7}{5}$
- B)  $1\frac{2}{3}$
- C)  $1\frac{2}{5}$
- D)  $\frac{5}{3}$
- E)  $1\frac{29}{75}$

5. Write  $5\frac{12}{13}$  as an improper fraction.
- A)  $\frac{72}{13}$   
 B)  $\frac{65}{13}$   
 C)  $\frac{17}{13}$   
 D)  $\frac{60}{13}$   
 E)  $\frac{77}{13}$
6. What is the least common denominator of  $\frac{7}{12}$  and  $\frac{23}{26}$ ?
- A)  $\frac{7}{156}$   
 B)  $\frac{1}{156}$   
 C) 156  
 D) 24  
 E) 312

Note: Answers may be found on Page 105.

## We're going to have to operate: Operations with numbers

This subtitle may sound like a really bad joke. OK, maybe it is, but many of the common mistakes made by students on the *ACT* math tests are due to simple mistakes in operations: addition, subtraction, multiplication, and division. More often than not, this is the result of rushing through an otherwise easy problem and arriving at one of the wrong answers. The test writers know what mistakes students commonly make, and they use this information when coming up with the wrong answer choices. This is where the “easy” problems get you. **Remember to take your time so you won't miss these questions!**

Also, **remember the all-important *order of operations***. (We'll review this as well.) Just as for a surgeon, the order in which you do things when performing mathematical operations is vitally important. We'll review some of the basics of addition, subtraction, multiplication, and division here. If you feel that you need more help or practice with any of these, consult your math teacher. Your ability to perform basic mathematical operations will be extremely important to your success on the test. You will be able to use your calculator on this part of the test, but your understanding of the basic mathematical operations will help prevent careless mistakes. Also, some of the more advanced calculators are capable of performing many of the more advanced operations as well, such as multiplying and dividing complex numbers. Consult the manual for your calculator to find out what its capabilities are.

### Addition

When adding two (or more) numbers together, the numbers to be added are called *addends* and the answer is the *sum*. So, in the problem below, 3,432 and 291 are the addends and 3,723 is the sum.

$$\begin{array}{r}
 3,432 \\
 + 291 \\
 \hline
 3
 \end{array}
 \qquad
 \begin{array}{r}
 \overset{x}{3},432 \\
 + 291 \\
 \hline
 23
 \end{array}
 \qquad
 \begin{array}{r}
 \overset{x}{3},432 \\
 + 291 \\
 \hline
 723
 \end{array}
 \qquad
 \begin{array}{r}
 \overset{x}{3},432 \\
 + 291 \\
 \hline
 3,723
 \end{array}$$

When adding integers, always be careful to keep the columns straight and line everything up with the units column. Addition is *commutative*. This means that the order in which we add the numbers does not change the final answer. Don't let this confuse you. It's something you already know  $2 + 3 = 3 + 2 = 5$ . Also, addition is *associative*. This means that we can group the numbers in any manner we choose and still get the same answer, e.g.  $(3 + 2) + 4 = 5 + 4 = 9 = 3 + 6 = 3 + (2 + 4)$ . Although this is not meant to be thorough enough to teach you addition, we'll look at a few things you need to remember. Most of this should already be familiar to you.

- When adding integers, remember the following:  
 Even + Even = Even ( $2 + 4 = 6$ )  
 Odd + Odd = Even ( $3 + 3 = 6$ )  
 Even + Odd = Odd ( $2 + 3 = 5$ )
- When adding negative and positive numbers, you may find it advantageous to rewrite the problem as a subtraction problem:

$$234 + 137 + -295 = (234 + 137) - 295 = 371 - 295 = 76$$

- When adding decimals, always line the columns up with the decimal point. The proper place for the decimal point in the final answer will be directly beneath the decimal points in the problem.

$$\begin{array}{r} \phantom{+} 1.324 \\ + 2.19 \\ \hline 3.514 \end{array}$$

- When adding fractions, always find a *common denominator*. There are many different ways that you can do this. Deciding which way is best really depends on what works best for you. Consider the problem  $5\frac{3}{12} + \frac{3}{10}$

You could first proceed by multiplying both the numerator and the denominator of each addend by the denominator of the other addend.

$$\frac{5}{12} \left( \frac{10}{10} \right) + \frac{3}{10} \left( \frac{12}{12} \right) = \frac{50}{120} + \frac{36}{120} = \frac{86}{120}$$

Now, you need to simplify the answer. We can pull out a common factor of 2.

$$\frac{86/2}{120/2} = \frac{43}{60}$$

Remember, as long as we do the same thing to both the numerator and the denominator, we have not changed the fraction; for example,  $86/120$  and  $43/60$  are *equivalent fractions*.

Going back to the original problem, we could also have found a common denominator by factoring the denominator of each addend and taking, as our common denominator, the union of all factors. This should give us what is known as the *least common denominator*.

$$\frac{5}{12} + \frac{3}{10} = \frac{5}{2 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 5}$$

Our common denominator will thus have two factors of 2, one 3, and one 5. We can then multiply the numerator and denominator of each addend by the necessary factors.

$$\frac{5}{12} \times \frac{5}{5} + \frac{3}{10} \times \frac{2 \cdot 3}{2 \cdot 3} = \frac{25}{60} + \frac{18}{60} = \frac{43}{60}$$

This answer needs no further simplification.

- When adding fractions written as mixed numbers, you don't have to rewrite them as improper fractions. Add the integer components and then add the fractions. Remember that you will still have to find a common denominator, and you may have to simplify the fractional component of the result. Consider the problem

$$11 \frac{3}{4} + 12 \frac{3}{5}$$

We will begin by adding the whole numbers and then adding the fractions, making sure to find a common denominator.

$$\begin{aligned} 11 \frac{3}{4} + 12 \frac{3}{5} &= (11 + 12) + \left( \frac{3}{4} + \frac{3}{5} \right) \\ &= 23 + \left( \frac{3}{4} \left( \times \frac{5}{5} \right) + \frac{3}{5} \left( \times \frac{4}{4} \right) \right) \\ &= 23 + \left( \frac{15}{20} + \frac{12}{20} \right) \\ &= 23 + \frac{27}{20} \end{aligned}$$

Now we'll have to rewrite this answer in simplest form.  $27/20$  is an improper fraction. 20 goes into 27 once, with a remainder of 7; thus,

$$= 23 + \frac{27}{20} = 23 + 1 \frac{7}{20} = 24 \frac{7}{20}$$

## Subtraction

When subtracting two numbers, the number we are subtracting from is the *minuend* and the result is called the *difference*. In the example problem below, 430 is the minuend, 220 the subtrahend, and 210 the difference.

$$\begin{array}{r} 430 \\ - 220 \\ \hline 210 \end{array}$$

Just like with addition, when subtracting integers you should always line everything up with the units column. Here are some of the things you need to remember with subtraction. Again, get more help if you feel that you need it.

- When subtracting integers, remember the following:
  - Even – Even = Even ( $6 - 4 = 2$ )
  - Odd – Odd = Even ( $5 - 3 = 2$ )
  - Even – Odd = Odd ( $8 - 5 = 3$ )
  - Odd – Even = Odd ( $7 - 4 = 3$ )
- When subtracting numbers some of which are negative and some of which are positive, you may find it advantageous to rewrite the problem as an addition problem:

$$234 - (-137) = 234 + 137 = 371$$

- When subtracting decimals, always line the columns up with the decimal point. The proper place for the decimal point in the final answer will be directly beneath the decimal points in the problem. Also, remember that you may borrow from the units to the left if needed. This is demonstrated in the problem below.

$$\begin{array}{r} 21.324 \\ - 2.19 \\ \hline 4 \end{array} \quad \begin{array}{r} \overset{2}{2}1.\overset{12}{32}4 \\ - 2.19 \\ \hline 34 \end{array} \quad \begin{array}{r} \overset{2}{2}1.\overset{12}{32}4 \\ - 2.19 \\ \hline .134 \end{array} \quad \begin{array}{r} \overset{111}{2}1.\overset{2}{3}24 \\ - 2.19 \\ \hline 9.134 \end{array} \quad \begin{array}{r} \overset{111}{2}1.\overset{2}{3}24 \\ - 2.19 \\ \hline 19.134 \end{array}$$

- When subtracting fractions, always find a common denominator. Consider the problem below.

$$\begin{aligned} \frac{7}{12} - \frac{13}{30} &= \left( \frac{7}{12} \times \frac{5}{5} \right) - \left( \frac{13}{30} \times \frac{2}{2} \right) \\ &= \frac{35}{60} \times \frac{26}{60} - \frac{9}{60} \times \frac{3}{20} \end{aligned}$$

- When subtracting fractions written as mixed numbers, you don't have to rewrite them as *improper fractions*. Subtract the integer components and then subtract the fractions. Remember that you will still have to find a common denominator, and you may have to simplify the fractional component of the result. Consider this problem:

$$12\frac{3}{5} - 11\frac{3}{4}$$

We will begin by subtracting the whole numbers and then subtracting the fractions, making sure to find a common denominator.

$$\begin{aligned} 27\frac{3}{5} - 11\frac{3}{4} &= (27 - 11) + \left(\frac{3}{5} - \frac{3}{4}\right) \\ &= 16 + \left(\frac{3}{5} - \frac{3}{4}\right) \end{aligned}$$

Were  $\frac{3}{5}$  not less than  $\frac{3}{4}$ , we could just go ahead and subtract the fractions and arrive at the answer. In this case, we will have to essentially borrow from the whole number.

$$\begin{aligned} 16 + \left(\frac{3}{5} - \frac{3}{4}\right) &= 15 + 1 + \left(\frac{3}{5} - \frac{3}{4}\right) \\ &= 15 + \left(\frac{8}{5} - \frac{3}{4}\right) \\ &= 15 + \left(\frac{32}{20} - \frac{15}{20}\right) = 15\frac{17}{20} \end{aligned}$$

## Multiplication

You will probably do much of the multiplication with your calculator. However, it is still a good idea to know what is going on. The answer to a multiplication problem is called the *product*. **Multiplication can be thought of as collective additions.** When we say  $4 \times 3$  we could view this as  $4 + 4 + 4$  or as  $3 + 3 + 3 + 3$ ; the answer is still 12. Multiplication is commutative, meaning that  $4 \times 3 = 3 \times 4$ . Here are some things to remember with multiplication:

- When multiplying integers, remember the following:
  - Even x Even = Even ( $2 \times 4 = 8$ )
  - Odd x Odd = Odd ( $3 \times 5 = 15$ )
  - Even x Odd = Odd ( $2 \times 3 = 6$ )
  - Odd x Even = Odd ( $3 \times 2 = 6$ )

- When multiplying integers with more than one digit, you must keep up with the place value. When multiplying 127 and 34, we begin by multiplying by 4.

$$\begin{array}{r} \overset{1\ 2}{127} \\ \times 34 \\ \hline 508 \end{array}$$

Now we multiply by 3, leaving a zero to the right because we are actually multiplying by 30.

$$\begin{array}{r} \overset{2}{127} \\ \times 34 \\ \hline 508 \\ 3810 \\ \hline 4318 \end{array}$$

- When multiplying positive and negative numbers, remember:
  - Positive x Positive = Positive
  - Negative x Negative = Positive
  - Positive x Negative = Negative
  - Negative x Positive = Negative
- When multiplying decimals, proceed just as if you were multiplying integers. The number of places to the right of the decimal point in the product is simply the sum of the number of places to the right of the decimal point in the numbers being multiplied. When we multiply 21.324 by 2.19, we will have  $3 + 2 = 5$  places to the right of the decimal point in the final answer.

$$\begin{array}{r} \overset{1\ 2\ 2\ 3}{21.324} \\ \times 2.19 \\ \hline 191916 \\ 213240 \\ 4264800 \\ \hline 46.69956 \end{array}$$

- Multiplication with fractions is much simpler than addition and subtraction. You do not need to find a common denominator. Just multiply the numerators and then multiply the denominators. You may have to simplify afterwards.

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$

When multiplying a series of fractions, it helps to recognize factors that you can cancel prior to multiplication. In the problem below, we can cancel the 3 and the 5 because they

appear in both the numerator and the denominator. We can even cancel the 2 in the numerator with a factor of 2 from the 4 in the denominator.

$$\frac{1}{3} \times \frac{2}{5} \times \frac{3}{4} \times \frac{5}{7} = \frac{1}{14}$$

By canceling these factors prior to multiplication, we arrive at an answer that is now in reduced form. Had we not done this and just multiplied across, we would have had to reduce  $\frac{30}{420}$ .

- When multiplying fractions written as mixed numbers, rewrite them as improper fractions. Consider this problem:

$$3\frac{3}{5} \times 4\frac{3}{4}$$

We will begin by rewriting the mixed numbers as improper fractions and multiplying across.

$$3\frac{3}{5} \times 4\frac{3}{4} = \frac{18}{5} \times \frac{19}{4} = \frac{171}{10}$$

## Division

**Division is closely related to working with fractions. The number being divided (numerator) is called the *dividend*, and the number we are dividing by (denominator) is called the *divisor*. The answer is called the *quotient*.** So,  $6 \div 2 = \frac{6}{2} = \frac{3}{1} = 3$ , where 6 is the dividend, 2 is the divisor, and 3 is the quotient. The divisor won't always divide evenly into the dividend (the quotient is not always a whole number). Four (4) goes into 6 once. The *remainder* is 2. This can also be expressed as  $6 \div 4 = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$ . Four (4) goes into 6 one and a half times. Thus, we can always express the answer as a mixed number with the fractional component written as the remainder divided by the divisor ( $\frac{2}{4} = \frac{1}{2}$  in the problem above).

- When dividing decimals, move the decimal point in the divisor all the way to the right and then in the dividend the same number of places.

$$2.34 \div 0.06 = 234 \div 6 = 39$$

- When dividing positive and negative numbers, remember:
  - Positive  $\div$  Positive = Positive
  - Negative  $\div$  Negative = Positive
  - Positive  $\div$  Negative = Negative
  - Negative  $\div$  Positive = Negative

- Division with fractions is much simpler than addition and subtraction. You can convert the problem into multiplication by flipping the divisor, and you do not need to find a common denominator.

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} = \frac{6}{4} = \frac{3}{2} = 1 \frac{1}{2}$$

- When dividing fractions written as mixed numbers, rewrite them as improper fractions:

$$4 \frac{3}{4} \div 3 \frac{1}{4} = \frac{19}{4} \div \frac{13}{4} = \frac{19}{4} \times \frac{4}{13} = \frac{19}{13} = 1 \frac{6}{13}$$

### Order of Operations

The order in which we apply operations is extremely important. Questions that test your knowledge of the correct order of operations frequently show up on the test. There is a simple mnemonic device, “PEMDAS,” that will assist you in remembering the proper order. Students are often taught to remember this using the phrase “Please Excuse My Dear Aunt Sally,” among others. The letters stand for

1. **P**arentheses
2. **E**xponents
3. **M**ultiplication and **D**ivision
4. **A**ddition and **S**ubtraction

Consider the problem  $3(2^4 - 2 \cdot 3)^2 - 2 \cdot 3$ . We begin by evaluating everything inside the parentheses, remembering the correct order of operations when doing so.

$$\begin{aligned} 3(2^4 - 2 \cdot 3)^2 - 2 \cdot 3 &= 3(16 - 2 \cdot 3)^2 - 2 \cdot 3 \\ &= 3(16 - 6)^2 - 2 \cdot 3 \\ &= 3(10)^2 - 2 \cdot 3 \end{aligned}$$

Now that we have evaluated everything inside the parentheses, we proceed to evaluate the exponents, followed by all multiplication and division, and then by all addition and subtraction.

$$\begin{aligned} 3(10)^2 - 2 \cdot 3 &= 3 \cdot 100 - 2 \cdot 3 \\ &= 300 - 6 \\ &= 294 \end{aligned}$$

Although the example has used integers, the rules concerning the order of operations should be applied for operations with any of the classes of numbers we have already discussed.

### Operations Practice Problems

1)  $\frac{187}{420} + \frac{91}{510} =$

A)  $\frac{278}{930}$       D)  $\frac{133,580}{214,200}$

B)  $\frac{278}{420}$       E)  $\frac{4,453}{7140}$

C)  $\frac{278}{510}$

2)  $5\frac{1}{3} - 2\frac{3}{4} =$

A)  $3\frac{7}{12}$       D)  $2\frac{1}{3}$

B)  $2\frac{7}{12}$       E)  $3\frac{3}{4}$

C)  $3\frac{5}{12}$

3)  $27.2347 - 13.395 =$

A) 14.1617      D) 14.8397

B) 25.8952      E) 13.1617

C) 13.8397

4)  $3.257 \times 0.749 =$

A) 2.430439      D) 0.2439493

B) 24.39493      E) 2.439493

C) 24.39493

5) What is the quotient of  $5\frac{3}{4}$  divided by  $2\frac{2}{3}$ ?

A)  $2\frac{8}{9}$     D)  $\frac{23}{4}$

B)  $\frac{46}{3}$     E)  $\frac{31}{7}$

C)  $2\frac{5}{32}$

6) Simplify  $\frac{4^3 - (8.5 - 2 \cdot 1.25)^2}{3 + \frac{3}{4} + (1 + \frac{1}{2})^2 - 1}$

$3 + \frac{3}{4} + (1 + \frac{1}{2})^2 - 1$

A) 4    D) -7.4801571

B)  $\frac{112}{13}$     E)  $4\frac{2}{3}$

C) 2

7)  $(6.31 - 2.4^2) \cdot 1.75 - 0.25 =$

A) 26.504175    D) 22.93215

B) 0.7125    E) 0.825

C) 0.55

8)  $\frac{4\frac{3}{4} - 2\frac{1}{4} + 1\frac{1}{3}}{3\frac{2}{3}} =$

A)  $\frac{147}{128}$     D)  $1\frac{1}{4}$

B)  $\frac{45}{64}$     E) 5

C) 2

Note: Answers may be found on Page 105.

## What's With All the Letters: Algebraic Expressions and Manipulation

Many of the questions, in the *ACT* math test may not include numbers at all. These questions (and others) will test your knowledge of basic algebraic concepts. You will need to be comfortable working with algebraic expressions and functions, solving for an unknown variable, factoring and simplifying algebraic expressions, and applying algebraic concepts to problem solving.

### Definitions and Basics

An **algebraic expression** is a collection of terms combined by addition, subtraction, or both in which letters or variables are used to represent numbers. **Terms** are made up of numbers or variables combined by multiplication or division. In order to achieve success on the exam, you will need to be able to apply the basic operations of arithmetic to algebraic expressions.

For example, like terms can be combined:

$$\begin{aligned}4x + 5x &= 9x \\5a + 6b + 2a - 3b &= 7a + 3b \\15x - 3y - (-6y) + 7x - 5 &= 22x + 3y - 5 \\(x - 5)(x + 2) &= x^2 - 3x - 10 \\ \frac{10ab}{2y} &= 5a\end{aligned}$$

### Definitions for algebraic exponents

$$\begin{aligned}x^3 &= x \cdot x \cdot x \\a^{-4} &= \frac{1}{a} \cdot \frac{1}{a} \cdot \frac{1}{a} \cdot \frac{1}{a} = \frac{1}{a^4} \\x^0 &= 1 \\x^{\frac{a}{b}} &= \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a \\y^{\frac{1}{2}} &= \sqrt{y}\end{aligned}$$

### Key concepts to remember:

- **Add exponents** when multiplying expressions with the same base:

$$\begin{aligned}x^3 \cdot x^4 &= (x \cdot x \cdot x)(x \cdot x \cdot x \cdot x) = x^7 \\x^a \cdot x^b &= x^{a+b}\end{aligned}$$

The same rule applies for negative exponents:

$$y^4 \cdot y^{-2} = (y \cdot y \cdot y \cdot y) \cdot \left(\frac{1}{y} \cdot \frac{1}{y}\right) = y^2$$

- **Subtract exponents** when dividing expressions with the same base:

$$\frac{x^5}{x^2} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x} = x^3$$

$$\frac{a^m}{a^n} = a^{m-n}$$

- **Multiply exponents** when a number raised to an exponent is raised to a second exponent:

$$(x^4)^3 = x^{4 \cdot 3} = x^{12}$$

$$(a^m)^n = a^{mn}$$

### Factoring

You are likely to see questions that ask you to evaluate or compare expressions that require factoring. The types of factoring included in the math section are

- Difference of two squares

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^2 - 16 = (x + 4)(x - 4)$$

$$c^2 - 100 = (c - 10)(c + 10)$$

- Finding common factors or grouping common terms

$$8a^2x^2 + 4a^3x = 4a^2x(2x + a)$$

$$3x + 6y = 3(x + 2y)$$

- Factoring quadratics and polynomials

$$x^2 - 7x + 12 = (x - 4)(x - 3)$$

$$\begin{aligned} -2x^2 - 6x - 4 &= -2(x^2 + 3x + 2) \\ &= -2(x + 1)(x + 2) \end{aligned}$$

$$x^2 + 2x + 1 = (x + 1)(x + 1) = (x + 1)^2$$

## Solving Equations

This is one of the most important components of the exam. Algebraic expressions will appear in many different forms, ranging from simple linear equations to complex word problems. Some questions may ask you to solve for a specific value of  $x$ , while others may ask you to solve for  $x$  in terms of (an) other variable(s).

To prepare you for the exam, we will first review solving basic linear equations, systems of linear equations and inequalities, and equations involving radical expressions.

### • Solving for One Variable in Terms of Another Variable

These types of questions are meant to test your understanding and use of the basic rules of arithmetic for algebraic expressions.

*Example 1: If  $2x + 3y = z$ , what is  $x$  in terms of  $y$  and  $z$ ?*

This question is asking you to isolate  $x$  by itself on one side of the equation.

First subtract  $3y$  from both sides.

$$2x = z - 3y$$

Then divide both sides by 2 to get the value of  $x$  in terms of  $y$  and  $z$ .

$$x = \frac{z - 3y}{2}$$

*Example 2: If  $2r = 5s$  and  $5s = 6t$ , what does  $r$  equal in terms of  $t$ ?*

Since you want to find  $r$  in terms of  $t$ , in the first equation find the value of  $r$ .

$$r = \frac{5}{2}s$$

Then find  $s$  in terms of  $t$ .

$$s = \frac{6}{5}t$$

Substitute the found value of  $s$  into the equation for  $r$ .

$$r = \frac{5}{2} \left( \frac{6}{5}t \right)$$

Thus,  $r = 3t$ .

### • Solving Quadratic Equations by Factoring

Don't be surprised if you are asked to solve quadratic equations that can be factored. You **will not** be asked to use the quadratic formula to solve these equations.

*Example 1: If  $x^2 + 12 = 7x$ , what are the two possible values of  $x$ ?*

First, subtract  $7x$  from both sides to get the standard quadratic equation.

$$\begin{aligned}x^2 - 7x + 12 &= 0 \\(x - 4)(x - 3) &= 0\end{aligned}$$

Therefore, either  $(x - 4) = 0$  or  $(x - 3) = 0$ . The values  $x = 4$  and  $x = 3$  satisfy the original equation.

*Example 2: How many different solutions are there for the equation*

$$2x + 6 = (x + 5)(x + 3)?$$

- A) 0
- B) 1
- C) 2
- D) 3

First multiply the binomials.

$$2x + 6 = x^2 + 8x + 15$$

Then reorganize the terms in order to get the standard quadratic equation.

$$\begin{aligned}x^2 + 6x + 9 &= 0 \\(x + 3)(x + 3) &= 0\end{aligned}$$

Solving for  $x$  gives a single solution of  $-3$ . Therefore, the answer to the question is B since there is only **one** solution,  $-3$ .

*Example 3: If  $\frac{x^2 + 7x + 12}{x + 4} = 5$ , for  $x \neq -4$ , then  $x = ?$*

- A) 1
- B) 2
- C) 3
- D) 5
- E) 6

First, factor the quadratic expression.

$$\begin{aligned}\frac{x^2 + 7x + 12}{x + 4} &= 5 \\ \frac{(x + 4)(x + 3)}{x + 4} &= 5\end{aligned}$$

Now we can cancel the  $(x + 4)$  terms to give  $x + 3 = 5$ . Thus,  $x = 2$ , and the answer is B.

## • Solving Algebraic Inequalities

### Definitions

An **inequality** is a statement that one quantity is greater than or less than another. Inequalities are shown using four symbols:

- Greater than:  $>$

$$4 > 1$$

$$1 > -3,$$

$$2 > \sqrt{2}$$

- Greater than or equal to:  $\geq$

$$\frac{8}{3} \geq 2$$

$$2 + 3 \geq 5$$

- Less than:  $<$

$$0 < 4$$

$$-6 < -5$$

- Less than or equal to:  $\leq$

$$6 - 2 \leq 5$$

$$\frac{5}{2} \leq 2.5$$

Solving inequalities works just like solving an equality, except for one important difference: **When you multiply or divide an inequality by a negative number, the inequality sign switches.**

*Example 1:* Solve the inequality  $2x - 5 > 9$ .

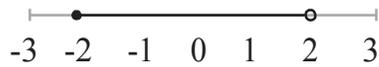
First, add 5 to both sides to give  $2x > 14$ . Then solve for  $x$ . The solution  $x > 7$  is the answer. In other words, “all values of  $x$  greater than 7” are solutions to this inequality.

*Example 2:* Solve the inequality  $3x + 5 < 5x - 9$ .

First, subtract 5 from both sides to give  $3x < 5x - 14$ . Then subtract  $5x$  from both sides to give  $-2x < -14$ . To solve for  $x$ , divide both sides by  $-2$  and switch the inequality sign to give  $x > 7$  as the solution.

You may also be given graphical representations of inequalities.

*Example 3:* Which of the following algebraic inequalities is represented by the graph below?



- A.  $-2 \geq y < 2$
- B.  $-2 < y > 2$
- C.  $-2 \leq y < 2$
- D.  $-2 < y \leq 2$

The graph represents a line whose solution  $y$  lies between  $-2$  and  $2$ . The filled point represents  $y = -2$ , and the open point represents  $y \neq 2$ . To answer the question, you must combine the two inequalities into one statement:  $-2 \leq y < 2$ . Therefore, the answer is C.

• **Solving Systems of Linear Equations**

You will be asked to solve systems of two or more linear equations or inequalities. Linear systems are equations that contain the same variables. Thus,  $a + 2b = 11$  and  $2a + b = 10$  are linear systems since they both contain the same variables,  $a$  and  $b$ .

To solve systems of linear equations, you should use the **substitution method**:

- Take one of the equations and find the value of one of the variables in terms of the other.
- Substitute the value found for the variable into the other equation.
- Solve for the second variable.
- Substitute that value into the original equation to solve for the first variable.

*Example:* For what values of  $x$  and  $y$  are the following equations true?

$$\begin{aligned} 2x + 4y &= 50 \\ 3x + 5y &= 66 \end{aligned}$$

In the first equation, solve for  $x$  in terms of  $y$ .

$$\begin{aligned} 2x + 4y &= 50 \\ 2x &= 50 - 4y \\ x &= 25 - 2y \end{aligned}$$

Then substitute for  $x$  in the other equation and solve for  $y$ .

$$\begin{aligned} 3x + 5y &= 66 \\ 3(25 - 2y) + 5y &= 66 \\ 75 - 6y + 5y &= 66 \\ 75 - y &= 66 \\ -y &= -9 \\ y &= 9 \end{aligned}$$

Substitute  $y = 9$  back into the original equation,  $2x + 4y = 50$ , in order to find the value of  $x$ .

$$2x + 4(9) = 50$$

$$2x + 36 = 50$$

$$2x = 14$$

$$x = 7$$

### • Solving Radical Expressions

The expression  $2\sqrt{x}$  is a radical expression because it involves a root – in this case, the square root of  $x$ .

The equation  $2\sqrt{x} + 7 = 19$  is radical because it involves a radical expression.

You can solve this equation in the same way you solved other linear equations:

$$2\sqrt{x} + 7 = 19$$

$$2\sqrt{x} = 12$$

$$\sqrt{x} = 6$$

$$x = 36$$

### • Solving Absolute Value Equations and Inequalities

Familiarity with both the concept and notation of absolute value will be helpful in solving math questions. The absolute value of a number is its distance from zero on the number line. The absolute value of the number  $x$  is denoted  $|x|$ . For example,  $|10| = 10$  and  $|-5| = 5$ . The absolute value of a number is never negative since it represents the value of the number regardless of charge.

You will be expected to work with expressions and solve equations that involve absolute value.

*Example 1:* Solve  $|2x - 3| > 5$ .

The first thing to do is clear the absolute value bars by splitting the inequality into two:

$$|2x - 3| > 5$$

$$2x - 3 < -5 \text{ or } 2x - 3 > 5$$

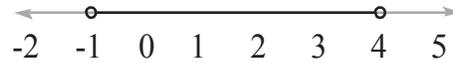
Then solve for  $x$  in each equation.

$$2x < -2 \text{ or } 2x > 8$$

$$x < -1 \text{ or } x > 4$$

Thus, the solution to  $[2x - 3] > 5$  consists of  $x < -1$  or  $x > 4$ .

On a graph, this would be represented as the number line shown below, with open points at  $-1$  and  $4$  and arrows pointing in opposite directions to show  $x < -1$  and  $x > 4$ .



### • Solving Rational Expressions

A **rational expression** is the quotient of two polynomials (written as a fraction). For example:

$$\frac{x - 1}{x - 2}$$

You will likely be asked to solve equations or inequalities involving such expressions.

*Example:* For what value of  $x$  is the following equation true?

$$\frac{4x}{2x-1} = 5$$

First, multiply both sides by  $2x - 1$  to put the equation in linear form.

$$\frac{4x}{2x - 1} (2x - 1) = 5(2x - 1)$$

$$4x = 10x - 5$$

Solve for  $x$ .

$$\begin{aligned} -6x &= -5 \\ x &= \frac{5}{6} \end{aligned}$$

### Functions

A function is a rule or formula that tells how to relate elements in one set (the domain) with the elements in another set (the range). There are many different types of function notations that you should be familiar with. Here are a few examples:

$$\begin{aligned} y &= x^2 \\ f(x) &= \sqrt{x - 3} \\ g(x) &= 2^x + \frac{1}{x} \end{aligned}$$

In each case, a specific value of  $x$  will give you a specific value of  $y$ ,  $f(x)$ , or  $g(x)$ .

*Example 1:* Given  $f(x) = x^2 + 2x - 1$ , find  $f(2)$ .

When you see a question like this, all you need to do is substitute the new  $x$  value into the function. In this case, substitute 2 for  $x$  in the function.

$$\begin{aligned} f(2) &= (2)^2 + 2(2) - 1 \\ &= 4 + 4 - 1 \\ &= 7 \end{aligned}$$

*Example 2:* Given  $f(x) = \begin{cases} 2x^2 - 1 & \text{for } x < 1 \\ x + 4 & \text{for } x \geq 1 \end{cases}$ , find  $f(0)$  and  $f(1)$ .

This is called a “piecewise” function. The function is split into two halves: the half that comes before  $x = 1$  and the half that goes from  $x = 1$  to infinity. Which half of the function that you use depends on the value of  $x$ . If we evaluate  $f(0)$ , we must use the first function, since  $0 < 1$ . Then  $f(0) = 2(0)^2 - 1 = -1$ . If we evaluate  $f(1)$ , we must use the second half of the function, since  $x \geq 1$ . Therefore,  $f(1) = (1) + 4 = 5$ .

*Example 3:* Given that  $f(x) = 3x^2 + 2x$ , find  $f(x + h)$ .

Everywhere there is an  $x$ , plug in  $x + h$ .

$$\begin{aligned} f(x) &= 3x^2 + 2x \\ f(x + h) &= 3(x + h)^2 + 2(x + h) \\ &= 3(x^2 + 2xh + h^2) + 2x + 2h \\ &= 3x^2 + 6xh + 3h^2 + 2x + 2h \end{aligned}$$

### Domain and Range

- The domain of a function is the set of all values for which the function is defined.
- The range of a function is the set of all values that are the output (result) of applying the function.

Example: What are the domain and range of  $f(x) = -\sqrt{-2x + 3}$  ?

In this example, the domain is all the values that  $x$  can be. The only rule of this function is that you cannot take the root of a negative number. Therefore, by definition,  $-2x + 3 \geq 0$ .

First, to find all acceptable values of  $x$ , solve for  $x$  in  $-2x + 3 \geq 0$ .

$$-2x + 3 \geq 0$$

$$-2x \geq -3$$

$$x \leq \frac{3}{2}$$

Then the domain is “all  $x \leq \frac{3}{2}$ .”

To find the range in this example, plug in the highest value for  $x$ , which is  $\frac{3}{2}$ . Thus,  $f(\frac{3}{2}) = 0$ . For any other value of  $x$ ,  $f(x)$  will be negative. Therefore, the range is  $f(x) \leq 0$ .

### Practice Problems for Manipulating Algebraic Expressions

1) If  $x = -3$ , then  $4x^2 - 3x - 7 = ?$

A)  $-34$

B)  $20$

C)  $34$

D)  $38$

E)  $52$

2) If  $3x + 5y = 6$  and  $2x + y = 4$ , what is  $x$ ?

A)  $2$

B)  $0$

C)  $-1$

D)  $4$

E)  $5$

3) Evaluate  $\sqrt{25a^2}$ .

A)  $5$

B)  $2a$

C)  $25$

D)  $5a$

E)  $5a^2$

4) Factor  $6x^2 - 12x + 6$  completely.

A)  $6(x - 1)(x + 2)$

B)  $-6(-x + 4)(x + 2)$

C)  $6(x - 1)(x - 1)$

D)  $3(2x - 1)(x - 2)$

E)  $2(3x + 2)(x + 1)$

5) What is a possible solution of  $[2x - 3] - 4 = 3$ ?

- A) 2
- B) -2
- C) -5
- D) 1
- E) 3

6) Solve the following equation:

$$\frac{x - 1}{15} = \frac{2}{5}$$

- A) 3
- B) 4
- C) 5
- D) 6
- E) 7

Note: Answers may be found on Page 105.

## How Things Change: Ratios, Proportions, and Variations

This section deals with ratios, proportions, and how a quantity changes with regard to another quantity (variation). The topics build upon each other. Proportions are merely statements of equality involving ratios, and variation problems will often be written as proportions. Whether you are aware of it or not, you use ratios every day. When you talk about the rate of travel of a vehicle being 45 mph, you are talking about the ratio of distance traveled to time traveled.

### Ratios and Proportions

Ratios are used to compare two quantities. They are typically written in one of two ways: as a fraction or with a colon (:). The ratio read as “4 to 10” could be written as  $\frac{4}{10}$  or 4:10. The ratio expressing the relationship between a day and a week would be written as  $\frac{1}{7}$  or 1:7 because there are seven days in a week.

A proportion is a mathematical statement equating two ratios. Two ratios are said to be equal if, when written in fractional form, the fractions are equivalent fractions. We will compare and solve ratios using *cross multiplication*. If the cross products are equal, the ratios are equal. Take a look at the example below.

$$\frac{x - 1}{2} = \frac{x + 2}{3}$$

Cross multiply and solve for x.

$$\begin{aligned}3(x - 1) &= 2(x + 2) \\3x - 3 &= 2x + 4 \\-2x \quad -2x & \\x - 3 &= 4 \\-3 \quad +3 & \\x &= 7\end{aligned}$$

The correct answer is  $x = 7$ . We can check our answer by plugging it back into the original proportion.

$$\begin{aligned}\frac{7 - 1}{2} &= \frac{7 + 2}{3} \\ \frac{6}{2} &= \frac{9}{3} \\ 3 &= 3\end{aligned}$$

The final statement,  $3=3$ , is a true statement. Therefore,  $x=7$  is the correct answer. If you noticed that  $6/2$  and  $9/3$  are equivalent fractions, you could have stopped at this point. Two ratios are equal if their fractional forms are equivalent fractions. Remember, you may have to rewrite the proportion in fractional form before you solve it.

$$\text{Solve for } x : x : 6 = 4 : 12$$

Begin by rewriting the problem in fractional form; then solve for x as we did above.

$$\begin{aligned}\frac{x}{6} &= \frac{4}{12} \\12x &= 6 \cdot 4 \\12x &= 24 \\/12 \quad /12 & \\x &= 2\end{aligned}$$

You probably could have solved this problem by observation. Recognizing that  $4/12$  is simply  $1/3$ , we know that the left-hand side must be equal to  $1/3$  as well. One-third of 6 is 2. You may find that simple observations such as this will provide significant time savings on the exam.

### Variations

Variations deal with explaining, in mathematical language, how one quantity changes with respect to one or more other quantities. The amount of flour needed in a bread recipe varies with the

number of loaves being made. The time it takes to paint a house varies with the number of people doing the work. The first is an example of *direct variation*; the second, of *inverse variation*.

In the example of direct variation given above, the amount of flour needed for the recipe will vary directly with the number of loaves being made. Making more loaves will require more flour; making fewer loaves will require less flour. In direct variation, the variables will move in the same direction. As one variable increases or decreases, the other variable will move in the same direction. These problems can be solved using proportions. Consider the example below:

If it takes 3 gallons of paint to cover 100 ft<sup>2</sup>,  
how many gallons of paint will be needed to cover 600 ft<sup>2</sup>?

The problem gives us the ratio of 3 gallons of paint to every 100 ft<sup>2</sup>. We can use this to set up a proportion problem.

$$\begin{aligned}\frac{3}{100} &= \frac{x}{600} \\ 100x &= 3 \cdot 600 \\ 100x &= 1800 \\ \frac{100x}{100} &= \frac{1800}{100} \\ x &= 18\end{aligned}$$

It will take 18 gallons of paint to cover the 600 ft<sup>2</sup>. The expression “y varies directly as x” could also be written:

$$y = kx$$

where k is called the *constant of variation*. In the previous problem, the number of gallons of paint varies directly with the square footage of wall that will be covered. The constant of variation is 3/100. We would then have

$$\begin{aligned}y &= \frac{3}{100}x \\ &= \frac{3}{100}600 \\ &= 18\end{aligned}$$

When two variables or quantities change in opposite directions, this is an example of *inverse variation*. In the example above about painting a house, the time required to paint the house *varies inversely with* the number of people painting. This means that the more people painting the house, the less total time it will take to paint. When we say “y varies inversely with x”, we can express this as

$$y = \frac{k}{x}$$

Again, **k is the constant of variation**. We can find k by simply multiplying the known values for the two variables. Consider the example below.

A particular hotel has a custodial staff of 12 employees, and they can typically clean all of the hotel rooms in 6 hours. If four members of the custodial staff are not at work today, how long will it take the remaining custodians to clean all of the hotel rooms?

In the example above, the total time taken to complete the job is inversely proportional to the number of workers. The constant of variation k is simply  $6 \cdot 12 = 72$ . We can thus solve the problem as follows:

$$\begin{aligned}y &= \frac{72}{x} \\ &= \frac{72}{8} \\ &= 9\end{aligned}$$

The correct answer is 9 hours. I hope you recognized that the number of workers present is 8. There are four custodians absent, so  $x = 12 - 4 = 8$ . Be sure to read the information provided in the problems carefully.

Two additional types of variation problems that we will discuss are *joint variation* and *combined variation*. These are just extensions of direct and inverse variation. When we say that “y varies jointly with x and z,” we have an example of *joint variation*. This can be written as

$$y = kxz ,$$

where k is again the constant of variation. The constant of variation is simply the ratio of y to x and z (x times z). Consider the problem below.

The variable y varies jointly with x and z.  
The value of y is 12 when x is 4 and z is 8.  
What is y when x = 6 and z = 10?

First, we want to be sure that we recognize the type of variation that is present. It is pretty simple for this problem: “y varies jointly with x and z.” We thus have the form:

$$y = kxy,$$

and we simply need to determine the value for the k.

$$\begin{aligned}k &= \frac{y}{xy} \\ &= \frac{12}{4 \cdot 8} \\ &= \frac{3}{8}\end{aligned}$$

Now, we simply plug this into the equation of variation and solve for our unknown.

$$\begin{aligned}k &= \frac{3}{8} xy \\ &= \frac{3}{8} \cdot 6 \cdot 10 \\ &= \frac{45}{2} \\ &= 22 \frac{1}{2}\end{aligned}$$

The correct answer is  $22 \frac{1}{2}$ .

*Combined variation* involves both direct and inverse variation. For example, if we say “y varies directly with x and inversely with z,” this is an example of combined variation. This can be written, similarly to the joint variations previously presented, as:

$$y = k \frac{x}{z}$$

Consider the following example.

It takes 2 hours for 3 people to paint 100 ft. of 6-ft. fencing.  
Assuming that each person is capable of painting at the same rate,  
how long will it take for 12 people to paint 1,800 ft. of the same fencing?

The total time (t) needed to paint a section of fencing is directly proportional to the length of the fence (l) and inversely proportional to the number of people (n) who are painting.

$$t = k \frac{l}{n}$$

We simply need to determine the constant of variation,  $k$ .

$$\begin{aligned}k &= \frac{tn}{1} \\ &= \frac{2 \cdot 3}{100} \\ &= \frac{6}{100}\end{aligned}$$

Now that we have  $k$ , we need to insert it into the equation of variation to get the answer.

$$\begin{aligned}t &= \frac{6}{100} \cdot \frac{1}{n} \\ &= \frac{6}{100} \cdot \frac{1800}{12} \\ &= 9\end{aligned}$$

The correct answer is 9 hours.

### **Ratios, Proportions, and Variation Practice Problems**

1) Which of the following ratios represent the relationship of hours to days?

- I. 24/1      IV. 1/24
- II. 1:24     V. 24:1
- III. 1 to 24

- A) I, III, and V      D) I and V
- B) II, III, and IV    E) II and IV
- C) II, III, and V

2) There are 14 boys and 16 girls in Tyler's class. Which ratio best represents the relationship between the number of boys and the number of students in Tyler's class?

- A) 7/8                      D) 14:16
- B) 7:10                    E) 8:15
- C) 7/15

3) The ratio of  $f(x)$  to  $g(x)$  is  $3:x + 2$ . If  $f(x) = x - 2$ , what is  $g(x)$  in terms of  $x$ ?

- A)  $3x - 6$                       D)  $3x^2 - 12$   
B)  $\frac{1}{3}(x^2 - 2)$                 E)  $\frac{1}{3}(x^2 - 4)$   
C)  $\frac{2}{3}x$

4) If the ratio of A to B is 3:4 and the ratio of B to C is 2:3, what is the value of A when C is 5?

- A) 10                                D) 5  
B)  $\frac{3}{2}$                                 E) 6  
C)  $\frac{5}{2}$

5) If  $y$  varies directly with  $x$  and  $y = 4$ , what is the value of  $y$  when  $x = \frac{7}{11}$ ?

- A)  $\frac{4}{11}$                                 D)  $\frac{44}{49}$   
B)  $2\frac{3}{4}$                                 E)  $1\frac{5}{49}$   
C)  $\frac{11}{18}$

6) If  $x$  varies inversely with  $y$  and  $y = 8$  when  $x = 3$ , what is the value of  $x$  when  $y = 12$ ?

- A)  $\frac{9}{4}$                                 D) 2  
B)  $\frac{1}{2}$                                 E)  $\frac{4}{9}$   
C)  $4\frac{1}{2}$

7) The variable  $s$  varies jointly with  $t$  and  $u$  such that  $s = 7$  when  $t = 4$  and  $u = 3$ . What is the constant of variation describing the relationship between  $s$  on the one hand and  $t$  and  $u$  on the other?

- A) 84                                D)  $\frac{21}{4}$   
B)  $\frac{12}{7}$                                 E)  $\frac{7}{12}$   
C)  $\frac{4}{21}$

8) The value of  $w$  varies directly with  $x$  and  $y$  and inversely with  $z$ . The constant of variation is  $\frac{13}{7}$ . What is the value of  $z$  when  $w = 13$ , and  $y = 3$ ?

- A)  $\frac{6}{7}$                                 D)  $\frac{7}{6}$   
B)  $\frac{7}{13}$                                 E)  $\frac{13}{7}$   
C)  $\frac{6}{13}$

Note: Answers may be found on Page 105.

## Playing the odds: Probability and Statistics

Some questions you are sure to encounter on the *ACT* math section will invoke your ability to calculate the likelihood (probability) of an event occurring and your ability to work with basic statistical measures (mean, median, and mode). We'll review here a few of the most important concepts that you will need to be successful on these questions.

### Probability

When answering these problems, keep in mind that the probability can be any number from 0 to 1 inclusively. If an event is not possible (cannot ever occur), the probability is 0. If an event is certain (will always occur), the probability is 1. You will never have a probability that is less than 0 or greater than 1. The probability formula can be used to calculate the probability of an event occurring.

$$\text{Probability} = \frac{\text{Number of times an event can occur}}{\text{Total number of possible outcomes}}$$

The probability is simply the ratio of the number of successful events divided by the total number of possible events. In a deck of 52 playing cards, the probability of pulling one of the 16 face cards is  $16/52$  or  $4/13$ . In contrast, the probability of drawing one of the four jacks is  $4/52$  or  $1/13$ .

### Dependent and Independent Events

Calculating the probability of a single event is typically pretty simple. However, you may be asked to calculate the probability that two (or possibly more) events will both occur. In order to answer such a question, you will need to understand the difference between *dependent* and *independent* events. Events are considered *dependent* if the outcome of one event affects the probability of the other event. To calculate the probability of dependent events, we have to use something known as *conditional probability*. Conditional probability is the probability that an event will occur given that one or more other events have occurred. Take the generic example of three dependent events (A,B, and C). The probability that all three events will occur is given by the following expression

$$P(A,B,C) = P(A) \cdot P(B|A) \cdot P(C|A,B).$$

Don't let this expression scare you.  $P(A)$  is simply the probability that event A will occur.  $P(B|A)$  is the probability that event B will occur given that event A has occurred. Likewise,  $P(C|A,B)$  is the probability that event C will occur if events A and B have occurred. The probability that all three events will occur,  $P(A,B,C)$ , is simply the product of these three terms. This can be generalized to include more than three events. This will make more sense to you with an example.

This weekend is the first round of the regional middle school soccer tournament. On Saturday morning, the team from Madris will play against the team from Dekalb. In the afternoon, Madris will play against the team from Fulton. On Sunday afternoon,

DeKalb will play against Fulton. In each game, either team has a 50 percent chance of winning the game. What is the probability that Madris will win both of its games and Fulton will lose both of its games?

To solve this problem, we will have to use conditional probabilities. The probability that Madris will win its first game is  $\frac{1}{2}$ . The probability that Madris will win its second game (against Fulton) is *independent* of the outcome of its first game. So, the conditional probability that Madris wins its second game given that it won its first game is still  $\frac{1}{2}$ . The probability that Fulton will lose its first game is  $\frac{1}{2}$ , but the conditional probability that Fulton will lose its first game given that Madris won both of their games is actually 1. Madris plays against Fulton in this game, so if we are given that Madris won this game then we know that Fulton lost this game. The probability that Fulton loses its second game is  $\frac{1}{2}$ . Notice that this is also conditional probability. The likelihood that Fulton will lose its second game is independent of whether Madris won its games and independent of whether Fulton lost its first game. The probability that all four events occur (Madris wins its games and Fulton loses its games) is just the product of these probabilities.

$$P(A,B,C,D) = P(A) \cdot P(B/A) \cdot P(C/A,B) \cdot P(D/ A,B,C)$$

$$= \frac{1}{2} \cdot \frac{1}{2} \cdot 1 \cdot \frac{1}{2} = \frac{1}{8}$$

The correct answer is  $\frac{1}{8}$ . This also clarifies another concept, that of *independent events*. Take the first two events in the above problem for example. Madris, winning its first game and Madris, winning its second game are independent events. Events are considered independent if the outcome of one event has no effect on the likelihood of the other. The probability of two independent events is simply the product of the respective probabilities for these events. The probability that Madris will win both of its games is  $\frac{1}{2} \cdot \frac{1}{2}$ , or  $\frac{1}{4}$ .

### Statistics

There won't be too many problems that cover statistics on the test, but you can be sure there will be at least one. It is highly unlikely that any will cover topics beyond the three most basic statistical measures; mean (arithmetic average), median, and mode. The mean of a set of  $n$  numbers is just the sum of the numbers divided by  $n$  (the number of items in the set). To find the mean or average of 15, 21, and 39, you would do the following:

$$\frac{15 + 21 + 39}{3} = \frac{75}{3} = 25$$

You may have to rely on your understanding of the properties of averages to arrive at an answer on the test when directly calculating the averages is difficult or time consuming. The average of two numbers is halfway between the two numbers on a number line. The average of 3 and 5 is 4. When you have a set of numbers evenly spaced, the average will simply be the middle number. The average of the first nine odd numbers will simply be the 5th odd number, or 9. When there

is an even number of terms in the set, the average is just the average of the two middle numbers. The average of the first 100 odd numbers is just the average of the 50th (99) and 51st (101) odd numbers. So, the average is 100.

Two concepts that often appear on the test deal with weighted averages and finding a missing number when an average is given. The average of a set of numbers is just the sum of the numbers divided by the number of terms in the set.

$$\text{Average} = \frac{\text{Sum of terms}}{\# \text{ of terms}}$$

Likewise, we can use the average and the number of terms to calculate the sum of the terms.

$$\text{Sum of terms} = (\text{Average}) \times (\# \text{ of terms})$$

This principle is central to both calculating a missing term and to the concept of a weighted average. If we are given the value for three of the four terms (17, 35, 39) and the average (34) we can calculate the value of the missing term.

$$\begin{aligned} \text{Sum of terms} &= 17 + 35 + 39 + x = 34 \times 4 = 136 \\ 91 + x &= 136 \\ x &= 45 \end{aligned}$$

If we are given that (in a class of 16 girls and 13 boys) the boys averaged 75 on the last exam and the girls averaged 83, we can calculate the class average using a weighted average. Note that the average is not just the average of the boys' and girls' test averages.

$$\begin{aligned} \text{Class Average} &= \frac{\text{Sum of Boy's Scores} + \text{Sum of Girl's Scores}}{\text{total \# of students}} \\ &= \frac{(\text{Boy's Average}) \times (\# \text{ of Boys}) + (\text{Girl's Average}) \times (\# \text{ of Girls})}{\# \text{ of Boys} + \# \text{ of Girls}} \\ &= \frac{75 \times 13 + 83 \times 16}{13 + 16} = \frac{975 + 1328}{29} = \frac{2303}{29} = 79.4 \end{aligned}$$

The average is slightly closer to 83 than it is to 75 because there are more girls in the class than boys.

## Probability and Statistics Practice Problems

1) If a pair of fair dice is tossed, what is the probability that the sum of the two numbers is 4?

- A)  $1/6$                       D)  $1/18$   
B)  $1/12$                      E)  $1/9$   
C)  $1/2$

2) If a pair of fair dice is tossed, what is the probability that the sum of the two numbers is an even number less than 10?

- A)  $1/3$                         D)  $7/18$   
B)  $1/2$                         E)  $2/3$   
C)  $4/11$

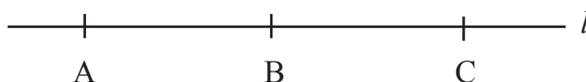
Note: Answers may be found on Page 105.

## Measuring Up: Geometry

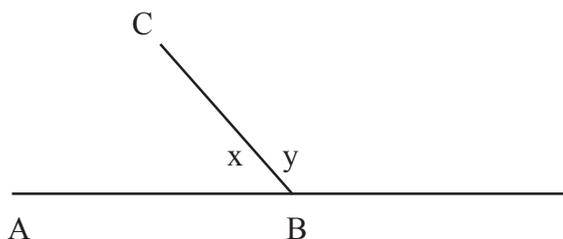
This section deals with the relationships between points, lines, and angles and the figures they form. The *ACT* has many questions involving the topics in this section. After algebra, this is the next most important section on the exam. Understanding the concepts will enable you to answer 30-40% of the questions that you will see on the exam.

### Definitions of Lines and Angles

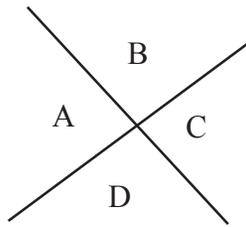
A line extends forever in either direction. The line below, called  $l$ , has three collinear points on it: A, B, and C. The part of the line between points A and B is called a line segment. In this instance, the line segment between points A and B will be referred to as either “segment AB” or simply  $\overline{AB}$ . A and B are the endpoints of segment AB.



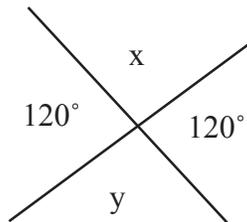
Angles are most often described by the points on the lines that intersect to form the angle or by the point of intersection itself. For instance, in the diagram below, angle  $ABC$  could be described as either  $\angle ABC$  or  $\angle x$ .



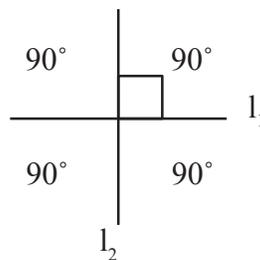
**A line forms an angle of  $180^\circ$ .** If that line is cut by another line, it divides the  $180^\circ$  into two pieces that together add up to  $180^\circ$  ( $\angle x + \angle y = 180^\circ$ ). Two angles that when added together make  $180^\circ$ , are called supplementary angles ( $\angle x$  and  $\angle y$  are **supplementary** to each other).



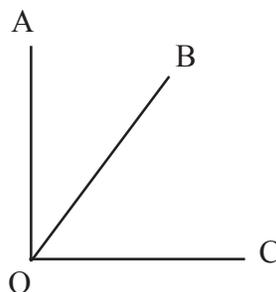
When two lines intersect, they form four angles, represented above by the letters A, B, C, and D.  $\angle A$  and  $\angle B$  together form a straight line, so they add up to  $180^\circ$ . The same is true for  $\angle C$  and  $\angle D$ ,  $\angle A$  and  $\angle D$ , and  $\angle B$  and  $\angle C$ . Since there are  $180^\circ$  above the line ( $\angle A + \angle B$ ), there are also  $180^\circ$  below the line ( $\angle C + \angle D$ ). Therefore, the sum of the four angles is  $360^\circ$ .  $\angle A$  and  $\angle C$  are opposite from each other and always equal to each other, as do  $\angle B$  and  $\angle D$ . These are known as **vertical angles**. In the figure below,  $\angle x = \angle y = 60^\circ$ .



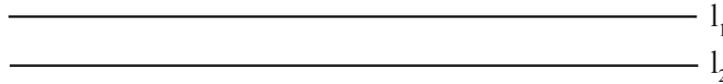
When two lines meet so that  $90^\circ$  angles are formed, the lines are said to be **perpendicular** to each other. The  $90^\circ$  angle is called a **right angle**. A right angle is represented by a little box at the point of intersection of the two lines. Perpendicular lines are represented as  $l_1 \perp l_2$ .



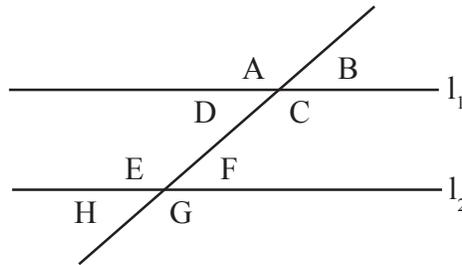
Two angles whose sum is  $90^\circ$ , or one right angle, are said to be **complementary**. For instance, in the figure below,  $\angle AOB$  is the complement of  $\angle BOC$ . Thus,  $\angle AOB + \angle BOC = 90^\circ$ .



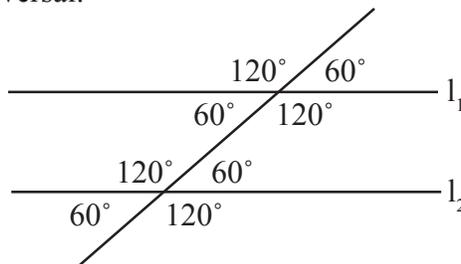
Two lines in the same plane that are equally distant from one another at all points are called parallel lines. Parallel lines never meet. Parallel lines are often represented as  $l_1 // l_2$ .



When parallel lines are cut by a third line (known as a transversal), eight angles are formed.

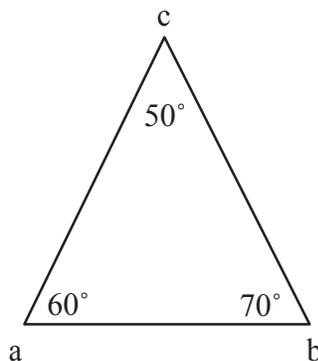


Based on what you've learned about parallel lines,  $\angle A = \angle C$  and  $\angle B = \angle D$ . Since the same transversal cuts line  $l_2$ , the four angles  $\angle E$ ,  $\angle F$ ,  $\angle G$ , and  $\angle H$  are in the same proportions as the angles above. Thus,  $\angle E = \angle A$  and  $\angle F = \angle B$ . The next figure shows an example of the relationship of parallel lines cut by a transversal.



### Triangles

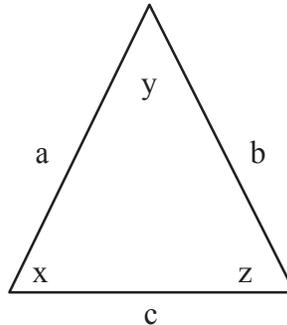
A triangle is a three-sided figure whose angles always add up to  $180^\circ$ . The largest angle of a triangle is opposite its longest side. On the exam, the triangle below will be represented as either "triangle abc" or  $\Delta abc$ .



On the exam, you will be expected to recognize specific types of triangles and understand the particular properties of each kind. Each kind is listed below with an explanation of the properties that you are expected to know.

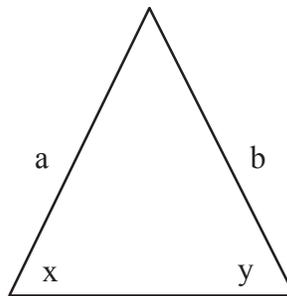
• **Equilateral Triangles**

The three sides of an equilateral triangle are equal in length ( $a = b = c$ ). The three angles are also equal, and therefore, since the three angles of a triangle must add up to  $180^\circ$ , each angle of an equilateral triangle must equal  $60^\circ$  ( $x = y = z = 60^\circ$ ).



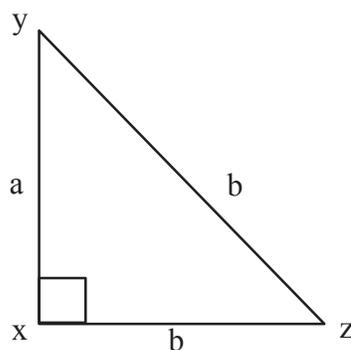
• **Isosceles Triangles**

An isosceles triangle is a triangle with two sides of equal length ( $a = b$ ). The angles opposite the equal sides are also equal ( $x = y$ ).



• **Right Triangles**

A right triangle is a triangle with a right angle. The other angles are, by definition, complementary angles ( $\angle Y + \angle Z = 90^\circ$ ). The longest side of a right triangle (the one opposite the  $90^\circ$  angle) is called the **hypotenuse** (YZ or side c is the hypotenuse in the figure below). The other sides are often referred to as **legs**.



Of all the triangles on the exam, right triangles appear the most often. Much information can be obtained from figures that contain right triangles due to the fact that their sides always exist in a particular proportion to each other. This proportion is better known as the **Pythagorean theorem**. (Note: Understanding this theorem is central to being successful on the exam.)

\* The Pythagorean theorem is expressed in the following equation:

$$a^2 + b^2 = c^2,$$

where  $c^2$  is always the square of the hypotenuse (the side opposite the right angle) and  $a^2 + b^2$  is the sum of the squares of the other sides.

If you know the lengths of any two sides of a right triangle, you can use this equation to find the length of the last side.

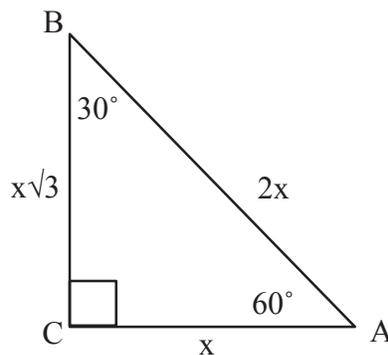
**Remember:** 1) the hypotenuse will always be the longest side, and 2) this works only for right triangles. In other words, if the triangle does not have a right angle, you **cannot** use the Pythagorean theorem.

There are several “special” right triangles that frequently appear on the exam:

### 30°-60°-90° Triangles

The lengths of a 30°-60°-90° triangle are in the ratio of 1:√3:2, as shown in the figure below.

Short leg =  $x$   
 Long leg =  $x\sqrt{3}$   
 Hypotenuse =  $2x$

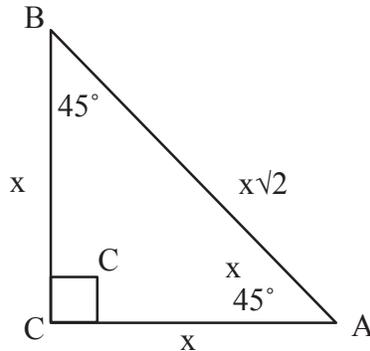


For example, if you know the length of the hypotenuse is 4 and the length of the short leg is 2, then the Pythagorean theorem gives you the length of the longer leg:

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c &= 4, a = 2 \\ 4^2 &= 2^2 + b^2 \\ 16 &= 4 + b^2 \\ 12 &= b^2 \\ 2\sqrt{3} &= b \end{aligned}$$

## 45°-45°-90° Triangles

A 45°-45°-90° triangle is an isosceles triangle containing a right angle. Thus, the two legs of the triangle (opposite the 45° angles) are equal. The sides of the triangle are in the ratio of  $1:1:\sqrt{2}$ , as shown in the figure below.



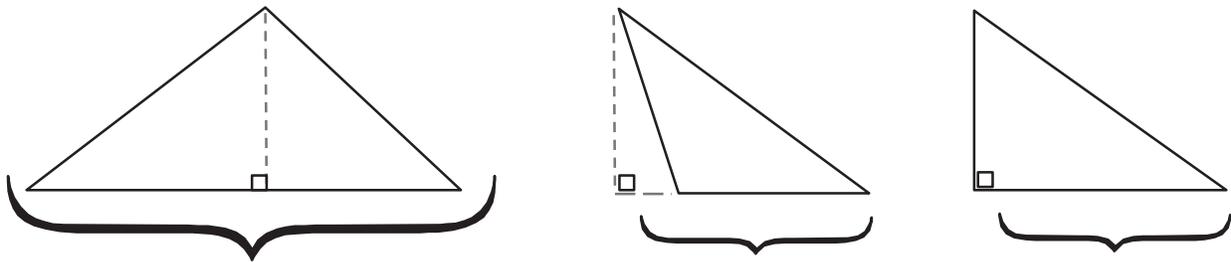
### • Area of a Triangle

The **area of a triangle** is defined as half the base (b) of the triangle times the height (h) of the triangle, which is represented by the formula below:

$$\text{area}\Delta = \frac{1}{2}bh$$

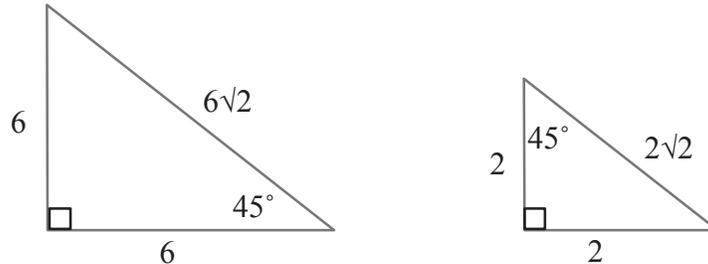
Height is the perpendicular distance from the base of the triangle to its highest point.

In the first two triangles below, the height is represented by a dashed line. The height of the triangle can be outside the triangle itself, as in the second example below. In the right triangle, the base and height are just the legs of the triangle.



## • Similar and Congruent Triangles

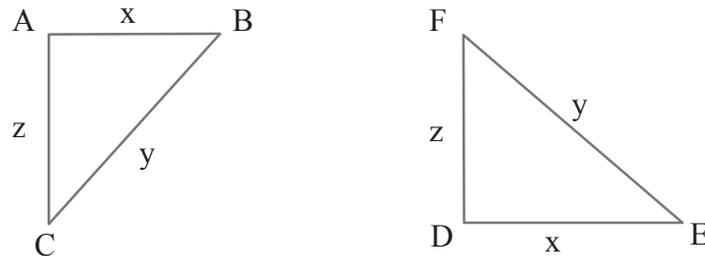
Two triangles are **similar** if their angles have the same degree measures. The triangles therefore have the same shape, and their sides will be in proportion. For example, the two triangles below are similar. Both are  $45^\circ$ - $45^\circ$ - $90^\circ$  right triangles, therefore; each side of one triangle is in proportion to the corresponding side of the other triangle. Here, the first triangle is three times as large as the second triangle since the sides are in a ratio of 3:1.



Two triangles are similar if:

- 1) Two pairs of corresponding angles have the same measure.
- 2) One pair of corresponding angles has the same measure, and the pairs of corresponding sides that form those angles have lengths that are in the same ratio.

**Congruent** triangles are triangles that have the same size and shape.



Each side of  $\triangle ABC$  has the same length as the corresponding side of  $\triangle DEF$ .

$$AB = DE = x$$

$$BC = EF = y$$

$$CA = FD = z$$

Each angle of  $\triangle ABC$  is equal to its corresponding angle in  $\triangle DEF$ .

Here's a quick checklist to determine if two triangles are congruent.

- 1) Each pair of corresponding sides has the same length.
- 2) Two pairs of corresponding sides each have the same length, and the angles formed by these sides have the same measure.
- 3) One pair of corresponding sides has the same length, and two pairs of corresponding angles each have the same measure.

**Note:** If any of these three details is true, then you're dealing with two congruent triangles.

## Quadrilaterals and Other Polygons

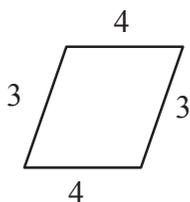
A **quadrilateral** is a four-sided polygon whose angle sum to  $360^\circ$ . There are several types of quadrilaterals that you will see on the exam. You will be expected to utilize the information that these figures offer in order to solve many types of questions.

The **perimeter** of any polygon is the sum of the lengths of its sides.

The most common four-sided figures are the rectangle and the square, followed by the parallelogram and the trapezoid.

### • Parallelograms

A parallelogram is a four-sided figure in which both pairs of opposite sides are parallel, both pairs of opposite angles are equal, and both pairs of opposite sides are equal in length.

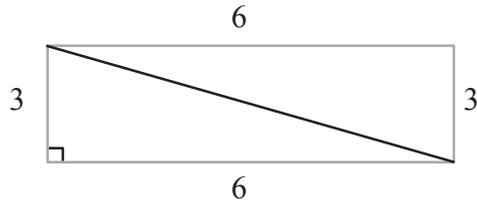


The area of a parallelogram is its base times its height, but due to its shape, the height of a parallelogram is not always equal to one of its sides. To find the height of a parallelogram, you must draw a perpendicular line from the base to the top of the figure.

$$\text{Area of a parallelogram} = \text{base} \times \text{height}$$

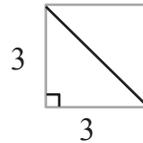
## • Rectangles and Squares

A rectangle is a parallelogram that has four interior angles that are each equal to  $90^\circ$ . Opposite sides of a rectangle are equal. The diagonal of a rectangle makes two equal right triangles. The Pythagorean theorem can be used to figure out details regarding the length of sides or the diagonal depending on what information you're given. The area of a rectangle is its length times its width. In the figure below, the area =  $6 \times 3$ , or 18.



*Area of a rectangle = length  $\times$  width*

A square is a type of rectangle whose four sides are equal in length. It is important to notice that the diagonal of a square makes two  $45^\circ - 45^\circ - 90^\circ$  triangles with the sides of the square. Thus, you can figure out the length of the sides from the length of the diagonal or the length of the diagonal from the length of a side.

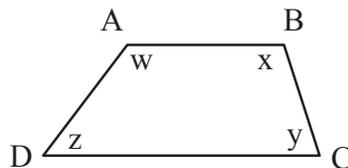


Since a square is a rectangle, the area of a square is also length times width. However, since each side  $s$  is the same, the area of a square can be represented as:

$$\text{Area of a square} = s^2$$

## • Trapezoids

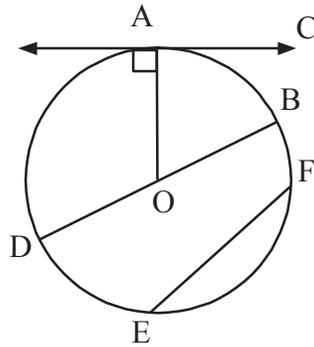
A trapezoid is a quadrilateral having one pair of parallel sides. Since  $AB \parallel DC$ ,  $x + y = 180^\circ$  and  $w + z = 180^\circ$ .



The easiest way to find the area of a trapezoid is to divide it into two triangles and a rectangle, figure out the areas of the individual pieces, and then add the results together to find the area of the whole figure.

# Circles

A circle represents all the points at a distance  $r$  from its center point. The measure of degrees around the point  $O$  is  $360^\circ$ .



## • Definitions

A **chord** is a line connecting two points on a circle ( $\overline{BD}$  and  $\overline{EF}$  are chords). The **diameter** of a circle is a line segment that passes through the center and has its endpoints on the circle, as in  $\overline{DOB}$ . All diameters of the same circle have equal length.

The **radius** of a circle is a line segment extending from the center of the circle to a point on the circle. All radii of the same circle have equal length ( $OA$  and  $OB$  are both radii). The radius is also half the diameter.

An **arc** is a part of a circle. An arc can be measured in degrees or in units of length. ( $AB$  is an arc.) If you form an angle by drawing radii from the ends of the arc to the center of the circle, the number of degrees in the arc equals the number of degrees in the angle formed.

A **tangent** to a circle is a line that touches the circle at only one point. A tangent is always perpendicular to the radius that contains the point of the line that touches the circle.  $AC$  is a tangent to circle  $O$ .

The **circumference** is the measure of the distance around the circle. It is equal to  $\pi$  times the diameter  $d$  (or  $\pi$  times twice the radius  $r$ ).

$$\text{Circumference} = \pi d$$

or

$$\text{Circumference} = 2\pi r$$

The area of a circle is equal to  $\pi$  times the square of the radius.

$$\text{Area of a circle} = \pi r^2$$

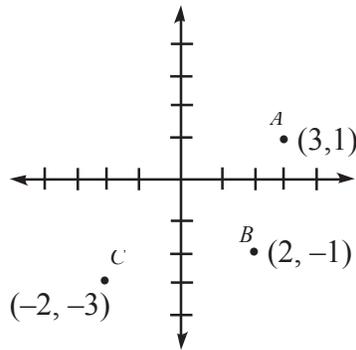
## Graphing and Coordinate Geometry

You will be expected to answer questions involving both linear and quadratic equations and their graphs. Therefore, you will need to understand the basics of the coordinate plane.

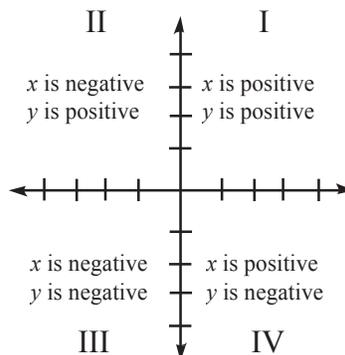
### • The Cartesian Grid

Every real point  $(x,y)$  has a place on this grid. For instance, the point A  $(3,1)$  can be found by counting over on the  $x$ -axis three places to the right of the origin  $(0,0)$  and then counting on the  $y$ -axis one place up from the origin.

**Note:** Always remember: moving up on a grid means  $x$  is becoming more positive, while moving left on a grid means  $y$  is becoming more positive. For instance, when moving from point C to point B, you must go from  $-2$  to  $2$  on the  $x$ -axis and from  $-3$  to  $-1$  on the  $y$ -axis.



The Cartesian grid can be broken up into four quadrants with respect to the  $x$ -axis and  $y$ -axis. The signs of  $x$  and  $y$  change depending on the quadrant in which the point lies. The quadrants are labeled from I to IV in a counterclockwise direction. In Quadrant I (top right), both  $x$  and  $y$  are positive. In Quadrant II (top left),  $y$  is positive whereas  $x$  is negative. In Quadrant III (bottom left), the values of both  $x$  and  $y$  are negative. Finally, in Quadrant IV (bottom right),  $x$  is positive while  $y$  is negative. The importance of this concept will become more apparent later when reflecting points or lines are discussed, since the signs in Quadrants I and III are exact opposites (the same is true for Quadrants II and IV).



### Finding the midpoint between two points

You will be expected to find the midpoint of line segments in the coordinate plane. The midpoint  $(x_m, y_m)$  is the average of the  $x$ 's and average of the  $y$ 's:

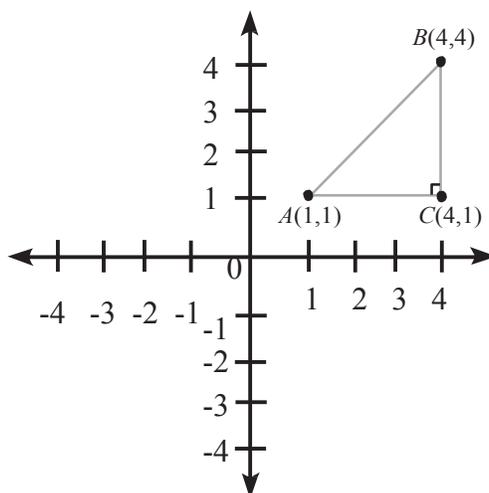
$$x_m = \frac{x_1 + x_2}{2} \quad y_m = \frac{y_1 + y_2}{2}$$

For instance, if  $\overline{AB}$  has endpoints A (4,5) and B (2,3), then the midpoint of  $\overline{AB}$  has the coordinates

$$\left(\frac{4+2}{2}, \frac{5+3}{2}\right) = (3, 4)$$

### Finding the distance between two points

The Pythagorean theorem can be used to find the distance between any two points in the coordinate plane. Take the figure below.



Points A, B, and C form a right triangle,  $\triangle ABC$ , where  $AC = 3$  and  $CB = 3$  (thus,  $\triangle ABC$  is an isosceles right triangle). Applying the Pythagorean theorem:

$$AB^2 = 3^2 + 3^2$$

$$AB^2 = 18$$

$$AB = \sqrt{18} = 3\sqrt{2}$$

Therefore, the distance between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  in the coordinate plane can be summarized by the formula,

$$\text{Distance between two points} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

• **Equations of lines (y-intercept form)**

The equation of a line can be found by using the formula

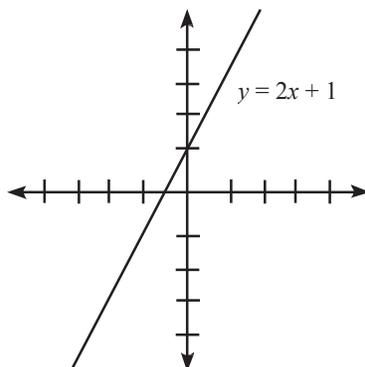
$$y = mx + b,$$

where  $x$  and  $y$  are represented by the point  $(x, y)$ ,  $m$  is the **slope** of the line (how sharply a line is inclining or declining), and  $b$  is the **y-intercept** (the point where a line crosses the y-axis).

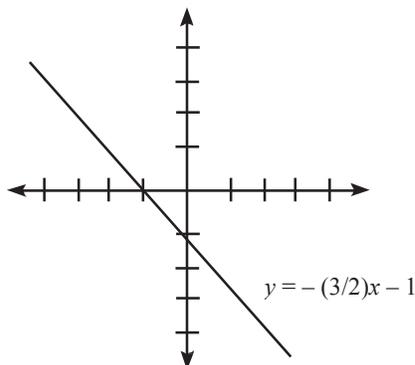
The **slope of a line** between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is defined as follows:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_1 - y_2}{x_1 - x_2}$$

Below is the graph of the line  $y = 2x + 1$ . The line crosses the y-axis at a point 1 above the origin ( $y = 1$ ). The slope is 2. An easy way to think of the slope is as the fraction  $\frac{2}{1}$ , where 2 represents the direction of movement on the y-axis and 1 represents the direction of movement on the x-axis. Thus, from  $y = 1$  on the y-axis, you would move up 2 and right 1.



If the equation of a line is  $y = -\frac{3}{2}x - 1$ , you would start by moving 1 down from the origin to  $y = -1$ . If you interpret the fraction as  $-\frac{3}{2}$ , then you would move 3 down and 2 to the right to give you the graph shown below.



## Key relationships

A **positive slope** represents an inclining line (y values increase from left to right).

A **negative slope** represents a declining line (y values decrease from left to right).

Parallel lines have **equal** slopes. For instance, the lines  $y = 3x + 1$  and  $y = 3x - 2$  are parallel lines because both lines have a slope of 3.

Two lines are **perpendicular** when the product of their slopes equals  $-1$ . The lines  $y = 2x + 1$  and  $y = -\frac{1}{2}x - 3$  are perpendicular lines because the product of their slopes equals  $-1$  ( $2 \cdot (-\frac{1}{2}) = -1$ ).

## • Graphs of Quadratic Functions

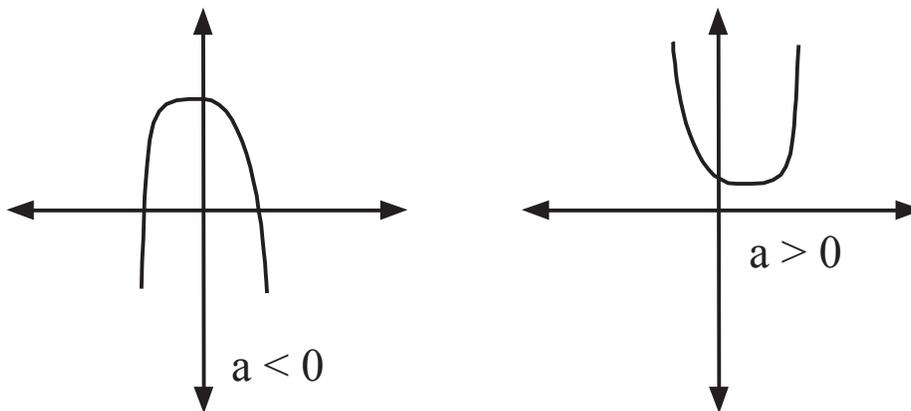
You will be expected to be able to identify some of the features of the graph of a quadratic equation, such as its highest or lowest point, its solutions and its direction.

The equation of a quadratic function is expressed as:

$$y = ax^2 + bx + c,$$

where a, b, and c are all constants and  $a \neq 0$ .

The graph of a quadratic function is called a **parabola**. A parabola is a U-shaped curve that can open upward or downward depending on the sign of a. If  $a > 0$ , then the graph will open upward. If  $a < 0$ , then the graph will open downward.



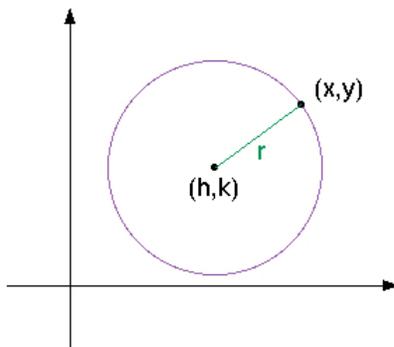
## • Graphs of Circles and Ellipses

It is a good idea for you to familiarize yourself with the equations of a circle and an ellipse.

**The equation for a circle is**

$$(x - h)^2 + (y - k)^2 = r^2,$$

where  $(h,k)$  is the center of the circle and  $r$  is the radius, as shown below.

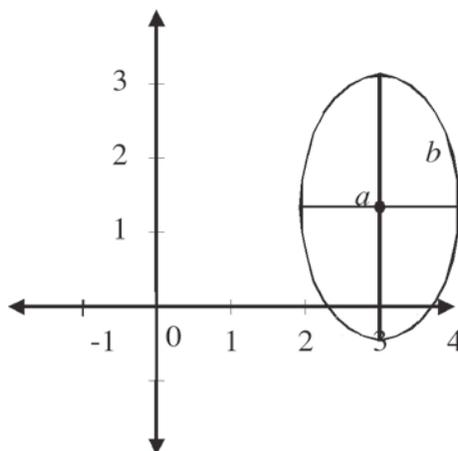


**The equation for an ellipse is**

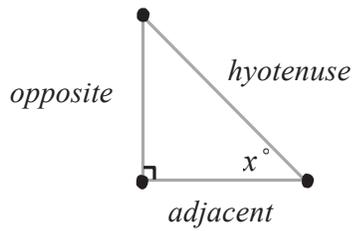
$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1,$$

where  $(h, k)$  is the center of the ellipse,  $2a$  is the horizontal axis (width) and  $2b$  is the vertical axis (height).

An example of an ellipse is shown below.



Basic trigonometry is based on the relationships between the sides of a right triangle and a corresponding angle.



$$\text{The sine (abbreviated as sin) of angle } x = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{The cosine (abbreviated as cos) of angle } x = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{The tangent (abbreviated as tan) of angle } x = \frac{\text{opposite}}{\text{adjacent}}$$

An easy way to remember the trigonometric functions is the acronym **SOHCAHTOA** (pronounced: SO-CAH-TOE-AH).

SOH = Sine is Opposite over Hypotenuse.

CAH = Cosine is Adjacent over Hypotenuse.

TOA = Tangent is Opposite over Adjacent.

### Other Important Trigonometric Functions and Relationships

$$\text{tangent} = \frac{\sin}{\cos}$$

$$\text{cosecant} = \frac{1}{\sin}$$

$$\text{secant} = \frac{1}{\cos}$$

$$\text{cotangent} = \frac{1}{\tan} = \frac{\cos}{\sin}$$

$$\sin^2\theta + \cos^2\theta = 1$$

## • Graphs of Trigonometric Functions

You will be expected to be able to identify the graph of a trigonometric function from a given equation or to find an equation from a given graph.

The general equation for a given trigonometric function is represented by (shown below for the sine of angle  $\theta$ )

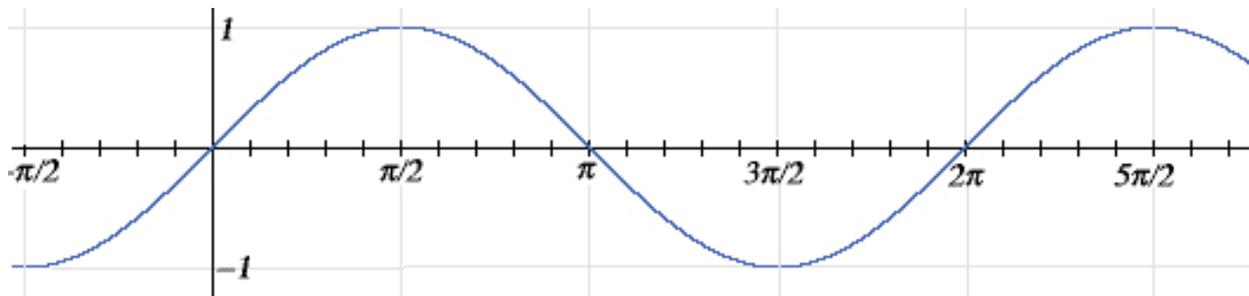
$$y = A \sin B\theta,$$

where  $A$  is the **amplitude** of the graph (how tall the graph is) and  $B$  is the **period** of the graph (how long it takes to get through a complete cycle).

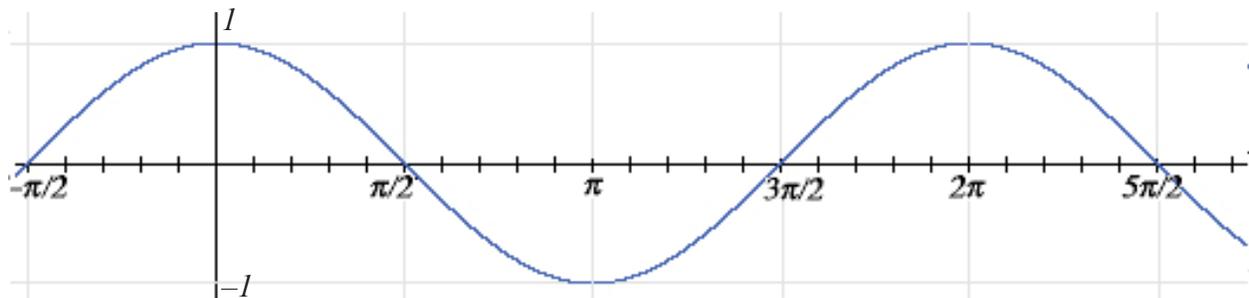
Thus, an increase in  $A$  also increases the amplitude or height of the graph (a direct relationship). Increases in  $B$  result in a decrease in the period (an inverse relationship).

Below are the graphs of the simple functions  $y = \sin x$  and  $y = \cos x$ , respectively:

$$y = \sin x$$



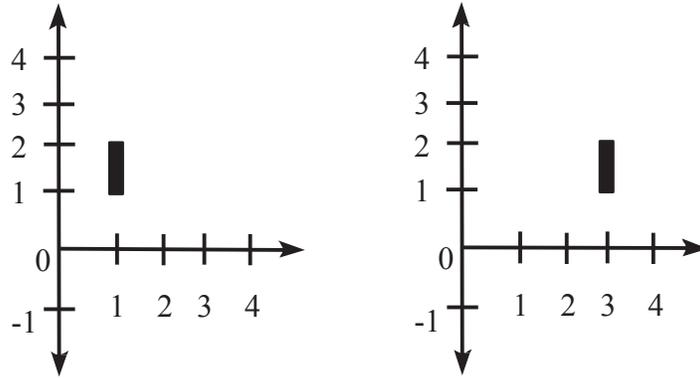
$$y = \cos x$$



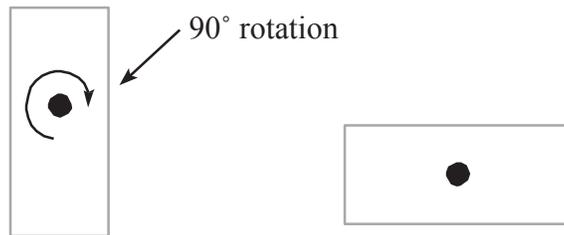
# Transformations

## • Important terms

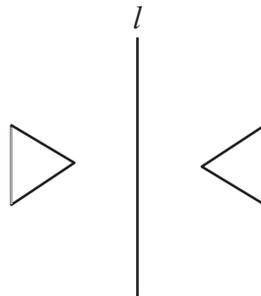
A **translation** is described as a linear movement that does not involve any rotations or reflections. In the figure below, the line segment has been translated 2 units to the right (in the positive x-direction).



When a figure is **rotated**, it is turned around a central point, or point of rotation. The first rectangle below has been rotated  $90^\circ$  to create the second rectangle.

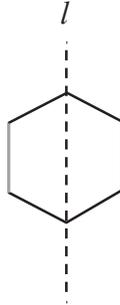


When an object is reflected, its mirror image is produced with respect to a line (called the line of reflection). The left triangle in the figure below has been reflected about line  $l$  to create the right triangle. The two are mirror images.



## • Symmetry

When a figure can be folded such that each half matches the other exactly, the figure is said to possess a degree of symmetry. The line on which the figure is folded to give the equal halves is called a **line** or **axis of symmetry**. Line  $l$  is one axis of symmetry in the figure below.



When a figure is rotated and the resulting figure is the same as the original figure, the figure is said to possess symmetry about a point (**point of symmetry**). A rotation of  $180^\circ$  of the figure below will yield the same figure.



**Note:** Symmetry about a point and symmetry about a line are different properties. A given figure may have either type of symmetry, both types of symmetry, or neither type.

## Geometry Practice Problems

- 1) What is the perimeter of a rectangle with a 7-inch width and a 16-inch length?
  - A) 32 inches
  - B) 63 inches
  - C) 23 inches
  - D) 46 inches
  - E) 54 inches
- 2) What is the radius of a circle with a 314-yard circumference?
  - A) 10 yards
  - B) 22 yards
  - C) 50 yards
  - D) 24 yards
  - E) 100 yards

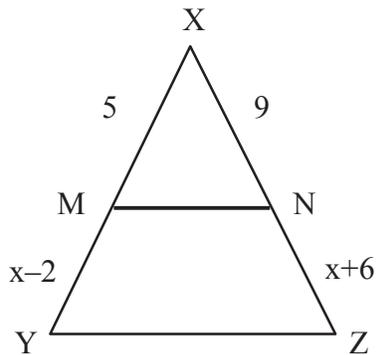
3) What is the slope of the equation  $2y + 17 = 8x$ ?

- A) 8
- B) -4
- C) 4
- D) 14
- E) 17

4) Three vertices of a parallelogram are at  $(2,1)$ ,  $(-1, -3)$ , and  $(6, 4)$ . Which of the following could be the coordinates of the remaining vertex?

- A)  $(0,3)$
- B)  $(3,0)$
- C)  $(-1,4)$
- D)  $(6,1)$
- E)  $(4,-1)$

5) In the triangle below,  $YZ \parallel MN$ .  $MX = 5$ ,  $NX = 9$ ,  $MY = x - 2$ , and  $NZ = x + 6$ . What is the length of  $YX$ ?



- A) 10
- B) 12
- C) 15
- D) 18
- E) 21

6) If  $\cos x = \frac{5}{13}$ , what is the value of  $\sin x$ ?

- A)  $\frac{12}{13}$
- B)  $\frac{5}{13}$
- C)  $\frac{13}{12}$
- D)  $\frac{13}{5}$
- E)  $\frac{12}{5}$

Note: Answers may be found on Page 105.

## **Is This a Foreign Language?**

### **- Mathematical Terminology and Word Problems**

Some problems in the math section will be presented as word problems. They require you to apply math skills to everyday situations. The hardest thing about doing word problems is translating them into math. The actual question is often pretty simple. These questions test your ability to set up an equation based on the information in the word problem. The best way to get comfortable with word problems is to practice solving them as much as possible.

Key points regarding word problems:

- Read the entire problem! Don't try to solve anything until you get a feel for the whole problem.
- List the information that is given in order to organize yourself.
- Label variables with what they stand for.
- Determine exactly what the problem is asking for. (What do you need to know?)
- Work out the answer.
- Double-check to make sure the answer makes sense. (Check your answer against the original word problem, not your equation.)

## Direct Translation into Math

You will be required to translate a verbal description of a mathematical relationship into math terms. Always read the problem carefully and double-check yourself. For instance:

“Nine less than the total of a number and 2” translates into  $(n + 2) - 9$ , or  $n - 7$ .

“The ratio of 9 more than  $x$  to 3” translates into  $\frac{x + 9}{3}$ .

“Sarah has four more dollars than Kevin” translates to  $S = K + 4$ .

“The average of the weights of three children is 80 pounds” translates to  $\frac{x + y + z}{3} = 80$ .

Certain words indicate mathematical operations. Below is a partial list:

Addition {  
increased by  
more than  
combined  
together  
total of  
sum  
added to

Subtraction {  
decreased by  
minus  
less  
difference between or of  
less than  
fewer than

Multiplication {  
of  
times  
multiplied by  
product of  
increased or decreased  
by a factor of

Division {  
per  
a  
out of  
ratio of  
quotient of  
percent (divide by 100)

Equals {  
is  
are  
was  
were  
will be  
gives  
yields  
sold for

“Per” and “a” sometimes mean “divided by,” as in “I paid \$3.00 *per* gallon” (or, “\$3.00 *a* gallon”).

*Example 1:* The product of two consecutive negative even integers is 24. Find the numbers.

First, evaluate the question. Since the two numbers are negative and nonconsecutive, you know that the two numbers are two apart (for example,  $-2$  and  $-4$ ) and, as a result, the second number is two greater than the first.

So, let’s call the first number  $n$  and the second number  $n + 2$ . The question states that the product of the two numbers is 24. Write the question in math terms using the variables you defined:

$$(n)(n+2) = 24$$

Solve for  $n$ .

$$\begin{aligned}n^2 + 2n &= 24 \\n^2 + 2n - 24 &= 0 \\(n - 4)(n + 6) &= 0 \\n = 4 \text{ and } n = -6\end{aligned}$$

Since the question stated that the numbers were negative, you can ignore the 4 and take  $n = -6$  as the solution. Substituting  $-6$  into  $n + 2$  gives  $-4$  as the second number.

*Example 2:* Gloria’s washing machine needs fixing. Since her machine is pretty old, she doesn’t want to spend more than \$100 for repairs. A service call will cost \$35, and the labor will be an additional \$20 per hour. What is the maximum number of hours that the repairperson can work and keep the total cost at \$100?

Let  $h$  = the maximum number of hours the repair can take.

Write out an equation based on the information given:

“\$35 plus \$20 per hour for  $h$  hours equals \$100.”

$$\begin{aligned}35 + 20h &= 100 \\20h &= 65 \\h &= \frac{65}{20} = \frac{13}{4} \text{ or } \frac{31}{4} \text{ hours}\end{aligned}$$

## Word Problem Practice Problems

1) A shipment of 100 CDs was just received at a record store. There is a 4% probability that one of the CDs was damaged during shipment, even though the package may not be cracked. If John buys a CD from this shipment, what are the odds that he is buying a damaged CD?

- A)  $\frac{1}{25}$
- B)  $\frac{4}{25}$
- C)  $\frac{96}{100}$
- D)  $\frac{1}{24}$

2) Peanuts sell for \$3.00 per pound. Cashews sell for \$6.00 per pound. How many pounds of cashews should be mixed with 12 pounds of peanuts to obtain a mixture that sells for \$4.20 per pound?

- A) 3
- B) 4
- C) 6
- D) 8

Note: Answers may be found on Page 105.

## If I Had to Guess: The Art of Estimating and Guessing

So what happens if you get stuck? Sometimes you may understand the problem but can't figure out how to solve it. You may have a general idea of what the answer should or shouldn't be, but you can't quite figure out how to get the correct answer. Here are a few tips to increase your chances of choosing the correct answer.

- 1) **Eliminate** choices that are blatantly incorrect. By eliminating choices, you increase your odds of selecting the correct answer.
- 2) **Work backwards.** When you're having trouble setting up an equation or interpreting a word problem, you can often work backwards from the answer choices. Pick one of the answer choices and see if it fits all the requirements set forth in the directions.
- 3) **Figures can be estimated** just by "eyeballing" them. Although figures are not technically drawn to scale, they represent a best representation of the desired result. You can sometime use this if you're stuck on a geometry question.
- 4) **Use your best guess.** If you are completely at a loss for an answer, use your best guess. When guessing, you should eliminate as many answer choices as possible and then take an "educated" guess from the remaining choices.

## Solutions to Practice Problems

### Properties of Numbers (Page 44-46)

- 1) A
- 2) D
- 3) B
- 4) B
- 5) D
- 6) C
- 7) C
- 8) E
- 9) D
- 10) C
- 11) A

### Fractions (Page 49-50)

- 1) E
- 2) C
- 3) D
- 4) C
- 5) E
- 6) C

### Operations (Page 59-60)

- 1) E
- 2) B
- 3) C
- 4) E
- 5) C
- 6) A
- 7) B
- 8) A

### Manipulating Algebraic Expressions (Page 70-71)

- 1) D
- 2) A
- 3) D
- 4) C
- 5) B
- 6) E

### Ratios, Proportions, and Variation (Page 76-77)

- 1) B
- 2) C
- 3) E
- 4) C
- 5) A
- 6) D
- 7) E
- 8) A

### Probability and Statistics (Page 81)

- 1) B
- 2) D

### Geometry (Page 99-100)

- 1) D
- 2) C
- 3) C
- 4) A
- 5) C

### Word Problems (Page 104)

- 1) A
- 2) D



## Chapter Five: Science Skills Review

The key point to understand about the *ACT Science Test* is that you are not being tested on your knowledge of science or scientific concepts, but rather on your ability to read and/or interpret graphs or data. The questions can be answered from the information provided in the passage.

The *ACT Science Test* requires many of the same skills that the *ACT Reading Test* does. Just like interpreting a reading passage, you will have to “read” through graphs, charts and scientific data in order to discover and understand the main idea of a passage.

On the *Science Test*, there are three passages that present scientific data in the form of charts or graphs (Data Representation), three passages based on specific scientific experiments (Research Summaries), and one passage in which two or more scientists state opposing views on the same issue (Conflicting Views). Accompanying each passage will be five to seven questions. Some passages will be very hard to understand, but they’ll be used to answer many easy questions.

The questions in the *Science Test* fall into three general categories:

- **Understanding** These questions ask you to basically reiterate parts of the passage. This could include looking up a value on a graph or explaining the underlying assumption behind the passage. The key to this type of question is familiarizing yourself with the passage. Recognize where in the passage the question is directing you for the answer.
- **Analysis** To answer these questions, you’ll need a deeper understanding of the information in the passage. You’ll be expected to recognize relationships and trends in the data. You may be asked to synthesize information from different parts of the passage in order to generate a prediction or an explanation.
- **Generalization** These questions expect you to put things into perspective (the so-called “big picture”). You may be asked about things not described in the passage, or you might be given new experimental conditions such that you must predict the outcome. A question on the impact of the result is also entirely possible.

Here is an outline of the steps necessary to answer any question on the Science Reasoning Test.

1. **Read the passage and underline key pieces of information as you read.** All the answers to the question will either be found in the passage or can be interpreted from the data in the passage. The key points you should underline will most likely represent answers to one or more of the questions being asked. In other words, determine what is being represented and identify the main idea of the passage.

- 2. Skim through the questions and choose the easiest questions to answer first.** The hardest part about the Science Test is that it is a timed exam. In order to pace yourself for this test, knocking out the easiest and least time-consuming questions first will enable you to focus on the tougher questions that require deeper understanding. Usually, the first two or three questions are easier than the last two or three. Don't waste time struggling with a hard question until you've answered all the easy ones. You get the same number of points for both, so focus on the "sure things" first.
- 3. Always use the process of elimination.** Some answer choices are blatantly wrong. When you come across these, cross them off and focus on the remaining choices. This will help you narrow things down so you can focus on the potential correct answer.
- 4. Always refer back to the passage to double-check yourself.** After you make an answer choice, go back to the passage and/or graphic representation to make sure your choice agrees with the details within the passage or graphic. If it doesn't agree, eliminate that choice, pick another one, and then double-check yourself again.
- 5. When in doubt, use your best guess.** As explained earlier in this chapter, if you're stuck, make an "educated" guess. Eliminate choices that are obviously wrong, and then take a guess. Your odds will have increased greatly by eliminating one or two choices.

In order for you to fully understand the nature of the *Science Test*, you will need to be able to recognize, read or interpret, and understand the different types of questions. Below, you will find some examples and explanations of each of the three types of passages.

### **Data Representation**

In order to interpret the data, you must first determine what is being represented. Once you understand that, you must determine what the depicted values mean. Are there any units of measurement? How does one unit of measurement relate to another (if there are others)? Finally, look for trends in the data. These kinds of passages are usually shorter than any of the other types.

Many graphics include legends. These are charts which refer to the specific information in the graphic. They may refer to symbols or colors used in the graphic. Make sure you understand what each symbol or color stands for and how each is indicated relative to the others.

Data Representation can be broken down into further subgroups:

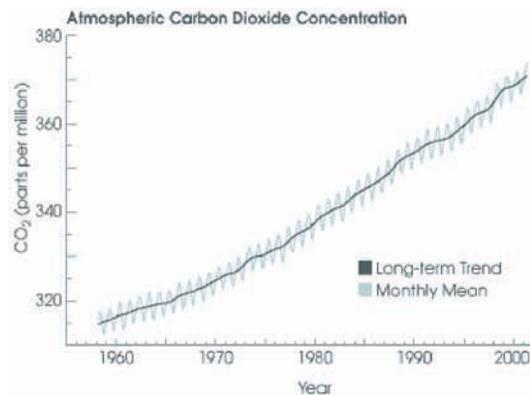
**1) Graphs and Charts.** There are many types of charts and graphs. They have actual values and units of measurement for precise and direct interpretation. The key to understanding these figures is determining what the variables (the "things" that are changing) are. There are two kinds of variables: independent (not affected by other factors or variables) and dependent (affected by other factors or variables). The dependent variables are often represented in some relation to one another. This is where you must observe the trends (how one variable relates to another). One of the most valuable uses for graphs is to "predict" data that are not measured on the graph.

- **Extrapolate:** extending the graph, along the same slope, above or below measured data.
- **Interpolate:** predicting data between two measured points on the graph.

Here are just a few common examples of the types of graphs and charts that you may see on the Science Test:

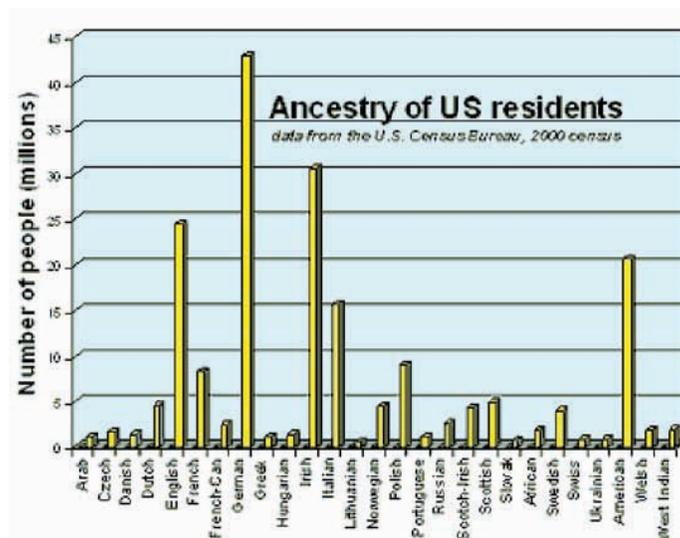
### *Line Graphs (Linear or Curved)*

A line graph is a good way to look at how one variable changes with respect to another variable. Just by looking at a line graph, you should be able to see whether or not a trend exists. Remember: A trend can be looked at over the complete graph or over a specific range within the graph. For instance, in the graph below, the trend observed is that as time has passed, the concentration of CO<sub>2</sub> has steadily increased.



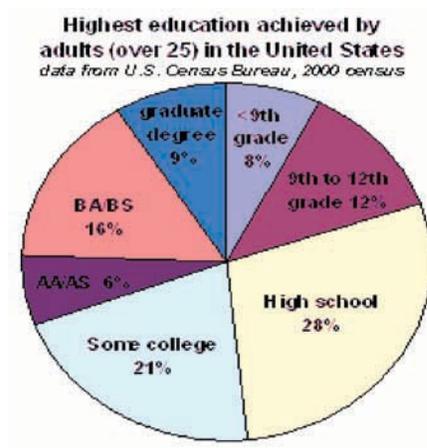
### *Bar Graphs*

Bar graphs are great for looking at differences amongst similar things. Bar graphs show stacks of numbers of things right next to each other that can be compared instantly. The height of each stack can tell you the number of things, either approximately by the numbers on the vertical axis or exactly by a number label on the stack. According to the bar graph below, Germans represent the ancestry of the greatest number of U.S. residents.



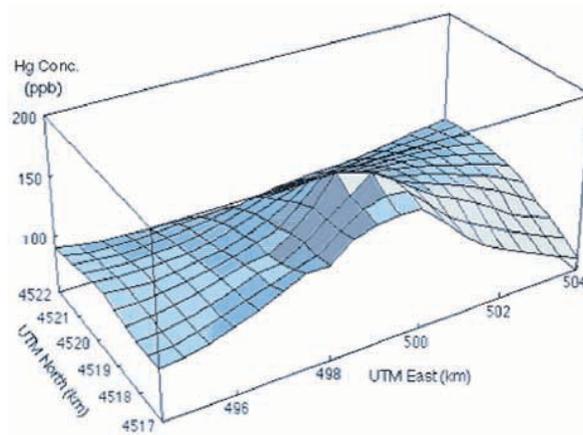
## Pie Charts

Think of a pie chart as a pizza pie, with each slice representing a percentage of the whole pie. For instance, in the chart below “high school” represents 28% of the whole pie.



## 3-D Plots

A three-dimensional plot can connect three variables together. In the plot below, there are three axes representing the three different variables.

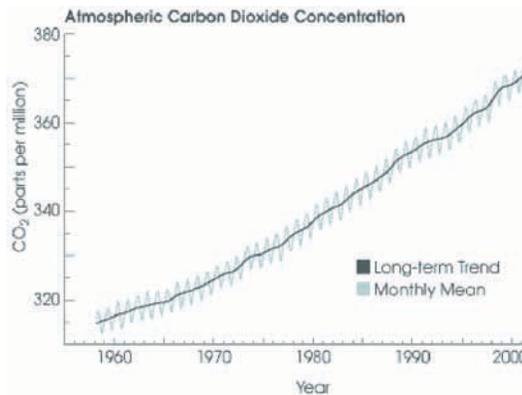


### Tables

Tables contain the raw data from an experiment. Tables of pure numbers can be difficult to interpret. It is up to you to put the raw data into some form (usually a chart or graph) that makes it easier for you to observe the trends.

Year	Jan.	Feb.	March	April	May
1958	-99.99	-99.99	315.71	317.45	317.5
1959	315.58	316.47	316.65	317.71	318.29
1960	316.43	316.97	317.58	319.03	320.03
1961	316.89	317.7	318.54	319.48	320.58
1962	317.94	318.56	319.69	320.58	321.01
1963	318.74	319.08	319.86	321.39	322.24
1964	319.57	-99.99	-99.99	-99.99	322.23
1965	319.44	320.44	320.89	322.13	322.16
1966	320.62	321.59	322.39	323.7	324.07
1967	322.33	322.5	323.04	324.42	325
1968	322.57	323.15	323.89	325.02	325.57

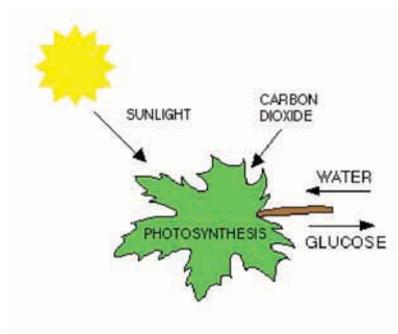
For instance, if you were to take the chart above and convert it into a graph, you would observe the figure below:



### Illustrations

Instead of

a chart or a graph, sometimes the test will give you an illustration that will have text within the picture, but usually not with any specific explanations. You are expected to be able to follow a chart or interpret a diagram. Below is an example of a basic illustration of photosynthesis. You should be able to get the main idea out of the picture.



**Key Tips:** Follow this short procedure to extract a lot of information from any graph. Although an infinite variety of data can appear in graphical form, this same procedure can apply when reading any kind of graph. **Make sure you understand the key or legend, if provided.**

**Describe the graph:** What does the title say? What is on the x-axis? What is on the y-axis? What are the units?

**Describe the data:** What is the numerical range of the data? What kinds of patterns can you see in the data?

**Interpret the data:** How do the patterns you see in the graph relate to other things you know?

**Sample Passage 1** (*Answers are on page 118.*)

The types of cancer most prevalent in the United States (or in any country of the world, for that matter) have not been constant. Changing lifestyles, changing behavior, and changing public awareness of the risk factors for cancer over the years have caused different forms of cancer to increase or decrease in importance. This table gives you a “snapshot” of the important forms of cancer, using data from 1999.

<b>Primary Site</b>	<b>Total</b>	<b>Females</b>	<b>Males</b>
All Sites	8,368,000	4,903,000	3,456,000
Brain	91,000	42,000	49,000
Breast	2,057,000	2,044,000	13,000
Cervix uteri	211,000	211,000	0
Colon	877,000	469,000	408,000
Corpus uteri	550,000	550,000	0
Hodgkin’s disease	158,000	74,000	84,000
Kidney & renal pelvis	204,000	81,000	123,000
Larynx	132,000	26,000	106,000
Leukemias	144,000	64,000	80,000
Lung	397,000	185,000	212,000
Melanomas of the skin	484,000	250,000	234,000
Non-Hodgkin’s lymphoma	300,000	150,000	150,000
Oral cavity/pharynx	214,000	80,000	134,000
Ovary	191,000	191,000	0
Pancreas	25,000	13,000	12,000
Prostate	1,017,000	0	1,017,000
Rectum	379,000	177,000	202,000
Stomach	77,000	35,000	42,000
Testis	130,000	0	130,000
Thyroid	214,000	161,000	53,000
Urinary bladder	601,000	158,000	443,000

### Estimated Prevalence

1. According to the data, which of the following is the most prevalent form of cancer?
  - A. pancreas
  - B. rectum
  - C. lung
  - D. colon
  
2. Women have a greater overall prevalence for cancer according to the data above. Which form of cancer truly accounts for that observation?
  - F. ovary
  - G. cervix
  - H. breast
  - J. thyroid
  
3. According to the data, which of the cancer types below also affected men?
  - A. ovary
  - B. corpus uteri
  - C. breast
  - D. cervix

### Research Summaries

Research summaries usually contain an introductory passage followed by a number of experiments. You don't have to know much science in order to attack these passages, but you are expected to understand how scientists go about getting and testing knowledge. This process is known as the **scientific method**.

The scientific method is the "tool" that scientists use to find the answers to questions. It is the process of thinking through the possible solutions to a problem and testing each possibility to find the best solution. The scientific method involves the following steps: doing research, identifying the problem, stating a hypothesis, conducting project experimentation, and reaching a conclusion.

Experimental design is often a subject of questions. Make sure you understand the variables involved in the experiment and how they relate to each other. You may also be questioned about the validity of results based on how the experiment was set up or conducted.

To succeed on the *ACT Science Test*, you must learn to understand how scientists think. In your science classes in school, your teachers normally explain general rules of science to you and then expect you to apply these rules to solve problems. This is considered specific thinking (solving a problem with a known rule of science). Some questions on the *Science Test* are based on this type of thinking; however, the majority of questions will test your ability to see the kinds of patterns in specific data that you must use to formulate your own general hypotheses, conclusions, or questions.

On the *Science Test*, you will be expected to understand how the experiments in a Research Summary passage are designed. You should jot down brief notes on each experiment as you read it. These notes should highlight the differences between each experiment. The notes will often refer to a procedure in the experiment. This is important because by noting the difference between the experiments, you will be able to quickly gauge to which experiment a question is referring.

You may also be asked to identify the control of the study (if one exists). A control is the baseline for comparison. The control is normally the independent system without adding or changing any variables.

A well-designed experiment involves:

- A control group
- A single variable that changes from test to test
- Other variables that remain the same from test to test

Carefully read the introductory paragraph. This paragraph often contains some small bit of information which you will need to answer a question. Read Experiment 1 and answer all questions pertaining to experiment 1. Do the same for Experiment 2 (and others if there are more than 2 experiments). Answer questions that compare the experiments last.

***Sample Passage 2 (Answers are on page 118.)***

Heat changes the properties of water. If we add enough heat to water in its solid form (ice), it will change its state of matter to a liquid. We call this melting. If more heat is added, the liquid will change to gas (water vapor). When enough water vapor forms so that the pressure of the vapor is equal to the pressure of the atmosphere above the water, the vapor can then push the air above the container away and allow vapor bubbles to be released. We call this boiling.

***Test 1***

At an altitude of 1000 feet, a beaker was filled about half full with distilled water. The beaker of water was then heated until the distilled water began to boil. A thermometer was suspended in the water to measure the temperature. The temperature observed was 210 °F.

***Test 2***

The experiment was repeated at an altitude of 800 feet, and the temperature was observed to be 212 °F.

***Test 3***

The experiment was repeated at an altitude of 4000 feet, and the observed temperature was 204 °F.

1. What pattern could be observed about the boiling points?
  - A. As elevation increases, boiling point decreases.
  - B. As elevation decreases, boiling point increases.
  - C. As elevation increases, the boiling point increases.
  - D. As elevation decreases, the boiling point decreases.
  
2. What should be the boiling point if the elevation is 7000 feet?
  - F. 214 °F
  - G. 210 °F
  - H. 205 °F
  - J. 199 °F
  
3. Boiling is the change between which two phases?
  - A. solid to liquid
  - B. gas to liquid
  - C. liquid to gas
  - D. liquid to solid

## Conflicting Viewpoints

On the *Science Test*, you will find one “Conflicting Views” passage, in which two or more scientists propose different theories about a particular scientific phenomenon. The two theories can either be differing interpretations of the same data or opinions of each scientist based on their own data. The key to these questions is to understand what theory is being proposed by a given scientist. Pay close attention to where and to what degree the scientists’ arguments differ.

It doesn’t matter which opinion is right or wrong. Your job is to understand each scientist’s position and the rationale behind it. The questions will focus on this and not on which opinion is valid.

Note: It doesn’t matter if you know a lot about the science being discussed. Everything that is relevant is present in the passage. Not all passages can be taken as true science or fact. Remember that this is a science *reasoning* test, not a science *concepts* test.

**Key tips:** Since you know the given arguments are going to conflict, figure out the points of difference and underline them. Once you know the points of difference, you need to understand the basis for those differences. This is the key to answering the questions, since there are three types of questions that you will see:

- 1) questions that will ask you specifically about a particular theory or scientist
- 2) questions that will compare arguments directly
- 3) questions that offer information to support or contradict arguments

If you understand the gist of each argument, you will have no problems tackling each question.

Tackle this section as you would a Research Summary. Read the introductory paragraph to identify the topic under discussion. Read the first scientist's explanation and answer the questions that pertain to that scientist. Do the same for the next scientist (and others, if given). Answer the questions that compare the scientist's viewpoints last.

***Sample Passage 3*** (Answers are on page 118.)

The United States desperately needs a liquid fuel replacement for oil in the future. The use of oil is projected to peak about 2007, and the supply is then projected to be extremely limited in 40–50 years. Alternative liquid fuels from various sources have been sought for years. The energy balance of ethanol is found by taking the amount of energy contained in a gallon of ethanol (roughly 76,000 Btu) and subtracting the amount of energy that goes into producing a gallon of ethanol.

***Scientist 1***

Energy outputs from ethanol produced using corn, switchgrass, and wood biomass were each less than the respective fossil energy inputs. The same was true for producing biodiesel using soybeans and sunflowers; however, the energy cost for producing soybean biodiesel was only slightly negative compared with ethanol production. Several physical and chemical factors limit the production of liquid fuels such as ethanol and biodiesel using plant biomass materials. These include the following:

- An extremely low fraction of the sunlight reaching America is captured by plants. On average, the sunlight captured by plants is only about 0.1%, with corn providing 0.25%. These low values are in contrast to photovoltaics that capture from 10% or more sunlight, or approximately 100-fold more sunlight than plant biomass.
- In ethanol production, the carbohydrates are converted into ethanol by microbes that, on average, bring the concentration of ethanol to 8% in the broth with 92% water. Large amounts of fossil energy are required to remove the 8% ethanol from the 92% water.
- For biodiesel production, there are two problems: the relatively low yields of oil crops, ranging from 1,500 kg/ha for sunflower to about 2,700 kg/ha for soybeans; sunflowers average 25.5% oil, whereas soybeans average 18% oil. In addition, the oil extraction processes for all oil crops is highly energy intensive as reported in this manuscript.

Therefore, these crops are poor producers of biomass energy.

***Scientist 2***

The energy balance of ethanol is 1.34:1, which means that ethanol yields 34% more energy than it takes to produce it, including growing the corn, harvesting it, and distilling it into ethanol. The positive ratio is due mostly to technological advances in the ethanol production process. Advances in the areas most critical in determining the net energy value (NEV): corn yields, changes in agricultural practices resulting in reduced energy inputs, and advances in the corn-to-ethanol conversion process. Corn yield plays a critical role in determining the energy balance of starch-based ethanol. In fact, a 1% increase in corn yield raises NEV by 0.37 percent. Importantly, with the exception of a few bad years, corn yields have been increasing over time since 1975. Ethanol plants are the largest fossil energy-consuming component in the corn-to-ethanol fuel cycle. Today's ethanol plants use far less energy than in the past. The majority of ethanol plants in production today have been extensively modernized, utilizing the latest

advances in ethanol production technology. Fertilizer accounts for about 45% of the energy required to grow corn. However, the use of fertilizer in grain production, which includes chemical inputs such as nitrogen, potash, and phosphate, has been in general decline since the early 1980s. Furthermore, biodiesel yields

3.2 units of fuel product energy for every unit of fossil energy consumed in its life cycle; in other words, the biodiesel life cycle produces more than three times the energy in its final fuel product than it uses in fossil energy. The *production* process of biodiesel and diesel is practically the same in terms of efficiency in the conversion of raw materials into fuel. The difference is that biodiesel is able to use renewable resources in its production – soybeans and rapeseed oils, or used frying oil and unwanted animal fats – while conventional diesel relies on fossil fuel resources. In fact, petroleum diesel’s life cycle yields only 0.83 units of fuel product per unit of fossil energy consumed.

1. What is the main idea of Scientist 1’s argument?

- A. Ethanol is an acceptable liquid fuel replacement for oil.
- B. The energy outputs from ethanol using biomass were more than the respective fossil energy inputs.
- C. Corn, soybeans, and sunflowers are poor producers of biomass energy.
- D. Technological advances have significantly lowered the energy requirements of producing ethanol.

2. Each of the statements below bolsters Scientist 2’s position EXCEPT:

- F. The use of fertilizer has decreased since the early 1980s.
- G. Ethanol plants have been thoroughly modernized and thus require less energy than previously needed.
- H. Photovoltaic cells capture 100-fold more sunlight than plant biomass.
- J. Corn yields have been increasing since 1975.

3. Which statement below would be most useful to Scientist 2 in countering Scientist 1’s argument?

- A. Technological advances have recently occurred in the ethanol producing industry.
- B. Corn yields have been increasing since 1975.
- C. All of the data in Scientist 1’s article are based on studies performed more than 15 years ago.
- D. Fertilizer accounts for about 45% of the energy required to grow corn.

## Answers to Sample Passages

### Sample Passage 1 (Page 112-113)

1. You should have chosen D. Of the group listed — pancreas (25,000), rectum (379,000), lung (397,000), and colon (877,000) – colon cancer has the largest number.
2. You should have chosen H due to the fact that out of the total number of cancer occurrences in women (4,903,000), almost 40% (2,044,000) is due to breast cancer.
3. You should have chosen C. A number of males (13,000) were afflicted with breast cancer, whereas men were not affected by any of the other types.

### Sample Passage 2 (Page 114-115)

1. You should have chosen D. This is a trend question. If you line up the data from the experiments, as you go from 800 to 4000 feet, the boiling point goes from 212 to 204 °C. Thus, as the elevation increases, the boiling point decreases.
2. The answer is J. The answer to this question requires extending the data outward beyond the given data points (extrapolation). According to the established trend, an elevation greater than 4000 feet should have a boiling point less than 204 °F since elevation and temperature are inversely proportionate.
3. The answer is C. By the definition explained in the introductory paragraph, the transition from water (liquid) to its vapor form (gas) is called boiling.

### Sample Passage 3 (Page 116-117)

1. The answer is C. Scientist 1's argument centers around biomass being poor compared to oil.
2. The answer is H. The statement about photovoltaics is the only statement that does not correspond with Scientist 2's argument. It actually is used by Scientist 1 to defend his point.
3. The answer is C. If all the data used by Scientist 1 were out of date, then his argument would be out dated. This would be the most damaging argument because the old data cannot be used to argue against the new data (for the argument to be valid, both scientists must have concurrent data).

## Chapter Six: ACT Practice Test

### Reading Test

40 questions, 35 minutes (Answers may be found on Page 187.)

**Directions:** On the answer sheet, fill in the oval that corresponds with the correct answer to each question.

**PROSE FICTION:** This passage is adapted from Eleanor H. Porter’s novel, *Just David*. The setting is a small cabin on a mountainside, with no road or other houses visible.

From the little lean-to kitchen the sound of the sputtering suddenly ceased, and at the door appeared a pair of dark, wistful eyes.

“Daddy!” called the owner of the eyes.

There was no answer.

“Father, are you there?” called the voice, more insistently.

From one of the bunks came a slight stir and a murmured word. At the sound the boy at the door leaped softly into the room and hurried to the bunk in the corner. He was a slender lad with short, crisp curls at his ears, and the red of perfect health in his cheeks. His hands, slim, long, and with tapering fingers like a girl’s, reached forward eagerly.

“Daddy, come! I’ve done the bacon all myself, and the potatoes and the coffee, too. Quick, it’s all getting cold!”

Slowly, with the aid of the boy’s firm hands, the man pulled himself half to a sitting posture. His cheeks, like the boy’s, were red—but not with health. His eyes were a little wild, but his voice was low and very tender, like a caress.

“David—it’s my little son David!”

“Of course it’s David! Who else should it be?” laughed the boy. “Come!” And he tugged at the man’s hands.

The man rose then, unsteadily, and by sheer will forced himself to stand upright. The wild look left his eyes, and the flush his cheeks. His face looked suddenly old and haggard. Yet with fairly sure steps he crossed the room and entered the little kitchen.

Half of the bacon was black; the other half was transparent and like tough jelly. The potatoes were soggy, and had the unmistakable taste that

comes from a dish that has boiled dry. The coffee was lukewarm and muddy. Even the milk was sour.

David laughed a little ruefully.

“Things aren’t so nice as yours, father,” he apologized. “I’m afraid I’m nothing but a discord in that orchestra to-day! Somehow, some of the stove was hotter than the rest, and burnt up the bacon in spots; and all the water got out of the potatoes, too,—though that didn’t matter, for I just put more cold in. I forgot and left the milk in the sun, and it tastes bad now; but I’m sure next time it’ll be better—all of it.”

The man smiled, but he shook his head sadly.

“But there ought not to be any ‘next time,’ David.”

“Why not? What do you mean? Aren’t you ever going to let me try again, father?” There was real distress in the boy’s voice.

The man hesitated. His lips parted with an indrawn breath, as if behind them lay a rush of words. But they closed abruptly, the words still unsaid. Then, very lightly, came these others:--

“Well, son, this isn’t a very nice way to treat your supper, is it? Now, if you please, I’ll take some of that bacon. I think I feel my appetite coming back.”

If the truant appetite “came back,” however, it could not have stayed; for the man ate but little. He frowned, too, as he saw how little the boy ate. He sat silent while his son cleared the food and dishes away, and he was still silent when, with the boy, he passed out of the house and walked to the little bench facing the west.

Unless it stormed very hard, David never went to bed without this last look at his “Silver Lake,” as he called the little sheet of water far down in the valley.

“Daddy, it’s gold to-night--all gold with the sun!” he cried rapturously, as his eyes fell upon his treasure. “Oh, daddy!”

It was a long-drawn cry of ecstasy, and hearing it, the man winced, as with sudden pain.

‘Daddy, I’m going to play it--I’ve got to play it!’ cried the boy, bounding toward the cabin. In a moment he had returned, violin at his chin.

**The man watched and listened; and as he watched and listened, his face became a battle-ground whereon pride and fear, hope and despair, joy and sorrow, fought for the mastery.**

**It was no new thing for David to “play” the sunset. Always, when he was moved, David turned to his violin. Always in its quivering strings he found the means to say that which his tongue could not express.**

**Across the valley the grays and blues of the mountains had become all purples now. Above, the sky in one vast flame of crimson and gold, was a molten sea on which floated rose-pink cloud-boats. Below, the valley with its lake and river picked out in rose and gold against the shadowy greens of field and forest, seemed like some enchanted fairyland of loveliness. And all this was in David’s violin, and all this, too, was on David’s uplifted, rapturous face.**

1. At the beginning of the passage, David wakes his father because:
  - A. supper is ready.
  - B. it’s time to watch the sunset.
  - C. something has scared David.
  - D. it’s time for breakfast.
  
2. We can infer that the father, most likely, closes his lips abruptly (when talking to David about the meal) because:
  - A. he is angry.
  - B. he has decided to change the subject.
  - C. he is shocked by the quality of the food.
  - D. he is trying not to laugh.
  
3. The author of this passage calls the boy’s eyes “wistful” (Paragraph 1, line 3) because he:
  - A. is wishfully yearning.
  - B. is terribly afraid.
  - C. is straining to see in the dark.
  - D. has been crying.
  
4. Which of the following does not provide evidence that the father is in poor health?
  - A. His son awakens him for supper.
  - B. He winces with pain when his son cries out joyfully.
  - C. His cheeks are red, and his eyes are a little wild.
  - D. He silently looks at the lake with his son.

5. Based on its use in the passage, the word “ruefully” (Paragraph 12) means:
- A. sorrowfully.
  - B. quietly.
  - C. uncomfortably.
  - D. excitedly.
6. The father’s comment “I think I feel my appetite coming back” (Paragraph 18, line 3) most likely serves to:
- A. show that the father is now hungry.
  - B. demonstrate the father’s desire to protect his son’s feelings.
  - C. show that the father’s health is improving.
  - D. suggest that the food tastes better than he expected.
7. Which of the following does the author use as a metaphor to illustrate the father’s internal conflict?
- A. His face looked suddenly old and haggard.
  - B. his voice was low and very tender, like a caress
  - C. his face became a battle-ground
  - D. the sky in one vast flame of crimson and gold, was a molten sea
8. Considering the information we learn about David in the passage, it would be most reasonable to infer that if the boy were moved by a beautiful painting, he would:
- A. tell his father about it.
  - B. write about it in his journal.
  - C. play his violin.
  - D. cry tears of joy.
9. The mood of the passage can best be described as:
- A. cheerful.
  - B. foreboding.
  - C. indignant.
  - D. regretful.
10. Which of the following words best characterizes David?
- A. sullen
  - B. defeated
  - C. talented
  - D. innocent

**HUMANITIES:** This passage is adapted from Washington Irving's *The Sketch Book: The Author's Account of Himself*.

But Europe held forth the charms of storied and poetical association. There were to be seen the masterpieces of art, the refinements of highly-cultivated society, the quaint peculiarities of ancient and local custom. My native country was full of youthful promise: Europe was rich in the accumulated treasures of age. Her very ruins told the history of times gone by, and every mouldering stone was a chronicle. I longed to wander over the scenes of renowned achievement--to tread, as it were, in the footsteps of antiquity--to loiter about the ruined castle--to meditate on the falling tower--to escape, in short, from the commonplace realities of the present, and lose myself among the shadowy grandeurs of the past.

I had, beside all this, an earnest desire to see the great men of the earth. We have, it is true, our great men in America: not a city but has an ample share of them. I have mingled among them in my time, and been almost withered by the shade into which they cast me; for there is nothing so baleful to a small man as the shade of a great one, particularly the great man of a city. But I was anxious to see the great men of Europe; for I had read in the works of various philosophers, that all animals degenerated in America, and man among the number. A great man of Europe, thought I, must therefore be as superior to a great man of America, as a peak of the Alps to a highland of the Hudson; and in this idea I was confirmed, by observing the comparative importance and swelling magnitude of many English travellers among us, who, I was assured, were very little people in their own country. I will visit this land of wonders, thought I, and see the gigantic race from which I am degenerated.

It has been either my good or evil lot to have my roving passion gratified. I have wandered through different countries, and witnessed many of the shifting scenes of life. I cannot say that I have studied them with the eye of a philosopher; but rather with the sauntering gaze with which humble lovers of the picturesque stroll from the window of one print-shop to another; caught sometimes by the delineations of beauty, sometimes by the distortions of caricature, and sometimes by the loveliness of landscape. As it is the fashion for modern tourists to travel pencil in hand, and bring home their portfolios filled with sketches, I am disposed to get up a few for the entertainment of my friends. When, however, I look over the hints and memorandums I have taken down for the purpose, my heart almost fails me at finding how my idle humor has led me aside from the great objects studied by every regular traveller who would make a book. I fear I shall give equal disappointment with

**an unlucky landscape painter, who had travelled on the continent, but, following the bent of his vagrant inclination, had sketched in nooks, and corners, and by-places. His sketchbook was accordingly crowded with cottages, and landscapes, and obscure ruins; but he had neglected to paint St. Peter's, or the Coliseum; the cascade of Terni, or the Bay of Naples; and had not a single glacier or volcano in his whole collection.**

11. In the first paragraph of the passage, the author compares America to a youth and Europe to:

- A. stories and poetry.
- B. a wise, old man.
- C. a moldering stone.
- D. a masterpiece of art.

12. The author's desire to go to Europe is partially based on:

- A. idealism kindled by the many books he has read.
- B. his desire to prove that Americans are not inferior to Europeans.
- C. his desire to elevate his position in society.
- D. curiosity stemming from previous trips abroad.

13. The word "baleful" (Paragraph 2, line 5) as it's used in this passage means:

- A. refreshing.
- B. beneficial.
- C. miserable.
- D. evil.

14. Which of the following is not an example of a poetic device used by the author of the excerpt?

- A. every moldering stone was a chronicle
- B. ruins told the history of times gone by
- C. to travel pencil in hand
- D. sometimes by the loveliness of landscape

15. The author states that he observes "the comparative importance and swelling magnitude of many English travellers among us, who, I was assured, were very little people in their own country." This comment serves to demonstrate:

- A. the arrogance of the English.
- B. the greatness of other men in Europe.
- C. the author's lack of confidence.
- D. the scarcity of great men in America.

16. Based on the author's comments in the second paragraph, we can infer that:
- A. he deems American things to be inferior to European things.
  - B. he has never seen a highland of the Hudson.
  - C. he has never met a great man.
  - D. he views American society as more important than that of Europe.
17. In the last paragraph, the author claims that his opportunity to travel has been a "good or evil lot" because:
- A. he has come to question the world's perceptions of greatness.
  - B. he failed to visit the most important sights in Europe.
  - C. he liked some aspects of his trip and detested others.
  - D. he has mixed feelings about whether or not he wants to return.
18. The author's "roving passion" (Paragraph 3, line 1) could also be called his:
- A. ever-changing desires.
  - B. desire to travel.
  - C. curiosity about Europe.
  - D. love for greatness.
19. The author compares himself to a landscape painter who doesn't paint the great landmarks because:
- A. he has no artistic ability.
  - B. there are no great landscapes left to be painted.
  - C. he feels he has wasted his time and failed to see what he went to see.
  - D. he prefers to write about the simpler pleasures of the countries he has visited.
20. An important lesson that the author has yet to learn is that:
- A. you never get a second chance to do the right thing.
  - B. greatness is in the eye of the beholder.
  - C. no one wants to read stories about print-shops in small towns.
  - D. he would have been better off staying at home.

**NATURAL SCIENCE:** This passage is adapted from “Innocent Murmurs,” a 2004 journal article by Dr. Thomas Biancaniello.

**Murmurs are sounds or noises made by blood as it flows through the heart and blood vessels of the body. Murmurs can be caused by abnormal flow patterns that occur when there are abnormalities of the heart valves, holes within the heart, or abnormal communications between blood vessels or between blood vessels and the heart. In infants and children, most murmurs originate through normal flow patterns, however, and are referred to as innocent, physiological, or normal murmurs. By innocent, we mean that there are no structural (anatomic) or functional (physiological) abnormalities of the heart and that the murmur comes from normal flow within a normal heart. Innocent is the preferred term because it strongly conveys that nothing is wrong, as opposed to the older term functional, which is not always clearly understood as being normal by parents and patients. Although murmurs termed innocent may be heard in virtually anyone, they are most often heard in childhood.**

**Although innocent murmurs may be prevalent mostly in childhood, the presence of murmur does not imply the presence of structural heart disease. Congenital heart defects, which are responsible for most of the heart problems in children in the developed world, are structural abnormalities caused by errors in the development of the heart while the child is still in the womb. The heart starts out as a single tube and, during the first eight weeks of pregnancy, divides into four chambers and forms four major valves. If an error or errors occur during this process, a congenital heart defect may result. Fortunately, less than 1% of infants are born with heart defects.**

**Parents are often quite concerned when informed by the pediatrician or family practitioner that a murmur has been detected. The primary care practitioner may be quite comfortable identifying innocent murmurs and explaining them to parents. Referral to a pediatric cardiologist is made either if the practitioner is concerned or unsure of the nature of the murmur or if the parent requires or requests further assurance. The evaluation of the child by the pediatric cardiologist will include a complete assessment of the cardiovascular system, not just listening for murmurs, because there are some serious cardiovascular abnormalities that do not have murmurs.**

**Looking for clues of heart disease, the pediatric cardiologist will obtain a focused history from the parents. For infants, this will include birth history, feeding patterns, breathing difficulties, color changes, growth pattern, and activity levels. For children, parents will be asked**

about activity capacity. For instance, can the child keep up with children of the same age while playing vigorously? Have there been complaints of shortness of breath or extra beats, skipped beats, racing of the heart (palpitations), or chest pain? Chest pain is a common complaint, but a cardiac cause is found in less than 1% of children complaining of chest pain. Has the child ever fainted? Although fainting occurs in about 15% of children before they reach 21 years of age, it is not usually caused by primary heart problems. The pediatric cardiologist will want to know whether fainting has occurred and under what circumstances in order to exclude cardiac causes.

An accurate family history is extremely important in assessing the child because congenital heart defects are 3 to 4 times more frequent in families in which a close relative has been born with a heart defect. In addition, the condition hypertrophic cardiomyopathy, a primary muscle disorder of the heart, is an inherited condition that may cause sudden unexpected death, especially during or after vigorous exercise in young people.

Physical examination of the child will include an assessment of general appearance, color, respiratory effort, and vital signs, including heart rate, respiratory rate, and blood pressure. The vital signs will be compared with age-established norms. The neck is evaluated for prominence of vessels and abnormal pulsation and is listened to for transmitted murmurs (bruits). The lungs are listened to for abnormalities of the breath sounds. The pulses in the arms and legs are checked. If the pulses are not equal, a narrowing of the main artery to the body (coarctation of the aorta) may be present, causing increased blood pressure in the upper body with lower blood pressure in the lower body. If this serious condition is not diagnosed, it can lead to considerable problems for the infant or child immediately or in the future.

Examination of the heart begins with observation and palpation of the chest for abnormal impulses that signify increased muscle activity of the ventricles or main pumping chambers. The heart is then listened to with the stethoscope—first, for the normal sounds of the valves closing (the lub-dub). Extra sounds may also be present from the filling of the ventricles (gallop sounds), which may signal a heart having difficulty keeping up with the demands placed on it.

21. The author's main purpose in writing this passage is to:
- A. help people prevent heart disease.
  - B. educate readers about the diagnosing of childhood heart disease.
  - C. outline risks and cures for heart disease.
  - D. encourage parents to take their children to pediatric cardiologists.
22. Based on the information in the first paragraph, the terms innocent and functional:
- A. mean the same thing and are used equally often.
  - B. are not the same but are often confused.
  - C. mean the same thing, but innocent is used more often.
  - D. have both been replaced by the word normal.
23. Which of the following statements are true of heart murmurs?
- I. They are most often heard in childhood.
  - II. They are usually caused by structural heart disease.
  - III. They may be heard in almost anyone.
- A. I and II only
  - B. II and III only
  - C. I and III only
  - D. I, II, and III
24. Based on the information provided in the second paragraph, we can infer that:
- A. the human heart is fully functional after eight weeks in the womb.
  - B. congenital heart defects establish themselves in the first eight weeks of pregnancy.
  - C. errors in development during the first eight weeks of pregnancy are common.
  - D. a human heart that develops properly during the first eight weeks in the womb will not encounter problems later in life.
25. Which of the following statements are true of congenital heart defects?
- I. They develop while the child is still in the womb.
  - II. They are the most common type of heart problem in children.
  - III. They are more common when a close relative has been born with a heart defect.
- A. I and II only
  - B. I and III only
  - C. II and III only
  - D. I, II, and III

26. Based on its use in the passage, the word “prevalent” (Paragraph 2, line 1) means:
- A. common.
  - B. prevented.
  - C. diagnosed.
  - D. dangerous.
27. Based on the information in the fourth paragraph, we can infer that a “focused history” is:
- A. a study of all aspects of a child’s life.
  - B. an account of a child’s previous health-related issues.
  - C. a careful look at a family’s tendency toward heart disease.
  - D. a list of symptoms the child has experienced.
28. Based on the information in the passage, which of the following statements is not true?
- A. Chest pain is not a reliable indicator of heart disease in children.
  - B. Fainting is not usually caused by primary heart problems.
  - C. Hypertrophic cardiomyopathy is an inherited condition.
  - D. All children with heart murmurs should be referred to pediatric cardiologists.
29. Which of the following is not part of a pediatric cardiologist’s physical examination of a child?
- A. determining the child’s ability to engage in physical activity
  - B. examining the child’s neck for conspicuous blood vessels
  - C. checking the child’s blood pressure
  - D. assessing the child’s appearance
30. “Gallop sounds,” as discussed in the last paragraph of the passage, are:
- A. normal sounds caused by the filling of the ventricles.
  - B. normal sounds which can be heard with a stethoscope.
  - C. abnormal sounds of valves closing.
  - D. abnormal sounds caused by the heart’s inability to meet the body’s needs.

**SOCIAL SCIENCE:** This passage is adapted from Edward Sapir's book *Language*.

Speech is so familiar a feature of daily life that we rarely pause to define it. It seems as natural to man as walking, and only less so than breathing. Yet it needs but a moment's reflection to convince us that this naturalness of speech is but an illusory feeling. The process of acquiring speech is, in sober fact, an utterly different sort of thing from the process of learning to walk. In the case of the latter function, culture, in other words, the traditional body of social usage, is not seriously brought into play. The child is individually equipped, by the complex set of factors that we term biological heredity, to make all the needed muscular and nervous adjustments that result in walking. Indeed, the very conformation of these muscles and of the appropriate parts of the nervous system may be said to be primarily adapted to the movements made in walking and in similar activities. In a very real sense the normal human being is predestined to walk, not because his elders will assist him to learn the art, but because his organism is prepared from birth, or even from the moment of conception, to take on all those expenditures of nervous energy and all those muscular adaptations that result in walking. To put it concisely, walking is an inherent, biological function of man.

Not so language. It is of course true that in a certain sense the individual is predestined to talk, but that is due entirely to the circumstance that he is born not merely in nature, but in the lap of a society that is certain, reasonably certain, to lead him to its traditions. Eliminate society and there is every reason to believe that he will learn to walk, if, indeed, he survives at all. But it is just as certain that he will never learn to talk, that is, to communicate ideas according to the traditional system of a particular society. Or, again, remove the new-born individual from the social environment into which he has come and transplant him to an utterly alien one. He will develop the art of walking in his new environment very much as he would have developed it in the old. But his speech will be completely at variance with the speech of his native environment. Walking, then, is a general human activity that varies only within circumscribed limits as we pass from individual to individual. Its variability is involuntary and purposeless. Speech is a human activity that varies without assignable limit as we pass from social group to social group, because it is a purely historical heritage of the group, the product of long-continued social usage. It varies as all creative effort varies--not as consciously, perhaps, but none the less as truly as do the religions, the beliefs, the customs, and the arts of different peoples. Walking is an organic, an instinctive, function (not, of course, itself an instinct); speech is a non-instinctive, acquired, "cultural" function.

There is one fact that has frequently tended to prevent the recognition of language as a merely conventional system of sound symbols, that has seduced the popular mind into attributing to it an instinctive basis that it does not really possess. This is the well-known observation that under the stress of emotion, say of a sudden twinge of pain or of unbridled joy, we do involuntarily give utterance to sounds that the hearer interprets as indicative of the emotion itself. But there is all the difference in the world between such involuntary expressions of feeling and the normal type of communication of ideas that is speech. The former kind of utterance is indeed instinctive, but it is non-symbolic; in other words, the sound of pain or the sound of joy does not, as such, indicate the emotion, it does not stand aloof, as it were, and announce that such and such an emotion is being felt. What it does is to serve as a more or less automatic overflow of the emotional energy; in a sense, it is part and parcel of the emotion itself. Moreover, such instinctive cries hardly constitute communication in any strict sense. They are not addressed to any one, they are merely overheard, if heard at all, as the bark of a dog, the sound of approaching footsteps, or the rustling of the wind is heard. If they convey certain ideas to the hearer, it is only in the very general sense in which any and every sound or even any phenomenon in our environment may be said to convey an idea to the perceiving mind. If the involuntary cry of pain which is conventionally represented by "Oh!" be looked upon as a true speech symbol equivalent to some such idea as "I am in great pain," it is just as allowable to interpret the appearance of clouds as an equivalent symbol that carries the definite message "It is likely to rain." A definition of language, however, that is so extended as to cover every type of inference becomes utterly meaningless.

31. The author of the passage states that the "naturalness of speech is but an illusory feeling," meaning that:

- A. the belief that learning to speak is natural is not based on fact.
- B. speech that is natural rather than rehearsed stirs the emotions.
- C. we generally don't stop to think about how speech is acquired.
- D. it is natural to express our feelings through speech.

32. According to the passage, which of the following statements are true of walking?

- I. Walking is a natural ability of human beings.
- II. Culture has a noticeable effect on a child's ability to walk.
- III. Walking results in part from muscular adaptations.

- A. I and II only
- B. I and III only
- C. II and III only
- D. I, II, and III

33. According to the information in the second paragraph, a child born without a society would probably:
- A. never learn to walk.
  - B. never learn to talk.
  - C. eventually learn to walk and to talk.
  - D. create his own spoken language.
34. According to the author, a newborn child transplanted into an alien society will:
- A. walk and talk differently than it would have in its own society.
  - B. walk the same but talk differently than if it had remained in its own society.
  - C. struggle with learning to talk but not with learning to walk.
  - D. be developmentally delayed in all areas.
35. Based on its use in the passage, the word “unbridled” (Paragraph 3, line 6) means:
- A. unspoken.
  - B. slight.
  - C. unrestrained.
  - D. sudden.
36. The author of the passage points out that speech is like religion and art in that it:
- A. is something we learn when we’re very young.
  - B. is practiced by humans only.
  - C. is completely voluntary.
  - D. varies from one society to another.
37. Which of the following is not true of the utterance of sounds caused by pain or joy?
- A. The sounds are non-symbolic.
  - B. The sounds are an involuntary response to emotional energy.
  - C. The sounds are not addressed to anyone in particular.
  - D. The sounds are considered to be a type of speech.
38. When the author states that the sound of pain or the sound of joy “does not stand aloof” (Paragraph 3, line 12), he means that:
- A. it is not always the same sound.
  - B. it has no recognizable meaning separate from the incident.
  - C. it does not indicate that some type of emotion is being felt.
  - D. it does not stand out as unusual.

39. Based on the information in the passage as a whole, we can infer that shouting “Oh!” when you step on something sharp is:

- A. a learned reaction.
- B. a basic form of speech.
- C. as natural as learning to walk.
- D. a result of societal influences.

40. The author’s primary purpose in writing this passage is:

- A. to show how speaking is different from walking.
- B. to prove that speaking is not a natural instinct.
- C. to argue that walking is a biological function of man.
- D. to suggest that every utterance of sound is a form of speech.

## English Test

75 questions, 45 minutes (Answers may be found on Page 190.)

**Directions:** On the answer sheet, fill in the circle that corresponds to the correct answer for each question.

### Passage 1

Before lunch, we divided into teams to build our catapults. Joey, Melissa, and Trevor were on one team, Katie, Tyler, and me were on the other. Our team set down in the grass to formulate a plan. Not one of our team members have ever built a catapult before, so we started out slow. Eventually, though, we figured up what we were doing and got the job done. For the most part, the catapults worked great and so everyone wanted to give it a try. The next hour was full of objects being flown through the air and plenty of cheering from the parents whom were watching. Flying 12 feet at most, we continued to feed baseballs to our catapult. The project made you realize how small-scale our catapults really have been. Although our team's catapult unquestionably threw the farthest. Eventually, we all decided to clean up and get ready to go home. Building catapults helped us learn more than a book.

1.
  - A. NO CHANGE
  - B. team; Katie, Tyler, and me
  - C. team, Katie, Tyler, and I
  - D. team; Katie, Tyler, and I
2.
  - A. NO CHANGE
  - B. sat down in the grass
  - C. set down on the grass
  - D. sat ourselves down on the grass

- 3.
- A. NO CHANGE
  - B. ever have built
  - C. has ever built
  - D. has ever been involved in building
- 4.
- A. NO CHANGE
  - B. off slow. Eventually,
  - C. out slowly, eventually,
  - D. slowly. Eventually,
- 5.
- A. NO CHANGE
  - B. figured out what we were doing
  - C. figured out what it was we were supposed to be doing
  - D. figured up what it was we were doing
- 6.
- A. NO CHANGE
  - B. great, and so everyone
  - C. great so everybody
  - D. great, so everyone
- 7.
- A. NO CHANGE
  - B. give them a try
  - C. give them tries
  - D. give it their best try
- 8.
- A. NO CHANGE
  - B. that were being flown
  - C. that were flying
  - D. flying
- 9.
- A. NO CHANGE
  - B. parents, whom
  - C. parents, which
  - D. parents who

- 10.
- A. NO CHANGE
  - B. Into our catapult we continued feeding baseballs, they flew 12 feet at most.
  - C. As we continued to feed baseballs to our catapult, flying 12 feet at most.
  - D. We continued feeding baseballs to our catapult, which threw them 12 feet at most.
- 11.
- A. NO CHANGE
  - B. was making you
  - C. made us
  - D. makes one
- 12.
- A. NO CHANGE
  - B. had been. Although
  - C. were; however,
  - D. are, although
- 13.
- A. NO CHANGE
  - B. teams' catapult
  - C. teams catapults
  - D. catapult for our team
- 14.
- A. NO CHANGE
  - B. threw further
  - C. threw the furthest
  - D. threw farther
- 15.
- A. NO CHANGE
  - B. reading a book
  - C. we would learn if we read a book
  - D. then any book

## Passage 2

Starting high school is nerve-racking enough with all the new surroundings; in addition, all upperclassmen seem to be a few hundred feet taller than you. Now, <sup>1</sup> its your turn to be the low man on the totem pole. Keep in mind, though, that you <sup>2</sup> will be an upperclassman after your freshman year. Seniority gets better and better year after year until you're the big man up top. When senior year comes, <sup>3</sup> you will become part of the most respected group of the school. Until that day, deal with upperclassmen the best you can, everyone else once had to also. (5) Just <sup>4</sup> how does one avoid and deal with a frightening confrontation with one of these <sup>6</sup> “creatures”? If an upperclassman insults you, makes fun of you, or he <sup>7</sup> embarrasses you in front of the whole student body, deal with it the best you can. <sup>7</sup> Because there is no way of escaping from it. High school should be a fun time <sup>8</sup> in your life. Upperclassmen should be given their two seconds of fun. Then laugh <sup>9</sup> it off and go on with life. One thing that you should not do is make rude jokes or <sup>10</sup> comments about an upperclassman to their face. This will most definitely anger <sup>11</sup> them and lessen their respect for you. Also, making fun of upperclassmen <sup>12</sup> usually just cause problems in the long run. <sup>13</sup> <sup>14</sup>

1.
  - A. NO CHANGE
  - B. hundred feet taller than you
  - C. hundreds of feet taller than you are
  - D. hundreds of feet taller then you are

- 2.
- A. NO CHANGE
  - B. its you're
  - C. it's your
  - D. it's you're
- 3.
- A. NO CHANGE
  - B. big person up top
  - C. big man on the top
  - D. big man on top
- 4.
- A. NO CHANGE
  - B. best way that you can, everyone else
  - C. best you can; everyone else
  - D. best way you can because everybody else in the world
5. To improve the flow of the paragraph, which transitional word/phrase should be added to the beginning of the sentence marked with (5)?
- A. Moreover,
  - B. As a matter of fact,
  - C. For now, however,
  - D. Similarly,
- 6.
- A. NO CHANGE
  - B. do you
  - C. does someone
  - D. can a person
- 7.
- A. NO CHANGE
  - B. you or he embarrasses you
  - C. you, or embarrasses you
  - D. you or causes you to be embarrassed
- 8.
- A. NO CHANGE
  - B. you can,
  - C. way you can. Because
  - D. you can because

- 9.
- A. NO CHANGE
  - B. to escape it
  - C. it is possible to escape it
  - D. for escaping from it
- 10.
- A. NO CHANGE
  - B. You should give upperclassmen their
  - C. Upperclassmen should get they're
  - D. You should let upperclassman have their
- 11.
- A. NO CHANGE
  - B. One thing that it would be better for you not to do is
  - C. It is not a good idea for you to
  - D. You should not
- 12.
- A. NO CHANGE
  - B. an upperclassman to his face
  - C. upperclassmen to their faces
  - D. upperclassmen to their face
- 13.
- A. NO CHANGE
  - B. make their respect less
  - C. make the respect they have less
  - D. lessen whatever respect they have
- 14.
- A. NO CHANGE
  - B. usually just cause a problem
  - C. usually just causes problems
  - D. usually just causes one problem or another
15. Which of the following sentences provides closure for this paragraph?
- A. If you remember this simple advice, you will not have to worry so much about any upperclassmen.
  - B. This advice will help you so you won't be afraid of upperclassmen.
  - C. Starting high school may not be easy, but at least you won't have to worry about upperclassmen if you follow this simple advice.
  - D. Starting high school is full of challenges, but the worst one, dealing with upperclassmen, will not be so bad if you remember this advice.

### Passage 3

Students need a working knowledge of grammar concepts and sentence structure or they  
can't hardly improve as writers. Teachers expected students to be able to edit their  
1 2  
writing for grammatical and mechanical errors, but students can't apply concepts, which  
3  
they don't understand. In order to speak intelligently about and make revisions in writing,  
4  
it is necessary for teachers and students to share a common grammar vocabulary. Writing  
lessons are more productive and students have a strong background in grammar.  
5  
Studies have been showing that more than forty percent of students entering college takes  
6 7  
remedial writing courses. In its National Curriculum Survey, the ACT found that college  
instructors believe that skills related to grammar and usage is important for entering  
8  
college freshmen, however, only 69 percent of the high school English teachers whom  
9 10  
participated in the survey teach grammar concepts at all. ACT vice president for  
development, whose name is Cynthia Schmeiser, recommends that grammar and usage  
11 12  
skills be continually reinforced at all grade levels.  
12

1.
  - A. NO CHANGE
  - B. in order to
  - C. if you want them to
  - D. or they won't be able to
  
2.
  - A. NO CHANGE
  - B. are always expecting
  - C. have expectations for
  - D. expect

- 3.
- A. NO CHANGE
  - B. concepts because
  - C. concepts that
  - D. concepts; after all,
- 4.
- A. NO CHANGE
  - B. speak intelligently about writing and to make revisions in it
  - C. speak with intelligence about writing, and to make revisions in writing
  - D. both speak about writing intelligently and make revisions in writing
- 5.
- A. NO CHANGE
  - B. productive, and
  - C. productive when
  - D. productive since
- 6.
- A. NO CHANGE
  - B. had shown
  - C. show
  - D. have been known to show
- 7.
- A. NO CHANGE
  - B. who enter college and take
  - C. entering college take
  - D. who are entering college are taking
- 8.
- A. NO CHANGE
  - B. usage are important
  - C. usage have been important
  - D. usage has been important
- 9.
- A. NO CHANGE
  - B. freshmen; however, only
  - C. freshmen, however; only
  - D. freshmen, and only

- 10.
- A. NO CHANGE
  - B. teachers, that
  - C. teachers which
  - D. teachers who
- 11.
- A. NO CHANGE
  - B. who's name is
  - C. her name being
  - D. (eliminate the underlined portion)
- 12.
- A. NO CHANGE
  - B. teachers continually reinforce grammar and usage skills
  - C. grammar and usage skills be something teachers continue to reinforce
  - D. teachers reinforce grammar and usage skills on a continuing basis
13. This passage best represents what type of writing?
- A. persuasive nonfiction
  - B. narrative nonfiction
  - C. descriptive fiction
  - D. expository fiction
14. The author of this excerpt wants to add the following sentence: "Placing more emphasis on grammar in high school could lower these numbers significantly." Where should the sentence go?
- A. after the first sentence of the second paragraph
  - B. before the last sentence of the second paragraph
  - C. after the first sentence of the first paragraph
  - D. nowhere in this paragraph
15. Which of the following sentences would be most effective at the end of the second paragraph?
- A. Apparently, Cynthia Schmeiser is a true expert in her field.
  - B. All teachers must make grammar a priority so that students will be more prepared for college.
  - C. There are many different ways that teachers have traditionally taught grammar.
  - D. I hope you all agree that grammar instruction is important.

#### Passage 4

William Wycherley's play *The Country Wife*, although we are certainly entertained by it,  
1  
serves as a vehicle for moral education. (2) It is the play's entertainment value itself that  
makes the moralistic comments more effective than other plays. What may initially seem  
3  
like licentious folly is in fact a very witty and very effective satire, it is anything but  
4  
preachy. Wycherley's audience was guilty of the libertine behavior he is striking out  
against; therefore, he must have known that a direct, finger-pointing approach would be  
5 6  
rejected. In addition, it would be too risky to attack the morals of King Charles II himself  
7 8  
in a direct manner. Instead, Wycherley offers his morals through satire and comedy—  
comedy that his audience could relate to and was understandable. Wycherley also knew  
9  
that if his play was not witty and risqué, he would not gain the attention of the audience,  
10 11  
who was so busy practicing immoral behavior even as the play was being progressed.  
12  
These people did not want to be lectured to; they wanted to be entertained. So  
Wycherley packages his message neatly in wit and humor. Thereby entertaining while he  
13  
teaches. In essence, his audience is swallowing his elixir with a spoonful of sugar.  
People often take sugar with their medicine to make it taste better. Otherwise, his play  
14  
would be doomed before his message ever had a chance of reaching immoral ears.

1.

- A. NO CHANGE
- B. certainly entertaining
- C. it is certainly entertaining
- D. most people find it to be entertaining

2. Which of the following transitional words/phrases can be inserted at the beginning of the sentence marked with (2) to improve the flow of the paragraph?
- A. In other words,
  - B. Although,
  - C. Besides that,
  - D. In fact,
- 3.
- A. NO CHANGE
  - B. than those of other plays
  - C. than that of other plays
  - D. then other plays'
- 4.
- A. NO CHANGE
  - B. satire, for
  - C. satire. Which
  - D. satire, and
- 5.
- A. NO CHANGE
  - B. against, therefore,
  - C. against, therefore;
  - D. against, and so therefore
- 6.
- A. NO CHANGE
  - B. viewers would have rejected a direct, finger-pointing approach
  - C. a direct, finger-pointing approach is one that viewers would have rejected
  - D. an approach that directly pointed fingers would be rejected.
- 7.
- A. NO CHANGE
  - B. would be to
  - C. would have been too
  - D. would of been too
- 8.
- A. NO CHANGE
  - B. King Charles II himself's morals
  - C. the morals of King Charles II himself
  - D. the morals belonging to King Charles II hisself

- 9.
- A. NO CHANGE
  - B. they could understand
  - C. would be understandable
  - D. understand
- 10.
- A. NO CHANGE
  - B. play had not been
  - C. play were not
  - D. play wasn't
- 11.
- A. NO CHANGE
  - B. audience who
  - C. audience; that
  - D. audience that
- 12.
- A. NO CHANGE
  - B. was progressing
  - C. progressed
  - D. had been progressing
- 13.
- A. NO CHANGE
  - B. humor, thereby entertaining
  - C. humor, therefore being entertaining
  - D. human. That way he can provide entertainment
14. What is the best way to revise the sentence?
- A. no revision necessary
  - B. place After all at the beginning
  - C. move it so it comes before the sentence that currently precedes it
  - D. remove it from the paragraph
15. Which sentence would make the most effective conclusion for the paragraph?
- A. All of Wycherley's plays teach important lessons.
  - B. As evidenced by *The Country Wife*, Wycherley knew that a subtle attack is often more effective than a direct one.
  - C. In conclusion, immoral ears were just the ears Wycherley wanted to affect.
  - D. In his play *The Country Wife*, Wycherley provides his audience with a much-needed moral education.

## Passage 5

All of life's experiences are more funner as long as their shared with a friend.

Coffee tastes better when shared with a friend, a walk is shorter when shared with a friend. Chocolate cake even has fewer calories when shared with a friend. True friends which are open-minded, thoughtful, and can be trusted are difficult to find. Finding a friend with these characteristics may be a difficult process. Once you find such a friend, they should be cherished and protected like a prized possession.

Appearance, race, gender, and age are not traits on which to base friendship. (11)

Friendship should be based on the characteristics that matter the most, like shared

interests, genuine personality traits, and similar values. A person who judges others based on appearance rather than on internal qualities are usually disappointed because appearances can be deceiving.

1.
  - A. NO CHANGE
  - B. lifes' experiences
  - C. of the experiences one has in life
  - D. lifes experiences
  
2.
  - A. NO CHANGE
  - B. is more fun
  - C. can be more funner
  - D. are more fun
  
3.
  - A. NO CHANGE
  - B. when
  - C. as long as they're
  - D. whenever their

- 4.
- A. NO CHANGE
  - B. friends, walks are
  - C. a friend; a walk is
  - D. a friend and walks are
5. What is the author's primary purpose for including this sentence?
- A. to add humor to the paragraph
  - B. to reveal a unique way to cut calories
  - C. to make readers think that friends can help them lose weight
  - D. to share another benefit of friendship
- 6.
- A. NO CHANGE
  - B. friends, which
  - C. friends who
  - D. friends, that
- 7.
- A. NO CHANGE
  - B. thoughtful, and trustworthy
  - C. and thoughtful as well as trustworthy
  - D. thoughtful, and capable of being trusted
8. What is the best way to revise this sentence?
- A. leave it as it is
  - B. combine it with the sentence before it
  - C. place the word usually at the beginning of it
  - D. remove it from the paragraph
- 9.
- A. NO CHANGE
  - B. you should cherish and protect them
  - C. cherish and protect them
  - D. you should cherish and protect him or her
- 10.
- A. NO CHANGE
  - B. for the basing of friendship
  - C. you should focus on when deciding on friendships
  - D. one typically bases friendship on

11. Which of the following transitional words/phrases would be most effective at the beginning of the sentence marked with (11)?
- A. Simply stated,
  - B. In addition,
  - C. Instead,
  - D. Moreover,
- 12.
- A. NO CHANGE
  - B. more important characteristics like
  - C. the most important characteristics, which include
  - D. characteristics that matter more, these include
- 13.
- A. NO CHANGE
  - B. qualities is usually
  - C. qualities usually find themselves
  - D. qualities usually are
- 14.
- A. NO CHANGE
  - B. appearances can deceive you
  - C. you can be deceived by appearances
  - D. deception is sometimes in appearances
15. Which of the following sentences would best fit at the end of the last paragraph?
- A. Sometimes you think you're getting one thing and end up getting another.
  - B. When you know what people are really like, you never get deceived.
  - C. Get to know all people regardless of what they look like, and you will be amazed at how many friends you will make.
  - D. Remember that a true friend will always be there for you, so choose your friends carefully.

## Mathematics Test

60 Questions, 60 Minutes (Answers may be found on Page 195.)

- 1) The average (arithmetic mean) of  $x$  and  $y$  is 5, the average of  $x$  and  $z$  is 8, and the average of  $y$  and  $z$  is 11. What is the value of  $z$ ?

- A) 2
- B) 5
- C) 7
- D) 14
- E) 28

- 2) For all integers  $n$ :

$$\boxed{n} = n^2 \quad \text{if } n \text{ is odd}$$

$$\boxed{n} = \sqrt{n} \quad \text{if } n \text{ is even}$$

What is the value of  $\boxed{16} + \boxed{9}$ ?

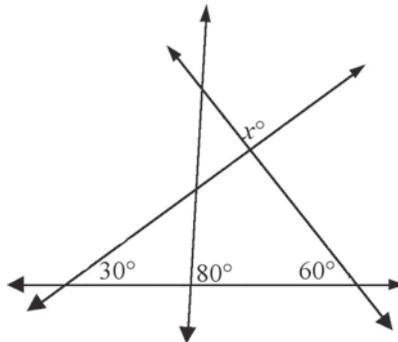
- A) 7
- B) 25
- C) 85
- D) 97
- E) 337

- 3) If  $3x + 7 = 12$ , what is the value of  $6x - 5$ ?

- A) 5
- B) 10
- C) 12
- D) 17
- E) 19

- 4) Find the value of  $x$ .

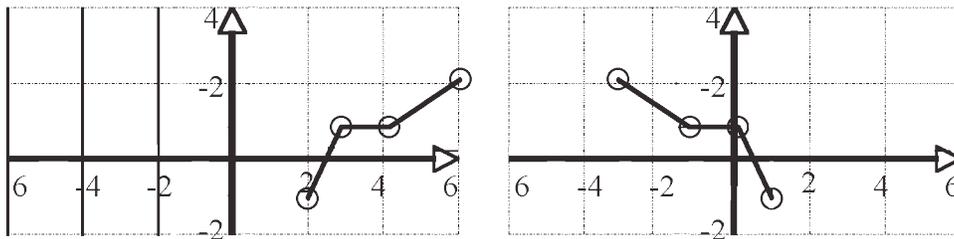
- A) 40
- B) 50
- C) 60
- D) 75
- E) 90



5) If 7 is less than 4 times a certain number is 8 more than the number, what is the number?

- A) -5
- B) -11
- C) 3
- D) 5
- E) 25

6) Given the graphs below of  $f(x)$  and its transformation, find an expression for the transformation in terms of  $f(x)$ .



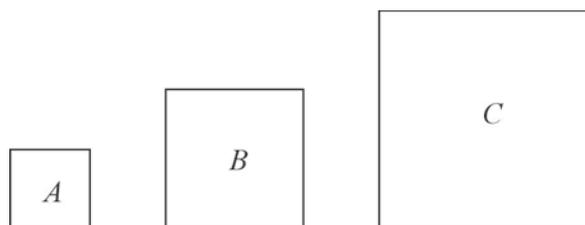
- A)  $f(x + 3)$
- B)  $f(x - 3)$
- C)  $f(-x - 3)$
- D)  $f(-x + 3)$
- E)  $f(-x) + 3$

7) Three nonzero numbers are represented by  $6 \times 2$ ,  $5 \times 2$ , and  $10 \times 2$ . What is the ratio of their sum to their product?

- A)  $\frac{7}{100}$
- B)  $\frac{7x^4}{100}$
- C)  $\frac{7}{100x^3}$
- D)  $\frac{7}{100x^4}$
- E)  $\frac{7}{100x^6}$

8) In the figure, the perimeter of square  $A$  is  $\frac{2}{3}$  the perimeter of square  $B$ , and the perimeter of square  $B$  is  $\frac{2}{3}$  the perimeter of square  $C$ . If the area of square  $A$  is 16, what is the area of square  $C$ ?

- A) 24
- B) 36
- C) 64
- D) 72
- E) 81



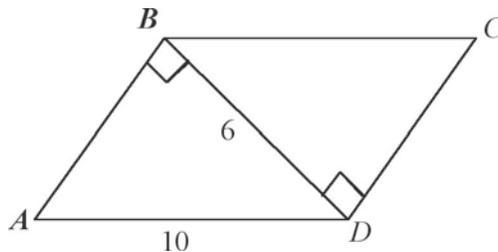
9) Over 12 games, a baseball team scored an average of 6 runs per game. If their average number of runs in the first 10 games was 5 runs per game and they scored the same number of runs in each of the last two games, how many runs did they score during the last game?

- A) 6
- B) 11
- C) 13
- D) 14
- E) 17

10) If  $x \neq 1$ , then  $\frac{x^2 - 2x + 1}{1 - x} = ?$

- A)  $x - 1$
- B)  $1 - x$
- C)  $-1$
- D)  $x + 1$
- E)  $-x - 1$

11) In the parallelogram ABCD,  $BD = 6$  and  $AD = 10$ . What is the area of ABCD?



- A) 24
- B) 30
- C) 48
- D) 60
- E) Cannot be determined from the information given.

12) What is the slope of a line perpendicular to the line represented by the equation  $2x - 8y = 16$ ?

- A)  $-4$
- B)  $-2$
- C)  $-14$
- D)  $14$
- E)  $4$

13) Bill and Tom live 150 miles apart. They each drive to the other's house along a straight road connecting the two houses. Bill drives at a constant rate of 30 miles per hour, while Tom drives at a constant rate of 50 miles per hour. If they leave their houses at the same time, how many miles are they from Tom's house when they meet?

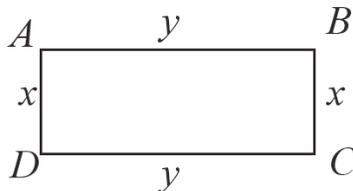
- A) 40
- B) 51.5
- C) 56.25
- D) 75
- E) 93.75

14) What is the equation of the line that passes through the points at (9,5) and (-3, -4)?

- A)  $y = -\frac{1}{12}x + \frac{23}{4}$
- B)  $y = \frac{4}{3}x$
- C)  $y = -\frac{7}{2}x - \frac{3}{4}$
- D)  $y = \frac{3}{4}x - \frac{7}{4}$
- E)  $y = -\frac{4}{3}x$

15) If the perimeter of the rectangle ABCD is equal to  $p$ , and  $x = \frac{1}{5}y$ , what is the value of  $y$  in terms of  $p$ ?

- A)  $\frac{p}{3}$
- B)  $\frac{5p}{12}$
- C)  $\frac{5p}{8}$
- D)  $\frac{5p}{6}$
- E)  $\frac{p}{12}$

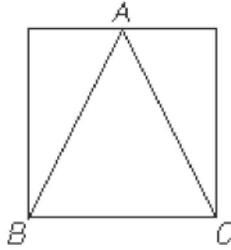


16) If  $b + 2(x - 4) = s$ , what is  $x + 2$  in terms of  $s$  and  $b$ ?

- A)  $\frac{s - b + 12}{2}$
- B)  $\frac{s - b + 6}{2}$
- C)  $\frac{12 - s + b}{2}$
- D)  $\frac{s - b}{2}$
- E)  $\frac{b - s}{2}$

- 17) If AB and AC, inscribed in a square as shown above, are each 10 centimeters in length, what is the area of the square (in centimeters)?

- A)  $40\sqrt{3}$
- B) 74
- C) 80
- D)  $50\sqrt{2}$
- E) 100



- 18) Which of the following represents the values of  $x$  that are solutions to the inequality  $[3x - 4] > 0$ ?

- A)  $-\frac{4}{3} < x < \frac{4}{3}$
- B)  $x < \frac{4}{3}$  or  $x > \frac{4}{3}$
- C)  $x > \frac{4}{3}$
- D)  $x > -\frac{4}{3}$
- E) all real numbers

- 19) If  $s^2 = 19$ , what is the value of  $(s + 1)(s - 1)$ ?

- A)  $\sqrt{19} - 1$
- B)  $\sqrt{19} + 1$
- C) 18
- D) 20
- E) 35

20)

$x$	$f(x)$
$-a$	$b$
$a$	$-b$
$2a$	$c$

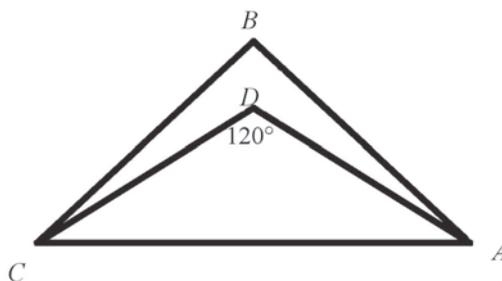
The table above shows some of the values for the function  $f$ . If  $f$  is a linear function, what is the value of the  $x$ -intercept in terms of  $a$ ,  $b$ , and  $c$ ?

- A)  $-2a$
- B)  $a - c$
- C)  $2a - b$
- D)  $b - c$
- E) 0

21) Sarah has at least one quarter, one dime, one nickel, and one penny. If she has three times as many pennies as nickels, the same number of nickels as dimes, and twice as many dimes as quarters, what is the least amount of money she could have?

- A) \$0.41
- B) \$0.48
- C) \$0.58
- D) \$0.61
- E) \$0.71

22) In the figure,  $\overline{DA}$  bisects  $\angle BAC$ , and  $\overline{DC}$  bisects  $\angle BCA$ . If  $\angle ADC = 120^\circ$ , what is the measure of  $\angle B$ ?



Note: Figure is not drawn to scale.

- A) 40
- B) 60
- C) 80
- D) 100
- E) 120

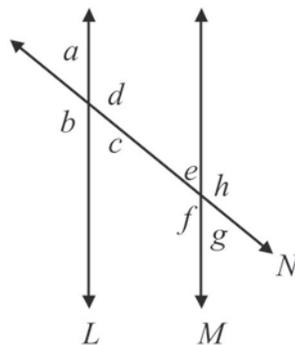
23) A triangle has base 9, and the other two sides are equal. If the side lengths are integers, what is the shortest possible side?

- A) 1
- B) 2
- C) 3
- D) 4
- E) 5

24) A bag contains only white and red marbles. The probability of selecting a red marble is  $\frac{1}{4}$ . The bag contains 200 marbles. If 50 white marbles are added to the bag, what is the probability of selecting a white marble?

- A)  $\frac{1}{3}$
- B)  $\frac{4}{5}$
- C)  $\frac{2}{5}$
- D)  $\frac{2}{3}$
- E)  $\frac{3}{5}$

25) In the figure, line  $L$  is parallel to line  $M$ , with angles  $a, b, c, d, e, f, g,$  and  $h$  as shown. Which of the following lists includes all of the angles that are supplementary to  $\underline{a}$ ?

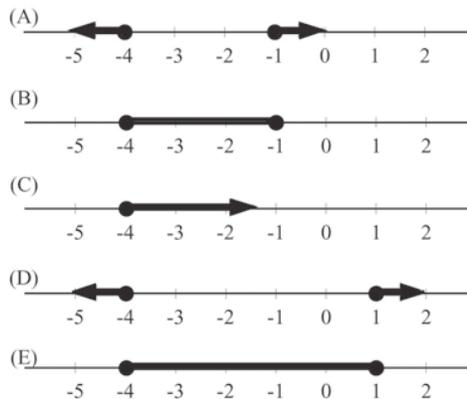


- A)  $b, d,$  and  $c$
- B)  $b, d, f,$  and  $h$
- C)  $c, e,$  and  $g$
- D)  $d, c, h,$  and  $g$
- E)  $e, f, g,$  and  $h$

26) The set  $B$  consists of all even integers between  $-34$  and  $2m$ . If the sum of these integers is 74, what is the value of  $m$ ?

- A) 19
- B) 36
- C) 37
- D) 38
- E) 40

27) Which graph represents the solution set of  $[5 + 2y] \geq 3$ ?



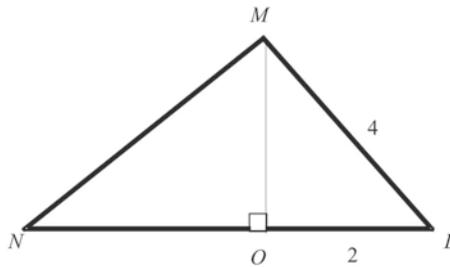
28) A 12-ton mixture consists of  $\frac{1}{6}$  sand,  $\frac{2}{6}$  gravel, and  $\frac{3}{6}$  cement. If  $x$  tons of cement are added, the mixture will contain 60% cement. How many tons of cement need to be added?

- A) 1.2
- B) 3
- C) 3.2
- D) 4
- E) 5.2

29) The points A, B, C, D, and E lie at  $(-4,0)$ ,  $(2,0)$ ,  $(0,4)$ , and  $(0,-5)$ , respectively. Which of the following line segments has the greatest length?

- A)  $\overline{AD}$
- B)  $\overline{BD}$
- C)  $\overline{AE}$
- D)  $\overline{AC}$
- E)  $\overline{CE}$

30) In the figure,  $\overline{MO} \perp \overline{LN}$ ,  $\overline{LO} = 2$ ,  $\overline{MO} = \overline{ON}$ , and  $\overline{LM} = 4$ . How long is  $\overline{MN}$ ?

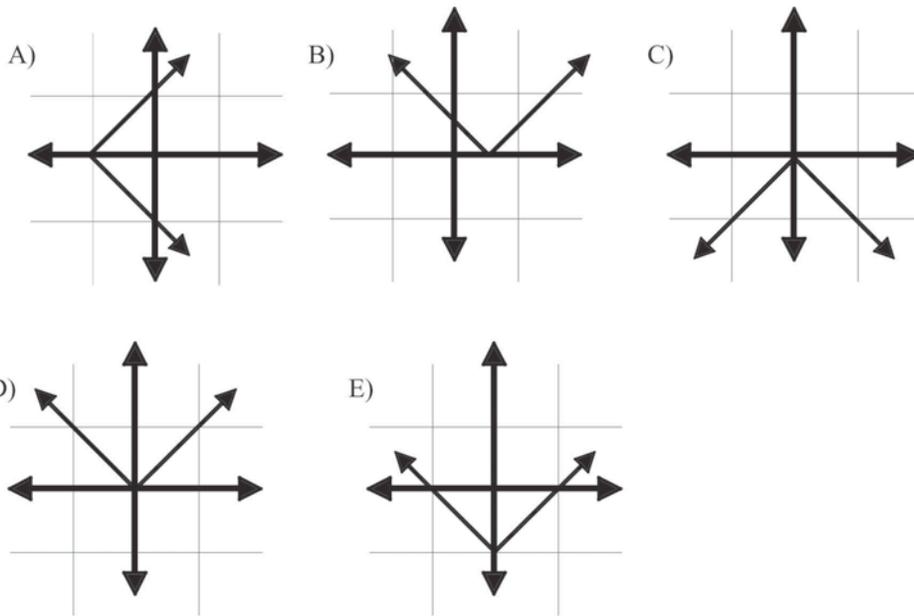


- A)  $2\sqrt{6}$
- B)  $2\sqrt{3}$
- C)  $3\sqrt{2}$
- D)  $3\sqrt{3}$
- E)  $3\sqrt{6}$

31) How many points of intersection exist if the equations  $(x - 5)^2 + (y - 5)^2 = 4$  and  $y = -x$  are graphed on the same coordinate plane?

- A) 0
- B) 1
- C) 2
- D) 3
- E) 4

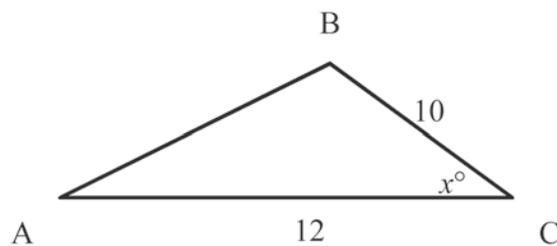
32) Which of the following graphs best represents  $[y] = x + 1$ ?



33) At what point  $(x,y)$  do the two lines with equations  $y = 3x - 1$  and  $10x - 3y = 9$  intersect?

- A) (6,17)
- B) (8,23)
- C) (12,35)
- D) (17,6)
- E) (35,12)

34) In the figure, what is the area of  $\triangle ABC$  in terms of  $x$ ?



- A)  $60 \cos x$
- B)  $120 \cos x$
- C)  $24 \tan x$
- D)  $60 \sin x$
- E)  $120 \sin x$

35) If  $16^{x^2-1} = 64^x$ , what is  $x$ ?

- A)  $\frac{2}{3}$
- B)  $-\frac{2}{3}$
- C)  $\frac{3}{2}$
- D)  $-\frac{3}{2}$
- E)  $-\frac{1}{2}$

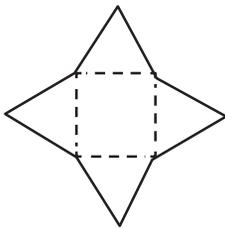
36) In the  $xy$ -plane, the equation of line  $l$  is  $y = -3(x + 2)^2 + 4$ . If line  $m$  is the reflection of line  $l$  in the  $y$ -axis, what is the equation of line  $m$ ?

- A)  $y = 3(x - 2)^2 - 4$
- B)  $y = -3(x - 2)^2 + 4$
- C)  $y = 3(x + 2)^2 + 4$
- D)  $y = 3(x + 2)^2 - 4$
- E)  $y = 3(2 - x)^2 + 4$

37) What is the value of  $(x + y)^2$  if  $x^2 + y^2 = 10$  and  $4xy = 8$ ?

- A) 4
- B) 10
- C) 14
- D) 18
- E) 100

38)



If the area of the square in the figure above is 81 and the perimeter of each of the 4 triangles is 30, what is the perimeter of the figure outlined by the solid line?

- A) 68
- B) 74
- C) 76
- D) 81
- E) 84

39) If  $rstv = 1$  and  $stuv < 0$ , which of the following must be true?

- A)  $r > 0$
- B)  $s < 1$
- C)  $t < 0$
- D)  $u \neq 0$
- E)  $v \neq 1$

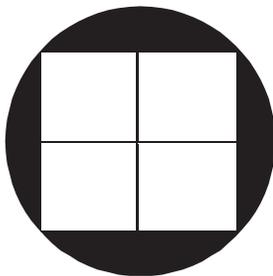
40) The center of a circle is  $M(0,2)$ , and the endpoint of one of its radii is  $A(-6,-4)$ . If  $\overline{AB}$  is a diameter, what are the coordinates of  $B$ ?

- A)  $(8,6)$
- B)  $(6,8)$
- C)  $(4,6)$
- D)  $(6,4)$
- E)  $(8,10)$

41) There are eight sections of seats in an auditorium. Each section contains at least 300 seats but not more than 400 seats. Which of the following could be the number of seats in the auditorium?

- A) 1600
- B) 2000
- C) 2200
- D) 2600
- E) 3400

42)



In the figure above, each square is tangent to the containing circle at only one point. If the area of each square is  $x^2$ , what is area of the shaded region in terms of  $x$ ?

- A)  $(\pi-2)x^2$
- B)  $(\pi-4)x^2$
- C)  $(4-\pi)x^2$
- D)  $(\pi-1)x^2$
- E)  $2(\pi-2)x^2$

43) How many complete tanks of water, each with a capacity of 3 cubic meters, are needed to fill an empty cylindrical tank whose height is 3 meters and whose base has a radius of 2 meters?

- A) 12
- B) 13
- C) 14
- D) 15
- E) 16

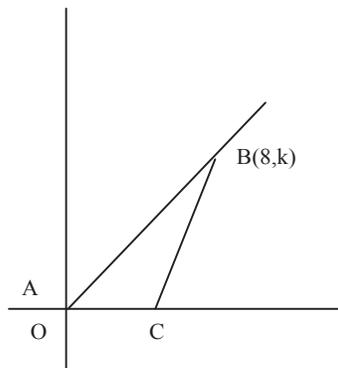
44) During a game, the green team scored one-eighth of its points in the first quarter, one-third in the second quarter, one-fourth in the third quarter, and the remaining points in the fourth quarter. If its total score for the game was 48, how many points did the green team score in the fourth quarter?

- A) 18
- B) 14
- C) 12
- D) 10
- E) 7

45) At a meat processing plant, chickens are accepted only if they weigh between 3.65 and 4.35 pounds. If the plant accepts a chicken weighing  $m$  pounds, which of the following describes all possible values of  $m$ ?

- A)  $[m - 4] = 0.35$
- B)  $[m + 4] = 0.35$
- C)  $[m - 4] < 0.35$
- D)  $[m + 4] < 0.35$
- E)  $[m - 4] > 0.35$

46) In the figure below, if  $\overline{AB} = 10$ , what is the value of  $k$ ?



- A) 6
- B) 8
- C) 10
- D) 12
- E) 18

47) 3, 7, -7, ...

The first term in the sequence of numbers shown above is 3. Each even-numbered term is 4 more than the previous term, and each odd-numbered term after the first is  $-1$  times the previous term. For example, the second term is  $3 + 4$ , and the third term is  $(-1) \times 7$ . What is the 155th term of the sequence?

- A)  $-7$
- B)  $-3$
- C)  $1$
- D)  $3$
- E)  $7$

48) Dwayne has a woodshop from which he builds and sells birdhouses. He sells each birdhouse for  $k$  dollars. Out of this, one-third is used to pay for the lumber and supplies, and he saves the rest of the money. In terms of  $k$ , how many birdhouses must Dwayne sell to save \$5000?

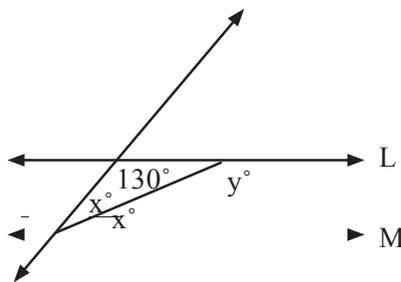
- A)  $\frac{15000}{k}$
- B)  $\frac{k}{15000}$
- C)  $\frac{k}{7500}$
- D)  $\frac{7500}{k}$
- E)  $7500k$

49) The sum of 12 consecutive integers is 5,250. What is the value of the greatest of these integers?

- (A) 440
- (B) 441
- (C) 442
- (D) 443
- (E) 444

50) If  $L$  is parallel to  $M$ , what is the value of  $y$ ?

- (A) 25
- (B) 50
- (C) 130
- (D) 155
- (E) 180

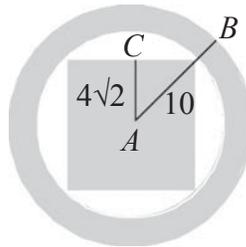


51) The table below gives values of the quadratic function  $h$  for selected values of  $x$ . Which of the following defines  $h$ ?

$x$	0	1	2	3
$h(x)$	1	2	5	10

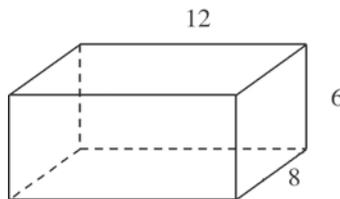
- A)  $h(x) = x^2 + 1$
- B)  $h(x) = x^2 + 2$
- C)  $h(x) = 2x^2 - 2$
- D)  $h(x) = 2x^2 - 1$
- E)  $h(x) = 2x^2 + 1$

52) In the figure below with point  $A$  as its center,  $AB = 10$  and  $AC = 4\sqrt{2}$ . What is the area of the shaded region?



- A)  $100\pi + 32$
- B)  $64\pi + 128$
- C)  $36\pi + 128$
- D)  $100\pi + 128$
- E)  $36\pi + 32$

53) What is the surface area of the rectangular prism below?



- A) 312
- B) 336
- C) 360
- D) 384
- E) 432

54) What is  $\cos \theta$  if  $\tan \theta = \frac{4}{3}$ ?

- A)  $\frac{3}{5}$
- B)  $\frac{3}{5}$
- C)  $\frac{4}{5}$
- D)  $\frac{5}{4}$
- E)  $\frac{5}{4}$

55) A coin was flipped 24 times and came up heads 14 times and tails 10 times. If the first and the last flips were both heads, what is the greatest number of consecutive heads that could have occurred?

- A) 7
- B) 9
- C) 10
- D) 13
- E) 14

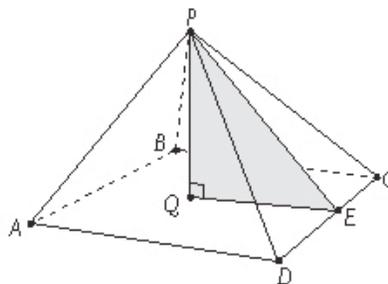
56) If a rectangle's length and width are both doubled, by what percent is the rectangle's area increased?

- A) 50
- B) 100
- C) 200
- D) 300
- E) 400

57) If a small cube (side = 2 inches) is filled twice and poured into a larger cube (side = 4 inches), what will be the height, in inches, of the water in the larger cube?

- A)  $\frac{1}{4}$ "
- B)  $\frac{1}{2}$ "
- C) 1"
- D) 2"
- E) 4"

58) Point  $Q$  lies at the center of the square base ( $ABCD$ ) of the pyramid below. The pyramid's height ( $PQ$ ) measures exactly one-half the length of each edge of its base, and point  $E$  lies exactly halfway between  $C$  and  $D$  along one edge of the base. What is the ratio of the surface area of any of the pyramid's four triangular sides to the surface area of the shaded triangle?



- A)  $3 : \sqrt{2}$
- B)  $\sqrt{5} : 1$
- C)  $\sqrt{3} : 3$
- D)  $2\sqrt{2} : 1$
- E)  $8 : \sqrt{5}$

59) Simplify  $\frac{\sin^2\theta}{1 - \cos^2\theta}$ .

- A)  $\sin \theta$
- B)  $\cos \theta$
- C)  $\tan \theta$
- D) 0
- E) 1

60) On the  $xy$ -coordinate plane, all of the following points lie on the circumference of a circle whose radius is 10 and whose center lies at  $(-1, 0)$  EXCEPT:

- A)  $(-6, -7)$
- B)  $(-2, 3\sqrt{11})$
- C)  $(-1, -10)$
- D)  $(7, 6)$
- E)  $(1, -4\sqrt{6})$

## Science Test

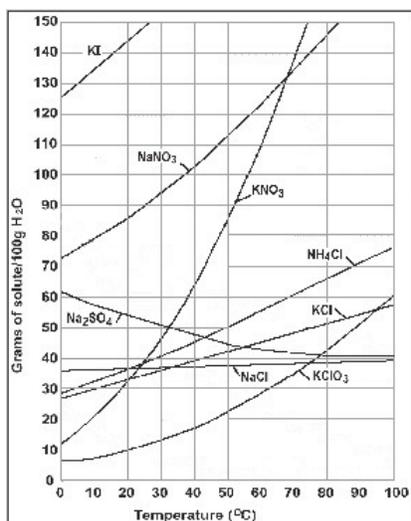
40 Questions, 35 Minutes (Answers may be found on Page 212.)

**Directions:** Each of the following seven passages is followed by several questions. After reading each passage, decide on the best answer to each question and fill in the corresponding oval on your answer sheet. You are allowed to refer to the passages while answering the questions. Calculator use is not allowed on this test.

### Passage I

Solubility refers to the ability for a given substance, the solute, to dissolve in a solvent. It is measured in terms of the maximum amount of solute dissolved in a solvent at equilibrium. The resulting solution is called a saturated solution. Under various conditions, the equilibrium solubility can be exceeded to give a so-called supersaturated solution.

The solubility of a given solute in a given solvent often depends on temperature. The chart below shows solubility curves for some typical inorganic salts.



1) How many grams of  $\text{NH}_4\text{Cl}$  can be dissolved in 200 g of water at  $70^\circ\text{C}$ ?

- A. 70
- B. 90
- C. 120
- D. 180

2) As the temperature of a solution of  $\text{NaNO}_3$  is increased, what trend in its solubility is observed?

- A. Decreasing solubility
- B. Increasing solubility
- C. No change in solubility
- D. Temperature does not affect its solubility.

3) At what temperature is the solubility of sodium chloride (NaCl) the same as the solubility of potassium chloride (KCl)?

- A) 22
- B) 30
- C) 34
- D) 40

4) What compound is least soluble at 40 °C?

- A) NaCl
- B) KCl
- C) Na<sub>2</sub>SO<sub>4</sub>
- D) NH<sub>4</sub>Cl

5) Which procedure will increase the solubility of KCl in water?

- A) stirring the solute and solvent mixture
- B) increasing the surface area of the solute
- C) raising the temperature of the solvent
- D) increasing the pressure on the surface of the solvent

### ***Passage II***

Lasers emitting visible light are the most common; the basic principles are applicable through much of the electromagnetic spectrum. The first stimulated emission of light was achieved in the microwave region of the spectrum, but now lasers are available that emit light in the ultraviolet, infrared, and x-ray spectral regions. Lasers range in continuous power output from less than a milliwatt to many kilowatts, and some produce trillions of watts in extremely short pulses.

### ***Experiment 1***

Research shows that a photon of light is absorbed by an atom in which one of the outer electrons is initially in a low-energy state. The energy of the atom is raised to the upper energy level and remains in this excited state for a period of time — typically less than 1 microsecond. The electron then drops to the lower state through the emission of a photon of light. The best energy output is achieved when, for every photon absorbed, another is emitted.

### ***Experiment 2***

If an atom or molecule lies in an energy state that is higher than the lowest, or ground level, state, it can spontaneously drop to a lower level without any outside stimulation. One possible result of dropping to a reduced energy state is the release of the excess energy (equaling the difference in the two energy levels) as a photon of light.

### ***Experiment 3***

Electrons can be bumped up to higher energy levels by the injection of energy — for example, by a flash of light. When an electron drops from an outer to an inner level, “excess” energy is given off as light. The wavelength or color of the emitted light is precisely related to the amount of energy released.

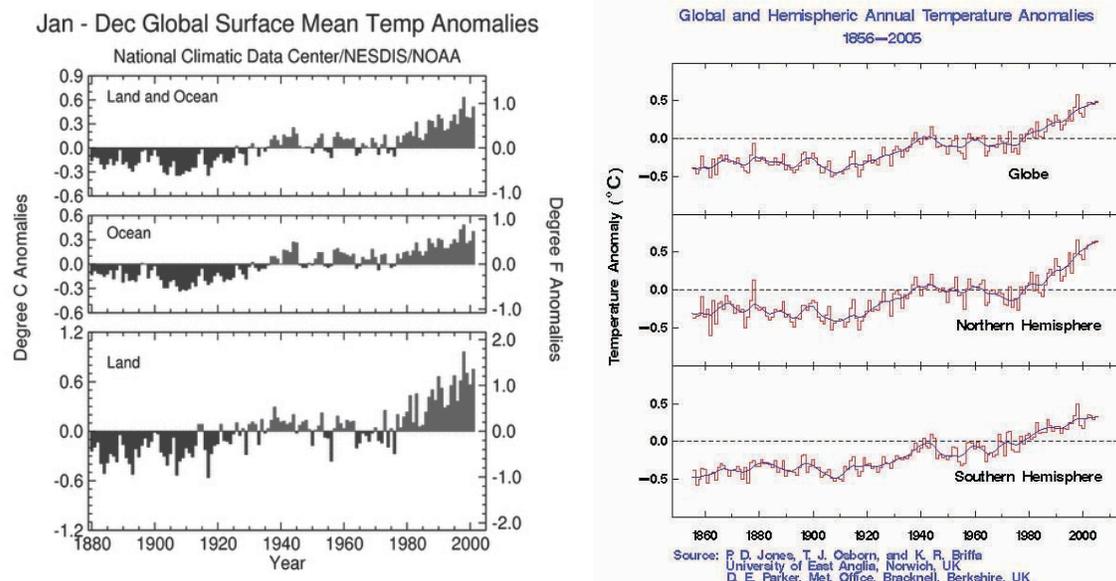
- 6) Which of the following is not supported by the experimental results?
- A) The amount of energy released when an electron drops energy levels correlates with the wavelength of the light emitted.
  - B) Lasers can emit only visible light.
  - C) Lasers can emit light in the ultraviolet, infrared, and x-ray spectral regions.
  - D) A photon of light can be absorbed by an atom.
- 7) Energy can be directly released from all of the following EXCEPT:
- A) an electron in an excited state
  - B) an outer electron in a higher energy state
  - C) an electron in its lowest energy state
  - D) a photon being emitted
- 8) Which experiment concerns atoms or molecules initially in a lowered energy state?
- A) Experiments 1 and 2
  - B) Experiments 2 and 3
  - C) Experiments 1 and 3
  - D) None of the experiments
- 9) Which experiment(s) DO(ES) NOT require any energy to be added in order for energy to be given off?
- A) Experiment 1
  - B) Experiment 2
  - C) Experiment 3
  - D) All of the experiments
- 10) What experiments offer the most promise for use in lasers (assuming the same atom or molecule)?
- A) Experiments 2 and 3
  - B) Experiments 1 and 2
  - C) Experiments 1 and 3
  - D) Unable to determine based on information given.

11) What is the most telling way to determine if an atom or molecule has the best potential for use in lasers?

- A) Measure the energy level of the initial atom or molecule.
- B) Measure the energy level of its excited state.
- C) Measure the amount of energy added to reach its excited state
- D) Measure the amount of energy given off after going from its excited state to a lower energy level.

### Passage III

The graphs below show annual mean global temperature anomalies over the period 1880-2001. The zero line represents the long-term mean temperature from 1880-2001, and the bars or lines show annual departures from that mean.



12) What overall trend, in terms of global temperature, is observed since the beginning of the 20<sup>th</sup> century?

- A) decreasing temperature
- B) increasing temperature
- C) no change
- D) increasing, then decreasing temperature

- 13) Which of the following statements is true based on the data in the graph?
- A) Ocean temperatures have greater anomalies than those observed on land.
  - B) Ocean temperatures have the same amount of anomalies as those observed on land.
  - C) Land temperatures have greater anomalies than those observed in the ocean.
  - D) Cannot be determined by the information given.
- 14) What hypothesis best explains the reason for the observed differences in the data between the ocean and land?
- A) Land absorbs heat from the oceans.
  - B) Water heats up and cools down faster than land.
  - C) There is more ocean than land on Earth's surface.
  - D) Land heats up and cools down faster than water.
- 15) According to the second graph, which hemisphere is experiencing the greater temperature anomalies since the 1980s?
- A) Northern
  - B) Southern
  - C) Both
  - D) Neither
- 16) Which year marks the change from negative °C anomalies to positive °C anomalies?
- A) 1880
  - B) 1900
  - C) 1940
  - D) 2000

#### ***Passage IV***

The developmental period for *Drosophila melanogaster* (fruit fly) varies with temperature, as with many ectothermic species. The shortest development time (egg to adult), 7 days, is achieved at 28 °C. Development times increase at higher temperatures (30 °C, 11 days) due to heat stress. Under ideal conditions, the development time is 8.5 days at 25 °C, 19 days at 18 °C and over 50 days at 12 °C. Under crowded conditions at 25 °C, development time increases to 12 days, and the emerging flies are smaller. Females lay some 400 eggs (embryos), about five at a time, into rotting fruit or other suitable material such as decaying mushrooms and sap fluxes. The eggs, which are about 0.5 mm long, hatch after 12-15 h (at 25 °C). The resulting larvae grow for about 4 days (at 25 °C) while molting twice (into 2nd- and 3rd-instar larvae), at about 24 and 48 h after eclosion. During this time, they feed on the microorganisms that decompose the fruit, as well as on the sugar of the fruit themselves. Then the larvae encapsulate in the puparium and undergo a four-day-long metamorphosis (at 25 °C), after which the adults emerge.

- 17) What trend best describes the information observed for the development of fruit flies over the temperature range between 12 °C and 28 °C?
- A) As the temperature increases, the development time decreases.
  - B) As the development time increases, the temperature decreases.
  - C) As the temperature increases, the development time decreases.
  - D) As crowding increases, the development time decreases.
- 18) About how long would it take for development to occur under extremely crowded conditions at 35 °C?
- A) 5 days
  - B) 8 days
  - C) 11 days
  - D) 18 days
- 19) A female fly lays 400 eggs, and half the eggs are allowed to develop at 12 °C while the other half are subjected to extreme crowding at 25 °C. What would you expect to observe in the two groups, based on their development conditions?
- A) The second group to hatch would be extremely small.
  - B) The first group would be normal sized.
  - C) The two groups would hatch at the same time.
  - D) The first group to hatch would be able to have offspring before the second group even hatched.
- 20) If a female fly that is smaller than normal due to development under crowded conditions were to lay eggs under ideal conditions, the flies that hatch would:
- A) be normal sized.
  - B) be smaller than normal flies.
  - C) take 12 days to develop.
  - D) take 50 days to develop.
- 21) A female fly lays 400 eggs. The eggs initially begin their development under ideal conditions but are forced into crowded conditions for the remainder of development. What can be said about the size of the flies?
- A) The flies will be smaller.
  - B) The flies will be normal sized.
  - C) Half will be normal sized and half will be smaller.
  - D) There is not enough information to determine the outcome.

## *Passage V*

Isopods (pillbugs or woodlice) demonstrate negative photo taxis. They have light receptors that are sensitive to general illumination, and they make directed movements away from areas of greater illumination toward dark areas. Isopods are classic examples of animals which orient to humidity gradients via kinesis (more specifically, orthokinesis); that is, there is a change in the general level of movement with a change in the stimulus intensity. Stress that orients via kinesis is due to a ***non-directed*** locomotory activity. Isopods increase their movements under dry conditions and wander quite randomly, decreasing their activity under preferred humidity. Because of this decrease in activity in areas of high humidity, isopods tend to accumulate or aggregate in damp places. Taxis, on the other hand, is a direct orientation of an organism in response to a stimulus. Kinesis indicates a variation in intensity of locomotor activity that is dependent on the intensity of the stimulation, not its direction. These exercises demonstrate taxis and kinesis separately and in combination.

### *Experiment 1*

A moist paper was placed under a paper box (opaque box) at one end of a tray and a dry paper under a second paper box at the other end of the tray. The trays were illuminated as evenly as possible with an available light source. Ten isopods were then introduced to the center of the tray and allowed to wander for 30 minutes. The isopods were observed during this period, and the number of exits from each box was recorded. At the end of 30 minutes, the number of animals that were still wandering and the number under each box were recorded. It was observed that the isopods initially began moving and then began to aggregate under the box with the moist paper.

### *Experiment 2*

The experiment was set up as in the first experiment, but the tray was covered with aluminum foil so that the isopods were completely in the dark. It was observed that the isopods were moving less initially, but ultimately aggregated under the moist box.

### *Experiment 3*

A moist paper was placed under a plastic box (transparent box), and a dry paper was placed under a paper box. It was observed that the isopods began to aggregate around the paper box.

### *Experiment 4*

A dry paper was placed under both a plastic box and a paper box. It was observed that the isopods aggregated under the paper box.

- 22) In which experiment does humidity NOT play a factor?
- A) Experiment 1
  - B) Experiment 3
  - C) Experiment 4
  - D) Experiment 2
- 23) Which experiment tests whether light or humidity is a stronger stimulus?
- A) Experiment 2
  - B) Experiment 4
  - C) Experiment 1
  - D) Experiment 3
- 24) What should be observed if a moist paper were placed under both the plastic and paper boxes in the presence of light?
- A) The isopods would aggregate under the plastic box.
  - B) The isopods would aggregate under the paper box.
  - C) The isopods will not move at all because of the high humidity.
  - D) The isopods will aggregate evenly between the two boxes.
- 25) All of the conditions for response below are tested in the experiments EXCEPT:
- A) light with no humidity difference
  - B) light with humidity difference
  - C) humidity with no light
  - D) dark with no humidity difference
- 26) In what conditions do isopods prefer to exist?
- A) low humidity and low light intensity
  - B) high humidity and high light intensity
  - C) high humidity and low light intensity
  - D) low humidity and high light intensity
- 27) To which stimulus should the isopods have a stronger response?
- A) Humidity
  - B) Light
  - C) Neither; the two elicit equivalent responses.
  - D) More information is needed.

## *Passage VI*

Until recently, people simply knew that dinosaurs went extinct — their fossils were found throughout the Mesozoic era but were not located in the rock layers (strata) of the Cenozoic era. So, we knew that dinosaurs went extinct some 64-66 million years ago, but that was all. Many wild ideas about how the dinosaurs were rendered extinct were presented over the years.

Few satisfactory answers to the mystery behind the extinction of dinosaurs were offered for a convincing mechanism for the “K-T extinction” (meaning the extinction of dinosaurs at the boundary between the Cretaceous period (K) and the Tertiary period (T)). However, it was noticed that at or near the K-T boundary (the layers of the earth mantle corresponding to this time frame) in several places around the globe, there exists a thin layer of clay with an unusually high iridium (a rare metal similar to platinum) content.

### *Scientist 1*

Major changes in the organization of the continental plates (continental drift) were occurring at the K-T boundary. This led to increased volcanism that could have created enough dust and soot to block out sunlight, producing the climatic change. In India during the Late Cretaceous period, huge volcanic eruptions were spewing forth floods of lava, which can be seen today at the K-T boundary (these ruptures in the Earth’s surface are called the Deccan traps). The chemical composition of the lava rocks in India shows that they originated in the Earth’s mantle, which is also relatively rich in iridium. This richness would explain the iridium layer.

### *Scientist 2*

The original hypothesis is the basis for several subsequent variations on the theme that a large extraterrestrial object collided with the Earth, its impact throwing up enough dust to cause the climatic change. The iridium layer is what proves that an asteroid impact could be responsible for the extinction — asteroids and similar extraterrestrial bodies are higher in iridium content than the Earth’s crust, so scientists figured that the iridium layer must be composed of the dust from the vaporized meteor. No crater was found, but it was assumed that one existed that was about 65 million years old and 100 kilometers (about 65 miles) in diameter.

28) Scientist 1 mentions all of the following as possible causes of dinosaur extinction EXCEPT:

- A) volcanoes
- B) continental drift
- C) global climate change
- D) celestial bodies

29) A major difference between the theories of Scientist 1 and Scientist 2 is that Scientist 2 believes that:

- A) a gradual climate change occurred, whereas Scientist 1 believes a rapid climate change occurred.
- B) an extraterrestrial catastrophe caused the climate change, whereas Scientist 1 believes continental drift was the cause for numerous volcanic eruptions.
- C) volcanoes are not powerful enough to cause a global climate change, whereas Scientist 1 believes that volcanoes are powerful enough.
- D) the iridium can only come from an extraterrestrial source, whereas Scientist 1 believes that asteroids do not contain enough iridium to account for the amount in the K-T boundary.

30) Which of the following, if true, could best defend Scientist 2's theory against criticism voiced by Scientist 1?

- A) To date, no reliable evidence for any celestial bodies that may have crossed paths with Earth has been found.
- B) The fossil record suggests that several marine species died out millions of years prior to the K-T extinction.
- C) Celestial bodies, such as asteroids, contain higher concentrations of iridium than are present in the Earth's mantle.
- D) Strong evidence exists for a series of volcanic eruptions during the K-T extinction.

31) Which discovery would support the hypothesis of Scientist 1?

- A) The presence of iridium was part of an unrelated phenomenon.
- B) Mass extinction occurred gradually over several million years.
- C) The K-T boundary was found to have millions of Deccan traps throughout various parts of the world.
- D) A massive crater was discovered recently on the Yucatan Peninsula in Mexico.

32) Both scientists agree that what was the underlying cause of the extinction of the dinosaurs?

- A) A massive asteroid collided with the Earth, killing all the dinosaurs.
- B) A massive climate change resulted in the extinction of the dinosaurs.
- C) Volcanic eruptions resulted in the death of the dinosaurs.
- D) Iridium was the cause for the massive extinction.

33. The following statement supports which Scientist's theory?

*"The marine fossil record does support a slightly rapid decline, while the terrestrial record (especially in North America) strongly suggests a more gradual decline...."*

- A) Scientist 1
- B) Scientist 2
- C) Both
- D) Neither

34) What data below could ultimately provide the best information to prove or disprove one of the arguments?

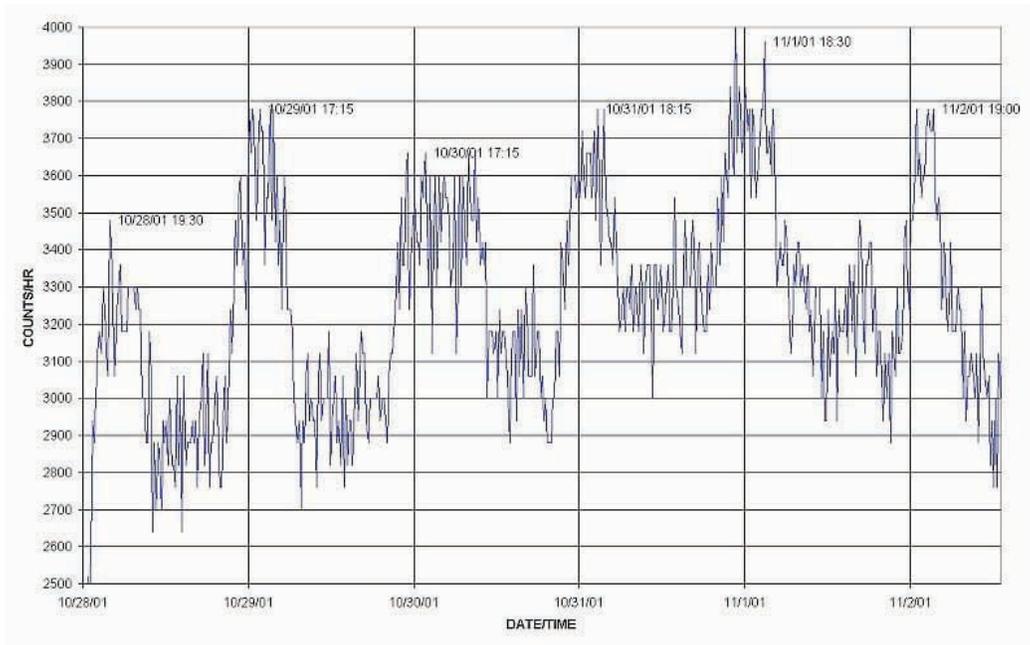
- A) a more detailed astronomical history
- B) a more detailed fossil history
- C) an accurate mapping of continental drift
- D) all of the above

### ***Passage VII***

A number of studies indicate that radon emissions occur from the earth. Some suggest that these emissions might serve as earthquake precursors. This study was undertaken to characterize the variability of radon emissions from the earth. The questions posed were: 1) Are there "bursts" or abrupt increases of radon gas? 2) Are these "bursts" random or periodic? 3) Are these abrupt increases distinguishable from the continual release of radiation from the earth? and 4) Is the sensor selected for this study suitable for this purpose?

### ***Experiment 1***

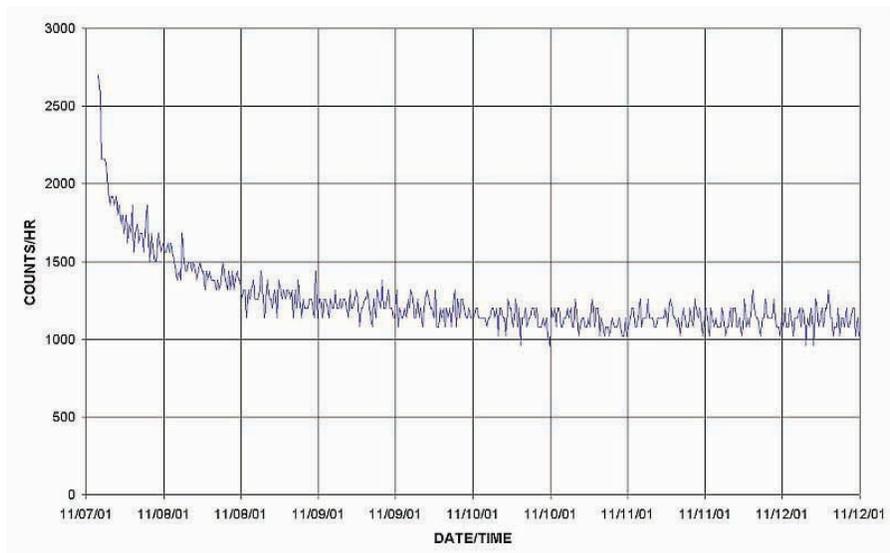
A 20- to 22-inch hole with a 6-inch diameter was prepared in the ground. A RM-60 Microroentgen Radiation Monitor (Aware Electronics Corp., P.O. Box 4299, Wilmington, Delaware 19807) was placed in the hole about 6 inches from the bottom. Because this sensor is sensitive to radiation from a variety of sources, we compared its response under a number of conditions to show that the radon signal was distinguishable from other sources of background radiation. The sensor was placed with the window facing down, 6 inches above the bottom. Data were collected using Aware software.



<u>Time Period (dates)</u>	<u>Number of Days</u>	<u>Average Interval Between Peaks (hours)</u>
10/22-11/2/01	11	24 hours 20 min.
11/14-11/19/01	5	24 hours 15 min.

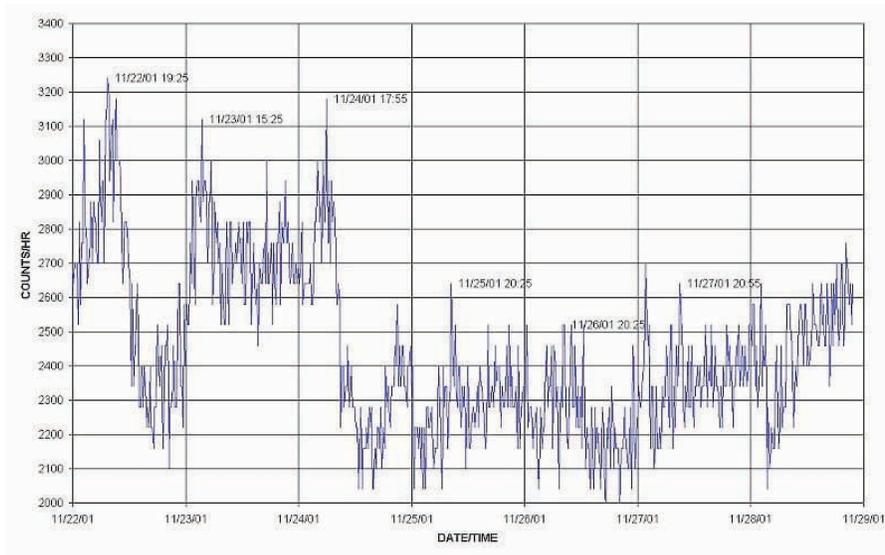
### *Experiment 2*

The sensor was suspended 5 feet above the ground in a small greenhouse close to the original hole described in experiment 1. It was observed that the counts diminished in amplitude to a very narrow band of 1100 to 1200 counts/hour (see figure below). No sign of periodicity was evident.



### Experiment 3

Using the initial experimental setup with the sensor in the hole, on November 24, 1 inch of rain fell between 8 a.m. and 8 p.m. No clearly defined sign of periodicity remained. Interestingly, the range in the amplitude of the counts per hour remained essentially the same, even though the periodicity of the peak emissions was not evident.



35) What information can be learned from Experiment 2?

- A) Heavy rain directly influences the signal.
- B) The periodicity was independent of the position of the sensor.
- C) The periodicity is directly related to the position of the sensor.
- D) The signal is independent of all outside factors.

36) It is well known that water-saturated soil restricts the exchange of gases between soil and the atmosphere. Thus, when soil gas is not being released under these conditions, observed radiation counts are reduced. Which experiment(s) would this information support?

- A) Experiment 1
- B) Experiment 2
- C) Experiment 3
- D) Experiments 2 and 3

37) The goal of which experiment was to answer each of the initially posed questions during its design?

- A) Experiment 2
- B) Experiment 1
- C) Experiment 3
- D) None

38) Which of the three experiments would be considered the control?

- A) Experiment 1
- B) Experiment 2
- C) Experiment 3
- D) There is no control.

39) The sensor was placed in a hole that was wet from rain. The soil then dried over the duration of the measurements. What result should be expected?

- A) exactly the same as previously observed
- B) a signal showing a gradual recovery to periodicity
- C) a signal similar to that observed in Experiment 2
- D) a signal similar to that observed in Experiment 1

40) What conclusion can be made based on the experiments?

- A) Periodicity observed in radiation counts is due to radon emissions from soils.
- B) Periodicity observed in radiation count is due to other gas emissions from soils.
- C) Radon emissions do not display a periodic cycle of 24 hours.
- D) The experiments failed to answer the initially posed questions.

## Chapter Seven: Answers to Practice Test

**Answers to Reading Test** (Test may be found on Page 123.)

1. A.

Although it seems as though it's morning (bacon, coffee, waking up), context clues within the passage tell us it's late in the day. Later in the passage, the father refers to the meal as supper.

2. B.

The topic of conversation changes abruptly from the preceding paragraph to the following one.

3. A.

You must read on further to determine how the boy feels here. He has tried so hard to prepare a meal and desperately wants his father to be pleased with it. The passage provides no evidence for the other choices.

4. D.

Although the other choices all point to physical or emotional strain, a man need not be unhealthy to look silently at a lake.

5. A.

The boy tries to laugh, but he feels sorrow because the entire meal is ruined. Also, he apologizes in the next paragraph. Choice C seems possible, but based on his attitude toward his father up to this point in the passage, we know that he feels comfortable around his father and just wants very much to please him.

6. B.

It is necessary to look at the preceding paragraphs to understand the meaning of this line. The father holds back any comments that might hurt the boy's feelings and "lightly" says he's eager to eat some of his supper. The fact that he doesn't eat much confirms that the comment is intended to spare the boy's feelings.

7. C.

Choices A and B are not metaphors, so they can be eliminated. Choice D, although a metaphor, does not refer to any type of struggle on the father's part. Choice C provides an outward sign of his internal struggle. His face is a battleground as the battle is waged inside him.

8. C.

When the boy is moved by the sunset, he plays his violin, which the author says is "no new thing."

9. B. Plenty of foreshadowing throughout the passage suggests that something bad will happen. Although the boy is cheerful, the reader realizes that it is only his innocence that allows him to feel this way.

10. D.

David shows his innocence in many ways throughout the passage, especially in his joy and optimism in spite of the obvious undercurrent of sadness. The father has clearly protected his son from sorrow to the best of his ability.

11. B.

He says specifically that Europe is “rich in the accumulated treasures of age.”

12. A.

In the first paragraph of the passage, the author speaks of the grand illusions he has of Europe. In the second paragraph, he mentions philosophical works that have led him to believe in the grandeur of Europe.

13. C.

The word “baleful” is used today to mean evil or sinister, but in the early 1800s it meant miserable as well. A close look at the context clues in the passage should make the meaning clear. Remember, in this section of the test, you will be asked to figure out the meanings of words *as they are used in passages*.

14. C.

Although the author uses all of these lines, only choice H does not include a poetic device. Choice F is a metaphor, choice G is personification, and choice J is an example of alliteration.

15. B.

The author reasons that, if these little men are so important, others in their country must be truly great. Don't be tricked by choice A. As readers, we detect arrogance in the Englishmen he speaks about, but the naive author is sincere, not sarcastic.

16. A.

In the paragraph, he compares men, animals, and even landscapes, pointing out that those in Europe are far superior.

17. B.

Although it's good that he was able to travel, he's frustrated by the fact that he returned home without seeing what he set out to see.

18. C.

He has a passion (or desire) for roving (or traveling). Evidence in the passage supports this desire.

19. C.

Rather than taking in the great sights, he enjoyed the small towns. Once he returns home, however, his heart almost fails him when he realizes that he missed out on the sights of the “regular traveller.”

20. B.

The things the author saw in his travels may not be considered important by “regular travelers,” but that doesn’t make them any less great. The author must learn that it’s okay to appreciate things that others deem unimportant.

21. B.

The article doesn’t address prevention or cures at all. Nor does the author suggest that parents should take their children to pediatric cardiologists.

22. C.

The author points out that *innocent* is used more often because people often misunderstand the term *functional*, even though it means the same thing.

23. C.

In the second paragraph, the author points out that murmurs do “not imply the presence of structural heart disease.”

24. B.

Since a congenital heart defect happens during the development of the heart, it must happen in the first eight weeks of pregnancy. Don’t be fooled by choice F. Although the heart is developed by the end of eight weeks, the author never suggests that it’s fully functional. In other words, it cannot yet function on its own outside the womb.

25. B.

The author states that less than 1% of babies are born with heart defects.

26. A.

In the preceding paragraph, we learned that murmurs are most often heard in childhood; hence, they are common. The author has told us that they are not usually dangerous; he’s not addressing diagnosis, and he never mentions prevention.

27. B.

The list of items constituting the focused history provides a clue for this answer.

28. D.

The author states that some children are referred, but he makes no comment about whether or not they should be.

29. A.

Although the pediatric cardiologist will ask about the child’s ability to engage in physical activity, it’s not a factor he *sees during his physical examination*.

30. D.

We know that these sounds are abnormal because the author calls them “extra,” or beyond the normal sounds. They are not the sounds of valves closing, however; those are the “lub-dub” sounds.

31. A.

An illusory feeling is one based on illusion, not on fact. Don't be tricked by choice C just because the author mentions that point in the first paragraph.

32. B.

The author states directly that walking is inherent, not culture dependent.

33. B.

The author states that "there is every reason to believe that he will learn to walk" but that it is "certain that he will never learn to talk." While the child may possibly create his own language, the author does not propose that idea in the passage.

34. B.

Since walking is not influenced by culture, the child's walking would be the same. Its speech, however, would be that of the new society. The author provides no evidence that the child would meet with any obstacles to language development.

35. C.

Joy that would lead us to cry out would not be unspoken or slight. A sudden twinge of sudden joy would be redundant.

36. D.

Although all of the choices may be true of speech, religion, and art, only choice J is addressed by the author.

37. D.

The author argues repeatedly that these sounds cannot be considered speech.

38. B.

*Aloof* means set apart. The author points out that these sounds don't have standardized meanings. "Oh!" can mean something different in every situation.

39. C.

Whereas the author doesn't consider such an utterance to be speech, he does acknowledge it as instinctive, like walking.

40. B.

Although he does compare walking and talking, he does so only to make his point that speech is not a natural instinct.

**Answers to English Test** (Test may be found on Page 138.)

**Passage 1**

1. D.

We need the semicolon to join the two independent clauses together. Also, *I* is the correct pronoun because it's used as the subject of the second clause.

2. B.

The correct verb is *sat* because there is no direct object. Although choice J uses *sat* as well, it adds a direct object (*ourselves*), making *sat* no longer correct.

3. C.

The subject (not one) is singular, so it requires the verb *has*. Although choice D also uses *has*, it is unnecessarily wordy.

4. D.

*Slowly* describes how the group started and must therefore be in its adverb form. Choice H uses *slowly* also, but it changes the two sentences into one comma splice.

5. B.

The correct idiomatic expression is *figured out*. Choice C also uses *figured out* but is too wordy.

6. D.

It is not necessary to use two coordinating conjunctions (*and* and *so*). We need the comma after *great* because we have a compound sentence.

7. B.

The pronoun *it* refers to the antecedent catapults, so it must be changed to *them*. To give something a *try* is an idiomatic expression, so *give them tries* wouldn't work.

8. D.

Choices F and G are both passive voice. Choice H is unnecessarily wordy.

9. D.

The correct word is *who* because it's used as the subject of *were watching*. *Which* can be a subject as well, but it refers to things, not people.

10. D.

Here we have a misplaced modifier. We weren't flying 12 feet; the balls were. Choice G changes the sentence into a comma splice. Choice H suggests that the catapults were flying 12 feet. Although the correct answer isn't usually the longest, choice J is an exception.

11. C.

It's important to be consistent with pronouns. Since the rest of the paragraph uses first person pronouns, this sentence should also.

12. C.

Choices F and G both provide a sentence fragment beginning with *Although*. Choice J uses the wrong verb tense to be consistent with this paragraph. Choice H provides a past tense verb and combines the two sentences into one correctly punctuated sentence.

13. A.

To make one team possessive, we add an apostrophe and an s after the word *team*.

14. D.

Since the sentence compares two catapults, we must use the comparative form (*-er*) rather than the superlative form (*-est*). Also, *farther*, not *further*, refers to physical distance.

15. B.

Choice B allows the sentence to be parallel (*building catapults, reading a book*). Choice C avoids the problem of parallel structure by adding a new clause, but the verb tense is wrong. It would have to be *would have learned*.

## Passage 2

1. B.

The correct word for comparison is *than*. Choice C also uses *than* but is needlessly wordy.

2. C.

*It's* is a contraction meaning *it is*, which is correct in this sentence. *You're* is a contraction meaning *you are*, which is not correct in this sentence.

3. D.

The correct idiomatic expression is *big man on top*.

4. C.

It is concise and corrects the comma splice. Hopefully, you've learned by now to recognize the excessively wordy choices like choice J.

5. C.

The previous sentence talks about what to do *until that day*, so it makes sense for this sentence to address what to do *for now*.

6. B.

The rest of the paragraph uses second person pronouns (like *you*), so we must be consistent here.

7. C.

To make the sentence structure parallel, we must eliminate the extra subject (*he*). Don't be fooled by D. It's wordy and it's passive voice.

8. D.

Choices F and H both provide a sentence fragment starting with *Because*. Choice G turns the fragment into a comma splice.

9. B.

It's clear; it's concise. Cut the extra words whenever possible.

10. B.

Read the choices carefully. Choice J would work, but upperclassMEN has been changed to upperclassMAN. Choice F, by the way, is passive voice. Choice H uses *they're* when it should be *their*.

11. D.

Whew, look at those other wordy choices!

12. C.

This one is all about pronoun/antecedent agreement. *Upperclassmen* agrees with *their* and with *faces*. Although the agreement is correct in choice G, look at the next sentence. The pronoun *them* indicates that you need the plural *upperclassmen* rather than the singular *upperclassman*.

13. A.

Keep it short and simple.

14. C.

The subject of the sentence is *making fun* (singular) and agrees with the verb *causes*. Choice J uses *causes*, but look at all those extra words!

15. C.

In order to bring the paragraph full circle, it's good to tie the closing line back in with the opening line. Choice C does just that without being wordy or awkward like the rest of the choices are.

### Passage 3

1. B.

It's clear and concise. It avoids the double negative of choice A, and it avoids the second person pronoun (*you*), which is not used anywhere else in the paragraph.

2. D.

Keep it short. Choice F uses the wrong verb tense.

3. C.

All of the other choices imply that the students can't apply *any* of the concepts because they don't understand *any* of them. Only choice C makes it clear that the concepts they can't apply are *only* the ones that they don't understand.

4. A.

The structure is parallel and clear. The other choices just add confusion.

5. C.

The lessons are more productive *when* students understand grammar. The conjunctions *and* and *since* both suggest that the students already *do* understand grammar.

6. C.

The studies still *show* it. All of the other choices have unnecessary words.

7. C.

Choice A has a subject/verb agreement problem. Choice B makes the sentence into a fragment. And certainly you recognized choice D as the wordy distracter.

8. B.

Choices F and J present a subject/verb agreement problem. And the skills haven't *been* important; they still *are* important.

9. B.

Choice A is a comma splice. Choice C puts the semicolon on the wrong side of *however*. Finally, the *and* in choice D shows agreement rather than contrast.

10. D.

The correct word is *who* because it acts as the subject of the verb *participated*. *Which* refers to things, not people. Although *that* can refer to groups of people, the unnecessary comma eliminates G as a viable choice.

11. D.

By eliminating the underlined portion altogether, we create a simple yet effective appositive.

12. B.

Choice F is passive voice. Although all of the other choices are active voice, G is the most clear and concise.

13. A.

The paragraph is certainly not made-up or imaginary, so we can eliminate C and D. Since the author is trying to convince readers to agree with his opinion rather than provide an account of events, we can also eliminate B.

14. A.

This sentence, when placed after the first sentence of the second paragraph, reinforces the impact that grammar instruction could have on this alarming statistic.

15. B.

The passage has told readers that grammar instruction is important. Now this final sentence reinforces the *reason* it's important. Choice C brings up a new topic; choice D introduces a first person pronoun not consistent with the rest of the passage; and choice A fails to bring closure to the paragraph.

#### Passage 4

1. B.

It's the clearest and most concise choice.

2. D.

The second sentence takes the idea of the first sentence one step further, making *in fact* an appropriate transition. Note that placing the word *although* at the beginning of this sentence would turn it into a fragment.

3. B.

As it is, the sentence compares comments and plays. Choice B emphasizes the fact that we're trying to compare comments in one play to *comments* in another play. Choice C won't work because that is singular and therefore cannot refer to the plural *comments*. Choice D uses *then* (next) rather than *than* (a comparative word).

4. B.

The conjunction *for* (rather than *and*) helps readers see the reason the satire is effective. Choice F is a comma splice, and choice H creates a fragment.

5. A.

The semicolon is necessary because there are two independent clauses. Choice C places the semicolon on the wrong side of *therefore*, and choice D has too many conjunctions.

6. B.

It's active voice, and it's clear. Choices F and J are passive voice. Although H is active voice as well, it's awkward.

7. C.

The helping verb *have* (creating present perfect tense) is necessary to be consistent with the rest of the paragraph.

8. C.

Since *hissself* isn't a word, we can eliminate choices F and J. You can also make *himself* possessive, so choice G doesn't work either.

9. D.

It is the only choice that makes the sentence parallel.

10. C.

This question tests your understanding of subjunctive mood. With the words *if* and *wish*, we always use the verb *were*, even though it may sound strange. (Example: I wish I wererich!)

11. D.

Use *that* to refer to groups of people as a whole. The semicolon (choice H) is not necessary, though, because we're not joining two independent clauses.

12. C.

Choice F is passive voice. Choice J is the wrong verb tense. Choice G would be acceptable, but the progressive tense is not necessary; therefore, go with the shorter choice.

13. B.

Choice A is a sentence fragment. Choices C and D are needlessly wordy.

14. D.

Oh, please do take the sentence out altogether! It doesn't have anything to do with the content of the paragraph.

15. B.

This sentence reinforces the content of the paragraph while bringing it full circle. Choices A and C are too broad and too narrow, respectively, and choice D merely restates the topic sentence.

## Passage 5

1. A.

We use an apostrophe and an *s* after the word *life* to make it possessive. Don't be fooled by the wordy choice C.

2. D.

*Funner* is not a word; besides, it is not necessary to add an *er* when you use the word *more*. Choice G doesn't work because it creates a subject/verb agreement problem.

3. B.

Although choice C uses the right form of *they're*, you don't need all those words when one will do the job.

4. C.

The semicolon is necessary to join the two independent clauses. Choices F and G are comma splices. Choice J would work if there were a comma after *friend*.

5. A.

The line is supposed to be funny because it's so obviously not true.

6. C.

Use *who* when referring to people. Although *that* can be used to refer to a group of people, this sentence is really talking about individual friends. Besides, the comma is unnecessary.

7. B.

None of the other choices makes the sentence parallel.

8. D.

The sentence is redundant because it merely repeats the sentence before it.

9. D.

Choice A is passive voice. Choices B and C are both active voice, but they create a pronoun/antecedent problem (*them/friend*).

10. A.

This sentence is just fine as is. The other choices are awkward or wordy.

11. C.

This sentence is intended to contrast with the one before it. Choice C offers the only contrasting transition.

12. B.

The author is comparing two groups of characteristics, so these are *more* important rather than the *most* important. Choice J uses *more*, but it is a comma splice.

13. B.

The singular subject (*person*) needs the verb *is* rather than *are*. Watch out for subjects that are widely separated from their verbs. They can be tricky.

14. A.

The original wording is clear and concise.

15. C.

The other choices are either awkward or too general.

**Solutions to Mathematics Test** (Test may be found on Page 153.)

1) D

$$\frac{x+y}{2} = 5 \quad \frac{x+z}{2} = 8 \quad \frac{y+z}{2} = 11$$

$$x+y = 10 \quad x+z = 16 \quad y+z = 22$$

Solve for  $x$ ; then substitute this expression in the next equation.

$$x = 10 - y$$

$$(10 - y) + z = 16$$

$$z = y + 6$$

Substitute this value in the final equation.

$$y + y + 6 = 22$$

$$2y + 6 = 22$$

$$2y = 16$$

$$y = 8$$

If  $y = 8$ , then  $8 + z = 22$ . Thus,  $z = 14$ , choice D.

2) C

Since  $\boxed{16}$  is even, you must use the second relationship. Thus,  $\boxed{16} = \sqrt[4]{16} = 4$ . Since  $\boxed{9}$  is odd,  $\boxed{9} = 9^2 = 81$ . Therefore,  $4 + 81 = 85$ .

3) A

$$3x + 7 = 12$$

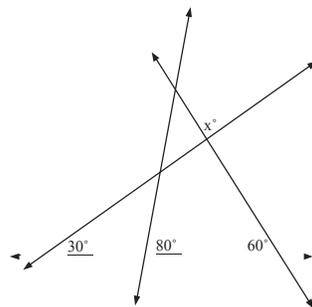
$$3x = 5$$

$$x = \frac{5}{3}$$

$$6\left(\frac{5}{3}\right) - 5 = 10 - 5 = 5$$

Therefore, the answer is A.

4) E



Two angles of triangle ABC sum to  $90^\circ$ , meaning the remaining angle is  $90^\circ$ . By definition of intersecting lines,  $x = 90^\circ$ .

5) D

Let "7 less than 4 times a number" =  $4x - 7$

Let "8 more than the number" =  $x + 8$

Set the two equal and solve for x.

$$4x - 7 = x + 8$$

$$3x = 15$$

$$x = 5$$

6) D

The second graph represents the inverse of the original function ( $-x$ ) followed by a translation of 3 units to the left ( $-x + 3$ ).

7) B

The perimeter of the rectangle ( $p$ ) =  $\frac{1}{5}y + \frac{1}{5}y + y + y = \frac{12}{5}y$

To express y in terms of p, multiply both sides by  $\frac{5}{12}$  to give  $y = \frac{5p}{12}$ . The answer is B.

8) E

The area of A = 16; therefore, one side of A = 4. If one side of A = 4, then the perimeter of A =  $4 + 4 + 4 + 4$ , or 16.

Perimeter of A =  $\frac{2}{3}$  (perimeter of B)

$16 = \frac{2}{3}$  (perimeter of B)

Perimeter of B = 24

Perimeter of B =  $\frac{2}{3}$  (perimeter of C)

$24 = \frac{2}{3}$  (perimeter of C)

Perimeter of C = 36

So, one side of C must equal  $\frac{36}{4} = 9$

Area of C =  $9^2 = 81$ .

9) B

Let x = total number of runs.

$\frac{x}{12} = 6$  runs per game for 12 games

$x = 72$  total runs

Now set up another relationship for the first 10 games.

Let y = number of runs in the first 10 games.

$\frac{y}{10} = 5$

$y = 50$  runs in the first 10 games

Number of runs in the last two games = (total runs) - (runs in the first 10 games)  
=  $72 - 50 = 22$  runs in the last two games

If you've scored the same number of runs, you're in the last two games, then  $\frac{22}{2} = 11$  runs per game in each of the last two games.

10) C

Factor the numerator:

$$x^2 - 2x + 1 = (x - 1)(x - 1)$$

Examine the denominator:

$$1 - x = -1(x - 1)$$

Solve the expression:

$$\frac{(x - 1)(x - 1)}{1(x - 1)} = -(x - 1) = 1 - x$$

11) C

The easiest way to determine the area is to find the area of one triangle; and double the result (both triangles are equal). To find the area of one of the triangles, you need to find the third side. Using the Pythagorean theorem:

$$a^2 + 6^2 = 10^2$$

$$a^2 + 36 = 100$$

$$a^2 = 64$$

$$a = 8$$

12) D

$$2x - 8y = 16$$

$$-8y = -2x + 16$$

$$y = \frac{1}{4}x - 2$$

Thus,  $m = \frac{1}{4}$  and the answer is J.

13) E

Remember: distance = rate x time or  $d = rt$

Let distance traveled by Bill =  $d_b = 30t$

Let distance traveled by Tom =  $d_t = 50t$

Thus,  $d_b + d_t = 150$  or  $30t + 50t = 150$

$$\text{So } t = \frac{15}{8}$$

To find Tom's distance:  $d_t = 50t$

$$d_t = 50\left(\frac{15}{8}\right)$$

$$d_t = 93.75$$

14) D

equation of a line:  $y = mx + b$

$$m = \frac{-4 - 5}{-3 - 9} = \frac{-9}{-12} = \frac{3}{4}$$

$$y = \frac{3}{4}x + b$$

Plug in the point (9,5) and solve for b:

$$5 = (\frac{3}{4})(9) + b$$

$$5 = \frac{27}{4} + b$$

$$b = -\frac{7}{4}$$

$$\text{Therefore: } y = \frac{3}{4}x + \frac{27}{4}$$

15) B

The perimeter of the rectangle (p) =  $\frac{1}{5}y + \frac{1}{5}y + y + y = \frac{12}{5}y$

To express y in terms of p, multiply both sides by  $\frac{5}{12}$  to give  $y = \frac{5p}{12}$ . The answer is B.

16) A

$$b + 2(x - 4) = s$$

Solve for x.

$$b + 2x - 8 = s$$

$$2x = s - b + 8$$

$$x = \frac{s - b + 8}{2}$$

$$x + 2 = \frac{s - b + 8}{2} + 2 = \frac{s - b + 8}{2} + \frac{4}{2} = \frac{s - b + 12}{2}$$

17) C

A is the midpoint of its side

Let  $x$  = half the length of a side

Let  $2x$  = length of a full side

Use the Pythagorean theorem to solve for  $x$ :

$$x^2 + (2x)^2 = 10^2$$

$$x^2 + 4(x)^2 = 100$$

$$5(x)^2 = 100$$

$$x^2 = 20$$

$$x = 2\sqrt{5}$$

$$\text{Full side length} = 2x = 4\sqrt{5}$$

$$\text{Area of square} = \text{side}^2 = (4\sqrt{5})^2 = 80.$$

18) B

Set up the two inequalities and solve:

$$3x - 4 > 0 \text{ and } 3x - 4 < 0$$

$$x > \frac{4}{3} \text{ and } x < \frac{4}{3}$$

19) C

$$(s + 1)(s - 1) = s^2 - 1$$

$$\text{Since } s^2 = 19, s^2 - 1 = 18$$

20) E

Since  $f(a) = -b$  and  $f(-a) = b$ , the graph of the function passes through the origin  $(0,0)$ , which happens to be the midpoint between the points  $(a, -b)$  and  $(-a, b)$ . Thus, the x-intercept is the value where  $f(x) = 0$ . That value is  $x = 0$ , since it satisfies the function.

21) D

$$\text{Let } Q = 2D$$

$$D = N$$

$$N = 3P,$$

where  $Q = \#$  of quarters,  $D = \#$  of dimes,  $N = \#$  of nickels, and  $P = \#$  of pennies.

If Sarah has one quarter, then she has 2 dimes, 2 nickels, and 6 pennies.

$$1 Q = \$0.25$$

$$2 D = \$0.20$$

$$2 N = \$0.10$$

$$6 P = \$0.06$$

Total of the least amount of money Sarah can have: \$0.61.

22) B

$\triangle DCDA$  is proportionate to  $\triangle DCBA$ .

Since  $DA$  is a bisector, you know that  $\angle DAC = \frac{1}{2} \angle BAC$

The same relationship is true for  $\angle DCA$  and  $\angle BCA$  ( $\angle DCA = \frac{1}{2} \angle BCA$ ).

The relationship between  $\angle B$  and  $\angle D$  should be inversely proportional due to the fact that as  $D$  approaches  $B$ ,  $\angle D$  would decrease inversely to the other angles. Thus,  $\angle B$  is  $60^\circ$  based on scaling  $\angle D$  by  $\frac{1}{2}$ .

23) E

Based on the triangle inequality rule, which states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side,

$$\text{Base} = 9$$

Let one of the other sides =  $x$

$$\text{Then } x + x > 9$$

$$2x > 9$$

$$x > \frac{9}{2} \text{ or } 4.5$$

Since the sides are integers, the shortest possible side would be  $\sim 5$ .

24) B

Probability (red) =  $\frac{1}{4}$  out of 200 marbles = 50 red marbles

Initial number of white marbles =  $200 - 50 = 150$  white marbles

Addition of 50 white marbles gives a total of 200 white marbles

Total number of marbles = 200 white + 50 red = 250 marbles

Probability of selecting a white marble =  $\frac{200}{250} = \frac{4}{5}$

25) B

By definition of intersecting lines,  $\angle a = \angle c$ ; thus, you can eliminate any choice that has  $\angle c$  in it (choices A, C, and D). Also,  $\angle a$  is supplementary to  $\angle d$  and  $\angle b$ . Thus,  $\angle b = \angle d$ .

By definition of parallel lines cut by a transversal,  $\angle b = \angle h = \angle f$ .

Thus  $\angle b$ ,  $\angle d$ ,  $\angle h$ , and  $\angle f$  must all be supplementary to  $\angle a$ . The choice is therefore B.

26) A

For the sum to add up to +74, you have to move from  $-34$  in the positive direction. Thus, the sum of all the negative even integers will be negative. You must have the same values on the positive end to cancel out the negative values. Thus, for a  $-34$  there must be a  $+34$ . As a result, from  $-34$  to  $+34$ , the sum is zero. The next two even positive integers are 36 and 38, whose sum is to 74.

$$\text{Therefore, } 2m = 38$$

$$m = 19$$

27) A

$$5 + 2y \geq 3 \text{ and } 5 + 2y \leq -3$$

$$2y \geq -2 \text{ and } 2y \leq -8$$

$$y \geq -1 \text{ and } y \leq -4$$

28) B

Prior to addition:

Amount of cement =  $\frac{1}{2}$  (total amount) =  $\frac{1}{2}$  (12) = 6 tons

Amount of granite and sand =  $12 - 6$  tons cement = 6 tons of granite and sand

You want 60% of cement after adding  $x$ .

After addition:

$$\text{Amount of cement} = x + 6$$

$$\text{Total amount of materials} = x + 12$$

You need to set up a percent ratio to solve the problem:

$$\frac{x + 6}{x + 12} = \frac{60}{100}$$

Solve for  $x$ :

$$100x + 600 = 60x + 720$$

$$40x = 120$$

$$x = 3 \text{ tons of cement}$$

29) C

You must use the distance formula to solve this problem. For all the line segments in the answers, you must find their lengths.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\AD &= \sqrt{(-4 - 0)^2 + (0 - 4)^2} = \sqrt{16 + 16} = \sqrt{32} = 4\sqrt{2} \\BD &= \sqrt{(-2 - 0)^2 + (0 - 4)^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5} \\AE &= \sqrt{(-4 - 0)^2 + (0 - (-5))^2} = \sqrt{16 + 25} = \sqrt{41} = \sqrt{41} \\AC &= \sqrt{(-4 - 2)^2 + (0 - 0)^2} = \sqrt{36 + 0} = \sqrt{36} = 6 \\CE &= \sqrt{(2 - 0)^2 + (0 - (-5))^2} = \sqrt{4 + 25} = \sqrt{29}\end{aligned}$$

Thus, AE is the longest segment and the correct answer choice is C.

30) A

First, find OM using the Pythagorean theorem.

$$\begin{aligned}4^2 &= 2^2 + x^2 \\16 &= 4 + x^2 \\12 &= x^2 \\x &= \sqrt{12} = 2\sqrt{3}\end{aligned}$$

Since  $MO = ON$ ,  $ON = 2\sqrt{3}$ .

Find MN using the Pythagorean theorem.

$$\begin{aligned}MN^2 &= (2\sqrt{3})^2 + (2\sqrt{3})^2 \\MN^2 &= 12 + 12 = 24 \\MN &= \sqrt{24} = 2\sqrt{6}\end{aligned}$$

31) C

Plug  $y = -x$  into the other equation, then solve for  $x$ .

$$\begin{aligned}(x - 5)^2 + (-x - 5)^2 &= 4 \\x^2 - 10x + 25 + x^2 + 10x + 25 &= 4 \\x^2 - 50 &= 4 \\x^2 &= 54 \\x &= 3\sqrt{6} \text{ and } -3\sqrt{6}\end{aligned}$$

32) A

The graph of  $[y]$  represent the graph in choice F. The equation given is equivalent to the equation  $x = [y] - 1$ , where the graph would be rotated  $90^\circ$  relative to the  $y$ -axis and then shifted to the left on the  $x$ -axis.

33) A

Plug one equation into the other and solve for x.

$$10x - 3(3x - 1) = 9$$

$$10x - 9x + 3 = 9$$

$$x + 3 = 9$$

$$x = 6$$

Plug x = 6 into original equation and solve for y.

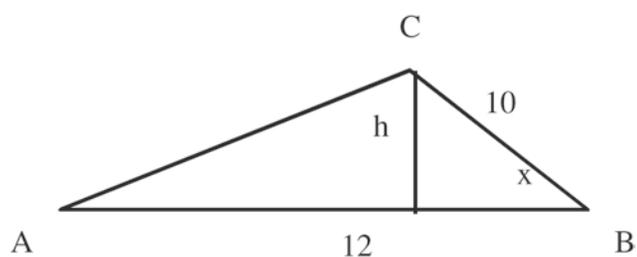
$$y = 3(6) - 1$$

$$y = 17$$

The point is (6,17).

34) D

Draw the triangle. Drop a perpendicular to the base.



Using trigonometric relationships:

$$\sin x = \frac{h}{10}$$

$$h = 10 \sin x$$

$$\text{Area of triangle} = \frac{1}{2}bh$$

$$= \frac{1}{2}(12)(10 \sin x)$$

$$= 60 \sin x$$

35) E

$$16^{x^2-1} = 64^x$$

$$(4^2)^{x^2-1} = (4^3)^x$$

$$4^{2x^2-2} = 4^{3x}$$

$$2x^2 - 2 = 3x$$

$$2x^2 - 3x - 2 = 0$$

Factor.

$$(2x + 1)(x - 2) = 0$$

$$x = -\frac{1}{2} \text{ or } x = 2$$

Because  $x = -\frac{1}{2}$ , the answer is E.

36) B

Definition for the reflection about the y-axis: for all points  $(x, f(x))$ , the reflected graph will contain points  $(-x, f(x))$ . Thus, a reflection about the y-axis is equivalent to translation of a point by  $2x$  units, where the line  $y = 0$  is the bisector of the distance between the original position and the new position.

Thus, in the original graph, the vertex is at  $y = 4$  and the graph is shifted right 2 units. In the reflected graph, the vertex occurs at  $y = 4$  but the graph is shifted left 2 units to give the equation  $y = -3(x - 2)^2 + 4$ , which is choice G.

37) C

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$\text{If } 4xy = 8, \text{ then } 2xy = 4$$

$$x^2 + y^2 + 2xy = 10 + 4 = 14$$

38) E

If area (square) = 81, then each side of the square is 9.

If perimeter of each  $\Delta = 30$ , then:

Perimeter of bold outlined figure = (sum of perimeters of all triangles) – (perimeter of square), since the dashed side of each triangle is a side of the square.

Since there are four triangles, each with a dashed side, that equals the perimeter of the square.

$$\text{The perimeter of the square is } 9 + 9 + 9 + 9 = 36.$$

$$\text{Thus, } 4(30) - 36 = 120 - 36 = 84$$

39) D

The only thing absolutely known about these two relationships is that they do not equal zero. Therefore,  $r, s, t, u,$  and  $v \neq 0$ . The correct choice is D.

40) B

Use the distance formula to first find the length of the radius.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(-6 - 0)^2 + (-4 - 2)^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$$

Then use the formula with the center and each of the answer choices to find which choice has the same length.

When you use this formula with (6,8):

$$\sqrt{(6 - 0)^2 + (8 - 2)^2} = \sqrt{36 + 36} = \sqrt{72} = 6\sqrt{2}$$

Thus G is the answer.

41) D

Let  $x$  represent number of seats in one section

$$300 \text{ seats} \leq x \leq 400 \text{ seats}$$

Minimum number of total seats = 300 seats  $\times$  8 sections = 2400 seats

Maximum number of total seats = 400 seats  $\times$  8 sections = 3200 seats

Therefore,  $2400 \leq \text{total number of seats} \leq 3200$ .

The only choice that satisfies the inequality is 2600, choice D.

42) E

Area of shaded region = area of circle – area of big square

Area of individual square =  $x^2$

Side of individual square =  $x$

First, find the value of the diagonal in one of the small squares, since this is the radius of the circle.

$$x^2 + x^2 = r^2$$

$$2x^2 = r^2$$

$$\text{Area of circle} = \pi r^2 = 2x^2\pi$$

$$\text{Area of big square} = 4x^2 \text{ (4 small squares each with area } x^2 \text{)}$$

$$\text{Area of shaded region} = 2x^2\pi - 4x^2 = 2x^2(\pi - 2)$$

The correct choice is K.

43) B

Volume of cylinder =  $\pi r^2 h$

$$= \pi(2)^2(3) = 12\pi \approx 39.68$$

To find number of full tanks to fill cylinder:

$$\# \text{ tanks} = \frac{39.68}{3} \approx 13.22 \text{ which means it takes over 13 full tanks}$$

44) B

Points in 1st quarter:  $\frac{1}{8}$  of 48 = 6

Points in 2nd quarter:  $\frac{1}{3}$  of 48 = 16

Points in 3rd quarter:  $\frac{1}{4}$  of 48 = 12

Points in first 3 quarters = 6 + 16 + 12 = 34

Points in 4<sup>th</sup> quarter = total points – points in first 3 quarters = 48 – 34 = 14.

45) C

Let  $m = \#$  of pounds

$$3.65 < m < 4.35$$

Because this is an inequality, you can eliminate choices A and B. Go to the remaining answer choices and solve the inequalities.

$$[m - 4] < 0.35$$

Split into two inequalities and solve.

$$\begin{aligned} m - 4 < 0.35 \text{ and } m - 4 > -0.35 \\ m < 4.35 \text{ and } m > 3.65 \text{ or } 3.65 < m < 4.35 \end{aligned}$$

46) A

If you drop a perpendicular down from point B to the x-axis, you will form a right triangle whose hypotenuse is 10 and whose base is 8. To find  $k$ , which is the length of the last leg, you will need the Pythagorean theorem.

$$\begin{aligned} 10^2 &= 8^2 + k^2 \\ 100 &= 64 + k^2 \\ k^2 &= 36 \\ k &= 6 \end{aligned}$$

47) A

Write out the 1<sup>st</sup> five terms:

$$\begin{aligned} 1^{\text{st}} \text{ term: } &3 \\ 2^{\text{nd}} \text{ term: } &7 \\ 3^{\text{rd}} \text{ term: } &-7 \\ 4^{\text{th}} \text{ term: } &-3 \\ 5^{\text{th}} \text{ term: } &3 \end{aligned}$$

You are at the beginning of a loop in which, after every four terms, the 5<sup>th</sup> term gives the 1<sup>st</sup> term. To figure out any term:

$$\frac{n^{\text{th}} \text{ term}}{4},$$

where the remainder will determine which of the initial four values the term will take.

For  $n = 155$ ,  $155/4 = 38 \frac{3}{4}$ , where the  $\frac{3}{4}$  represents that the loop has completed three of the four terms, or equals the 3<sup>rd</sup> term ( $-7$ ).

48) D

Of Dwayne's sales, he pays  $\frac{k}{3}$  for lumber and supplies. That leaves  $\frac{2k}{3}$  left for savings.

To save \$5000, Dwayne must sell:

$$\frac{5000}{\frac{2k}{3}} = 5000 \cdot \frac{3}{2k} = \frac{15000}{2k} = \frac{7500}{k}$$

49) D

This problem involves an arithmetic series, so you need the equation for the sum of an arithmetic series:

$$\text{Sum of arithmetic series} = \frac{n}{2} (2a_1 + (n - 1)d)$$

You are given that the sum = 5250, the number of terms (n) = 12 and d (difference between terms) = 1. You will need to find the 1<sup>st</sup> term,  $a_1$ , in order to find the 12<sup>th</sup> term.

Plug the given values into the equation and solve for  $a_1$ .

$$5250 = \frac{12}{2} (2a_1 + (12 - 1)(1))$$

$$5250 = 6(2a_1 + 11)$$

$$875 = 2a_1 + 11$$

$$2a_1 = 864$$

$$a_1 = 432$$

Next, you need the equation for the n<sup>th</sup> term of an arithmetic series:

$$a_n = a_1 + (n - 1)d$$

Plug in the values n = 12, d = 1, and  $a_1 = 432$  to find  $a_{12}$ .

$$a_{12} = 432 + (12 - 1)(1) = 432 + 11 = 443.$$

50) D

By definition of parallel lines cut by a transversal,  $x + y = 180$  and  $x + x + 130 = 180$ .

Solve for x.

$$2x + 130 = 180$$

$$2x = 50$$

$$x = 25$$

Plug x into the next equation to find y.

$$x + y = 180$$

$$25 + y = 180$$

$$y = 155$$

51) A

Start with  $x = 0$ . When  $x = 0$ ,  $h(x) = 1$ . Therefore, 1 must be added somewhere in the equation. Look at the choices and eliminate B, C, and D because none of them involves adding 1. Plug  $x = 1$  into the remaining equations to see which one satisfies the table.

52) C

The area of the outer circle is  $100\pi$ .

To find the area of the inner circle, you need to find the length of its radius. The radius of the inner circle is the diagonal of the square.

Use the Pythagorean theorem to find the length of the diagonal. The diagonal of a square forms two  $45^\circ - 45^\circ - 90^\circ$  right triangles, so the legs of the triangle formed with the square will be equal ( $4\sqrt{2}$ ).

$$\begin{aligned}(4\sqrt{2})^2 + (4\sqrt{2})^2 &= c^2 \\ 32 + 32 &= c^2 \\ 64 &= c^2 \\ c &= 8\end{aligned}$$

Thus, the area of the inner circle is  $64\pi$ .

One side of the square equals  $4\sqrt{2} + 4\sqrt{2} = 8\sqrt{2}$

The area of the square is  $(8\sqrt{2})^2 = 128$

Area of the shaded region (A) = (area of outer circle) – (area of inner circle) + (area of square)

$$A = (100\pi - 64\pi) + 128 = 36\pi + 128$$

53) E

There are six faces to the prism. Within the six faces, there are three pairs of equal faces.

Area of first face of one pair:  $6 \times 8 = 48 \times 2$  faces = 96

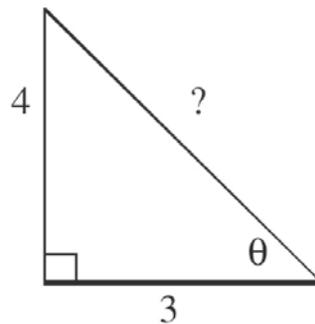
Area of first face of second pair:  $12 \times 8 = 96 \times 2$  faces = 192

Area of first face of third pair:  $6 \times 12 = 72 \times 2$  faces = 144

Total surface area =  $96 + 192 + 144 = 432$ . Thus, the answer is E.

54) A

Draw a right triangle with sides corresponding to  $\tan \theta = \frac{4}{3}$ .



Find the hypotenuse using the Pythagorean theorem.

$$\begin{aligned}3^2 + 4^2 &= \text{hyp}^2 \\ 9 + 16 &= \text{hyp}^2 = 25 \\ \text{hyp} &= 5 \\ \text{Therefore, } \cos \theta &= \frac{3}{5}\end{aligned}$$

55) D

If 1<sup>st</sup> flip = heads and last flip = heads, out of 14 flips, only a maximum of 14 - 1 flips that come up heads can be consecutive. The maximum number of consecutive heads must correspond to the maximum number of consecutive tails.

Flips 1 – 13 = Heads (13 consecutive)

14 – 23 = Tails (10 consecutive)

24 = Heads

56) D

Let x and y represent the length and width of a rectangle, respectively. If you double those sides, you get 2x and 2y.

The area of the initial figure is xy

The area of the new figure is 2a x 2b = 4ab.

To find the percentage of increase, set up a ratio:

$$\frac{4ab}{ab} = \frac{x}{100}$$

$$4 = \frac{x}{100}$$

$$x = 400$$

Percent increase = 400 – 100 = 300% increase

57) C

Volume of small cube = 2 x 2 x 2 = 8

Volume of large cube = 4 x 4 x 4 = 64

Twice volume of small cube = 16

Set up a ratio:

$$\frac{16}{64} = \frac{1}{4} \text{ full of water}$$

$$\frac{1}{4} \text{ of height} = 4 \left(\frac{1}{4}\right) = 1$$

58) D

Let CD = x

Let CE = ED = QE = PQ =  $\frac{x}{2}$

$$\text{Area of DPQE} = \frac{1}{2} (\text{QE})(\text{PQ}) = \left(\frac{1}{2}\right) \left(\frac{x}{2}\right)^2 = \frac{x^2}{8}$$

$$\text{Area of DPCD} = \frac{1}{2} (\text{CD})(\text{PE})$$

To find PE use Pythagorean theorem

$$\text{PE}^2 = \text{QE}^2 + \text{PQ}^2$$

$$= \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^2 = \frac{2x^2}{4} = \frac{x^2}{2}$$

$$\text{PE} = \sqrt{\frac{x^2}{2}} = \frac{x}{\sqrt{2}} = \frac{x\sqrt{2}}{2}$$

$$\text{Area of } \Delta\text{PCD} = \frac{1}{2} x \left(\frac{x\sqrt{2}}{2}\right) = \frac{x^2\sqrt{2}}{4}$$

$$\text{Ratio of } \Delta\text{PCD} : \Delta\text{PQE} = \frac{\Delta\text{PCD}}{\Delta\text{PQE}} = \frac{x^2\sqrt{2}/4}{x^2/8} = x^2\sqrt{2}/4 \cdot \frac{8}{x^2} = 2\sqrt{2}/1 \text{ or } 2\sqrt{2} : 1.$$

59) E

Use trigonometric identity:  $\sin^2\theta + \cos^2\theta = 1$

Thus,  $\sin^2\theta = 1 - \cos^2\theta$

$$\text{So } \frac{\sin^2\theta}{1 - \cos^2\theta} = 1$$

60) A

Use the distance formula to solve the question:

$$d = \sqrt{(x^2 - x_1)^2 + (y^2 - y_1)^2}$$

Then find which of the given points is not at a distance of 10 from the point (-1,0).

The only point that does not work is (-6,-7).

$$d = \sqrt{(-6 - (-1))^2 + (-7 - 0)^2} = \sqrt{(-5)^2 - 7^2} = \sqrt{25 + 49} = \sqrt{74} \neq 10.$$

### Solutions to Science Test (Test may be found on Page 173).

1. C

60 g of  $\text{NH}_4\text{Cl}$  will dissolve in 100 g of water at 70 °C. So if you had 200 g of water at 70 °C, you could dissolve double the amount:  $2 \times 60 \text{ g} = 120 \text{ g}$ .

2. B

For  $\text{NaNO}_3$ , as temperature increases, the solubility increases.

3. C

Look on the chart and find the temperature at which the KCl line and the NaCl line cross. That occurs at 34 °C.

4. A

Look at the chart and find 40 °C. Trace upward along the 40 °C line until you reach the first line that appears in the choices. The first line you meet will be for NaCl.

5. C

The only thing that directly affects the solubility according to the data in the chart is increasing the temperature. Solubility of a given compound is an innate property that changes with respect to temperature.

6. B

In the introductory paragraph, it states "...lasers are available in the ultraviolet, infrared, and x-ray spectral regions." Thus, lasers do not emit only visible light.

7. C

An electron in a lowest energy state will not be able to drop any further to give off energy. Electrons in higher energy states can drop to lower levels to give off energy. Photons being emitted are by definition being accompanied by a release of energy.

8. C

In experiment 1, a low-energy electron is excited to an excited state. In experiment 2, the electron lies in a higher energy state but spontaneously drops to a lower energy level. Thus, it began in a higher energy state. In experiment 3, an electron is excited by adding energy. Thus, the answer is experiments 1 and 3.

9. B

Experiment 2 involves a spontaneous drop from higher level to a lower level. No energy was added for this to occur.

10. D

We cannot say which offers the most promise because we do not know how much energy each experiment generates or consumes.

11. D

The best way is to measure the amount of energy emitted. This will offer a critical gauge in determining which would be the most efficient laser.

12. B

The observed trend is increasing temperature as time passes.

13. C

The maximum observed anomalies for land are much higher than for the ocean. The ocean does not top anomalies at 0.6 above the zero line, whereas on land, the observed anomalies are approaching 1.2° C above the zero line.

14. D

This is a hypothetical question having its basis in the science of heating and cooling. One should realize that it takes a lot longer to heat water than land. Thus, there will be a more gradual heating and cooling and fewer observed anomalies.

15. A

The Northern Hemisphere is observing anomalies above the +0.5° C line, whereas the Southern Hemisphere is still below that line.

16. C

Around 1940, there was a change to the net positive anomalies (above the zero line) from the previous net negative anomalies (below the zero line).

17. A

Development time decreases as the temperature increases. At 12° C, development takes 50 days. At 28° C, development takes 7 days.

18. D

Under a combination of heat stress (35° C) and crowding, you should expect the development time to be longer than both the development times for heat stress and crowding taken independently (11 and 12 days, respectively). Thus, the expected development time would be greater than 11 or 12. So, the answer is 18 days, or choice J.

19. D

The group at 12 °C would take 50 days to develop, whereas the group under crowding at 25° C would develop in 12 days. Thus, the first group to hatch would be the small flies and the second group to develop would be normal flies. Given that there is a difference of 38 days between the development of the two groups, if members of the first group had any offspring, the offspring would be able to fully develop before the other group finished their development.

20. A

The conditions under which the parent developed are irrelevant to the conditions in which its offspring develop. The size of the parent is due to environmental pressure, not genetics.

21. D

The passage never states at what point during development that crowding plays a factor in the development. It could be the early stages or it could be the later stages. There just isn't enough information to determine the outcome.

22. C

Experiment 4 tests the response to light alone. Both papers being dry means that humidity has been removed as a variable.

23. D

Experiment 3 does test humidity vs. light. Aggregation at the dry but opaque box means that the isopods would rather be shielded from light than exist in a moist environment.

24. B

An experiment with two moist papers translates into a situation in which the humidity is very high. This means that the isopod movement will be slowed, but this won't interfere with their desire to avoid the light. Thus, they will aggregate at the opaque paper box.

25. D

None of the experiment's tests dark with no humidity difference. Such an experiment would be set up in the dark with either two moist papers or two dry papers.

26. C

The isopods avoid light but aggregate toward the moist paper in the absence of light. Thus, they prefer high humidity (moist paper) and low light intensity (opaque box).

27. B

Experiment 3 shows that the response to light dominates over the response to humidity.

28. D

Scientist 1's argument revolves around a major climate change caused by factors on the planet, not by any celestial bodies.

29. B

Scientist 2's argument centers around a celestial body colliding with the planet, whereas Scientist 1's argument centers on volcanic activity and continental drift as the underlying causes of the climate change.

30. C

If celestial bodies are shown to have higher iridium concentrations, it could be used by Scientist 2 to address the discovery of the high iridium content at the K-T boundary.

31. C

This discovery would mean that during that time period there were numerous volcanic eruptions, many of which could potentially occur at the same time, thus casting large amounts of ash, smoke, and other matter into the atmosphere. Scientist 1 could use this as evidence to support his argument.

32. B

Both scientists agree on a massive climate change. They disagree on how that change came about.

33. A

You would expect a more gradual decline in the scenario of Scientist 1. Continental drift occurred over a long period of time, so a gradual climate change could be expected. In Scientist 2's scenario, if a celestial body hit the planet, one would expect a more rapid change to occur (immediate impact followed by the aftermath).

34. D

All of the given data would give a better picture of what actually may have occurred.

35. C

Experiment 2 shows that when the sensor is above the hole (not within it), no periodicity is observed. Thus, the position of the sensor directly relates to the observance of periodicity.

36. C

Experiment 3 is the only experiment that involves moisture as a variable.

37. B

Experiment 1 answered the initial question posed in the introductory paragraph. The experiment was properly able to measure the radon emissions separated from other emissions and observe periodicity.

38. A

Experiment 1 was the basic experiment to prove the validity of the design; thus, it is the control.

39. B

You should expect to see no periodicity initially (as in Experiment 2) but then begin to see a gradual recovery (as observed in Experiment 3) before a return to periodicity (as in Experiment 1).

40. A

The overall experiment proves that the periodicity observed is due to the radon emissions. The sensor is designed to isolate radon emissions from all other emissions (stated in the introductory paragraph). Thus, if radon were not responsible for the periodicity, the overall experiment would be invalid because the experiment design would not answer the initially posed questions.

## Chapter Eight: Additional Resources

You may find these additional resources helpful as you prepare for the ACT and move toward college planning:

- **Club Z! Tutoring ([www.ClubZ.com](http://www.ClubZ.com))**
- ***The Hipp List* by IVY1600, Inc. ([www.ivy1600.com](http://www.ivy1600.com))**
- ***Tooth and Nail: A Novel Approach*, by Charles Harrington Elster and Joseph Elliott**
- **American College Testing ([www.act.com](http://www.act.com))**
- **My College Options ([www.mycollegeoptions.com](http://www.mycollegeoptions.com))**
- **The National Association for College Admission Counseling ([www.nacacnet.org](http://www.nacacnet.org))**
- ***The Road to College: A High School Student's Guide to Discovering Your Passion, Getting Involved, and Getting Admitted*, by Joyce E. Suber and the Staff of The Princeton Review**



## About the Authors

**Dawn Burnette**, author of the verbal section of *The Essential Guide to the ACT*, is a National Board Certified Teacher who has taught high school English for fifteen years. She is also certified by the State of Georgia in gifted education for grades K-12. Dawn holds a B.A. in English Education and Journalism from Lenoir-Rhyne College and an M.A. in English Education from Georgia State University.

In addition to her work on this guide, Dawn has published several works, including *Daily Grammar Practice* (a program for helping students in first grade through college to understand, remember, and apply grammar concepts); *The Burnette Writing Process* (an individualized, web-supported writing approach for grades 6-12); *Vocabulary: A Novel Solution* (a literature-based vocabulary program); and *DGP Plus: Building Stronger Writers* (writing strategies for helping students transfer grammar concepts to writing).

A finalist for 2007 Georgia Teacher of the Year and recipient of a National Council of Teachers of English Teacher of Excellence Award, Dawn has spoken at conferences and to school systems all around the country.

**Stefan France**, author of the mathematics and science sections of *The Essential Guide to the ACT*, graduated from Duke University in 2000 with a B.S. in Chemistry. Stefan has earned numerous academic awards for his outstanding scholarship and abilities in math and science.

After obtaining his Ph.D. in Chemistry from The Johns Hopkins University in 2005, he joined Emory University as a postdoctoral associate. In 2007, he joined the faculty at Georgia Institute of Technology as an assistant professor of chemistry. He has also served as a math tutor for middle and high school students at ClubZ!.

**Joyce Suber**, author and editor of *The Essential Guide to the ACT*, is a seasoned educator. A graduate of the University of Illinois, she holds a B.A. degree in Sociology and earned a 4.0 in the graduate Educational Psychology program at the National College of Education (National-Louis University). Joyce is certified by the State of Illinois to teach grades K-12, with concentrations in Language Arts and Social Studies (grades 6-12), and has taught high school English in both public and independent schools.

In 2007, Joyce completed a book published by The Princeton Review (Random House) entitled *Road to College: A High School Student's Guide to Discovering Your Passion, Getting Involved, and Getting Admitted*. She has served on numerous boards and committees of professional organizations in the field of college admission counseling. She has also spoken at numerous conferences and schools throughout the United States.

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**Diane Darling** is currently in the 11<sup>th</sup> grade, but she is attending the University of West Georgia full-time in a dual enrollment program, taking college courses while earning her high school diploma. She currently has a 4.0 GPA and consistently scores in the top 95% of her class on both the SAT and ACT exams.

**Erin Gard** is currently in the 11<sup>th</sup> grade and is on track to graduate early and begin her college career. She scored in the top 98% of her class on the ACT exam.

# The Essential Guide to the ACT - Practice Test

Name: \_\_\_\_\_

<b>Reading Test</b> (Test on page 123)	<b>English Test</b> <b>Passage 1 (p. 138)</b>
1. A B C D E	1. A B C D E
2. A B C D E	2. A B C D E
3. A B C D E	3. A B C D E
4. A B C D E	4. A B C D E
5. A B C D E	5. A B C D E
6. A B C D E	6. A B C D E
7. A B C D E	7. A B C D E
8. A B C D E	8. A B C D E
9. A B C D E	9. A B C D E
10. A B C D E	10. A B C D E
11. A B C D E	11. A B C D E
12. A B C D E	12. A B C D E
13. A B C D E	13. A B C D E
14. A B C D E	14. A B C D E
15. A B C D E	15. A B C D E
16. A B C D E	
17. A B C D E	
18. A B C D E	
19. A B C D E	
20. A B C D E	
21. A B C D E	
22. A B C D E	
23. A B C D E	
24. A B C D E	
25. A B C D E	
26. A B C D E	
27. A B C D E	
28. A B C D E	
29. A B C D E	
30. A B C D E	
31. A B C D E	
32. A B C D E	
33. A B C D E	
34. A B C D E	
35. A B C D E	
36. A B C D E	
37. A B C D E	
38. A B C D E	
39. A B C D E	
40. A B C D E	



# The Essential Guide to the ACT - Practice Test

Name: \_\_\_\_\_

<b>English Test Passage 2 (p. 141)</b>	<b>English Test Passage 3 (p. 144)</b>	<b>English Test Passage 4 (p. 147)</b>
1. A B C D E	1. A B C D E	1. A B C D E
2. A B C D E	2. A B C D E	2. A B C D E
3. A B C D E	3. A B C D E	3. A B C D E
4. A B C D E	4. A B C D E	4. A B C D E
5. A B C D E	5. A B C D E	5. A B C D E
6. A B C D E	6. A B C D E	6. A B C D E
7. A B C D E	7. A B C D E	7. A B C D E
8. A B C D E	8. A B C D E	8. A B C D E
9. A B C D E	9. A B C D E	9. A B C D E
10. A B C D E	10. A B C D E	10. A B C D E
11. A B C D E	11. A B C D E	11. A B C D E
12. A B C D E	12. A B C D E	12. A B C D E
13. A B C D E	13. A B C D E	13. A B C D E
14. A B C D E	14. A B C D E	14. A B C D E
15. A B C D E	15. A B C D E	15. A B C D E



# The Essential Guide to the ACT - Practice Test

Name: \_\_\_\_\_

## English Test Passage 5 (p. 150)

- 1. A B C D E
- 2. A B C D E
- 3. A B C D E
- 4. A B C D E
- 5. A B C D E
- 6. A B C D E
- 7. A B C D E
- 8. A B C D E
- 9. A B C D E
- 10. A B C D E
- 11. A B C D E
- 12. A B C D E
- 13. A B C D E
- 14. A B C D E
- 15. A B C D E

## Mathematics Test (Test on page 153)

- 1. A B C D E
- 2. A B C D E
- 3. A B C D E
- 4. A B C D E
- 5. A B C D E
- 6. A B C D E
- 7. A B C D E
- 8. A B C D E
- 9. A B C D E
- 10. A B C D E
- 11. A B C D E
- 12. A B C D E
- 13. A B C D E
- 14. A B C D E
- 15. A B C D E
- 16. A B C D E
- 17. A B C D E
- 18. A B C D E
- 19. A B C D E
- 20. A B C D E
- 21. A B C D E
- 22. A B C D E
- 23. A B C D E
- 24. A B C D E
- 25. A B C D E
- 26. A B C D E
- 27. A B C D E
- 28. A B C D E
- 29. A B C D E
- 30. A B C D E
- 31. A B C D E
- 32. A B C D E
- 33. A B C D E
- 34. A B C D E
- 35. A B C D E
- 36. A B C D E
- 37. A B C D E
- 38. A B C D E
- 39. A B C D E
- 40. A B C D E
- 41. A B C D E
- 42. A B C D E
- 43. A B C D E
- 44. A B C D E
- 45. A B C D E
- 46. A B C D E
- 47. A B C D E
- 48. A B C D E
- 49. A B C D E
- 50. A B C D E
- 51. A B C D E
- 52. A B C D E
- 53. A B C D E
- 54. A B C D E
- 55. A B C D E
- 56. A B C D E
- 57. A B C D E
- 58. A B C D E
- 59. A B C D E
- 60. A B C D E



## The Essential Guide to the ACT - Practice Test

Name: \_\_\_\_\_

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### Science Test

(Test on page 173)

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- |     |   |   |   |   |   |     |   |   |   |   |   |
|-----|---|---|---|---|---|-----|---|---|---|---|---|
| 1.  | A | B | C | D | E | 26. | A | B | C | D | E |
| 2.  | A | B | C | D | E | 27. | A | B | C | D | E |
| 3.  | A | B | C | D | E | 28. | A | B | C | D | E |
| 4.  | A | B | C | D | E | 29. | A | B | C | D | E |
| 5.  | A | B | C | D | E | 30. | A | B | C | D | E |
| 6.  | A | B | C | D | E | 31. | A | B | C | D | E |
| 7.  | A | B | C | D | E | 32. | A | B | C | D | E |
| 8.  | A | B | C | D | E | 33. | A | B | C | D | E |
| 9.  | A | B | C | D | E | 34. | A | B | C | D | E |
| 10. | A | B | C | D | E | 35. | A | B | C | D | E |
| 11. | A | B | C | D | E | 36. | A | B | C | D | E |
| 12. | A | B | C | D | E | 37. | A | B | C | D | E |
| 13. | A | B | C | D | E | 38. | A | B | C | D | E |
| 14. | A | B | C | D | E | 39. | A | B | C | D | E |
| 15. | A | B | C | D | E | 40. | A | B | C | D | E |
| 16. | A | B | C | D | E |     |   |   |   |   |   |
| 17. | A | B | C | D | E |     |   |   |   |   |   |
| 18. | A | B | C | D | E |     |   |   |   |   |   |
| 19. | A | B | C | D | E |     |   |   |   |   |   |
| 20. | A | B | C | D | E |     |   |   |   |   |   |
| 21. | A | B | C | D | E |     |   |   |   |   |   |
| 22. | A | B | C | D | E |     |   |   |   |   |   |
| 23. | A | B | C | D | E |     |   |   |   |   |   |
| 24. | A | B | C | D | E |     |   |   |   |   |   |
| 25. | A | B | C | D | E |     |   |   |   |   |   |









