

Project Proposal AESE 404 | Joshua Kraus

This semester I would like to simulate and fly a solid propellant rocket that holds parallel about a vertical axis without passive aerodynamic stabilization such as fins. Fin restoring moment scales with dynamic pressure, so it vanishes at low speed and in thin air (e.g. space). TVC moment scales with thrust times the lever arm and does not depend on airspeed making it the primary method to control space-faring vehicles. The goal of this project is to demonstrate high controllability in regimes that are too slow or too thin for fins, and to address the mechanical, computational, and control challenges of TVC at model scale. To do this I am repurposing hardware and firmware from the drone and aircraft industries, namely the Pixhawk 6X running ArduPilot. To my knowledge, these open source stacks have no native mode for guiding a rocket by gimbaling a motor. So I will build the estimator and control law myself on the flight controller's addressable hardware and compare the result against ArduPilot's own estimator of position. The end goal is a program and control law that flies the vehicle with no user input.

The vehicle is slated to be about four feet (1.2 m) long and three inches (7.6 cm) in diameter, with a liftoff mass of a little over two pounds (about 1 kg). The airframe likely should start as cardboard, with the goal of moving to carbon fiber or fiberglass depending on the university rocketry team's manufacturing capability this year. The sensors, batteries, and peripherals are to sit in the nosecone to keep the CG forward and offset the instability of a finless body. Propulsion is an AeroTech I49N-P, a reloadable, non monolithic 38 mm motor with about 11 lbs of average thrust, 16 lbs of peak thrust at liftoff, and a 6.5 s burn (AeroTech n.d.). That gives a liftoff thrust to weight ratio of roughly 6.

The entire motor assembly gimbals about a fixed pivot at the base of the rocket, roughly 1 ft from the CG. Two orthogonal control arms, each driven by a high voltage servo, move the motor through a cone of up to $\sim \pm 10^\circ$ in the ideal linkage geometry, with the usable range to be confirmed on the bench. Each servo horn is 1 in long and drives a 4 in carbon pushrod attached to the motor casing 4 in from the pivot. The horn sits perpendicular to the pushrod at neutral, so motor angle goes roughly as the sine of servo angle and gain is highest at center. Testing the mechanism to measure this gain map and the servo frequency response is a primary concern, since both feed the actuator model in the equations of motion. Moving the motor also shifts the CG, although almost negligibly, and adds inertial coupling, which should be quantified and either modeled or justified as negligible as well.

The Pixhawk 6X provides three IMUs and barometers, and an external MPU-6050 is added for bench testing. Sensors will be modeled with noise and bias, and a Kalman filter + PID based controller will provide attitude, altitude, and vertical speed estimate and control. GPS is a stretch goal that would add waypoint guidance during the vertical ascent.

The ultimate deliverable of this project is controlled ascent demonstration and the hardware and code capability to control any similar scale vehicle, with the ambition of applying it to a hopper or landing vehicle for the university rocketry team's liquid propulsion rocket.

Development will follow a simulation first workflow. The equations of motion, sensor and estimator models, open loop simulation, and control design are to be built and tested in simulation, starting with control law scheduled by flight regime. The software is then to be bench tested on the gimbal and

ultimately static fired to determine response characteristics of the mechanism under flight-like load. From this point changes will likely be incorporated into the software, simulated and then taken either to static fire or directly to attempted flight in 2027.

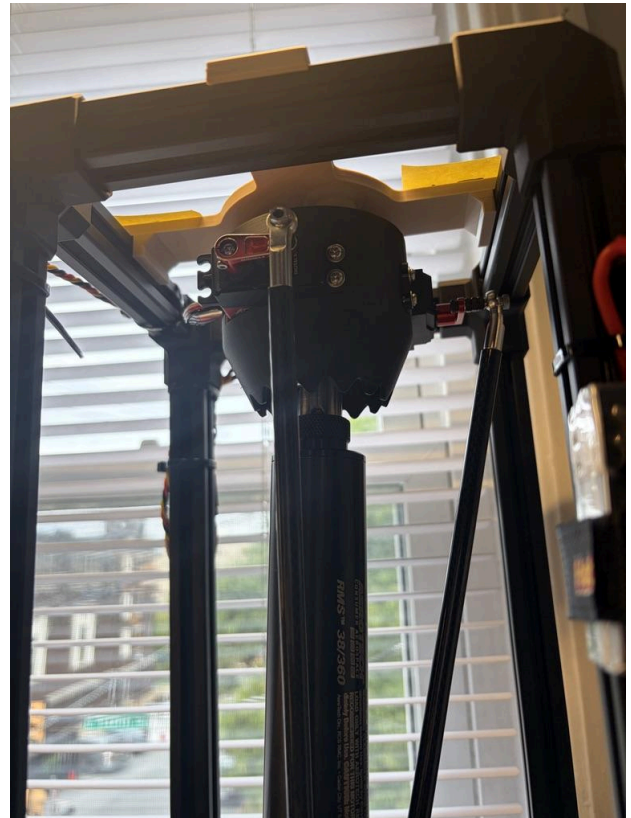
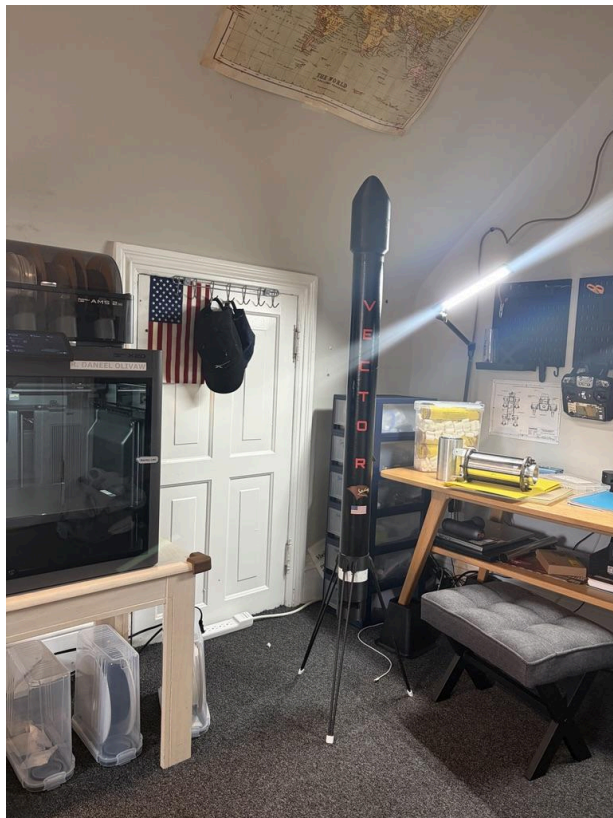
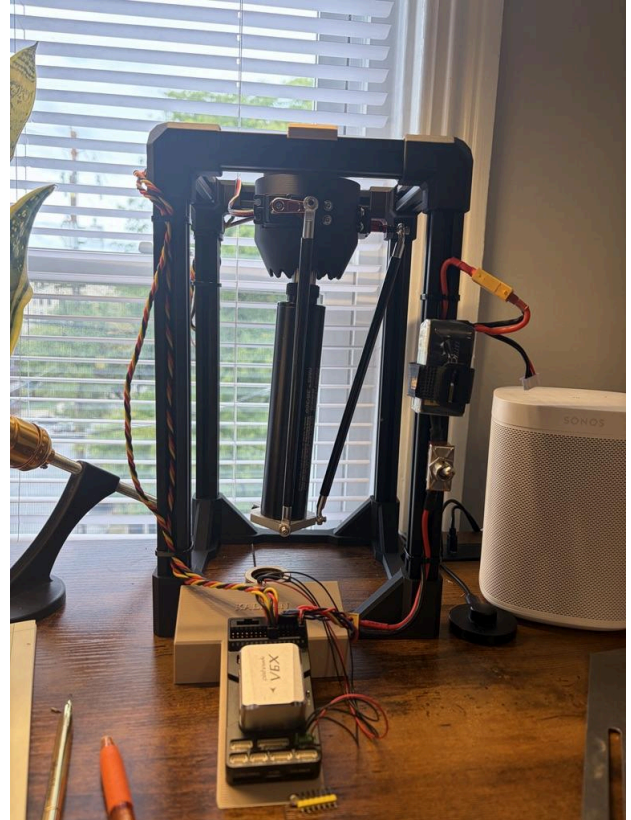
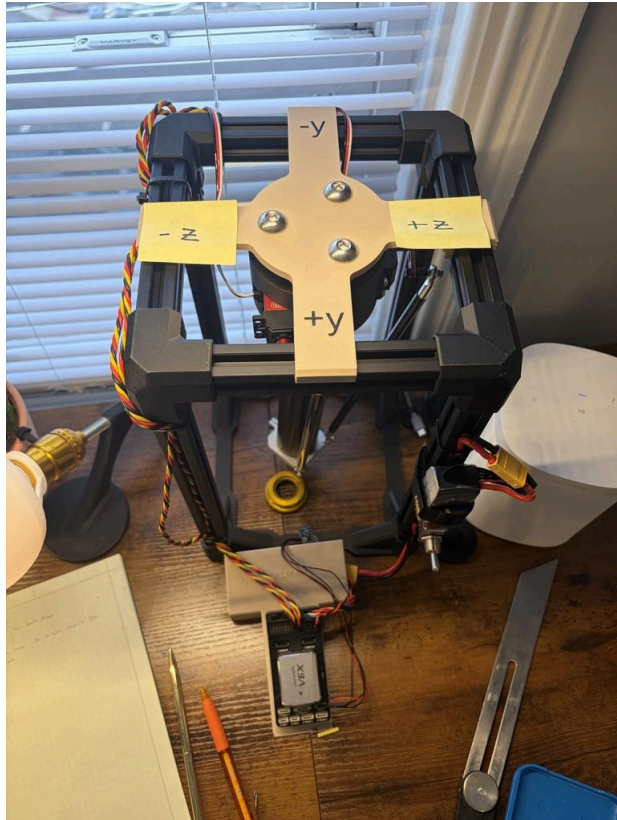
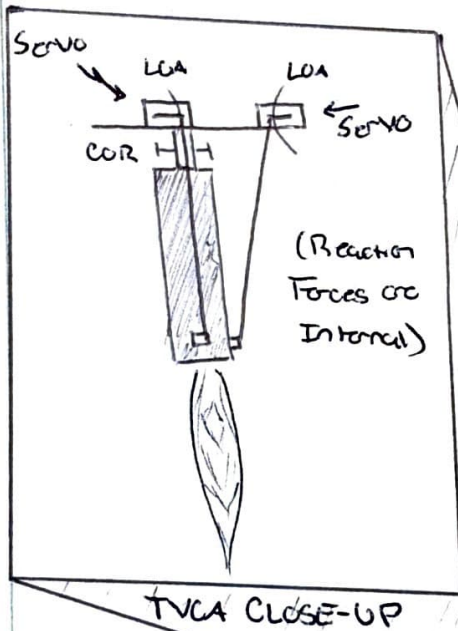
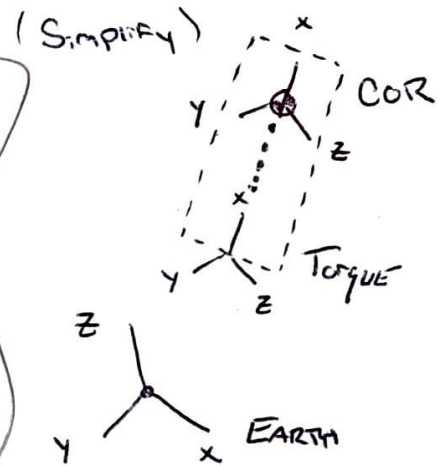
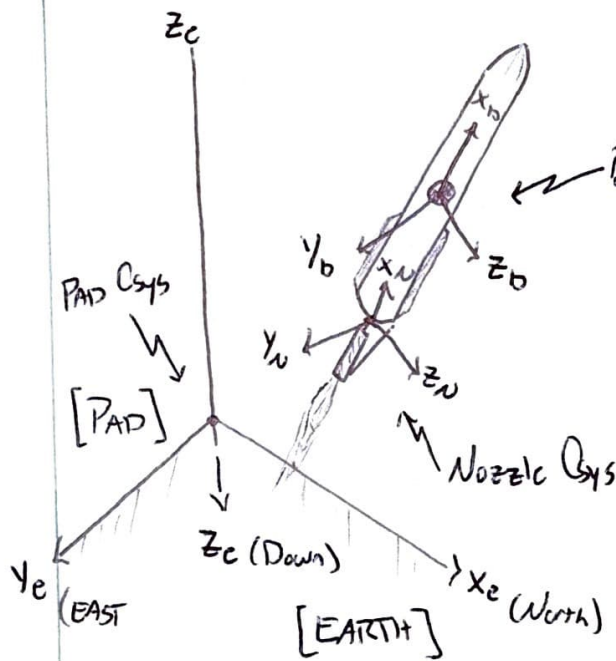


FIG 01: FRAMES



T_E - Thrust of Exhaust
 R_w - Relative Wind
 w - Weight
 W_m^* - Inertial Mass of Motor as it burns

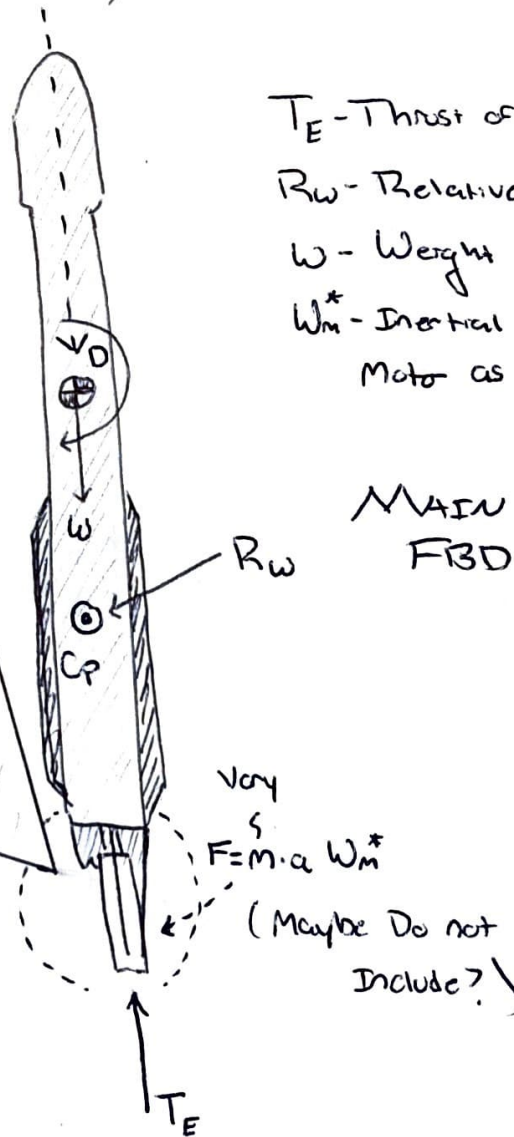
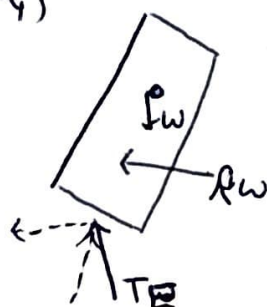


Fig 02, FBD

(Simplify)

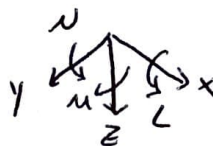


UNRELATED NOTES TO LEARN SUBJECT FOR HW (THIS WAS FOR A PHONE)

$$\underline{F} = m \cdot \underline{a} \quad \underline{T} = I \underline{\alpha} \quad H = I \underline{\Omega}$$

$$\downarrow \quad \downarrow$$

$$\frac{d\underline{V}}{dt} \quad \frac{d\underline{h}}{dt}$$



$$[LMN] = \text{Roll Pitch Yaw}$$

Transport Thm $m \left(\frac{d\underline{V}}{dt} \right)_E = m \left(\left(\frac{d\underline{V}}{dt} \right)_B + \underline{\omega}_B \times \underline{V}_B \right)$

Accel $\downarrow$

$$\begin{bmatrix} \ddot{u} \\ \ddot{v} \\ \ddot{w} \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} \text{ Any View}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

→ Convert to Skew Sym!!

$$\rightarrow \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ q & p & 0 \end{bmatrix} \cdot \begin{bmatrix} u \\ v \\ w \end{bmatrix} \left\{ \begin{array}{l} x_{\text{Force}} = m \cdot \left(\frac{du}{dt} + q \cdot w - r \cdot v \right) \\ y_{\text{Force}} = m \cdot \left(\frac{dv}{dt} + r \cdot u - p \cdot w \right) \\ z_{\text{Force}} = m \cdot \left(\frac{dw}{dt} + p \cdot v - q \cdot u \right) \end{array} \right.$$

Can do similar with Torques / Moments →

$$\underline{T} = \left(\frac{d\underline{H}}{dt} \right)_E \quad H = I \underline{\omega}, \quad \underline{\omega} = [p \ q \ r]^T$$

$$\left(\frac{d\underline{H}}{dt} \right)_E = \left(\frac{d\underline{H}}{dt} \right)_B + \underline{\omega}_B \times \underline{H}_B$$

rate of change

$$[LMN]^T \quad \downarrow \quad H = I \underline{\omega} \text{ so } \frac{d\underline{H}}{dt} = I \left(\frac{d\underline{\omega}}{dt} \right) = I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}$$

$$I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{xy} & I_{zz} \end{bmatrix} \quad \leftarrow \text{Symmetr!}$$

$$\underline{\omega}_B/E = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}$$

$$\underline{H}_B = I \cdot \underline{\omega}_B = \begin{bmatrix} I_{xx} & 0 & I_{xz} \\ 0 & I_{yy} & 0 \\ I_{xz} & 0 & I_{zz} \end{bmatrix} \cdot \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

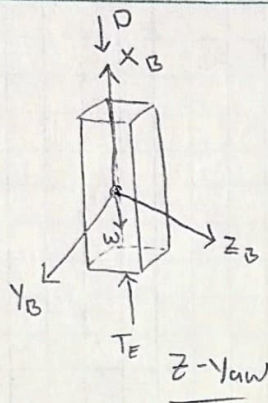
All tog. →

$$\begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} I_{xx} \cdot \dot{p} + I_{xz} \cdot \dot{r} \\ I_{yy} \cdot \dot{q} \\ I_{zz} \cdot \dot{r} + I_{xz} \cdot \dot{p} \end{bmatrix} + \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \cdot \begin{bmatrix} I_{xx} & 0 & I_{xz} \\ 0 & I_{yy} & 0 \\ I_{zx} & 0 & I_{zz} \end{bmatrix} \cdot \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\begin{aligned} L &= I_{xx} \cdot \dot{p} + I_{xz} \cdot \dot{r} + q(r(I_{zz} - I_{yy}) + p \cdot I_{xz}) \\ M &= I_{yy} \cdot \dot{q} + p \cdot r(I_{xx} - I_{zz}) + (r^2 - p^2) I_{xz} \\ N &= I_{zz} \cdot \dot{r} + I_{xz} \cdot \dot{p} + p \cdot q(I_{yy} - I_{xx}) - q \cdot r I_{xz} \end{aligned} \quad \left. \vphantom{\begin{aligned} L \\ M \\ N \end{aligned}} \right\} \text{Moments!}$$

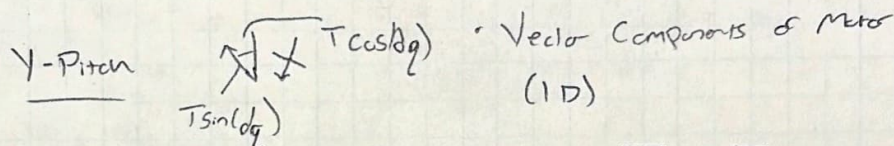
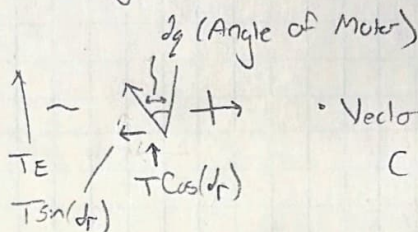
~ END of Notes ~

Eq of Motion From FBD's



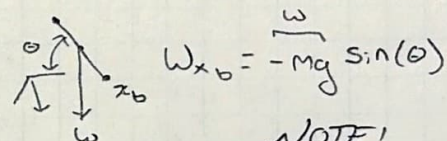
$\sum F_x = \text{Weight, Thrust, Aero}$
 ↑
 Varies w Angle Varies w angle

Will keep Symbolic Since I have no good numbers
 Now maybe can do an Appx of a cyl?
 (Constant Force Appx)



So in 2 axis we can pull both x deflections $T_{x_b} = T \cos(d_g) \cos(d_r)$

Say $\theta = \text{Angle From NED Frame to Nose Point}$
 Axis (x_b)



NOTE!

For now drag is constant as $\underline{D_x}$ (→ C_p & Force of drag possible to Calculate Roughly From Open Rocket)

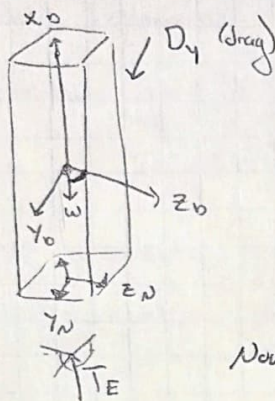
Lecture has $\underline{F_x} = m(\dot{V}_x + gV_z - rV_y)$

$$m(\dot{V}_x + gV_z - rV_y) = T \cos(d_r) \cos(d_g) - mg \sin(\theta) + D_x$$

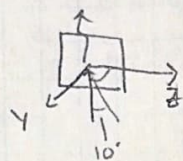
$$\dot{V}_x = \left(T \cos(d_r) \cos(d_g) - mg \sin(\theta) + D_x \right) \frac{1}{m} - gV_z - rV_y$$

↑ deflection of gimbal ↑ Pitch ↑ Yaw Spin Rates ← Current Velo in this direction

Now moving to $y \rightarrow$



Say $d_f = 10^\circ$ $T = 49 \text{ N}$ $d_r = 0$



Then $x = T \cos(10) \approx 48.3 \text{ N}$
 $z = T \sin(10) \approx -8.5 \text{ N}$
 $y = 0!$

Now say $d_r = 10^\circ$ then $T = 48.3$

$x = 48.3 \times \cos(10) = 47.6 \text{ N body}$

$y = 48.3 \cdot \sin(10) = 8.4$

$z_{\text{STATIC}} = -8.5 \text{ N!}$

Pitch $\rightarrow$ YAW
 (per NASA TB-03)

Whats left



So for y , $T_y = T \sin(d_r) \cdot \underbrace{\cos(d_f)}_{\text{Commanded}}$

gravity is simply $+mg \cdot \underbrace{\sin(\phi) \cos(\theta)}_{\substack{\text{Whats avail.} \\ \text{Projected}}}$

Drag we can keep symbolic

So $F_y = T \sin(d_r) \cos(d_f) + mg \sin(\phi) \cos(\theta)$

↑
Yaw

↑
Pitch

↑

↑
x to NED



Rot angle ϕ

$\omega_x = P$

$\omega_z = R$

So $\dot{V}_y = (T \sin(d_r) \cos(d_f) + mg \sin(\phi) \cos(\theta) + D_y) \frac{1}{m} + \underbrace{P V_z}_{\text{Spin rate x}} - \underbrace{R V_x}_{\text{Spin rate z}}$

z can be found the same way. So it follows its

$\dot{V}_z = \underbrace{(-T \sin(d_f))}_{\text{no yaw}} + mg \cos(\phi) \cos(\theta) + D_z \frac{1}{m} - P V_y + R V_x$

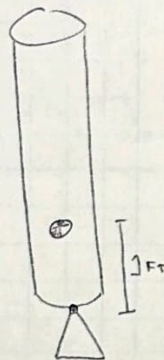
in
Thrust to
Note
direction

Pitch $\rightarrow$ Yaw. Yaw only gets what pitch leaves of x

OK new Rotations

↳ We have
$$\left. \begin{aligned} T_x &= I_x \dot{p} + qr(I_z - I_y) \\ T_y &= I_y \dot{q} + pr(I_x - I_z) \\ T_z &= I_z \dot{r} + pq(I_y - I_x) \end{aligned} \right\} \text{From lecture slides}$$

Need a moment arm, which I don't have, so I will call it l_F
From the gimbal pivot point to CG



Well for T_x , right along axis so CG produce
Roll (possible manufacturing Assym may contribute)
Will assume on axis so

- Small Aero
- Assym
- uneven mass distro

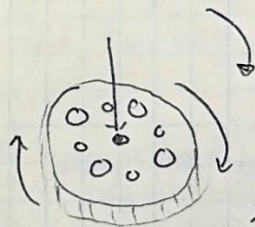
} Calling all this M_{Roll}

Prob will have $M_{roll}=0$ in simulation

▼ Prob want a Small reaction Wheel or Fins to
Counter Roll...

So Solving For $\dot{p}$ $\leadsto T_x = M_{roll}$

$$\dot{p} = \frac{M_{roll}}{(I_x - qr(I_z - I_y)) \frac{1}{I_x}}$$

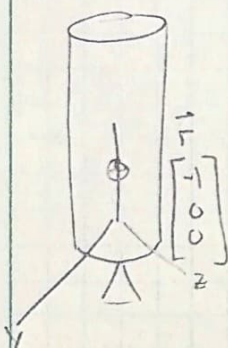


Can size
For MoI
about a cyl,

Which is small
given dia

OK so $\text{moment} = \vec{r} \times \vec{F}$ Where $\vec{r}$ = Vector From CG To Pivot point

I am going to Assume ~ 1 foot Since this is What it eyeball looked like During a Markup so $\rightarrow$



The Full Thrust Vector is the Center of all the Translation Parts From before so

$$\vec{T}_{All} = [T \cos(d_r) \cos(d_\phi), T \sin(d_r) \cos(d_\phi), -T \sin(d_\phi)]$$

So for y $(\vec{r} \times \vec{F})_y = \underbrace{r_z F_x - r_x F_z}_{\substack{\downarrow \\ 0} \quad \substack{\downarrow \\ 1}} =$

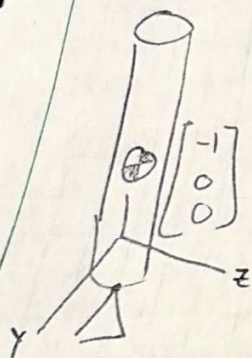
Having 1 as a reminder its a holder $\rightarrow$
$$= (-)(-1)(-T \sin(d_\phi)) = (1)T \sin(d_\phi) \checkmark$$

- grav is at C_g , Aero I don't know C_p yet so will leave as Variable

So $T_y = -2T \sin(d_\phi) + M_{Aerodynamic, y}$, using Skeleton

$$\ddot{\theta} = \frac{(-2T \sin(d_\phi) + M_{Aerodynamic, y})}{I_y - (P_r(I_x - I_z) \frac{1}{I_y})}$$

New T_z The Yaw Moment



Following the pattern from pitch I expect the

$$(r \times F)_z = r_x F_y - r_y F_x$$

$\downarrow \quad \downarrow \quad \downarrow$
 $-1 \quad 0 \quad 0$

From Translation so

$$= (-1)(T \sin(\delta r) \cos(\delta q))$$

$\Rightarrow$ Plug with Sincelot then

Again, no CP yet

$$T_z = -1 T \sin(\delta r) \cos(\delta q) + M_{Aerodynamic, z}$$

$$\ddot{r} = \left(-1 T \sin(\delta r) \cos(\delta q) + M_{Aerodynamic} \right) \frac{1}{I_z - \left(P_g(I_y - I_x) \frac{1}{I_z} \right)}$$

Assumptions

$\hookrightarrow$ Mass is constant, reasonable since prop weight while it will go down is not a large percent of the total mass. Conservative since we will have better control arm as core goes up away from thrust point

C_g is 1 ft away from pivot, needs to be locked at end defined based on real hardware/CAD and flagged

Flat, non rotating Earth, for simplification makes sense and flight will not be very high

- Rigid body
 - gimbal angles are treated as direct inputs

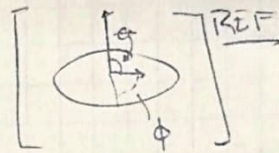
} For modeling case, will flag to characterize on bench

State is to be described by position in earth Frame, Velocity in body Frame attitude as roll pitch and yaw Euler angles ϕ, θ, ψ and the Tinge rotation rates p, q, r . Gimbal deflection angles $d\phi$ and $d\theta$ are Control inputs. Vehicle Params are Mass, I_x, I_y, I_z , gimbal to C_g distance and C_p to C_g distance. Thrust will come from the published Thrust Curve by AeroTech. Aerodynamic terms or Symbolic until C_D and C_p are produced from either FEA or open rocket

So $\dot{x} = f(x, u)$ gimbal angle

$\dot{P} = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}$ sensor/Pos state for Matlab

$\dot{P} = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}$ (Control = $(0, 0, 0)$)



$$\dot{V}_x = T \cos(d\theta) \cos(d\phi) - Mg \sin(\theta) \frac{1}{m} - q V_z + r V_y$$

$$\dot{V}_y = T \sin(d\theta) \cos(d\phi) + Mg \sin(\phi) \frac{1}{m} + p V_z - r V_x$$

$$\dot{V}_z = -T \sin(d\phi) + Mg \cos(\phi) \frac{1}{m} - p V_y + q V_x$$

$$\dot{p} = \frac{M_{roll}}{I_x} - q r (I_z - I_y) \frac{1}{I_x}$$

$$\dot{q} = -\frac{I_z}{I_y} \sin(d\phi) \frac{1}{I_y} - p \cdot r (I_x - I_z) \frac{1}{I_y}$$

$$\dot{r} = -\frac{I_z}{I_z} \sin(d\theta) \cos(d\phi) \frac{1}{I_z} - p \cdot q (I_y - I_x) \frac{1}{I_z}$$

$$\dot{\phi} = p + q \sin(\phi) \tan(\theta) + r \cos(\phi) \tan(\theta)$$

$$\dot{\theta} = q \cos(\phi) - r \sin(\phi)$$

$$\dot{\psi} = q \sin(\phi) \sec(\theta) + r \cos(\phi) \sec(\theta)$$

($D=0$) For now
 $M=0$ since I
 don't have
 $T = \text{Thrust}$

So the State Space is described by the Vector of $\dot{x}$ which is answered by solving the above with inputs of x , and u vectors.

One obvious limit here is if $\theta = 90^\circ$ since $\sec(90^\circ) = \text{Undefined}$

(this has been known, and is why quaternions were invented, but this has not been scraped into this proposal)