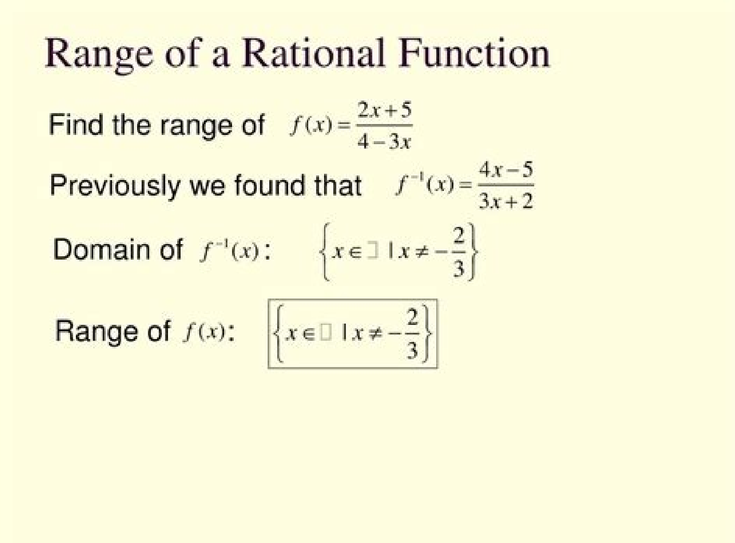
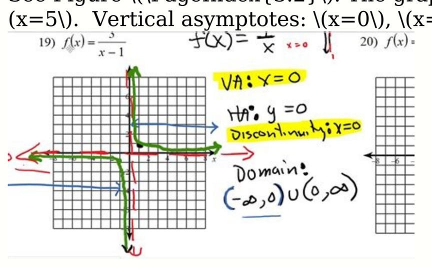
Domain and range of rational function graph

How do you find the domain and range of a rational function graph. Find the domain and range of the rational function graphed below. How to find domain and range of rational function graph. Domain and range of rational functions. Finding the intercepts asymptotes domain and range from the graph of a rational function. Finding the intercepts asymptotes domain and range from the graph of a rational function calculator.

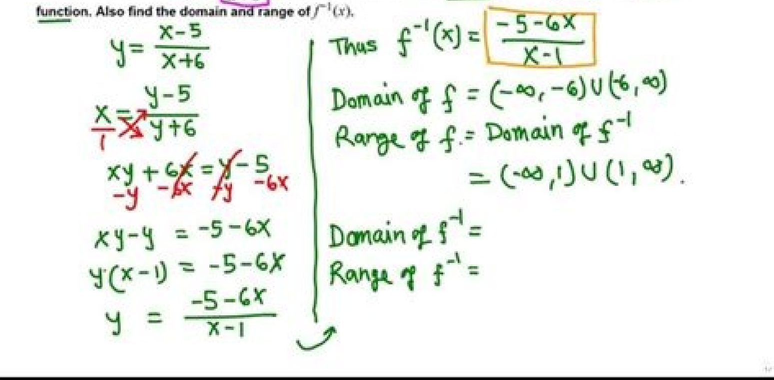
Learn Rational Function with tutors mapped to your child's learning needs.30-DAY PROMISE | GET 100% MONEY BACK**T&C ApplyMath worksheets and visual curriculum In order to continue enjoying our site, we ask that you confirm your identity as a human. Thank you very much for your cooperation. In order to continue enjoying our site, we ask that you confirm your identity as a human. Thank you very much for your cooperation. A vertical asymptote represents a value at which a rational function is undefined, so that value is not in the domain of the function. A reciprocal function cannot have values in its domain that cause the denominator to equal zero. In general, to find the domain of a rational function, we need to determine which inputs would cause division by zero. Definition: DOMAIN OF A RATIONAL FUNCTION The domain of a rational function includes all real numbers except those that cause the denominator to equal zero. How To: Given a rational function, find the domain. Set the denominator equal to zero. Solve to find the x-values that cause the denominator to equal zero. The domain is all real numbers except those found in Step 2. Example $\PageIndex{1}$: Finding the Domain of a Rational Function Find the domain of $f(x)=\dfrac{x+3}{x^2−9}$. Solution Begin by setting the denominator equal to zero and solving. $x^2-9=0$ $x^2=9$ $x=\pm 3$ The denominator is equal to zero when $x=\pm 3$. The domain of the function is all real numbers except $x=\pm 3$. Analysis A graph of this function, as shown in Figure $\PageIndex{1}$, confirms that the function is not defined when $x=\pm 3$. There is a vertical asymptote at $x=3$ and a hole in the graph at $x=−3$. We will discuss these types of holes in greater detail later in this section. Figure $\PageIndex{1}$ Try It $\PageIndex{1}$ Find the domain of $f(x)=\dfrac{4x}{5(x−1)(x−5)}$. Answer The domain is all real numbers except $x=1$ and $x=5$. By looking at the graph of a rational function, we can investigate its local behavior and easily see whether there are asymptotes. We may even be able to approximate their location. Even without the graph, however, we can still determine whether a given rational function has any asymptotes, and calculate their location. The vertical asymptotes of a rational function may be found by examining the factors of the denominator that are not common to the factors in the numerator. Vertical asymptotes occur at the zeros of such factors. How To: Given a rational function, identify any vertical asymptotes of its graph Factor the numerator and denominator. Note any restrictions in the domain of the function. Reduce the expression by canceling common factors in the numerator and the denominator. Note any values that cause the denominator to be zero in this simplified version. These are where the vertical asymptotes occur.



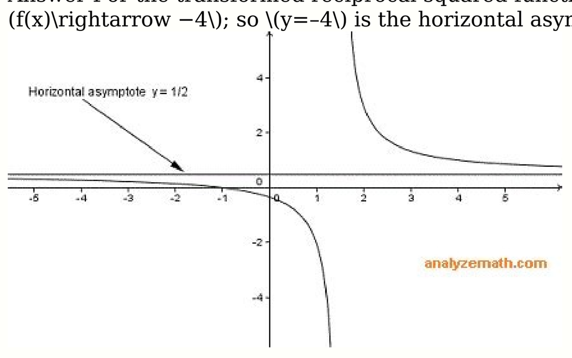
Note any restrictions in the domain where asymptotes do not occur. These are removable discontinuities, or “holes.” Example $\PageIndex{2}$: Identifying Vertical Asymptotes Find the vertical asymptotes of the graph of $k(x)=\dfrac{5+2x^2}{2−x−x^2}$. Solution First, factor the numerator and denominator. $$k(x)=\dfrac{5+2x^2}{2−x−x^2} \nonumber$$ $$=\dfrac{5+2x^2}{(2+x)(1-x)} \nonumber$$ To find the vertical asymptotes, we determine where this function will be undefined by setting the denominator equal to zero: $$(2+x)(1−x)=0 \nonumber$$ $$x=−2, \; x=1 \nonumber$$ Neither $x=–2$ nor $x=1$ are zeros of the numerator, so the two values indicate two vertical asymptotes. The graph in Figure $\PageIndex{2}$ confirms the location of the two vertical asymptotes.
Figure $\PageIndex{2}$. Occasionally, a graph will contain a hole: a single point where the graph is not defined, indicated by an open circle. We call such a hole a removable discontinuity. For example, the function $f(x)=\dfrac{x^2−1}{x^2−2x−3}$ may be re-written by factoring the numerator and the denominator. $$f(x)=\dfrac{(x+1)(x−1)}{(x+1)(x−3)} \nonumber$$ Notice that $x+1$ is a common factor to the numerator and the denominator. The zero of this factor, $x=−1$, is the location of the removable discontinuity. Notice also that $(x–3)$ is not a factor in both the numerator and denominator. The zero of this factor, $x=3$, is the vertical asymptote. See Figure $\PageIndex{3.1}$. [Note that removable discontinuities may not be visible when we use a graphing calculator, depending upon the window selected.] Figure $\PageIndex{3.1}$. REMOVABLE DISCONTINUITIES OF RATIONAL FUNCTIONS A removable discontinuity occurs in the graph of a rational function at $x=a$ if a is a zero for a factor in the denominator that is common with a factor in the numerator. We factor the numerator and denominator and check for common factors. If we find any, we set the common factor equal to 0 and solve. This is the location of the removable discontinuity.
This is true if the multiplicity of this factor is greater than or equal to that in the denominator. If the multiplicity of this factor is greater in the denominator, then there is still an asymptote at that value. Example $\PageIndex{3}$: Identifying Vertical Asymptotes and Removable Discontinuities for a Graph Find the vertical asymptotes and removable discontinuities of the graph of $k(x)=\dfrac{x−2}{x^2−4}$. Solution Factor the numerator and the denominator. $$k(x)=\dfrac{x−2}{(x−2)(x+2)} \nonumber$$ Notice that there is a common factor in the numerator and the denominator, $x–2$. The zero for this factor is $x=2$. This is the location of the removable discontinuity. Notice that there is a factor in the denominator that is not in the numerator, $x+2$. The zero for this factor is $x=−2$. The vertical asymptote is $x=−2$.
See Figure $\PageIndex{3.2}$. The graph of this function will have the vertical asymptote at $x=−2$, but at $x=2$ the graph will have a hole. Figure $\PageIndex{3.2}$. Try It $\PageIndex{3}$ Find the vertical asymptotes and removable discontinuities of the graph of $f(x)=\dfrac{x^2−25}{x^3−6x^2+5x}$. Answer Removable discontinuity at $x=5$. Vertical asymptotes: $x=0$, $x=1$. While vertical asymptotes describe the behavior of a graph as the output gets very large or very small, horizontal asymptotes help describe the behavior of a graph as the input gets very large or very small. Recall that a polynomial’s end behavior will mirror that of the leading term.



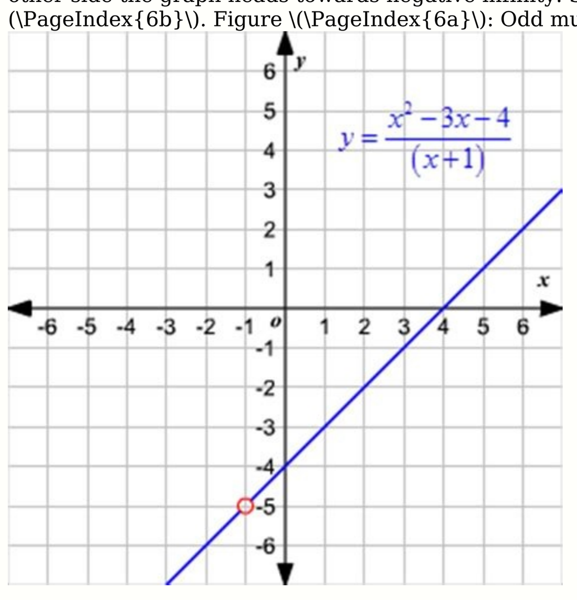
Likewise, a rational function’s end behavior will mirror that of the ratio of the function that is the ratio of the leading terms. There are three distinct outcomes when checking for horizontal asymptotes: Case 1: If the degree of the denominator > degree of the numerator, there is a horizontal asymptote at $y=0$. Example: $f(x)=\dfrac{4x+2}{x^2+4x−5}$ In this case, the end behavior is $f(x)≈\dfrac{4x}{x^2}=\dfrac{4}{x}$. This tells us that, as the inputs increase or decrease without bound, this function will behave similarly to the function $g(x)=\dfrac{4}{x}$, and the outputs will approach zero, resulting in a horizontal asymptote at $y=0$. See Figure $\PageIndex{4a}$. Note that this graph crosses the horizontal asymptote. Figure $\PageIndex{4a}$: Horizontal asymptote $y=0$ occurs when the degree of the numerator is < degree of the denominator. Case 2: If the degree of the denominator < degree of the numerator by one, we get a slant asymptote.
Example: $f(x)=\dfrac{3x^2−2x+1}{x−1}$ In this case, the end behavior is $f(x)≈\dfrac{3x^2}{x}=3x$. This tells us that as the inputs increase or decrease without bound, this function will behave similarly to the function $g(x)=3x$.
As the inputs grow large, the outputs will grow and not level off, so this graph has no horizontal asymptote. However, the graph of $g(x)=3x$ looks like a diagonal line, and since f will behave similarly to g, it will approach a line close to $y=3x$. This line is a slant asymptote. To find the equation of the slant asymptote, divide $\dfrac{3x^2−2x+1}{x−1}$. The quotient is $3x+1$, and the remainder is 2. The slant asymptote is the graph of the line $g(x)=3x+1$. See Figure $\PageIndex{4b}$.
Figure $\PageIndex{4b}$: Slant asymptote occurs when the degree of the numerator = 1+ degree of the denominator. Case 3: If the degree of the denominator = degree of the numerator, there is a horizontal asymptote at $y=\frac{a_n}{b_n}$, where a_n and b_n are respectively the leading coefficients of the numerator and denominator of the rational function. Example: $f(x)=\dfrac{3x^2+2}{x^2+4x−5}$ In this case, the end behavior is $f(x)≈\dfrac{3x^2}{x^2}=3$. This tells us that as the inputs grow large, this function will behave like the function $g(x)=3$, which is a horizontal line. As $x\rightarrow \pm \infty$, $f(x)\rightarrow 3$, resulting in a horizontal asymptote at $y=3$. See Figure $\PageIndex{4c}$. Note that this graph crosses the horizontal asymptote. Figure $\PageIndex{4c}$: Horizontal asymptote $y=a/b$ occurs when the degree of the numerator = degree of the denominator. Notice that, while the graph of a rational function will never cross a vertical asymptote, the graph may or may not cross a horizontal or slant asymptote. Also, although the graph of a rational function may have many vertical asymptotes, the graph will have at most one horizontal (or slant) asymptote. It should be noted that, if the degree of the numerator is larger than the degree of the denominator by more than one, the end behavior of the graph will mimic the behavior of the reduced end behavior fraction. For instance, if we had the function $$f(x)=\dfrac{3x^5−x^2}{x+3} \nonumber$$ with end behavior $$f(x)≈\dfrac{3x^5}{x}=3x^4 \nonumber$$ the end behavior of the graph would look similar to that of an even polynomial with a positive leading coefficient. $x\rightarrow \pm \infty, f(x)\rightarrow \infty$ HORIZONTAL ASYMPTOTES OF RATIONAL FUNCTIONS The horizontal asymptote of a rational function can be determined by looking at the degrees of the numerator and denominator. Degree of numerator is less than degree of denominator: horizontal asymptote at $y=0$. Degree of numerator is greater than degree of denominator by one: no horizontal asymptote; slant asymptote. Degree of numerator is equal to degree of denominator: horizontal asymptote at ratio of leading coefficients. Example $\PageIndex{4.1}$: Identifying Horizontal and Slant Asymptotes For the functions listed, identify the horizontal or slant asymptote. $g(x)=\dfrac{6x^3−10x}{2x^3+5x^2}$ $h(x)=\dfrac{x^2−4x+1}{x+2}$ $k(x)=\dfrac{x^2+4x}{x^3−8}$ Solution For these solutions, we will use $f(x)=\dfrac{p(x)}{q(x)},\space q(x)≠0$. $g(x)=\dfrac{6x^3−10x}{2x^3+5x^2}$: The degree of $p =$ degree of $q=3$, so we can find the horizontal asymptote by taking the ratio of the leading terms. There is a horizontal asymptote at $y =\frac{6}{2}$ or $y=3$. $h(x)=\dfrac{x^2−4x+1}{x+2}$: The degree of $p=2$ and degree of $q=1$. Since $p>q$ by 1, there is a slant asymptote found at $\dfrac{x^2−4x+1}{x+2}$. $k(x)=\dfrac{x^2+4x}{x^3−8}$: The degree of $p=2$ < degree of $q=3$, so there is a horizontal asymptote $y=0$. Example $\PageIndex{4.2}$ Identifying Horizontal Asymptotes In the sugar concentration problem earlier, we created the equation $C(t)=\dfrac{5+t}{100+10t}$. Find the horizontal asymptote and interpret it in context of the problem. Solution Both the numerator and denominator are linear (degree 1). Because the degrees are equal, there will be a horizontal asymptote at the ratio of the leading coefficients. In the numerator, the leading term is t, with coefficient 1.
In the denominator, the leading term is 10t, with coefficient 10. The horizontal asymptote will be at the ratio of these values: $t\rightarrow \infty,\space C(t)\rightarrow \frac{1}{10}$ This function will have a horizontal asymptote at $y=\frac{1}{10}$. This tells us that as the values of t increase, the values of C will approach $\frac{1}{10}$. In context, this means that, as more time goes by, the concentration of sugar in the tank will approach one-tenth of a pound of sugar per gallon of water or $\frac{1}{10}$ pounds per gallon. Example $\PageIndex{4.3}$: Identifying Horizontal and Vertical Asymptotes Find the horizontal and vertical asymptotes of the function $f(x)=\dfrac{(x−2)(x+3)}{(x−1)(x+2)(x−5)}$ Solution First, note that this function has no common factors, so there are no potential removable discontinuities. The function will have vertical asymptotes when the denominator is zero, causing the function to be undefined. The denominator will be zero at $x=1,–2,$and 5, indicating vertical asymptotes at these values. The numerator has degree 2, while the denominator has degree 3. Since the degree of the denominator is greater than the degree of the numerator, the denominator will grow faster than the numerator, causing the outputs to tend towards zero as the inputs get large, and so as $x\rightarrow \pm \infty$, $f(x)\rightarrow 0$. This function will have a horizontal asymptote at $y =0.$ See Figure $\PageIndex{4.3}$. Figure $\PageIndex{4.3}$.
Try It $\PageIndex{4}$ Find the vertical and horizontal asymptotes of the function: $f(x)=\dfrac{(2x−1)(2x+1)}{(x−2)(x+3)}$ Answer Vertical asymptotes at $x=2$ and $x=–3$ horizontal asymptote at $y =4$. INTERCEPTS OF RATIONAL FUNCTIONS A rational function will have a y-intercept at $f(0),$ if the function is defined at zero. A rational function will not have a y-intercept if the function is not defined at zero. Likewise, a rational function will have x-intercepts at the inputs that cause the output to be zero.
Since a fraction is only equal to zero when the numerator is zero, x-intercepts can only occur when the numerator of the rational function is equal to zero. Example $\PageIndex{5}$: Finding the Intercepts of a Rational Function Find the intercepts of $f(x)=\dfrac{(x−2)(x+3)}{(x−1)(x+2)(x−5)}$. Solution We can find the y-intercept by evaluating the function at zero $f(0)=\dfrac{(0−2)(0+3)}{(0−1)(0+2)(0−5)}$ $=−\dfrac{6}{10}$ $=−\dfrac{3}{5}$ $=−0.6$ The x-intercepts will occur when the function is equal to zero. Notice the function is zero when the numerator is zero. $0=\dfrac{(x−2)(x+3)}{(x−1)(x+2)(x−5)}$ $0=(x−2)(x+3)$ $x=2, x=−3$ The y-intercept is $(0,–0.6)$, the x-intercepts are $(2,0)$ and $(–3,0)$. See Figure $\PageIndex{5}$. Figure $\PageIndex{5}$. Try It $\PageIndex{5}$ Given the reciprocal squared function that is shifted right 3 units and down 4 units, write this as a rational function. Then, find the x- and y-intercepts and the horizontal and vertical asymptotes.



Answer For the transformed reciprocal squared function, we find the rational form. $f(x)=\dfrac{1}{(x−3)^2}−4=\dfrac{1−4(x−3)^2}{(x−3)^2}=\dfrac{1−4(x^2−6x+9)}{(x−3)(x−3)}=\dfrac{−4x^2+24x−35}{x^2−6x+9}$ Because the numerator is the same degree as the denominator we know that as $x\rightarrow \pm \infty$, $f(x)\rightarrow −4$; so $y=–4$ is the horizontal asymptote.



Next, we set the denominator equal to zero, and find that the vertical asymptote is $x=3$, because as $x\rightarrow 3$, $f(x)\rightarrow \infty$. We then set the numerator equal to 0 and find the x-intercepts are at $(2.5,0)$ and $(3.5,0)$. Finally, we evaluate the function at 0 and find the y-intercept to be at $(0,−\frac{35}{9})$.
In Example $\PageIndex{10}$, we see that the numerator of a rational function reveals the x-intercepts of the graph, whereas the denominator reveals the vertical asymptotes of the graph. As with polynomials, factors of the numerator may have integer powers greater than one. Fortunately, the effect on the shape of the graph at those intercepts is the same as we saw with polynomials. The vertical asymptotes associated with the factors of the denominator will mirror one of the two toolkit reciprocal functions. When the degree of the factor in the denominator is odd, the distinguishing characteristic is that on one side of the vertical asymptote the graph heads towards positive infinity, and on the other side the graph heads towards negative infinity. See Figure $\PageIndex{6a}$. In contrast, when the degree of the factor in the denominator is even, the distinguishing characteristic is that the graph either heads toward positive infinity on both sides of the vertical asymptote or heads toward negative infinity on both sides. See Figure $\PageIndex{6b}$. Figure $\PageIndex{6a}$: Odd multiplicity Figure $\PageIndex{6b}$: Even multiplicity For example, the graph of $f(x)=\dfrac{(x+1)^2(x−3)}{(x+3)^2(x−2)}$ is shown in Figure $\PageIndex{6c}$.



At the x-intercept $x=−1$ corresponding to the $(x+1)^2$ factor of the numerator, the graph "bounces", consistent with the quadratic nature of the factor. At the x-intercept $x=3$ corresponding to the $(x−3)$ factor of the numerator, the graph passes through the axis as we would expect from a linear factor. At the vertical asymptote $x=−3$ corresponding to the $(x+3)^2$ factor of the denominator, the graph heads towards positive infinity on both sides of the asymptote, consistent with the behavior of the function $f(x)=\dfrac{1}{x^2}$. At the vertical asymptote $x=2$, corresponding to the $(x−2)$ factor of the denominator, the graph heads towards positive infinity on the left side of the asymptote and towards negative infinity on the right side, consistent with the behavior of the function $f(x)=\dfrac{1}{x}$. Figure $\PageIndex{6c}$. Howto: Given a rational function, sketch a graph. Evaluate the function at 0 to find the y-intercept.
Factor the numerator and denominator. For factors in the numerator not common to the denominator, determine where each factor of the numerator is zero to find the x-intercepts. Find the multiplicities of the x-intercepts to determine the behavior of the graph at those points. For factors in the denominator, note the multiplicities of the zeros to determine the local behavior. For those factors not common to the numerator, find the vertical asymptotes by setting those factors equal to zero and then solve. For factors in the denominator common to factors in the numerator, find the removable discontinuities by setting those factors equal to 0 and then solve. Compare the degrees of the numerator and the denominator to determine the horizontal or slant asymptotes. Sketch the graph. Example $\PageIndex{6}$: Graphing a Rational Function Sketch a graph of $f(x)=\frac{(x+2)(x−3)}{(x+1)^2(x−2)}$. Solution We can start by noting that the function is already factored, saving us a step. Next, we will find the intercepts. Evaluating the function at zero gives the y-intercept: $f(0)=\dfrac{(0+2)(0−3)}{(0+1)^2(0−2)}=3$ To find the x-intercepts, we determine when the numerator of the function is zero. Setting each factor equal to zero, we find x-intercepts at $x=–2$ and $x=3$. At each, the behavior will be linear (multiplicity 1), with the graph passing through the intercept. We have a y-intercept at $(0,3)$ and x-intercepts at $(–2,0)$ and $(3,0)$. To find the vertical asymptotes, we determine when the denominator is equal to zero. This occurs when $x+1=0$ and when $x–2=0$, giving us vertical asymptotes at $x=–1$ and $x=2$. There are no common factors in the numerator and denominator. This means there are no removable discontinuities. Finally, the degree of denominator is larger than the degree of the numerator, telling us this graph has a horizontal asymptote at $y =0$. To sketch the graph, we might start by plotting the three intercepts.
Since the graph has no x-intercepts between the vertical asymptotes, and the y-intercept is positive, we know the function must remain positive between the asymptotes, letting us fill in the middle portion of the graph as shown in Figure $\PageIndex{6a}$. Figure $\PageIndex{6a}$. The factor associated with the vertical asymptote at $x=−1$ was squared, so we know the behavior will be the same on both sides of the asymptote. The graph heads toward positive infinity as the inputs approach the asymptote on the right, so the graph will head toward positive infinity on the left as well. For the vertical asymptote at $x=2$, the factor was not squared, so the graph will have opposite behavior on either side of the asymptote. See Figure $\PageIndex{6b}$. After passing through the x-intercepts, the graph will then level off toward an output of zero, as indicated by the horizontal asymptote. Figure $\PageIndex{6b}$. Try It $\PageIndex{6}$ Given the function $f(x)=\dfrac{(x+2)^2(x−2)}{2(x−1)^2(x−3)}$, use the characteristics of polynomials and rational functions to describe its behavior and sketch the function.
Answer Horizontal asymptote at $y=\frac{1}{2}$.
Vertical asymptotes at $x=1$ and $x=3$.
y-intercept at $(0,\frac{4}{3})$.
x-intercepts at $(2,0)$ and $(–2,0)$. $(–2,0)$ is a zero with multiplicity 2, and the graph bounces off the x-axis at this point. $(2,0)$ is a single zero and the graph crosses the axis at this point. Figure $\PageIndex{6}$. Now that we have analyzed the equations for rational functions and how they relate to a graph of the function, we can use information given by a graph to write the function. A rational function written in factored form will have an x-intercept where each factor of the numerator is equal to zero. (An exception occurs in the case of a removable discontinuity.) As a result, we can form a numerator of a function whose graph will pass through a set of x-intercepts by introducing a corresponding set of factors. Likewise, because the function will have a vertical asymptote where each factor of the denominator is equal to zero, we can form a denominator that will produce the vertical asymptotes by introducing a corresponding set of factors. WRITING RATIONAL FUNCTIONS FROM INTERCEPTS AND ASYMPTOTES If a rational function has x-intercepts at $x=x_1,x_2,...,x_n$, vertical asymptotes at $x=v_1,v_2,…,v_m$, and no $x_i=$ any v_j, then the function can be written in the form: $f(x)=a\dfrac{ {(x−x_1)}^{p_1} {(x−x_2)}^{p_2}⋯{(x−x_n)}^{p_n} }{ {(x−v_1)}^{q_1} {(x−v_2)}^{q_2}⋯{(x−v_m)}^{q_n}}$ where the powers p_i or q_i on each factor can be determined by the behavior of the graph at the corresponding intercept or asymptote, and the stretch factor a can be determined given a value of the function other than the x-intercept or by the horizontal asymptote if it is nonzero. Given a graph of a rational function, write the function.
Determine the factors of the numerator. Examine the behavior of the graph at the x-intercepts to determine the zeroes and their multiplicities. (This is easy to do when finding the “simplest” function with small multiplicities—such as 1 or 3—but may be difficult for larger multiplicities—such as 5 or 7, for example.) Determine the factors of the denominator. Examine the behavior on both sides of each vertical asymptote to determine the factors and their powers. Use any clear point on the graph to find the stretch factor. Example $\PageIndex{7}$: Writing a Rational Function from Intercepts and Asymptotes Write an equation for the rational function shown in Figure $\PageIndex{7e}$. Figure $\PageIndex{7e}$. Solution The graph appears to have x-intercepts at $x=–2$ and $x=3$. At both, the graph passes through the intercept, suggesting linear factors. The graph has two vertical asymptotes. The one at $x=–1$ seems to exhibit the basic behavior similar to $\dfrac{1}{x}$, with the graph heading toward positive infinity on one side and heading toward negative infinity on the other. The asymptote at $x=2$ is exhibiting a behavior similar to $\dfrac{1}{x^2}$, with the graph heading toward negative infinity on both sides of the asymptote. See Figure $\PageIndex{7s}$. We can use this information to write a function of the form $f(x)=a\dfrac{(x+2)(x−3)}{(x+1){(x−2)}^2}$ To find the stretch factor, we can use another clear point on the graph, such as the y-intercept $(0,–2)$. $−2=a\dfrac{(0+2)(0−3)}{(0+1){(0−2)}^2}$ $-2=a\dfrac{−6}{4}$ $a=\dfrac{−8}{−6}=\dfrac{4}{3}$ This gives us a final function of $f(x)=\dfrac{4(x+2)(x−3)}{3(x+1){(x−2)}^2}$. Figure $\PageIndex{7s}$. Rational Function $\qquad$ $f(x)=\dfrac{P(x)}{Q(x)}=\dfrac{a_px^p+a_{p−1}x^{p−1}+...+a_1x+a_0}{b_qx^q+b_{q−1}x^{q−1}+...+b_1x+b_0},\space Q(x)≠0$ Key Concepts We can use arrow notation to describe local behavior and end behavior of the toolkit functions $f(x)=\frac{1}{x}$ and $f(x)=\frac{1}{x^2}$. See Example $\PageIndex{1}$. A function that levels off at a horizontal value has a horizontal asymptote. A function can have more than one vertical asymptote. See Example. Application problems involving rates and concentrations often involve rational functions. See Example.
The domain of a rational function includes all real numbers except those that cause the denominator to equal zero. See Example. The vertical asymptotes of a rational function will occur where the denominator of the function is equal to zero and the numerator is not zero. See Example. A removable discontinuity might occur in the graph of a rational function if an input causes both numerator and denominator to be zero. See Example. A rational function’s end behavior will mirror that of the ratio of the leading terms of the numerator and denominator functions. See Example, Example, Example, and Example. Graph rational functions by finding the intercepts, behavior at the intercepts and asymptotes, and end behavior. See Example. If a rational function has x-intercepts at $x=x_1,x_2,...,x_n$, vertical asymptotes at $x=v_1,v_2,…,v_m$, and no $x_i=$ any v_j, then the function can be written in the form Contributor The domain of a function f x is the set of all values for which the function is defined, and the range of the function is the set of all values that f takes. A rational function is a function of the form f x = p x q x , where p x and q x are polynomials and q x ≠ 0 . The domain of a rational function consists of all the real numbers x except those for which the denominator is 0 . To find these x values to be excluded from the domain of a rational function, equate the denominator to zero and solve for x . For example, the domain of the parent function f x = 1 x is the set of all real numbers except x = 0 . Or the domain of the function f x = 1 x − 4 is the set of all real numbers except x = 4 . Now, consider the function f x = x + 1 x − 2 x − 2 . On simplification, when x ≠ 2 it becomes a linear function f x = x + 1 . But the original function is not defined at x = 2 .
This leaves the graph with a hole when x = 2 .
One way of finding the range of a rational function is by finding the domain of the inverse function. Another way is to sketch the graph and identify the range. Let us again consider the parent function f x = 1 x . We know that the function is not defined when x = 0 . As x → 0 from either side of zero, f x → ∞ .
Similarly, as x → ± ∞ , f x → 0 . The graph approaches x -axis as x tends to positive or negative infinity, but never touches the x -axis. That is, the function can take all the real values except 0 . So, the range of the function is the set of real numbers except 0 . Example 1: Find the domain and range of the function y = 1 x + 3 − 5 .
To find the excluded value in the domain of the function, equate the denominator to zero and solve for x . x + 3 = 0 ⇒ x = − 3 So, the domain of the function is set of real numbers except − 3 . The range of the function is same as the domain of the inverse function. So, to find the range define the inverse of the function. Interchange the x and y . x = 1 y

+ 3 − 5 Solving for y you get, x + 5 = 1 y + 3 ⇒ y + 3 = 1 x + 5 ⇒ y = 1 x + 5 − 3 So, the inverse function is f − 1 x = 1 x + 5 − 3 . The excluded value in the domain of the inverse function can be determined byequating the denominator to zero and solving for x . x + 5 = 0 ⇒ x = − 5 So, the domain of the inverse function is the set of real numbers except − 5 . That is, the range of given function is the set of real numbers except − 5 . Therefore, the domain of the given function is { x ∈ ℝ | x ≠ − 3 } and the range is { y ∈ ℝ | y ≠ − 5 } . Example 2: Find the domain and range of the function y = x 2 − 3 x − 4 x + 1 . Use a graphing calculator to graph the function. When you factor the numerator and cancel the non-zero common factors, the function gets reduced to a linear function as shown. y = x + 1 x − 4 x + 1 = x + 1 x − 4 x + 1 = x − 4 So, the graph is a linear one with a hole at x = − 1 . Use the graph to identify the domain and the range. The function is not defined for x = − 1 . So, the domain is { x ∈ ℝ | x ≠ − 1 } or − ∞ , − 1 ∪ − 1 , ∞ .

The range of the function is { y ∈ ℝ | y ≠ k where y − 1 = k } . For x ≠ − 1 , the function simplifies to y = x − 4 . The function is not defined at x = − 1 or the function does not take the value − 1 − 4 = − 5 . That is, k = − 5 . Therefore, the range of the function is { y ∈ ℝ | y ≠ − 5 } or − ∞ , − 5 ∪ − 5 , ∞ . Asymptotes of a rational function:

An asymptote is a line that the graph of a function approaches, but never touches.

In the parent function f x = 1 x , both the x - and y -axes are asymptotes. The graph of the parent function will get closer and closer to but never touches the asymptotes. To find the vertical asymptote of a rational function, equate the denominator to zero and solve for x . If the degree of the polynomial in the numerator is less than that of the denominator, then the horizontal asymptote is the x -axis or y = 0 .

The function f x = a x , a ≠ 0 has the same domain, range and asymptotes as f x = 1 x . Now, the graph of the function f x = a x − b + c , a ≠ 0 is a hyperbola, symmetric about the point b , c .

The vertical asymptote of the function is x = b and the horizontal asymptote is y = c . Considering a more general form, the function f x = a x + b c x + d has the vertical asymptote at x = − d c and the horizontal asymptote at y = a c . More generally, if both the numerator and the denominator have the same degree, then horizontal asymptote would be y = k where k is the ratio of the leading coefficient of the numerator to that of the denominator. If the degree of the denominator is one less than that of the numerator, then the function has a slanting asymptote. Example 3: Find the vertical and horizontal asymptotes of the function f x = 5 x − 1 . To find the vertical asymptote, equate the denominator to zero and solve for x . x − 1 = 0 ⇒ x = 1 So, the vertical asymptote is x = 1 Since the degree of the polynomial in the numerator is less than that of the denominator, the horizontal asymptote is y = 0 .