Incomplete interest rate pass through: a recursive partial equilibrium approach[1]

Martín Cherkasky[2] — CEPAL, BA Office

Damian Pierri[3] — IIEP-BAIRES (UBA-CONICET)

Luis Trajtenberg[4] — UBA and Ministry of Economy

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Abstract

The empirical literature provides robust evidence on the incomplete pass through (IPT) of monetary policy instruments on market rates. However, the coefficients found are not similar across regions, ranging between 0.25 and 0.75. Also, the theoretical literature has not provided a comprehensive framework which can explain this fact satisfactorily. This paper proposes a recursive partial equilibrium model that generates a PT coefficient between 0.5 and 0.6. The model is calibrated for Latin American economies where the active (passive) rate is above (below) the policy rate and the spread is stable. These facts allow explaining the IPT: after a tightening in the monetary policy, banks switch the composition of their assets from loans to central bank notes. This change allows them to reduce the effects on earnings caused by the tightening provided that there is an IPT in the passive rate and that this rate is below the policy rate. The stability of the spread explains the PT in the active rate. In order to obtain the IPT coefficients, this paper derives a structural estimation of the supply elasticity of deposits and demand elasticity of loans. The former ranges between 3.0 and 4.0 and the latter between -1.5 and -2.0. These results suggest that the efficiency of the monetary policy might be affected more by an incomplete PT than by the lack of sensibility of loans and deposits.

Keywords: financial intermediation, incomplete pass through, interest rates.

JEL Classification: E43, E47.

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[2] martincherkasky@gmail.com

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[4] luis_trajtenberg@hotmail.com

1 Introduction

The purpose of this paper is to study the pass through of monetary policy rates on market interest rates. This result is relevant to evaluate the effectiveness of monetary policy regimes that use an interest rate as an instrument. Evidence suggests that both active and passive rates do not fully respond to changes in the policy rate (de Bondt, et. al., 2005). However, pass through coefficients differ greatly across regions, ranging between 0.25 and 0.75 (Sander and Kleimeier, 2002, Heinemann and Schüler, 2002, respectively), and there is no consensus on the determinants of the phenomenon (see section 2.1). This paper shows that for economies with a specific pattern of market and policy rates, the pass through coefficient is between 0.5 and 0.6.

The empirical literature is not able to obtain robust results even using extremely flexible methodologies. This is the case of the asymmetric cointegration technique proposed by Enders and Siklos (2001), which allows estimating switching pass through coefficients. Grigoly and Mota (2015) applied this method successfully to the Dominican Republic. Nevertheless, this paper shows that these results cannot be used in other Latin American economies like Argentina, Chile and Colombia.

The lack of robustness of the reduced form (empirical) models suggests the presence of identification problems as, depending on the case of study, the potentially omitted variables may have different effects on the estimated coefficients. To circumvent this problem, this paper proposes a structural approach based on a recursive partial equilibrium model. This model allows identifying separately the factors explaining the behavior of supply and demand of loans and deposits using a smooth function of state variables. This approach is based on the fact that the only robust variable in the reduced form models is the liquidity of banks, which depends on the ratio of deposits to loans, and that the hypothesis of multiple pass though coefficients was rejected.

The theoretical literature has already proposed models to understand the problem at hand. However, these models address the behavior of only 1 market rate at the time and do not provide an endogenous mechanism which explains the incomplete pass through (see for instance Kobayashi, 2008). This paper contributes to the theoretical literature by developing a model that can explain the behavior of both market rates, active and passive, in response to a policy change. Further, the papers endogenously derive the incomplete pass through by modeling the interaction between the supply of deposits (by patient families), the demand of credit (by impatient families) and their counterpart (by a financial intermediary).

In order to obtain realistic predictions, the model is calibrated using stylized facts for 3 of the most representative Latin American economies: Argentina, Chile and Colombia. These facts are: i) the spread between active and passive rates is stable through time, ii) active rates are above passive rates, iii) passive rates are below the policy rate. Facts i) to iii) not only allows taking the model to the data, they also affect the mechanism behind the incomplete pass through. Suppose that the central bank decides to raise the policy rate. Then, under a standard parametrization, financial intermediation falls as there is an increase in the supply of deposits and a decrease in the demand for credit. Because fact ii) holds, the profits of financial institutions go down. In order to dampen this effect, as fact iii) holds, banks can invest the excess of liquidity in central bank notes obtaining a return equal to the policy rate. If there is an *incomplete pass through* on passive rates, the return on liquidity rises, balancing the effect of the policy rate on profits. The incomplete pass through on the active rate is then explained by fact i).

The cornerstone of the mentioned mechanism is fact iii)[5] as it allows banks to increase the return on liquidity after a monetary tightening by means of the incomplete pass through. However, it was claimed that after a hike in the monetary policy rate, follows a drop in intermediation which causes the initial reduction in profits. This effect is derived, as in the empirical literature (see for instance Maudos and De Guevara, 2004), from a negative (positive) demand (supply) elasticity. The calibrated values for these parameters are around -1.5 and -2.0 for the demand and 3.0 and 4.0 for the supply.

Overall the results of the model suggest that the ineffectiveness of the monetary policy in Latin American economies (see Werner and Jacome, 2016) may be due to an incomplete interest rate pass through. In particular, those economies that do not remunerate savings through high interest rates generate incentives for financial

[5] Theoretically it is possible to obtain an incomplete pass through even if the policy rate is below the passive rate as this strategy allows banks to improve their situation after a reduction in the intermediation levels. However, fact iii is essential to match the order of magnitude of the pass through coefficients obtained in the empirical literature.

institutions to reduce the pass through on passive rates; affecting the performance of the monetary policy.

The remaining of the paper is organized as follows: section 2 present a review of the literature and the main characteristics of the monetary system in Argentina, Chile y Colombia. Section 3 addresses the lack of robustness of the empirical literature to answer the question at hand. Section 4 describes the model, calibrates it and presents the main results. Section 5 concludes.

2 Literature review and Data

2.1 Relation with the literature

Economists have found multiple explanations behind the lack of effectiveness of central bank policies; among them is the rigidity of market rates after changes in the monetary policy instruments.

The Stiglitz and Weiss (1981) argument on the impact of asymmetric information in credit markets can be used to explained the rigidity of active rates: as higher interest rates implies an adverse selection among borrowers, banks prefers to ration credits and keep the interest rate constant. Rotenberg and Saloner (1987) and Lowe and Rohling (1992) also note the rigidity of active rates but use different channels to explain it: the former argued in favor of the existence of menu costs in financial institutions and the latter claimed that borrowers faced fixed cost when they decide to change from one bank to another.

Hannan y Berger (1991) claimed that the collusive behavior in an oligopolistic banking system may explain the incomplete pass through in passive rates. After a monetary policy hike, banks can interact with each other in order to keep savings rate depressed and at the same time raising active rates so as to maximize the intermediation margin.

These arguments can be used to interpret the coefficients in a reduced form regression for active and passive rates separately as in Grigoli and Mota (2015). However, they have not been included in an integrated framework which allows explaining the incomplete pass through in *both* rates simultaneously. This paper provides the first attempt to derive a partial equilibrium model for the simultaneous strictness observed in active and passive rates.

Kobayashi (2008) derived a general equilibrium model for the active rate which assumes that financial intermediaries follow a Calvo pricing rule. As will be shown below, empirical evidence suggests that the only robust drivers of active and passive rates are the liquidity of banks and the policy rate. Thus, a general equilibrium framework looks unnecessary to address the problem at hand. Further, it has consequences as it forced the author to impose a Calvo rule to model the incomplete pass through. Contrarily, this paper derives an endogenous mechanism which allows accounting for the stickiness in the market rates.

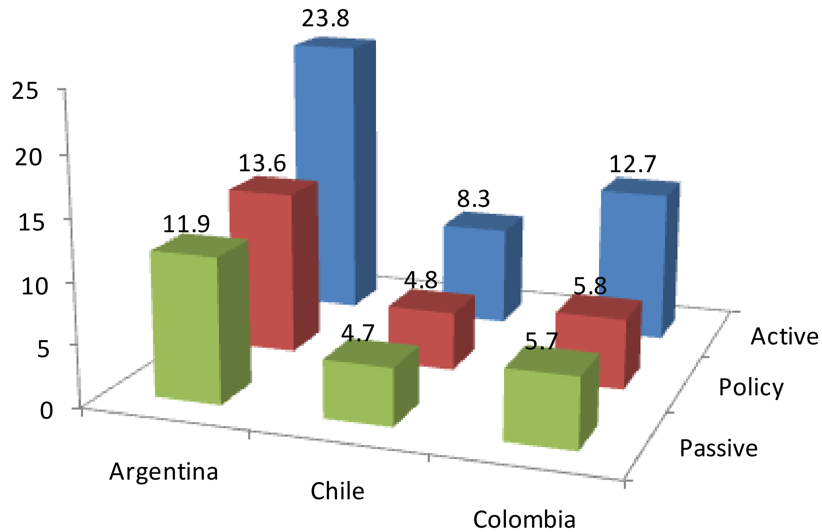
2.2 Data: Argentina, Chile and Colombia 2005-2015

Since 2016 Argentina, Chile and Colombia follow an inflation targeting using a short term interest rate as an instrument. However, until 2015 Argentina used monetary aggregates as an instrument, in particular private M2, and the short term interest rate was an intermediate target. That is, the results in this paper do not depend on the monetary regime. They only require a particular relationship between market rates and policy rates. While the former are just active and passive rates, the latter is the interest rate earned by banks on their liquidity.

The rates used are similar to the ones reported by the IMF in the International Financial Statistics data base. The active and passive rates, RA and RPF respectively, are the weighted average of loans and term deposits rates. The policy rates (RL) are rather different across countries: for Argentina is the 90 days Central Bank note rate (called “Lebac”) scaled down to 30 days, for Chile is the 1 day interbank loans rate and for Colombia is an average of the short term (1 to 14 days) Central Bank REPO rate.

Even though the disparity among rates, they share a common trend: in the three mentioned countries the active rate is above the policy rate which is in turn above the passive rate as depicted by the following figure:

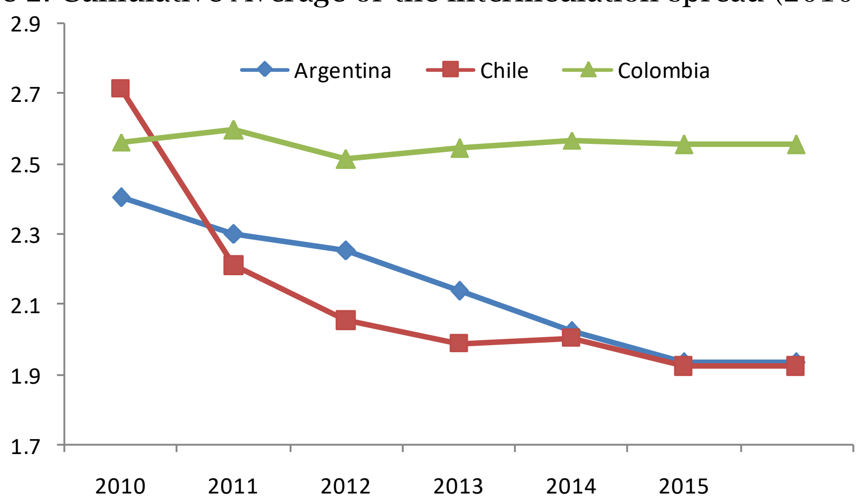
Figure 1: Average of Active, passive and policy rates between 2005 and 2015.



Source: Prepared using data from the Central Bank of Argentina, Chile and Colombia[6]

It is also worth noticing that the spread between the active and passive rates is quite stable across time. Figure 2 below illustrates that fact.

Figure 2: Cumulative Average of the intermediation spread (2010-2015).



Source: Prepared using data from the Central Bank of Argentina, Chile and Colombia

Notice that the three averages tend to stabilize in the last 2 periods, implying convergence in a law of large numbers sense. This fact will be essential to design the characteristics of the model in section 4. We need to derive a theoretical structure that

[6] The average excludes 2009 due to the abnormal behavior implied by the financial crises.

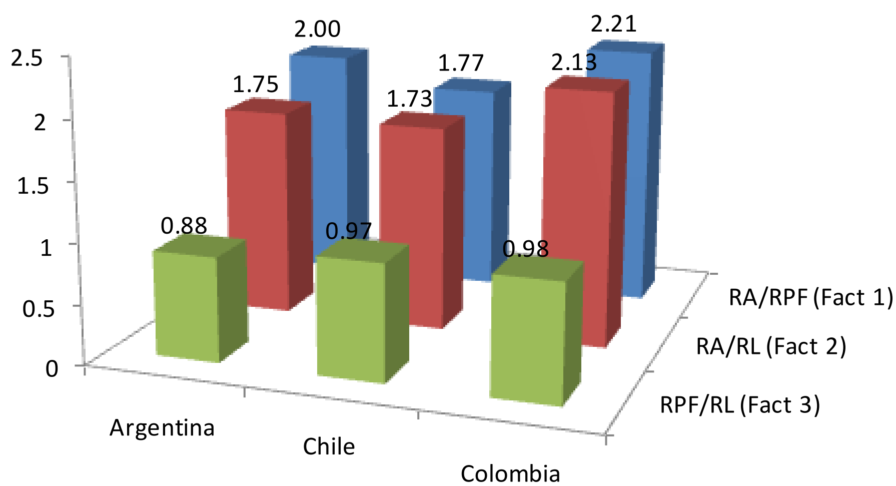
is able to generate simulations that converge as in figure 2. Moreover, the limiting value of these series must reflect the equilibrium of the model in the long run. The recursive model presented below satisfies these requirements.

Figures 1 and 2 can be used to illustrate the following stylized facts, which are summarized by figure 3 below:

1) The cumulative average of the intermediation spread, measured as the ratio of active to passive rate, rapidly converges to a steady value strictly above 1.
2) The active rate is above the policy rate
3) The passive rate is below the policy rate

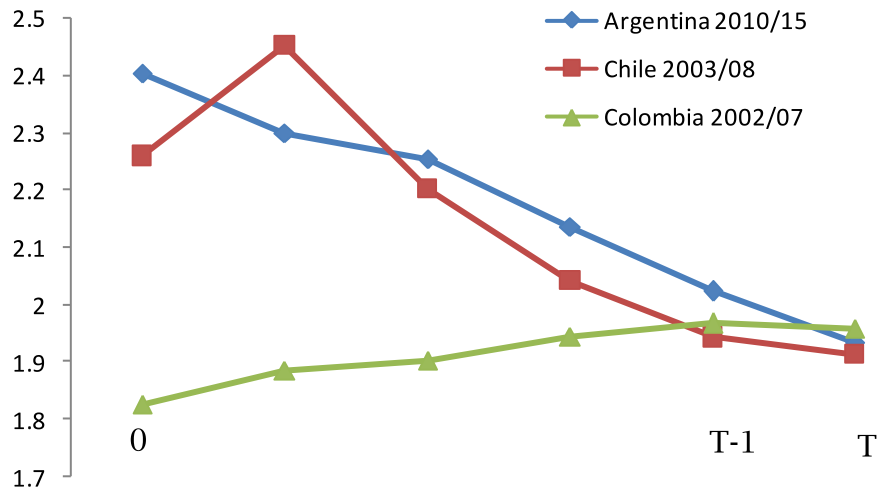
Fact 1 is depicted in figure 2. The figure below illustrates the other 2:

Figure 3: Stylize facts Argentina, Chile y Colombia (2010-2015).



The distinctive feature of these 3 countries will be clear by observing the evolution over time of the cumulative average of the intermediation spread (fact 1). As this average converge for different time periods to the same level, it will be possible to fit the 3 countries with the same parameters of the model. Moreover, facts 2 and 3 can be matched by comparing directly the simulated and the observed time series as long as the former is shown to be stable after “throwing away” a few initial observations.

Figure 4.1: Fact 1 for Argentina, Chile y Colombia.



Figures 4.2- 4.3. Facts 2 and 3 for Argentina, Chile and Colombia in selected periods.

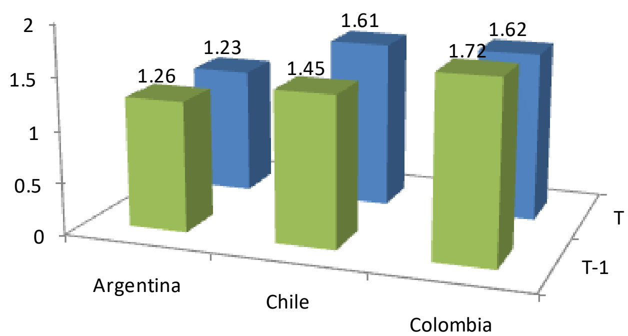


Fig. 4.2: Fact 2. Active Rate/Policy Rate

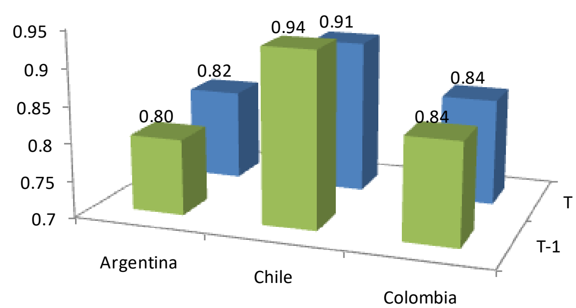


Fig. 4.3: Fact 3. Passive Rate/Policy Rate

In section 4 it will be shown that it is possible to obtain 1 set of parameters for the three economies and still get a good fit in terms of the relative error. Of course the fit of the model can be improved by computing 3 different sets of parameters but the pass through coefficients will be very similar to each other as the intermediation spread is the same for the three economies.

3 Empirical Results

In order to find evidence to support the incomplete pass through hypothesis we use the methodology proposed by Grigoli and Mota (2015). This approach not only will allow us to estimate a reduced form version of the pass through coefficients, which can be compared with their structural counterpart, but also provides information about the determinants of the market rates besides the policy rate.

3.1 Methodology

The method can be considered a 2 step approach. First, we test for cointegration. Second, we estimate an error correction model. The methodology is sufficiently flexible to capture discontinuities in the proposed long term relationship. These “jumps” gives rise to an asymmetric adjustment which is introduced in the model by a step function.

Let us first consider a linear model for the evolution of the market rate:

$$i_t = \beta_0 + \beta_1 RL_t + \beta_2 \cdot X_t + \mu_t \quad (2.1)$$

Where i_t stands for either the active (RA_t) or passive (RPF_t) rate and X_t is a vector of covariates. Then, using the residuals of the estimated model, denoted without lost of generality μ_t, we can test for asymmetric cointegration using 2 different specifications. The first one was proposed by Enders and Siklos (2001) and is called TAR model:

$$\Delta\mu_t = I_t\rho_1\mu_{t-1} + (1 - I_t)\rho_2\mu_{t-1} + \sum_{l=1}^{q}\gamma_l\Delta\mu_{t-l} + v_t \quad (2.2)$$

Where I_t is an indicator function of the form:

$$I_t = \begin{cases} 1, & \mu_{t-1} \geq \tau \\ 0, & \mu_{t-1} < \tau \end{cases}$$

Enders and Granger (1998) proposed a slightly modified version of 2.2 called M-TAR model:

$$\Delta\mu_t = M_t\rho_1\mu_{t-1} + (1 - M_t)\rho_2\mu_{t-1} + \sum_{l=1}^{q}\gamma_l\Delta\mu_{t-1} + v_t \quad (2.3)$$

Where M_t is just a different indicator function:

$$M_t = \begin{cases} 1, & \Delta\mu_{t-1} \geq \tau \\ 0, & \Delta\mu_{t-1} < \tau \end{cases}$$

Note that it is possible to test 2 different hypothesis using equations 2.2 and 2.3: first, the null of no cointegration, $\rho_1 = \rho_2 = 0$. Then, if this hypothesis is rejected, it is possible to test for asymmetric adjustment under the null $\rho_1 = \rho_2$.

After testing for cointegration, it is possible to estimate the long term pass through coefficients controlling for cyclical fluctuations using an error correction model:

$$\Delta i_t = \alpha_0 + \alpha_1\Delta RL_t + \alpha_2 \cdot \Delta X_t + \rho\mu_{t-1} + \varepsilon_t \quad (2.4)$$

Finally, in order to understand the asymmetry induced by equations 2.2-2.3 note that, under asymmetric cointegration, equation 2.4 becomes:

$$\Delta i_t = \eta + \sum_{i=0}^{n}\gamma_i\Delta RL_{t-i} + \sum_{k=0}^{m}\gamma_k \cdot \Delta X_{t-k} + J_t\rho_{1,j}\mu_{t-1} + (1 - J_t)\rho_{2,j}\mu_{t-1} + \varepsilon_t \quad (2.4')$$

Where $J = I$ or $J = M$ depending on the model we are using (TAR or M-TAR respectively).

Equation 2.4') can be used to provide an intuition behind the null hypothesis H_0: $\rho_1 = \rho_2 = 0$ (no cointegration) and H_0: $\rho_1 = \rho_2$ (symmetric cointegration). Take for instance the first one: if it holds, it implies that the differences between the observed value of the dependent variable and the long run equilibrium proposed in equation 2.1), $i_{t-1} - \beta_0 + \beta_1 RL_{t-1} - \beta_2 \cdot X_{t-1} = \mu_t$, do not affect the short run dynamic behavior of the dependent variable as described by 2.4').

Finally, note that equations 2.2 and 2.3 simply allow for a discontinuity or "jump" in the (linear) model used to estimate the impact of deviations from the long run relationship on the short run movements in the market rates[7].

3.2 Results

The tables below present the results of the cointegration tests for the TAR and M-TAR model. As in Grigoli and Mota (2015) the threshold parameter τ was set at zero.

Table 1: Cointegration tests

Argentina

	TAR Model		M-TAR Model	
	$\rho_1=\rho_2=0$	$\rho_1=\rho_2$	$\rho_1=\rho_2=0$	$\rho_1=\rho_2$
RA	2,03	2,3	1,37	1
RPF	2,53*	5,04**	0,01	0

Significance Levels 1% (***), 5% (**) y 10% (***).

Chile

	TAR Model		M-TAR Model	
	$\rho_1=\rho_2=0$	$\rho_1=\rho_2$	$\rho_1=\rho_2=0$	$\rho_1=\rho_2$
RA	20,89***	2,36	20,41***	1,62
RPF	19,23***	0,01	24,19***	7,71***

Significance Levels 1% (***), 5% (**) y 10% (***).

Colombia

	TAR Model		M-TAR Model	
	$\rho_1=\rho_2=0$	$\rho_1=\rho_2$	$\rho_1=\rho_2=0$	$\rho_1=\rho_2$
RA	9,28***	3,75*	13,88***	12,05***
RPF	2,49*	1,02	2,16	0,38

Significance Levels 1% (***), 5% (**) y 10% (***).

For Argentina evidence suggests the presence of asymmetric cointegration only for the passive rate (RPF) but we cannot reject the null of no cointegration for the active rate (RA) using neither a TAR nor a M-TAR model.

[7] It is possible to estimate 2.4'. However, section 3.2 will present weak evidence in favor of asymmetric cointegration. Thus, the estimated version of equation 2.4' is omitted for expositional purposes as the validity of the coefficients remains an open question.

For Chile, however, there is robust evidence for cointegration on both rates but none in favor of an asymmetric behavior.

Finally, in Colombia there is evidence in favor of asymmetric cointegration for the active rate but we cannot reject the null of no cointegration for the passive rate.

Section A.1 in the appendix contains the estimation results for the error correction model, equation 2.4). Note that none of the short term coefficients associated with the policy rate (α_1) are significant. Moreover, the pass through coefficients estimated in the long term relationship (β_1 in equation 2.1) and the speed of adjustment coefficient in the error correction model (ρ in equation 2.4) are generally significant and suggest the presence of an incomplete pass through. In particular, note that β_1 ranges between 0.366 and 1.069 suggesting that the incomplete pass through holds in the long run and ρ ranges between 0 and 1 implying that innovations in the long run relationship, may be due to a monetary policy surprise, have a less than proportional impact on the market rates. Tables A.1 and A.2 in the appendix shows that the short run components in equations 2.4 and 2.4') are generally not significant. Thus, we will only discuss the long run components as specified in equation 2.1).

Even though the estimated coefficients are significant, their value could be misleading due to the lack of a robust long term relationship. Section A.1 in the appendix also contains the estimation results for equations 2.1 and 2.4 adding real wages and the volume of traded durable goods as controls (see Tables A.1). The former captures a measure of disposable income and the latter a proxy of credit demand; both for households. In these tables not only the range of the pass through coefficient rises (from 0.366/1.069 to -0.013/1.044) but also the average effect of the policy rate on market rates varies inversely with the liquidity ratio coefficient (loans/deposits); suggesting the presence of simultaneity among these variables or at least that the policy rate has a direct effect on loans and deposits.

More to the point, Table 1 suggests that the policy rate and the passive rate are not related in the long run in Colombia. The same is true for the active and policy rates in Argentina. In Chile, although policy and market rates are cointegrated, it is not clear if there is an asymmetric adjustment in the sense of equation 2.4').

Finally, the sing of the coefficients associated with the vector of covariates X_t vary along with the country. In particular, an increase in the liquidity of banks rises the passive rate in Argentina but lowers it in Colombia. This result is robust to the introduction of additional covariates in X_t. A model which identifies supply and demand of loans and deposits could generate both results. Suppose an increase in deposits associated with a hike in the rate of growth of consumption[8]. In a standard Euler equation setting, this is compatible with a higher passive interest rate. However, the liquidity of banks depends on the demand for credit. At the same time the behavior of loans depend on the elasticity of the demand for credit, which can be different for each country depending on the data used to calibrate the model. If there is an aggregate shock which generates simultaneously an increase in the supply of deposits and the demand of credit, the behavior of the liquidity ratio is undefined and the passive rate will be higher; causing either a positive or a negative correlation between these 2 variables that can explained the difference in the sign of estimated coefficients.

The lack of statistical significance together with non-robust coefficients may be associated with omitted variables or a misspecified model. Besides, the different signs of the coefficients associated with the vector of covariates suggest that the liquidity of banks and interest rates are determined simultaneously. Thus, in order to control for these problems, we must develop a structural approach taking into account the decisions of banks, lenders and borrowers. The next section addressees these issues.

4 A recursive partial equilibrium model

The purpose of this section is to derive a parsimonious model to match stylized facts 1 to 3, especially figures 4.1 to 4.3. This can be done by means of a recursive partial equilibrium model as such a theoretical framework is enough to match the mentioned data. In particular, from the model it is possible to derive: i) a (markovian) stochastic process. This process will be shown to have a long run equilibrium set, called ergodic, which is distributed according to a probability measure called invariant. Both elements

[8] This could be generated by a change in the distribution over time of exogenous disposable income.

define a steady state for the model as, once payoff relevant variables enter to this set, they will never leave it. ii) A law of large numbers which guarantee the convergence of simulated series to the steady state.

The model is the first step. Then, once an appropriate initial condition is chosen, i) and ii) allows obtaining a good approximation of stylized facts 1 to 3 using the same set of parameters for the three economies. This is possible because, as shown in figure 4.1, the cumulative average of the intermediation spread converges to the same value for the three economies. The model developed in this section is used to match this value with the steady state of the economy, i), by means of the law of large numbers, ii). Moreover, it will be shown that, after "throwing away" a few observations, the simulated time series for RA/RL and RPF/RL oscillate within a small interval, which can be interpreted as the ergodic set of the stochastic process generated by the model. Figure 4.2 and 4.3 shows that the observed time series for RA/RL and RPF/RL also fluctuate within a small range *for the three countries exactly when the cumulative average for the intermediation spread converges to the steady state* (time period "T" in the three figures); allowing us to match facts 2 and 3 using the same set of parameters.

Besides, the simple partial equilibrium structure is not only appropriate to match the empirical evidence in section 3 but also provides an endogenous explanation for the incomplete pass through, which will be explained in detail below. To our knowledge, the literature has not provided a theoretical structure which *generates endogenously the incomplete pass through on both active and passive rates*.

4.1 Structure of the economy and contribution to the literature

The economy is supposed to be infinite horizon, closed and with incomplete markets. There are 2 types of households: type 1, called patient, who consume and save in fixed term deposits D which in turn earn an interest rate RPF. The agent of type 2, called impatient, also consume but borrow by means of a loan B paying an interest rate RA. Both types of agents receive an exogenous income denoted W, that evolves in time following markov process, and government transfers.

Besides, there is a bank in the economy that collects savings and grants loans. The bank can invest liquidity $D - B$ in central bank notes L that earn an interest rate RL. This rate follows an exogenous markov process.

The recursive equilibrium notion used in this paper, borrowed from Mehra and Prescott (1980), assumes common knowledge across the different type of agents. Further, as in Kobayashi (2008), there is only 1 financial institution. Thus, the asymmetric information arguments as in Stiglitz and Weiss (1981) and the oligopolistic behavior in Hannan and Berger (1991) cannot be used to generate the observed rigidity in active and passive rates respectively. In this model the incomplete pass through *on both rates* is a consequence of the portfolio decisions of the bank that faces an interest rate profile as the one described in figure 1.

In order to keep the model analytically and numerically tractable we assume a *partial equilibrium* structure. The cost of this assumption is reflected in the exogeneity of policy rates and household's income. However, the key fact to identify the incomplete pass through is the relationship among interest rates, as depicted in figure 3, and not the monetary policy regime. Further, real wages do not appear to be statistically relevant according to the results in section 3. Thus, assuming an exogenous monetary policy and income stream is not relevant to identify the causes of the problem at hand.

The model is presented in *recursive form and in infinite horizon* as this structure allows deriving a stationary stochastic process that has a well defined *long run equilibrium* (i.e. a steady state) which can be used to match *jointly* stylized facts 1 to 3 for the three countries. DSGE models as in Kobayashi (2008) are not appropriate to capture the long run behavior of oscillating time series as they assume the existence of a non-stochastic or constant steady state that is used to compute the model using a local algorithm. This type of assumption can lead to severe biases in the computed time series as discussed in Feng, et. al. (2014).

It is possible to extend the recursive structure to general equilibrium as in Arellano (2008). However, the results in Kubler and Schmedders (2002) suggest that a continuous policy function defined on a minimal state space as in Arellano may not always exist in an incomplete market framework. This type of functions is essential to prove the existence of a well defined steady state. Contrarily, as it will be discussed below, in a partial equilibrium framework a continuous policy function arises naturally from standard assumptions; implying the existence of an empirically meaningful steady state.

4.2 The Model

Assume without loss of generality that there is just 1 household of each type[9]. Also, suppose that each of the exogenous state variables, wages W and policy rate R_L, can take a finite number of values in each time period and follow a Markov process so that the evolution of exogenous uncertainty can be (jointly) characterized by a transition matrix.

Then, the type 1 household solves the following recursive problem:

$$V1(A_2, A_1, W, R_L) = Max_{D \geq 0} u_1(A_1 - D - T_1) + \beta_1 E_{W', R_L' | W, R_L} V_1(A_2', A_1', W', R_L')$$

Subject to

$$A_1' = W' + (1 + R_{PF})D$$

$$R_{PF} = F_{1,E}^{R_{PF}}(A_2, A_1, W, R_L)$$

$$A_2' = F_{1,E}^{A_2}(A_2, A_1, W, R_L)$$

$$T_1 = \alpha T(A_2, A_1, W, R_L)$$

Where A_i is the wealth level of agent of type i, W is current income, D is the stock of deposits, R_{PF} is the passive interest rate, R_L is the policy rate and $F_{1,E}^{R_{PF}}, F_{1,E}^{A_2}$ is the law of motion perceived by the agent of type 1 for the passive rate and the wealth of agent 2 respectively. Finally, T_1 is a lump sum tax / transfer. Taking expectations as given the solution to this problem is a policy function $D(A_2, A_1, W, R_L)$.

[9] It is possible to solve the problem assuming that there is a continuum of each type of household and normalize the total amount of them to [0,1]. This will generate a couple of additional state variables, a_1 and a_2. However, in equilibrium, as in Mehra and Prescott, we will have $a_i = A_i$ with $i = 1,2$. For expositional purposes, we choose to assume that there is just 1 household of each type. As there is no idiosyncratic uncertainty within each type, this assumption is without loss of generality.

The conditional expectation $E_{w',R_L'|w,R_L}(.)$ is taken with respect to the distribution of the vector $[w', R_L']$ conditional on $[w, R_L]$. This distribution is represented by the row of a transition matrix which will be defined below.

The agent of type 2 solves the following problem:

$$V2(A_2, A_1, W, R_L) = Max_{B\geq 0} u_2(A_2 + B - T_2) + \beta_2 E_{w',R_L'|w,R_L} V_2(A_2', A_1', W', R_L')$$

Subject to

$$A_2' = W' - (1 + R_A)B$$

$$R_A = F_{2,E}^{R_A}(A_2, A_1, W, R_L)$$

$$A_1' = F_{2,E}^{A_1}(A_2, A_1, W, R_L)$$

$$T_2 = (1 - \alpha)T(A_2, A_1, W, R_L)$$

Where R_A is the active rate, B the demand of loans, $F_{2,E}^{R_A}, F_{2,E}^{A_1}$ are the law of motion perceived by the type 2 agent for the active rate and the wealth of the agent of type 1 respectively: The interpretation of the rest of the variables is straightforward. As before the solution to this problem is a policy function $B(A_2, A_1, W, R_L)$.

Note that both type of agents have to for expectation about the evolution of aggregate wealth, characterized by the vector $[A_1, A_2]$, and market interest rates. These expectations depends on the aggregate state of the economy $[z, R_L]$, where $z \equiv [A_2, A_1, W]$, as the financial intermediary takes into account the demand for loans $B(z, R_L)$ and the supply of deposits $D(z, R_L)$ to define its optimal pricing policy as follows:

$$Max_{R_{PF},R_A\geq 0} R_A B(R_A, R_L, z) - R_{PF} D(R_{PF}, R_L, z) + [D(R_{PF}, R_L, z) - B(R_A, R_L, z)]R_L$$

Subject to

$$R_{PF} \leq R_A$$

There are 2 things to note about the intermediary's problem: first, it implicitly contains the asset market equilibrium equation $B + L = D$ which implies that deposits can be applied either to loans or CB notes. Second, the bank takes B and D as depending on market rates and this must be reflected by household's expectations $F_{1,E}^{R_{PF}}$ and $F_{2,E}^{R_A}$ in order to allow the supply of deposits and the demand for credit to respond to different interest rate levels. From a practical perspective this means that the bank is able to influence expectations directly by changing the pricing function. Moreover, from the characterization of the above problem it will be possible to derive an explicit pricing function for both market rates which, in equilibrium, will be fully anticipated by households.

Taking first order conditions with respect to market rates it follows that:

$$F_1(R_A, R_L, z) = 0 \ (4.1)$$

$$F_2(R_{PF}, R_L, z) = 0 \ (4.2)$$

Where F_1 and F_2 are the first order conditions associated with R_A and R_{PF} respectively. These equations implicitly assume that $\frac{\partial B}{\partial R_A}$ and $\frac{\partial D}{\partial R_{PF}}$ are well defined, a fact which will follow from the characterization of households' problem, and that $\frac{\partial B}{\partial R_A} < 0$ as solutions are supposed to be interior. Stylized fact 1, depicted in figure 4.1, implies that the model must generate an interior solution in order to match the data. Thus, $\frac{\partial B}{\partial R_A} < 0$ *is an identification assumption* and thus must be imposed a priori.

Now it is possible to define the equilibrium notion for this model:

<u>Definition 1 (Partial Recursive Equilibrium):</u> an equilibrium for this economy is a collection of value V_1, V_2, policy B, D pricing R_A, R_{PF}, expectation functions $F_{1,E}^{R_{PF}}, F_{1,E}^{A_2}$, $F_{2,E}^{R_A}, F_{2,E}^{A_1}$ and a tax policy such that

i) V_1, D *solve the problem of the type 1 agent given* $F_{1,E}^{R_{PF}}, F_{1,E}^{A_2}$ *and* T.

ii) V_2, B *solve the problem of the type 2 agent given* $F_{2,E}^{R_A}, F_{2,E}^{A_1}$ *and* T.

iii) R_A, R_{PF} *satisfy equations 11 and 12 respectively*

iv) Expectations functions satisfy:

$$F_{2,E}^{A_1} = W' + (1 + R_{PF})D$$

$$F_{1,E}^{A_2} = W' - (1 + R_A)B$$

$$F_{1,E}^{R_{PF}} = R_{PF}, \ F_{1,E}^{R_A} = R_A$$

v) *Asset market clears:* $B + L = D$

vi) *Given* $\alpha, T' + L^{'} - (1 - R_L)L = 0$ *with* $T_1^{'} = \alpha T', T_2^{'} = (1 - \alpha)T'$

As regards the canonical definition of recursive (competitive) equilibrium (see for instance Mehra and Prescott, 1980), the above definition differs in the absence of an aggregate state and in the existence of exogenous prices. As regards the latter, this characteristic is inherited from the partial equilibrium nature of the model. As regards the former, it is an omission which can be done without loss of generality. In this type of economies it is accustomed to define an aggregate state Y which influences market prices directly. Then, assuming a unit mass of identical agents each of whom has control over an individual state y, it is possible to set $y = Y$ in order to solve for the equilibrium prices. Assuming the existence of a unit mass of each type of household in the above definition and expanding the state space to include a_i, the individual wealth of a type i agent, both definitions would be equivalent if we set a_i=A_i in equilibrium. For the sake of simplicity, this step is omitted and we use directly the definition above.

One of the virtues of the model presented above is the existence of a well behaved steady state. This property depends on the existence of an ergodic set which is distributed according to a stationary measure called invariant. To guarantee the existence of this steady state, it is desirable that the laws of motions generated by the model should be continuous; a condition that could be hard to get for pricing functions in general equilibrium (see for instance Kubler and Schmedders 2002). Fortunately, in a partial equilibrium framework the continuity of R_A and R_{PF} (on A_i for each W, R_L) follows naturally from the second order conditions for the banks problem (equations 11 and 12).

From equation vi) in the definition above, which is the consolidated Government budget equation, it is clear that fiscal policy does not have an exogenous component. The stock of CB notes L can be though as a REPO with no collateral that is determined by the excess of liquidity $D - B$. For instance, the bank can invest $D^{'} - B^{'} > 0$ in L' obtaining a net return of R_L. In this case, $L' > 0$ is a liability for the Government which simply makes transfers taxes or collects taxes depending on its net debt position defined by $L^{'} - (1 + R_L)L$. Given these restrictions, any sequence of exogenous Government expenditure would require an additional source of founding which would turn the model highly untractable; at least from a numerical point of view. Further, as we are dealing with an endowment economy, public expenditure will only have a locative effect on private expenditure; a role that can be played partially by net transfers T and their distribution α. There is room for a more meaningful fiscal policy but, taking into account the nature of the question at hand, it is outside the scope of the paper.

In order to understand the dependence of policy functions on the distribution of wealth $[A_1 \ A_2]$ take the case of the patient household with wealth level A_1. Its savings decisions are affected by the passive rate R_{PF} which in turn depends on the asset market equilibrium condition, equation v). This equation is affected by the amount of loans granted, which in equilibrium equals B; the policy function of impatient household with wealth level A_2. Thus, A_2 affects the decisions of the patient family through the pricing functions of the financial intermediary. Also the fiscal policy explains this dependence. Note that the tax rate depends on $[A_1 \ A_2]$ as can be seen from equation vi):

$$D - B - \left(1 - R_{L,(-1)}\right)L_{(-1)} = -T$$

Where $R_{L,(-1)}$ and $L_{(-1)}$ are the policy rate and stock of CB notes in the previous period respectively. As D and B depends on $[A_1 \ A_2 \ W \ R_L]$ so it does T; affecting the disposable income of both types of households. A similar argument can be done for the household of type 2.

4.3 Facts 1 to 3 from a recursive point of view.

The purpose of this section is to relate the model described above with styled facts 1 to 3. To that end we will use the characterization of the households' and bank's problem.

Equations 4.1 and 4.2 can be written, for interior solutions, as:

$$R_A\left[1+\frac{1}{\epsilon_t^B(R_L,z)}\right]=R_L \quad (4.1’)$$

$$R_{PF}\left[1+\frac{1}{\epsilon_t^D(R_L,z)}\right]=R_L \quad (4.2’)$$

Where $\epsilon_t^B(R_L,z)$ and $\epsilon_t^D(R_L,z)$ are elasticity of the demand for loans and supply of deposits with respect of the active and passive rates respectively. In order to derive these elasticities, it is possible to apply the implicit function theorem on the first order conditions that characterized the households problem in order to obtain dB/dR_A and dD/dR_{PF} [10]:

$$u_1^{'}\big(A_1-D(R_L,z)-T_1(R_L,z)\big)=$$

$$\beta_1 E_{(.)}[u_1^{'}\{W^{'}+(1+R_{PF})D(R_L;z)-D(R_L^{'};z')-T_1(R_L^{'};z')\}(1+R_{PF})] \quad (4.3)$$

$$u_2^{'}\{A_2+B(R_L,z)-T_1(R_L;z)\}=$$

$$\beta_2 E_{(.)}[u_2^{'}\{W^{'}-(1+R_A)B(R_L,z)+B(R_L^{'};z^{'})-T_1(R_L^{'};z')\}(1+R_A)] \quad (4.4)$$

Where $E_{(.)}\equiv E_{(W',R_L^{'}|W,R_L)}$ is the conditional expectation taking with respect to the transition matrix that characterizes the evolution of exogenous variables.

Note that in equations 4.3 and 4.4 we are using the equilibrium condition iv) in definition 1 (i.e. $F_{1,E}^{R_{PF}}=R_{PF}$ and $F_{1,E}^{R_A}=R_A$). Thus, $\epsilon_t^B(R_L,z)$ and $\epsilon_t^D(R_L,z)$ are obtained by assuming that the bank can change slightly its pricing policy given states $[R_L,z]$ and households immediately take into account this movement adapting their expectations. Consequently, both elasticities are well defined for each possible state variable $[R_L,z]$.

Now we are in position to match facts 1 to 3 using the model above.

Fact 1). The cumulative average of the intermediation spread converges to a steady value strictly above 1.

Dividing 4.1’) over 4.2’) and taking logs we obtain a linear expression for the evolution of the intermediation spread:

[10] The technical conditions which guarantee the existence of these differentials are discussed, for instance, in Milgrom and Segal (2002) and Santos (1991).

$$ln(R_A) - ln(R_{PF}) = ln\left(1 + \frac{1}{\epsilon^D(R_L, z)}\right) - ln\left(1 + \frac{1}{\epsilon^B(R_L, z)}\right)$$

Then, taking cumulative average and limits, the above expression implies that fact 1, in logs, can be approximated with

$$lim_{T\to\infty}[\textstyle\sum_{t=0}^{T} ln\, f(\epsilon_t^B)]T^{-1} - lim_{T\to\infty}[\sum_{t=0}^{T} ln\, f(\epsilon_t^D)]T^{-1} \quad (4.5)$$

Where $\epsilon_t^i = \epsilon^i([RL_t, z_t])$ with $i = D, B$ and $f(x) = \frac{1}{1+x}$.

Provided that $\epsilon_t^i \neq 0$ almost surely (in a measure to be defined below) and that an ergodic invariant measure exists (which will be shown below), 4.5 can be generated by means of a strong law of large numbers for markovian processes.

Fact 2). The active rate is above the policy rate.

Equation 4.1') implies that $R_A > R_L$ if $\epsilon_t^B(R_L, z) < -1$.

Fact 3). The passive rate is below the policy rate.

Equation 4.2') implies that $R_{PF} < R_L$ if $\epsilon_t^D(R_L, z) > 0$.

Equation 4.5 requires the existence of an ergodic invariant measure for the Markov process generated by the model. In order to match stylized facts 1 to 3 it is necessary to compute the model. The next section addresses these issues.

5 Computation and simulation

This section derives formally the markovian stochastic process associated with the model presented above, describes the algorithm used to compute it and presents the structural estimation results. The theoretical results, although novel as an application to this branch of the literature, are based on standard results borrowed from the stochastic processes and heterogeneous agents literature. The algorithm differs from the Krussel Smith method as the model does not have heterogeneous uncertainty and from the methods used in recursive general equilibrium models like Arellano (2008) due to the partial equilibrium nature of the economy described in section 4. The results presented in this paper are, to our knowledge, the first attempt to use structural estimation methods to compute the (incomplete) pass through coefficient.

5.1 Preliminaries

The purpose of this section is to derive a compact state space that contains $[R_L, z]$ and a first order stochastic process that describes the evolution over time of these variables. While the first property will allow us to solve the model using standard (numerical) methods the second will guarantee that equation 4.5 holds for the simulated series and that the levels of the ratios in figures 4.2 and 4.3 can be interpreted as realizations on the ergodic set of the model. Then, equipped with these tools, it is possible to compute the model and find empirically meaningful parameters by matching simulated moments with facts 1 to 3.

The following assumption will be useful to derive the results in this section:

Assumption 1:

i) $W \in \{W_{min}, \dots, W_{max}\}$ *and* $R_L \in \{RL_{min}, \dots, RL_{max}\}$. *That is, each of the exogenous states belongs to a finite set.*

ii) $0 \leq B \leq W_{min}$

iii) $\beta_1(1 + R_{PF}) \geq 1$

iv) $0 \leq L \leq L_{min}$ *with* $L_{min} > 0$

v) $u_1(c_1) = \frac{-(c_1-b)^a}{a}$ *with* $a, b > 0$ *and* a *is an even number.*

Under assumption 1-i) the vector $[W, R_L]$ can be assumed to follow a markov chain with transition matrix p; a restriction that is analytically convenient as will be seen in the proof of the proposition below.

Assumptions 1-ii) to 1-iv) impose bounds on loans granted, the passive interest rate and central banks notes. The first one is the "natural borrowing limit" as in Aiyagari (1994). The second one is a lower bound on the passive rate: provided that the type 1 household is assumed to be patient, β_1 can be made arbitrarily closed to 1. Then, assumption 1-iii) means only that the passive rate must be positive. Assumption 1-iv) implies that L is an asset for the bank (i.e. $L \geq 0$) and that the central bank faces a borrowing limit (i.e. $L \leq L_{min}$). The first part of this assumption guarantees that liquidity of the bank is positive (i.e. $D \geq B$); a fact that is supported by data[11]. The second part implies that the CB faces a borrowing limit similar to the one restricting the decisions of the household of type 2.

Assumptions 1-iii) and 1-v) must be understood jointly and are put in place in order to bound assets and consumption for the type 1 household. To see this fact we must take several consecutive steps.

First, from the characterization of Bellman's problem we have:

$$\frac{\partial V_1(R_L,z)}{\partial A_1} \geq E_{(.)}\left[\frac{\partial V_1(R_L^{'},z')}{\partial A_1}(1+R_{PF})\beta\right] \geq E_{(.)}\left[\frac{\partial V_1(R_L^{'},z')}{\partial A_1}\right] \quad (5.1)$$

The first inequality follows from the first order condition for the Bellman's problem of the type 1 agent and the second one from assumption 1-iii). Further, it will be shown that the left hand side of equation 5.1 converges to a constant V (see the appendix A.2 for a detailed discussion) and, under assumption 1-v), $V \in [0, b]$. This fact coupled with a standard envelope theorem (see Stokey, Lucas and Prescott page 266) implies that the consumption of the type 1 household, C_1, converges to a non-negative constant $\bar{C}$.

The convergence of C_1 is a consequence of combining the preference structure in assumption 1-v) and a convergence theorem for martingales. The former is standard in the quadratic dynamic programming literature (see Sargent and Hansen 2005) and the latter is borrowed from the Hugget (1993) style models (Sargent and Ljungqvist 2000).

Second, contrarily to the partial equilibrium Hugget style models, the convergence of the left hand side of equation 5.1 to a constant does not imply that consumption must be unbounded. In the canonical version of the Hugget model, the presence of i.i.d aggregate shocks and a constant interest rate for savings imply that the derivative of the value function with respect to assets and the marginal utility of consumption must both converge to zero. This is a consequence of the budget constraint for households: as the only state variable is the stock of assets, both consumption and assets must be

[11] The ratio D/B, computed as the average between 2003 and 2016, is 1.65, 1.11 and 1.16 for Argentina, Chile and Colombia respectively.

constant if they are finite. This fact contradicts the households' budget equation in the presence of aggregate uncertainty (see Sargent and Ljungqvist page 357 for a detailed discussion). Contrarily, in the model presented in section 4 the state space contains more than 1 variable and interest rates are endogenous. In particular, the budget equation for the household of type 1 is:

$$C_{1,t} = \left(1 + R_{PF,t-1}\right)D_{t-1} + W_t - D_t - T_{1,t} \quad (5.2)$$

Equation 5.1 implies that $C_{1,t} \to \bar{C}$ and equation 5.2 means that $\bar{C}$ can be finite provided that D and R_{PF} are bounded as wages are contained in a finite set by assumption 1-i) and taxes are bounded by assumption 1-i) and 1-iv) provided that equilibrium condition vi) in definition 1 holds. In this type of equilibrium, *D and R_{PF} must oscillate along with W. This fact makes DSGE models unsuitable to answer the question at hand* as the non-stochastic steady state (i.e. one that has constant endogenous variables) is not compatible with the compactness of the equilibrium set.

Third, from the discussion above, assumption 1-v) implies $0 \leq \bar{C} \leq b$. Then, it will be shown that assumptions 1-ii) to 1-iv) together with 1-i) implies:

$$\bar{D} \leq min\{(b + (1 + RL_{max})\bar{L})\beta_1; \bar{L} + W_{min}\} \quad (5.3)$$

Forth, both market rates can be bounded using assumptions 1-i) and 1-iii) using standard arguments (see Duffie, et. al. 1994).

The discussion above can be summarized by the following proposition:

Proposition 1: under assumptions 1-i) to 1-v) the endogenous variables in definition 1 satisfy the following restrictions:

a) $B \in [0, W_{min}]$

b) $D \in [0, \bar{L} + W_{min}]$

c) $L \in [0, \bar{L}]$

d) $R_{PF} \in \left[\frac{1}{\beta_1} - 1, \bar{R}_{PF}\right]$

e) $R_A \in [0, \bar{R}_A]$

Proof: see appendix A.2

Besides the compactness of the state space, we are interested in a Markov process with an appropriate steady state (i.e. an ergodic set and an invariant measure). The convenience of this type of (long run) equilibrium can be seen in equation 5.2: an invariant measure is the stationary distribution which describes the behavior of state variables in the long run. Equation 5.2 implies that this distribution has to assign positive probability to more than 1 point in the equilibrium (or ergodic) set. This fact can be achieved by means of an invariant measure. Not only this type of equilibrium is compatible with an appropriate equilibrium set, it also allows matching stylized fact 1 generating equation 4.5 out of simulated series.

The easiest way to obtain such a process is to derive a *continuous* function g that maps $(R_L, z) \rightarrow (A_1^{'} A_2^{'})$. The virtue of this type of function is that allows deriving a *Feller* process for $[R_L, z]$ which is well known to have an ergodic invariant measure over compact sets. In order to derive g we will use: i) the policy functions B, D and the laws of motion for wealth $A_1^{'} A_2^{'}$ from the households' problem. ii) The pricing functions obtained from the banks problem, equations 4.1') and 4.2'). iii) The equilibrium condition for expectations, equation iv) in definition 1.

For expositional purposes it is convenient to invest a little bit in notation: let $x \equiv [A_1\ A_2]$ and $s \equiv [W\ R_L]$ denote the endogenous and exogenous states respectively. From the policy functions and the laws of motion for wealth of both types of agents we have:

$$[z^{'} R_L^{'}] = [A_2^{'} A_1^{'}\ s'] = [A_2(x,s) A_1(x,s) s^{'}] \equiv [g(x,s) s^{'}] \quad (5.4)$$

Note that equation 5.4 implies that g is a vector value function that has as a first and second coordinates, respectively, $\mathrm{A}_2^{'} = \mathrm{W}^{'} - [(1+\mathrm{R_A})\mathrm{B}](\mathrm{R_L}, \mathrm{z})$ and $\mathrm{A}_1^{'} = \mathrm{W}^{'} + [(1 +$ RPF)D(RL,z). In these last 2 expressions we have used the pricing functions 4.1')-4.2') and the equilibrium expectations equations in definition 1 to obtain the market interest rates as a function of states.

The second order conditions applied to equations 4.1 and 4.2 imply that the pricing functions for the financial intermediary are continuous on x for each s[12]. The continuity of the policy functions B and D are then guaranteed by standard dynamic programming results (see Stokey, Lucas and Prescott, 1998, page 263). Using these results proposition 1 implies that g is continuous on each coordinate (on x for each s).

Note that proposition 1 guarantees the compactness of $X \times S$, with $[x, s] \in X \times S$. Equipped with this result, the following operator defines a Feller Markov process on the product space (see Stokey, Lucas and Prescott page 284):

$$P[(x,s), A \times B] = \begin{cases} p(s,B) & si \;\; g(x,s) \in A \\ 0 & si \;\; g(x,s) \notin A \end{cases}$$

The operator P can be used to describe the evolution over time of the (unconditional) distribution of $[x, s]$, namely $\{\mu_t\}_{t=0}^{\infty}$, as it satisfies:

$$\mu_{t+1}(A \times B) = \int P[(x,s), A \times B]\mu_t(dx \times ds) \; (5.5)$$

The Feller property guarantees that operator P is continuous in an appropriate sense (see Futia 1982); a property which guarantees the existence of a fixed point over compact state spaces. A fixed point of 5.5 is also kwon as an invariant measure as it satisfies:

$$\mu(A \times B) = \int P[(x,s), A \times B]\mu(dx \times ds) \; (5.5')$$

Equation 5.5' can be seen as a long run equilibrium notion as it implies the stationarity of the distribution of payoff relevant variables. Further, note that μ can assign positive probability to more than 1 point in the equilibrium set as required by equation 5.2.

Finally, a Feller Markov process has an ergodic invariant measure (see also Futia 1982). This type of process generates cumulative averages which satisfy:

[12] Market interest rates are continuous due to the implicit function theorem applied to equations 4.1 and 4.2. In general, it is sufficient to show that second order conditions hold globally to guarantee that the solutions to a monopolists' problem are continuous. The numerical solutions to the Bellmans' problem of both types of agents, computed in section 5.3, are both linear. Under these type of solutions, second order conditions are satisfied if $D'_{R_{PF}} > 0$ and $B'_{R_{\acute{A}}} < 0$. These 2 conditions will also be satisfied under the parametrization used in section 5.3.

$$lim_{T\to\infty}[\sum_{t=1}^{T} q([x_t, s_t])]T^{-1}=E_\mu(q) \ (5.6)$$

Equation 5.6 holds $\nu_{[x_0,s_0]}$-almost surely for any $[x_0, s_0]$ with μ-positive probability (see Varadhan 2000). The measure $\nu_{[x_0,s_0]}$ defines the Markov process $(\Omega, \sigma, \nu_{[x_0,s_0]})$, see Stokey, Lucas and Prescott (page 224), q is an $[X \times S]$-measurable function, E_μ is the expectation with respect to an invariant measure μ and Ω is the space of sequences $[X \times S]^\infty$ with an appropriate sigma algebra σ.

Finally, note that equation 4.5 can be generated using equation 5.6 as $\ln(f(\epsilon_t^i))$ is a measurable function provided that $\epsilon_t^i \neq 0$ for $i = B, D$ and any t[13].

5.2 Algorithm

Besides the implications discussed in section 5.1, proposition 1 allows us computing the model using a slightly modified version of Judd's (1998) value function iteration algorithm (VFIA). This proposition defines a proper "box" that includes endogenous and exogenous variables: assumption 1-i) and implications a) to e) of proposition 1 defines a compact subset of $\mathcal{R}_+^7$ which contains all payoff relevant variables.

The distinctive feature of this algorithm resides in the presence of prices. In particular the algorithm is endowed with an iterative procedure which involves the pricing equation of the bank and the expectation functions of each type of agent *which are supposed to converge to a fixed point.* Numerically, for the parametrizations used, the algorithm converges. However, contrarily to the standard version of the VFIA (see Judd 1998), there is no available formal proof for the convergence of this type of procedure and we must rely in numerical results. The lack of theoretical results for the convergence of this type of algorithms is connected with the absence of existence proofs for the equilibrium in definition 1. For the case of uncountable agents and idiosyncratic shocks Miao (2006) showed the existence of this type of equilibrium. Unfortunately, Miao's results cannot be applied to this framework.

The syntax of the code is the following[14]:

[13] We are assuming that the partial derivatives of the policy functions are measurable.
[14] The code is available under request to damian.pierri@gmail.com.

1) Compute $F_{1,E}^{R_{PF}}, F_{1,E}^{A_2}, F_{2,E}^{R_A}, F_{2,E}^{A_1}$ as a function of states $[R_L, z]$ using an arbitrary functional form.
2) Using 1), solve the dynamic programming problem of the 2 types of households.
3) Using B and D obtained from 2) we obtain the elasticities involved in the pricing functions of the bank, equations 4.1') and 4.2'). Using these price functions we update $F_{1,E}^{R_{PF}}, F_{1,E}^{A_2}, F_{2,E}^{R_A}, F_{2,E}^{A_1}$.
4) Using the results in 3, we solve again the dynamic programming problems of the agents of type 1 and 2.
5) We update $F_{1,E}^{R_{PF}}, F_{1,E}^{A_2}, F_{2,E}^{R_A}, F_{2,E}^{A_1}$ using the results in 4).

If these results "match" the ones obtained in 3, we stop the procedure; otherwise we keep on loping on 3 to 5.

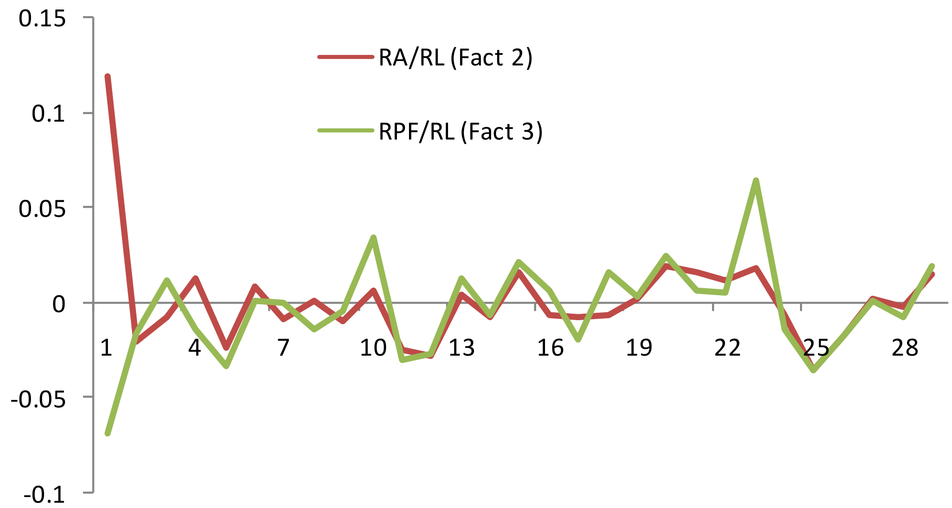
5.3 Results

The purpose of this section is to match facts 1 to 3, depicted in figures 4.1 to 4.3, using the model characterized in sections 4 and 5.1. It will be computed by means of the algorithm described above.

As suggested by figure 4.1, the spread between the active and passive rate will be computed as a cumulative average of T observations. Contrarily, RA/RL and RPF/RL will be computed as the average of the simulated series obtained for T and $T-1$. We simulate these ratios for different values of T but the results did not change significantly.

Remarkably, the results in section 4 and 5.1, which guarantee the existence of an ergodic invariant measure, insure the law of large numbers in equation 4.5 holds and that the payoff relevant variables lie in a "stable" or ergodic set. In particular, the cumulative average of the simulated intermediation spread RA/RPF can be associated with its expected value taken with respect to the steady state distribution of the model. A similar argument can be made for RA/RL and RPF/RL. However, the length of the series used could be smaller as shown in the figure below.

Figure 5.1: Simulated series path against its cumulative average ($\frac{y_t - \bar{Y}_T}{\bar{Y}_T}$)



As suggested by figure 5.1, after removing the first observation, the simulated series converges to its cumulative average as $\frac{y_t - \bar{Y}_T}{\bar{Y}_T} \approx 0$ for $t > 1$, where y_t is either $\frac{RA_t}{RPF_t}$ or $\frac{RA_t}{RPF_t}$ and $\bar{Y}_T$ their length-T cumulative average. Then, $y_{T-1} \approx y_T \approx \bar{Y}_T$ which, by means of the law of large numbers in equation 5.6, implies that either y_{T-1} or y_T could be associated with the expected value of y taking with respect to the steady state distribution.

Fact 1 in figure 4.1 will be matched using the cumulative average of the simulated series for RA/RPF between $t = 1$ and $t = T$ and facts 2 and 3, in figures 4.2 and 4.3 respectively, will be matched using the average for RA/RL and RPF/RL over $t = q$ and $t = T$ with $q > 1$. In both cases equation 4.5 and its analogous for RA/RL and RPF/RL guarantee that these averages are arbitrarily close to the respective mean obtained using the steady state distribution. That is, the appropriately computed data averages matched the long run distribution of the model.

The table below presents the results of the match.

Table 2: Calibration results[15]

	Argentina 2015	Chile 2008	Colombia 2006	Benchmark
RA/RPF (Fact 1)	1.999	1.915	2.003	2.004
RA/RL (Fact 2)	1.226	1.607	1.719	1.634
RPF/RL (FACT3)	0.824	0.908	0.837	0.817
Relative Error	0.003	0.002	0.001	

The last row in table 2 shows the cumulative numerical error. The columns associated with each country reflect figures 4.1 to 4.3 (facts 1 to 3 respectively). These values were computed exactly as their numerical counterpart according to the procedure described above. The benchmark column is obtained from the following parameters.

Table 3: Parameters

β_1	0.99	ϵ_1	$[\mathbf{3}, \mathbf{4}]$
β_2	0.97	ϵ_2	$[-\mathbf{2}, -\mathbf{1}, \mathbf{5}]$
u_1	$-(c_1 - 0.2)^2/2$	R_L	$\{0.05; 0.07\}$
u_2	$-(c_2 - 0.1)^2/4$	W	$\{1.0; 1.1\}$

Remarkably the values for the elasticity of deposits supply ϵ_1 and loans demand ϵ_2 reflect that these variables are quite sensitive to interest rate changes. Further, as can be seen in section 4.3, $\epsilon_2 < -1$ and $\epsilon_1 > 0$ guarantee that facts 2 and 3 hold. This is confirmed by simulation results in table 2.

In order to compute a policy experiment we simulate the effects of a permanent increase in the policy rate as follows: let $\left[R_{L,min}, \dots, R_{L,max}\right]$ denote the initial vector of possible policy rates before the shock. Then, an increase of x% in these rates will be represented by replacing $\left[R_{L,min}, \dots, R_{L,max}\right]$ with $(1 + \frac{x}{100})\left[R_{L,min}, \dots, R_{L,max}\right]$. The nature of the shock is designed in order to avoid spurious results due to the

[15] Each period in the simulation were considered a month. As data is available annually and averages in figure 4.1 converge after 6 years, we take T=72. Then we compute the average of the simulated series as follows: $[\sum_{t=1}^{72} y_t]72^{-1} = \sum_{j=0}^{6}\left[[\sum_{m=1}^{12} y_m^j]12^{-1}\right]6^{-1}$ where y_m^j is the value of the series at month m and years j and $t = 12j + m$.

fact that the bank in assumed to maximize profits each period without taking into the entire path of loans and deposits. If we allow interest rates to change in only 1 state or change differently across them, different realizations of the Markov process would imply a different shock in this 1 period problem.

The table below contains the results of the policy experiment.

Table 4. Policy Experiment (PE): increase of 1% in the set of policy rates

	Benchmark	PE (↑ R_L 1%)	Variation
Spread	2.021	2.016	**-0.2%**
RA/RL	1.674	1.684	**0.6%**
RPF/RL	0.835	0.839	**0.5%**

As we are interested in the percentage increase of RA and RPF after a 1% increase in the policy rate, we construct the PE column using the original set of policy rates and then compute the variation as:

$$\frac{(RA(\widetilde{RL})/RL)-(RA(RL)/RL)}{(RA(RL)/RL)} \times 100 = \frac{RA(\widetilde{RL})-RA(RL)}{RA(RL)} \times 100 \quad (5.7)$$

$RA(\widetilde{RL})$ is the active rate computed using $(1+\frac{1}{100})[R_{L,min}, \dots, R_{L,max}]$ and $RA(RL)$ using the original set $[R_{L,min}, \dots, R_{L,max}]$. The variation in RPF was computed analogously.

In order to interpret these results we must take into account assumption 1-i) which implies that policy rates lie in a finite set. Thus, using equations 4.1') and 4.2'), equation 5.7) and its analogous for RPF can be written as:

$$\frac{RA(\widetilde{RL})-RA(RL)}{RA(RL)} \times 100 = \frac{M_2(\widetilde{RL})(1+\frac{1}{100})-M_2(RL)}{M_2(RL)} \times 100 \quad (5.7')$$

Where $M_2(\widetilde{RL}) = \epsilon_2(\widetilde{RL})/[1+\epsilon_2(\widetilde{RL})]$ is the mark up for the active rate after the shock. An analogous expression holds for the variation in the passive rate.

Table 4 shows that the active and passive rates raised 0.6% and 0.5% respectively after a 1.0% increase in the set of policy rates. According to equation 5.7'), this fact means that the mark up for the active and passive rates raised 0.4% and 0.2%. In other words, the incomplete pass through is explained by a less than proportional increase in the mark ups.

In order to obtain an intuition behind this result we can compute the variation in the benefits obtained by the bank before and after the shock, $\Delta\pi = \pi(\tilde{R}_L) - \pi(R_L)$:

$$\Delta\pi =$$
$$\underbrace{\left[\left(R_A(\tilde{R}_L) - R_A(R_L)\right) - \left(\tilde{R}_L - R_L\right)\right]B(\tilde{R}_L)}_{I} + \underbrace{\left[B(\tilde{R}_L) - B(R_L)\right](R_A(R_L) - R_L)}_{II} -$$
$$\underbrace{\left[\left(R_{PF}(\tilde{R}_L) - R_{PF}(R_L)\right) - \left(\tilde{R}_L - R_L\right)\right]D(\tilde{R}_L)}_{III} - \underbrace{\left[D(\tilde{R}_L) - D(R_L)\right](R_{PF}(R_L) - R_L)}_{IV} \quad (5.8)$$

The 0.2% drop in the intermediation spread can be explained by the difference in absolute value in the elasticieties of demand of credit and supply of deposits: as $|\epsilon_1| > |\epsilon_2|$, the bank chooses to increase the active rate *less than* the passive rate in order to minimize the drop in the intermediation spread (and in profits according to II in equation 5.8). The stability of the intermediation spread can be explained by equation 4.5: as it is computed as a cumulative average before and after the shock, the values in table 4 can be associated with the steady state distribution of the model. The small change computed in the spread suggests that the long run properties of the economy are almost unaffected by monetary policy.

Equation 5.8 can also be used to explain the incomplete pass through intuitively: as $\epsilon_1 > 0$ and $\epsilon_2 < 0$, the policy shock implies that $II < 0$ and $IV < 0$ *if facts 2 and 3 hold simultaneously.* Suppose that an increase in the policy rate as the one described above takes place. Then the incomplete pass though strategy implies a reduction in the amount of credits granted and, as fact 2 implies $R_A(R_L) - R_L > 0$, reduces profits. However, as long as fact 3 holds, $R_{PF}(R_L) - R_L < 0$. Then, the increase in the policy rate also raises the amount of deposits in equilibrium; implying that IV partially compensates the decrease in profits generated by II.

Terms II and IV give incentive to the bank to set $R_{PF}(R_L) < R_L$ if $R_A(R_L) > R_L$ if the bank decides to increase both market rates jointly. Terms I and III explains the incomplete pass through given this interest rate profile: term I reflects the decrease in profits generated by a less than proportional increase in the active rate. This drop must the compensated by a less than proportional increase in the passive rate, implying that $III < 0$. Thus, the incomplete pass though may be seen as arising from a portfolio strategy of the bank which tries to buffer the reduction in profits associated with less intermediation.

There may be other profitable strategies for the bank. However, the parameters in table 3 and assumption 1 can be used to understand why the bank prefers an incomplete pass through strategy. It should be clear that this result is entirely explained by the magnitude of these parameters as, from a qualitative point of view, the other strategies can also be used to reduce the variance in profits.

Suppose that the bank decides to increase more than proportionally the active rate. Taking into account the wide range of possible elasticieties, equation 5.7 implies that this choice is feasible. The stability of the mark up implies then that the passive rate must also rise more than the policy rate. However, assumption 1-iv), which is back up by data in section 5.1, and in particular $L \geq 0$ implies $D > B$. Then, the increase in profits generated by I will be more than compensated by III; implying that this choice is not profitable.

It is possible that the bank to reduce both rates by a similar amount in order to keep spreads almost constant. Then, $I < 0$ and $III < 0$. Assumption 1-iv) implies that this may be a profitable choice. However, $|\epsilon_1| > |\epsilon_2|$ together with $\epsilon_1 > 0$ and $\epsilon_2 < 0$ imply under facts 2 and 3 the effects on the other 2 terms, $II > 0$ and $IV > 0$, may more than compensate the increase in profits associated with $I < 0$ and $III < 0$. In particular, as interest rates do fall down after the policy shock, the drop in D must be big enough to turn this strategy unprofitable.

Finally, as the intermediation spread drops 0.2%, it is possible for the bank to reduce the active rate and increase the passive rate. But this strategy implies that only $I < 0, III > 0, IV > 0$ and $II > 0$ implying that 3 out of four terms reduces profits. Thus, the bank can do better by raising both rates less than the policy rate.

6 Conclusions

This paper investigates the determinants of the incomplete pass though of both market rates; active and passive.

The results obtained imply that after a 1% increase in the active policy rate, for instance going from 5,00% to 5,05%, the active rate only raises 0.6% and the passive 0.5%; implying a reduction in the intermediation spread. This fact implies that the effectiveness of the monetary policy can be affected by the incomplete pass though of market rates even if credit demand is highly elastic. The structurally estimated parameters in table 3 imply that the elasticity of the demand for loans is well above unity. In this context, banks may prefer to buffer the reduction in profits associated with the drop in intermediation after a monetary tightening by lowering the mark up charged over the cost of founding.

Among the determinants of the incomplete pass though is a small interest rate for deposits. In particular, this interest rate may be well below the policy rate as observed in Argentina, Chile and Colombia. Thus if a Central Bank wants to increase the effectives of its monetary policy, it must raise the return for deposits. This measure not only will raise the intermediation levels, it also will generate the incentives for banks to move market rates in line with the policy rates.

Finally, the stability of the intermediation spread can be taken as evidence of the long run neutrality of monetary policy. However, the model allows interpreting this stylized fact differently as DSGE models: it is the long run distribution of payoff relevant variables that is unaffected by monetary policy and not a particular value of some of these variables.

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Appendix

Section A.1 Additional Empirical Results

	Argentina		Chile		Colombia	
Equation 2.1	**RA**	**RL**	**RA**	**RL**	**RA**	**RL**
β_0	13.060***	-0.426	3.620***	-0.236	15.352***	4.519***
	(1.92)	(1.47)	(0.73)	(0.28)	(0.93)	(0.47)
$\beta 1$	0.366***	0.500***	0.771***	0.986***	1.069***	0.860***
	(0.09)	(0.07)	(0.05)	(0.02)	(0.03)	(0.02)
Ratio Loans/Deposits	0.105	0.097*	0.042***	0.006	-0.097***	-0.041***
X	(0.05)	(0.04)	(0.01)	(0.00)	(0.01)	(0.01)
Dummy FX Controls	2.056*	1.205				
	(0.96)	(0.73)				
N	141	141	141	141	141	141
R-Square	0.596	0.755	0.828	0.975	0.908	0.958
Equation 2.4						
α_0	20.595***	7.896***	10.344***	4.116***	13.673***	6.233***
	-0,27	-0,32	(0.15)	(0.15)	(0.22)	(0.17)
$\alpha 1$	0.125	0.058	-0.591	-0.112	0.566	0.333
	(0.18)	(0.22)	(0.52)	(0.51)	(0.86)	(0.66)
Ratio Loans/Deposits	0.425	0.701	0.347*	0.418**	-0.097	-0.079
X	(0.39)	(0.44)	(0.14)	(0.14)	(0.19)	(0.15)
Dummy FX Controls	7.260***	7.327***				
	(0.52)	(0.63)				
ρ	0.942***	0.919***	0.583*	0.902	0.845**	0.768
	(0.08)	(0.12)	(0.23)	(0.57)	(0.28)	(0.44)
N	140	140	140	140	140	140
R-Square	0,735	0,604	0,099	0,103	0.073	0.029

Significance Levels 1% (***), 5% (**) y 10% (***).

Table A.1

The ratio of loans to deposits captures inversely the liquidity of banks. The dummy variable controls for restrictions to the access to the FX market in Argentina since 2011. These controls affected severely the behavior of market rates as indicated by the significance of the coefficients.

The Tables below shows the results of estimating equations 2.1 and 2.4 adding more controls and equation 2.7 for 2 different specifications for the cutoff function (TAR and M-TAR).

The results confirm the evidence presented above: the only significant relationship is in the long and the cointegration equation is simetrical. As regards the controls, only real wages and liquidity seems to be significant.

The first 2 tables below are labeled A.1 as they also estimate equations 2.1 and 2.4.

		Argentina		Chile		Colombia	
	Equation 2.1	**RA**	**RL**	**RA**	**RL**	**RA**	**RL**
	β_0	34.219***	13.609***	6.220***	-0.679*	17.055***	6.786***
		(1.60)	(1.52)	(0.81)	(0.34)	(1.69)	(0.82)
	β1	-0.013	0.249***	0.700***	0.998***	1.044***	0.826***
		(0.06)	(0.05)	(0.05)	(0.02)	(0.04)	(0.02)
	Ratio Loans/Deposits	0.781***	0.546***	0.075***	-0.000	-0.084***	-0.024**
		(0.05)	(0.05)	(0.01)	(0.00)	(0.02)	(0.01)
(β_2)	Real Wage	-0.434***	-0.288***	-0.044***	0.008*	-0.022	-0.029**
x		(0.02)	(0.02)	(0.01)	(0.00)	(0.02)	(0.01)
	Dummy FX Controls	0.775	0.356				
		(0.53)	(0.51)				
	N	141	141	141	141	141	141
	R-Square	0.878	0.884	0.859	0.976	0.909	0.961
	Equation 2.4						
	α_0	20.665***	8.007***	10.338***	4.084***	13.673***	6.237***
		(0.28)	(0.33)	(0.16)	(0.16)	(0.22)	(0.17)
	α1	0.086	-0.005	-0.592	-0.073	0.567	0.341
		(0.18)	(0.22)	(0.53)	(0.52)	(0.86)	(0.67)
	Ratio Loans/Deposits	0.412	0.679	0.348*	0.424**	-0.097	-0.087
		(0.39)	(0.44)	(0.14)	(0.14)	(0.19)	(0.15)
(α_2)	Real Wage	-14.280	-22.820	3.350	16.724	-0.076	-2.046
x		(14.72)	(17.72)	(26.30)	(25.71)	(7.00)	(5.30)
	Dummy FX Controls	7.194***	7.221***				
		(0.52)	(0.63)				
	ρ	0.942***	0.919***	0.576*	0.855	0.845**	0.769
		(0.08)	(0.12)	(0.23)	(0.57)	(0.28)	(0.44)
	N	140	140	140	140	140	140
	R-Square	0.737	0.609	0.099	0.106	0.073	0.031

Significance Levels 1% (***), 5% (**) y 10% (***).

		Argentina		Chile		Colombia	
	Equation 2.1	**RA**	**RL**	**RA**	**RL**	**RA**	**RL**
	β_0	25.668***	8.685***	2.996	2.594***	16.446***	5.180***
		(2.30)	(2.31)	(1.85)	(0.73)	(2.55)	(1.23)
	β1	-0.087	0.206***	0.692***	0.983***	1.038***	0.812***
		(0.05)	(0.05)	(0.05)	(0.02)	(0.04)	(0.02)
	Ratio Loans/Deposits	0.821***	0.568***	0.086***	-0.005	-0.080***	-0.011
		(0.05)	(0.05)	(0.01)	(0.01)	(0.02)	(0.01)
	Real Wage	-0.334***	-0.230***	-0.020	-0.023**	-0.018	-0.020*
(β_2)		(0.03)	(0.03)	(0.02)	(0.01)	(0.02)	(0.01)
x	Durable Expenditure	-0.000***	-0.000**	-0.007	0.009***	-0.002	-0.005
		(0.00)	(0.00)	(0.01)	(0.00)	(0.01)	(0.00)
	Dummy FX Controls	0.861	0.405				
		(0.50)	(0.50)				
	N	141	141	117	117	141	141
	R-Square	0.897	0.891	0.849	0.976	0.909	0.962
	Equation 2.4						
	α_0	20.731***	8.057***	10.717***	4.440***	13.686***	6.245***
		(0.28)	(0.34)	(0.17)	(0.18)	(0.22)	(0.17)
	α1	0.050	-0.031	-0.503	0.109	0.637	0.380
		(0.18)	(0.22)	(0.51)	(0.52)	(0.87)	(0.67)
	Ratio Loans/Deposits	0.476	0.705	0.410**	0.483**	-0.104	-0.093
		(0.39)	(0.44)	(0.14)	(0.15)	(0.19)	(0.15)
	Real Wage	-13.175	-22.050	3.844	19.581	1.818	-0.898
		(14.68)	(17.76)	(28.30)	(29.22)	(7.51)	(5.70)
(α_2)	Durable Expenditure	-2.949	-2.058	-1.615	-0.665	-1.111	-0.672
x		(2.06)	(2.46)	(1.22)	(1.29)	(1.58)	(1.20)
	Dummy FX Controls	7.117***	7.167***				
		(0.52)	(0.63)				
	ρ	0.964***	0.931***	0.792***	0.087	0.848**	0.757
		(0.08)	(0.13)	(0.23)	(0.62)	(0.28)	(0.44)
	N	140	140	116	116	140	140
	R-Square	0.741	0.611	0.172	0.115	0.076	0.033

Significance Levels 1% (***), 5% (**) y 10% (***).

	Equation 2.4'	Argentina RA	Argentina RL	Chile RA	Chile RL	Colombia RA	Colombia RL
	η	20.804***	8.124***	10.330***	4.080***	13.683***	6.251***
		(0.25)	(0.33)	(0.15)	(0.16)	(0.22)	(0.17)
	γ_i	0.263	0.380	-0.204	0.039	0.657	0.592
		(0.17)	(0.21)	(0.48)	(0.52)	(0.86)	(0.69)
	Ratio Loans/Deposits	0.291	0.177	0.369**	0.432**	-0.121	-0.080
		(0.34)	(0.38)	(0.13)	(0.14)	(0.19)	(0.15)
(γ_k)	Real Wage	-16.428	-19.936	15.164	19.024	-0.730	-0.912
X		(13.29)	(16.44)	(23.96)	(25.92)	(6.95)	(5.31)
	Dummy FX Controls	6.996***	7.033***				
		(0.47)	(0.59)				
	ρ_1	1.099***	1.060***	1.005***	0.551	1.117*	1.007
		(0.10)	(0.12)	(0.28)	(0.73)	(0.44)	(0.65)
	ρ_2	1.022***	1.072***	1.137***	0.617	0.894*	1.095
		(0.11)	(0.29)	(0.27)	(0.83)	(0.36)	(0.61)
	N	140	140	140	140	140	140
	R-Square	0.787	0.668	0.224	0.098	0.096	0.046

	Equation 2.4'	Argentina RA	Argentina RL	Chile RA	Chile RL	Colombia RA	Colombia RL
	η	21.885***	8.569***	10.056***	3.843***	13.658***	5.418***
		(0.34)	(0.42)	(0.18)	(0.21)	(0.32)	(0.25)
	γ_i	0.307	0.379	0.247	0.290	0.659	0.286
		(0.16)	(0.20)	(0.50)	(0.53)	(0.90)	(0.64)
	Ratio Loans/Deposits	0.145	0.064	0.322*	0.427**	-0.129	-0.082
		(0.32)	(0.38)	(0.13)	(0.14)	(0.19)	(0.14)
(γ_k)	Real Wage	-16.797	-23.555	13.182	19.614	-0.799	-0.531
X		(12.48)	(16.40)	(23.39)	(25.56)	(6.98)	(4.97)
	Dummy FX Controls	6.295***	6.751***				
		(0.47)	(0.61)				
	ρ_1	0.821***	0.946***	1.591***	1.485	1.061	4.258***
		(0.09)	(0.13)	(0.28)	(0.76)	(0.63)	(0.86)
	ρ_2	1.748***	1.547***	0.308	-0.989	0.944*	-1.295
		(0.18)	(0.33)	(0.35)	(1.05)	(0.41)	(0.69)
	N	140	140	140	140	140	140
	R-Square	0.812	0.674	0.261	0.119	0.095	0.162

Tables A.2

Section A.2 Proof of proposition 1

Iterating over the sequential version of the type 2 household's budget constraint we obtain:

$$lim_{T\to\infty}\left(\sum_{s=0}^{T}\frac{w_{t+s}}{1+R_{A,t+s}}-\sum_{s=0}^{T}\frac{c_{t+s}+T_{2,t+s}}{\left(1+R_{A,t+s}\right)^{s}}+\frac{B_{T+1}}{\prod_{s=0}^{T}\left(1+R_{A,t+s}\right)}\right)\geq\left(1+R_{A,t-1}\right)B_{t-1}$$

Thus, imposing assumption 1-i) and $lim_{T\to\infty}\frac{B_{T+1}}{\prod_{s=0}^{T}(1+R_{A,t+s})}=0$, the more restrictive level of debt for a finite active rate is obtained by setting consumption and taxes to zero and W_{t+s} to the smallest possible value. That is,

$$lim_{T\to\infty}\left(\sum_{s=0}^{T}\frac{w_{t+s}}{1+R_{A,t+s}}\right)\geq\frac{w_{min}}{1-(1/(1+\bar{R}_{A}))}\geq w_{min}\geq\left(1+R_{A,t-1}\right)B_{t-1}\geq B_{t-1}$$

The first inequality follows from letting $\bar{R}_A$ be the upper bound for the active rate and the remaining expressions can be obtained from simple algebraic operations.

In order to analyze the case of unbounded active rate note that the implicit function theorem applied to equation 4.4) implies $dB/dR_A<0$. Thus $lim_{R_A\to\infty}B(R_L,z,R_A)=0$ as the optimization problem for the household of type 2 requires $B\geq 0$, where $B(R_L,z,R_A)$ is the demand for credit faced by the bank. Equation 4.1') implies that if $dB/dR_A<0$ then $R_A>R_L$. Taking into account that definition 1 implies that $B+L=D$ and $L\geq 0$ by assumption 1-iv), the bank will always prefer to reduce R_A in order to obtain a positive level of intermediation (i.e. $B>0$). That is, $B+L=D$ and $R_A>R_L$ imply $(R_A-R_L)B+R_LD>R_LD$. Consequently the active rate must be bounded above.

In order to bound D we must first take care of C_1. Using the first order condition for the maximization problem of the type 1 household we get equation 5.1:

$$\frac{\partial V_1(R_L,z)}{\partial A_1}\geq E_{(.)}\left[\frac{\partial V_1(R_L^{'},z')}{\partial A_1}\left(1+R_{PF}(R_L,z)\right)\beta\right]\geq E_{(.)}\left[\frac{\partial V_1(R_L^{'},z')}{\partial A_1}\right]\ (5.1)$$

The second equality follows from assumption 1-iii). Benveniste and Scheinkman's formula (see Stokey, Lucas and Prescott ch. 4 and 9) implies $\frac{\partial V_1(R_L,z)}{\partial A_1}=\frac{\partial u_1}{\partial c}$. Then 5.1 implies that $\frac{\partial V_1(R_L,z)}{\partial A_1}$ follows a non negative martingale and with probability 1 converge to a real number V. Under assumption 1-v), $V \in [0, b]$. Then $V = \frac{\partial u_1(\bar{c})}{\partial c}$. Moreover, $\bar{c}$ satisfies the budget constraint of the type 1 household as stated in equation 5.2. Consequently, under assumption 1-v) we get $0 \leq \bar{c} \leq b$. Finally, using $B + L = D$, the budget constraint of the public sector, $T_{t+1} + L_{t+1} = (1 - RL_t)L_t$ and assumptions 1-i) to 1-iv) we get:

$$\bar{D} \leq min\{(b + (1 + R_{L,max})\bar{L})\beta; \bar{L} + w_{min}\}$$

Finally, we must bound R_{PF}. Under assumption 1-v) u_1 is bounded above. The arguments in Duffie, et. al (1994, section 3) imply that $0 < \underline{c_1} \leq c_{1,t}$. Then,

$$\frac{\partial u_2}{\partial c_2} \in \left[0, -\left(\underline{c_1} - b\right)^{a-1}\right]$$

Then, using equation 4.3 for interior solutions we obtain:

$$\frac{u_1^{'}(c_{1,t})}{1 + R_{PF,t}} = E_{(.)}[u_1^{'}(c_{1,t+1})]$$

As marginal utility can be 0, it is possible for $R_{PF,t}$ to be unbounded. However, as the arguments above imply that $R_{A,t}$ is bounded for any t assumption 1-i) imply that the bank will always choose a finite passive rate. Of course, this argument holds for interior solutions as a corner in the problem of the type 1 household implies a trivial intermediation problem in equilibrium with $B = 0$ under assumption 1-iv).

■