

# Memory, Collateral and Emerging Market Crisis

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# Research Question

Is there a connection between the long and short run in a small open economy exposed to BOP crises?

We need a stochastic ergodic steady state

## Message

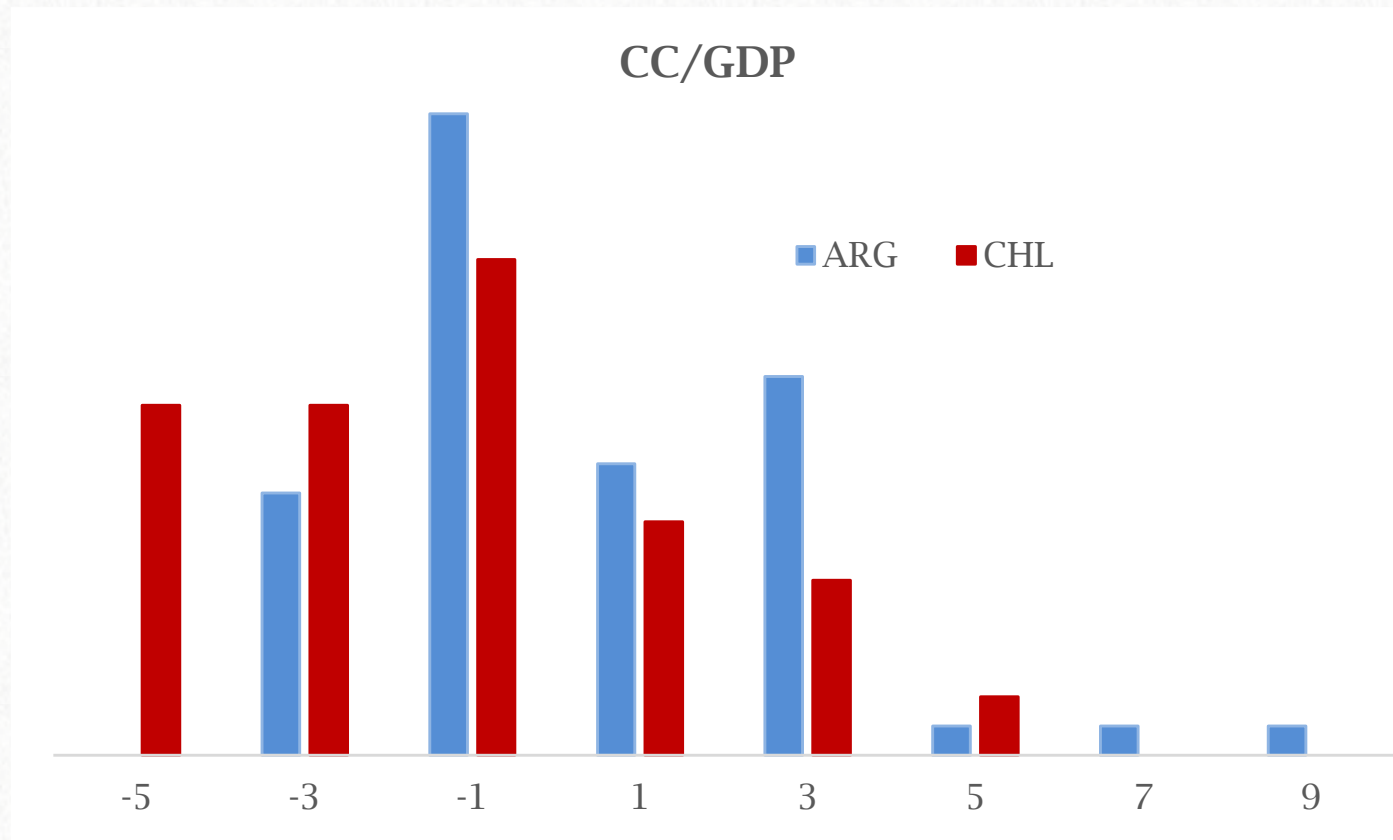
Using different equilibrium concepts, you can match distinct stylized facts with the same model. All are useful, but you can safely estimate one of them.

We need different types of computable equilibria

# Motivation

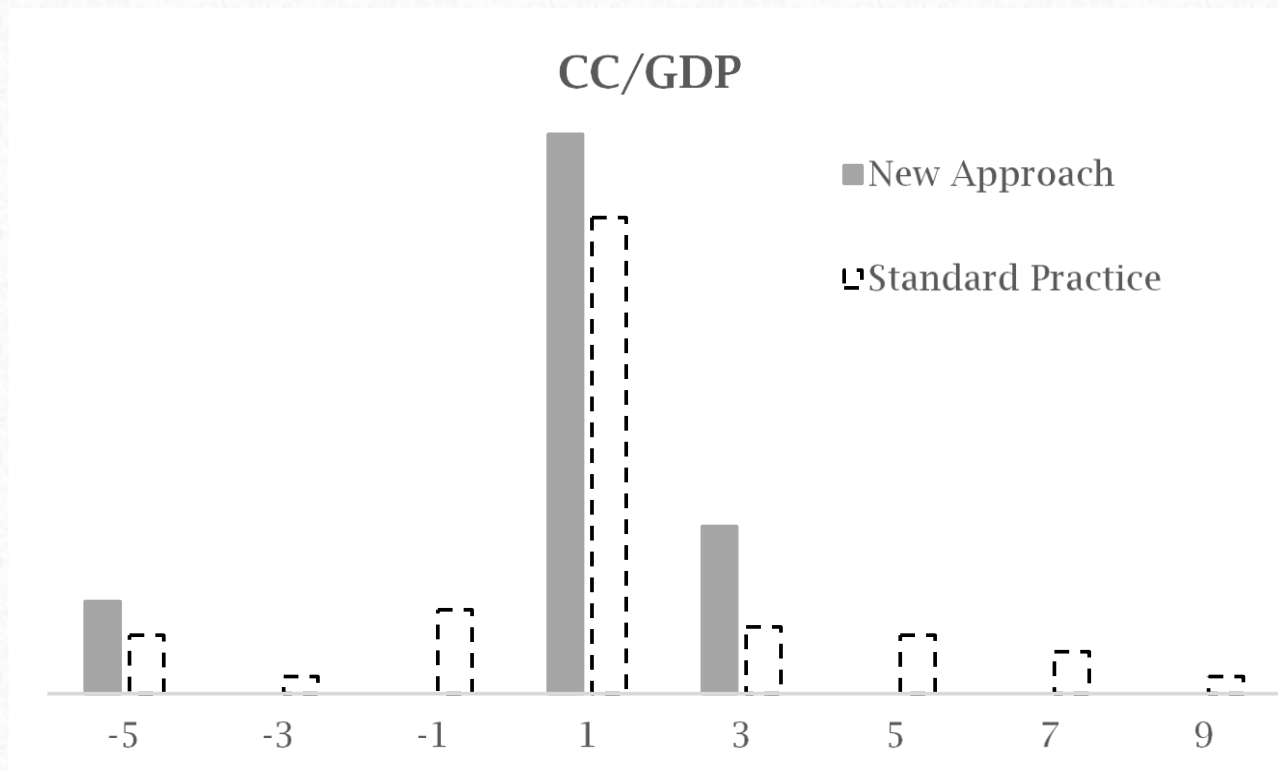
## Borrowing and Crises 1961-2016

- CHL: more borrowing, tilted to the left
- ARG: stronger crises, heavy right tails

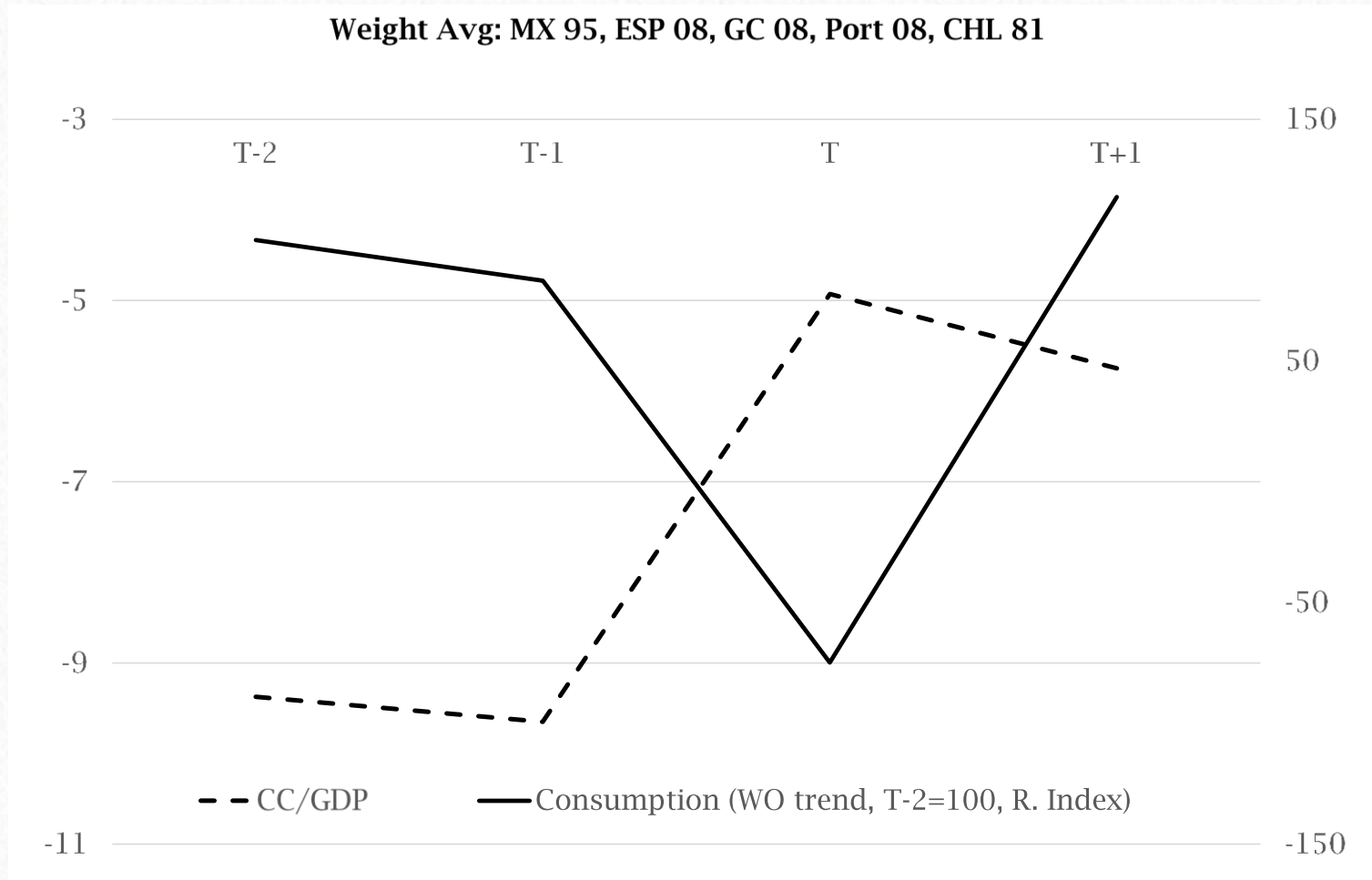


# Different equilibrium, same parameters

- New Approach: a stochastic steady state generates stable paths and more borrowing: frequency  $\times$  CC/GDP smaller (Avg CC/GDP).
- Standard Practice: stationary equilibria may contain unstable paths, which generate extreme values



# Anatomy of a Sudden Stop



# Answers and challenges

- Due to the disruptive nature of a crises and multiple equilibria (SG-U, 2020), we may lose continuity (Kubler and Schmedders, 2002).
- Standard tools used to show the existence of a steady state (SLP, chapters 7 to 12) don't work. You can be lucky, but you miss deep insights.
- We show that it is possible to:
  - replicate the anatomy of a crises and
  - match (long run) stylized facts with a stochastic steady state.
- Are they connected?
  - **YES!!**

# Contributions

- We take the 2 ruling approaches in the BOP crises literature (Bianchi 2011 and SG-U 2020) and get the best out of them:
  - A computable recursive equilibrium (Biachi)
  - A flexible sequential characterization (SG-U)
- A toolkit to deal globally with the short and the long run in models of BOP crises and financial accelerators.
  - A new equilibrium concept which:
    - Generate a stable distribution
    - Is computed globally and efficiently
    - Can be estimated safely.
    - Captures the anatomy of the crises.

# The Model

- We apply our results to a workhorse model Bianchi (2011).
- Additional assumptions to ensure compactness.
- 2 goods: tradable and non-tradable
- 1 non-contingent real asset with fixed price
- 1 price: RER. An increase implies appreciation
- Endowments: non-tradable, fixed. Tradable, i.i.d



# The model

- The budget equation is given by:

$$c_t^T + p_t c_t^N + d_t = y_t^T + p_t y_t^N + \frac{d_{t+1}}{R} \quad (1)$$

- Distinctive feature in the literature: collateral

$$d_{t+1} \leq \kappa(p_t y_t^N + y_t^T) \quad (2)$$

- The objective function is

$$E_0[\sum_t \beta^t U(A(c_t^T, c_t^N))] \quad (3)$$

- Small open economy with a rep. agent who solves:

$$\text{Max}_{\{c_t^T, c_t^N, d_{t+1}\}} \quad (3) \text{ subject to } (1) + (2)$$

- Preferences are given by 2 nested components: a CRRA function  $U(\cdot)$ , a CES aggregator  $A(c^T, c^N)$ . Then,  $U(A(c^T, c^N))$  gives the instantaneous return.

# Equilibrium Definitions

- Sequential Competitive equilibrium (SCE):
  - Optimization: Max (3) s.t. (1) + (2)
  - Feasibility:
    - $c_t^N = y_t^N$
    - $c_t^T = y_t^T + d_{t+1}/R - d_t$
- SCE gives functions  $[c_t, d_{t+1}, p_t](y_0, \dots, y_t)$ , **FULL memory**.

# Equilibrium Definitions

- Minimal state space recursive equilibrium (MSSRE)
  - Optimization: pick  $c^T, c^N, d' \in G(y, d, Y, D; C^T)$  with

$$V(y, d, Y, D; C^T) = \text{Max } U(A(c^T, c^N)) + \beta E[V(y', d', Y', \Phi(Y, D; C^T); C^T)]$$

- Feasibility is the same wrt the SCE.
  - Rac. Exp.:  $C^T(Y, D) = c^T(y, d, Y, D)$  when  $y = Y, d = D$
- $G$  budget equation,  $\Phi = D'$  and  $C^T$  aggregate consumption.
- MSSRE gives functions  $[c, d', p](Y, D)$  with **NO memory**.

# Equilibrium Definitions

- Let  $m$  be the derivative of the value function of a MSSRE without imposing the rational expectations condition.
- $y, d, m$  belong to a compact set  $K$
- $p = \psi(c^T, c^N)$  characterizes intra-temporal optimality.

- Generalized Markov Equilibrium (GME) **New!!**

- Pick  $y', d', m' \in K$  with

$$\left[ U_{c^T} \left( y - d + \frac{d'}{R} \right) - \beta RE(m') \right] \left[ d' - \kappa \left( \psi \left( y - d + \frac{d'}{R}, y^N \right) y^N + y \right) \right] = 0$$

- Compute  $d$  then  $c, p = \psi(\cdot, \cdot), m$
  - Continue iterating backwards.
  - Feasibility is included in optimality.
- GME gives  $[c, d', p](Y, D, m)$  with **PARTIAL memory**.

# Equilibrium Selection

- **MSSRE** notions cannot replicate all the phases of the crises and match the long run.
- However, we show they generate computable and multiple self-fulfilling stationary rational expectations equilibrium.
- MSSRE is too restrictive: when we connect the RE with the SCE we are making an implicit assumption:

$$V'_d(Y', g(Y, D)) = U_{c^T}(y' - g(Y, D) + \mathbf{g}(g(Y, D))/R)$$

$$U_{c^T}(y - D + g(Y, D)/R) = \beta RE[U_{c^T}(y' - g(Y, D) + \mathbf{g}(g(Y, D))/R)]$$

- Where  $g$  is the PF for debt and  $V'_d$  is an envelope.

# Equilibrium Selection

- Picking  $m$  implies using  $D'$  as an additional state variable.
- If we allow for an **expanded state space** (> minimal):

$$U_{cT}(y - D + D'/R) = \beta RE[U_{cT}(y' - D' + \mathbf{h}(Y, D, D')/R)]$$

- We get more **flexibility and memory**.
- **Change the type of equilibria depending on the question.**
- Using this flexibility, we show that a canonical model:
  - **replicates the short and long run during an EMC**
  - **represents the interaction between the 2 of them.**

# What is new? GME

- We propose a Markov equilibrium notion, Generalized Markov Equilibria (GME), which:
  - is constructed directly from the SCE. We pick selections **(h)** and characterize them (ergodic, etc.)
  - captures a larger fraction of the SCE than the MSS.
  - GME Contains MSS.
  - has a larger state space.
  - it is computed backwards as in GE theory.
  - uses the primal, no multipliers, as in DGMM (94).
  - **have more “memory”, it captures more SCE.**
  - **is more “flexible”,  $h$  “looks” at the data.**

# What is new? Ergodic SS

- We show the existence of a stochastic steady state which can be characterized by stationary moments.
- These moments (mean, variance, etc.) can be approximated using a law of large number.
- The model can be tested using structural estimation methods safely. **This is new in BOP crises models!**
- This type of steady states is called **ergodic**.
  
- We show that there is 1 GME that is ergodic even if there are multiple discontinuous SCE/MSSRE/GME.
- **Ergodicity is a selection (h) mechanism.**
- **We show you how to find the ergodic selection h.**

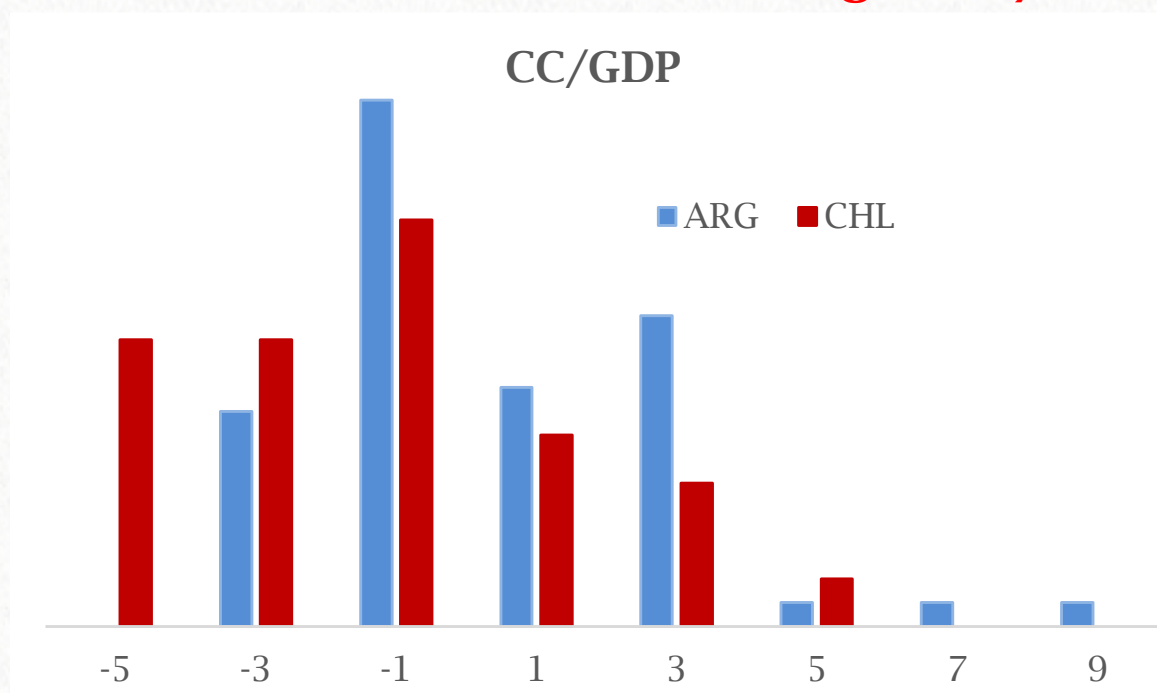


# What is new? Ergodic SS

- Going back to the initial figures ... simulations for a stationary and ergodic equilibrium are different quantitatively and qualitatively.
- Only the ergodic equilibrium can be estimated / calibrated safely using a law of large numbers.
- Ergodic paths are **stable**:
  - Orbit around a point, which gives stability
  - We get rid of unstable paths **which generate big crises** (heavy right tails in CC/GDP distribution)
- The steady state, called invariant measure, is:
  - **connected with the frequency of crises. The IM is constructed as the sum of probabilities of hitting a set *avoiding* a crisis.**
  - **More borrowing *defines* dynamic stability.**
  - **disciplines the short run dynamics.**

# What is new? Ergodic SS

- **Choose the equilibrium depending on data.**
  - **Argentina** has a heavy right tale; the country is not in a stable steady state. **MSSRE maybe suitable.**
  - **Chile** is less financially constrained. We solve the model and find that, with the same parameters, the variance of consumption in the ergodic equilibrium is 13% lower than the stationary. **Ergodic GME maybe suitable and estimated / calibrated globally.**



# How to compute a GME?

- The GME proceeds backwards.
- The 3 phases of the crises have  $t+2$  periods with  $\{\tilde{y}_0, \dots, \tilde{y}_{t+1}\} = \tilde{y}^{t+1}$ .
- We define states as  $k_s = [d_s, m_s, y_s]$ ,  $s = 0, \dots, t + 1$ .  $m_s$  is an envelope. Recover prices from intratemporal optimality and consumption from BC and feasibility:  $c_s^T(y_s, d_s, d_{s+1}), p_s(c_s)$ .
- Because the sequential equilibrium is compact,  $k_s \in K$ , any  $s$ .
- Starting from  $t+1$ , we can:
  - A) Fix  $k_{t+3}(y_{t+2})$  in  $K$  for each possible  $y_{t+2} \in Y$  (defines the selection and the thus the transition function)
  - B) For  $\tilde{y}_{t+1}$  choose  $k_{t+2}(\tilde{y}_{t+1})$  (typically binding collateral)
  - C) Compute  $k_{t+1}$
- Continue until  $t=0$  pairwise ( $k_+ \rightarrow k$ ). Link with envelopes: a MSS problem could be discontinuous / non-stationary but it is still concave, compact and Euler equations are necessary for optimality.

# How to get ergodicity?

- Get the MSSRE for the unconstrained problem (i.e., the collateral is not binding),  $d(d, y)$  when  $d = D, y = Y$ .

- Find a point “\*”, which gives stability and generate BOP crises, **short and long run are connected.**

- “\*” is the earliest possible hit to collateral with equality:

$$d(d_*, y_{lb}) = \kappa(y_{lb} + p[c(d_*, y_{lb})]y^N)$$

- **We can partition the state space wrt to  $d_*$  and characterize path which:**

- **Hit the collateral constraint at  $t = \tau$**

- **Have**

- $c_\tau > c(d_*, y_{lb})$  if  $d_\tau > d_*$  or

- $d_{\tau+1} > d(d_*, y_{lb})$  if  $d_\tau < d_*$

- **Generate a Sudden Stop by reverting to  $c(d_*, y_{lb})$**

- This is optimal as:

$$\begin{aligned} \beta RE(m'(d_*, y_{lb})) &= U_{c^T}(c(d_*, y_{lb}), y^N) > U_{c^T}(c_\tau, y^N) \\ &\geq \beta RE(m'(d_\tau, y_\tau)) \end{aligned}$$

# Characteristics of GME

- We compute:
  - an ergodic,
  - a stationary
  - and a non-stationary GME.
- The algorithm is efficient because it exploits the speed of the contained MSS equilibrium between crises.
- We found that the:
  - ergodic equilibrium has smoother consumption paths (i.e., is less financially constrained) wrt to the stationary / non-stationary equilibria and generates a crisis endogenously.
  - non-stationary equilibrium matches a wide range of macroeconomic crises.

# Characteristics of SCE and MSSRE

- We provide sufficient conditions to show existence of the SCE and MSSRE.
- These conditions imply compactness, non-homotheticity of preferences and bounded marginal utilities.
- For the MSSRE we derive a:
  - monotone 2 step operator. As there are 2 regimes for  $d_{t+1}$  wrt  $d$  (increasing when unconstrained, decreasing when constrained), we need 2 steps.
  - constructive existence proof
  - algorithmic procedure: time iteration based on the primal. Computes multiple stationary equilibria.
  - robust comparative statics.

# MSSRE, existence and computation

- The constructive existence proof contains a theoretically based initial condition and an updating rule for a convergent algorithm.
- A MSSRE replicates the spiralized recession of an EMC.
- **For the MSSRE we found 2 stationary equilibria:**
  - one with low/under borrowing and
  - one with high/over borrowing.
- **Under rational expectations these equilibria are self-fulfilling.** Coordination is based on observable variables (i.e., the initial condition), not on sunspots.

# Takeaway Points

- Choose the equilibrium based on observations:
  - Sustained borrowing suggest ergodicity. The ergodic distribution is constructed using mainly smooth consumption paths. The economy is in a stable steady state.
  - Otherwise, the traditional approach works.
- The ergodic distribution is defined by the frequency of crises (i.e., hits to the collateral).
  - We connect ergodicity with financial frictions.
  - In the steady state, more crises, less consumption smoothing.
  - Avg time spent in the unconstrained regime is smaller the more frequent are crises.
- The ergodic equilibrium can be estimated using structural estimation techniques.



# Future Research

- Our theoretical results apply to models with:
  - ***1 dynamic equation,***
  - inequality / equality constraints,
  - price dependent / independent constraints
  - representative agents
  - intra-temporal optimality conditions.
  
- We can handle economies with
  - **Non-contingent debt**
  - Labor (up to GHH preferences)
  - Collateral constraints
  - 2 sectors
  - Non-homothetic preferences

Thank you!!