# Memory, Collateral and Emerging Market Crisis

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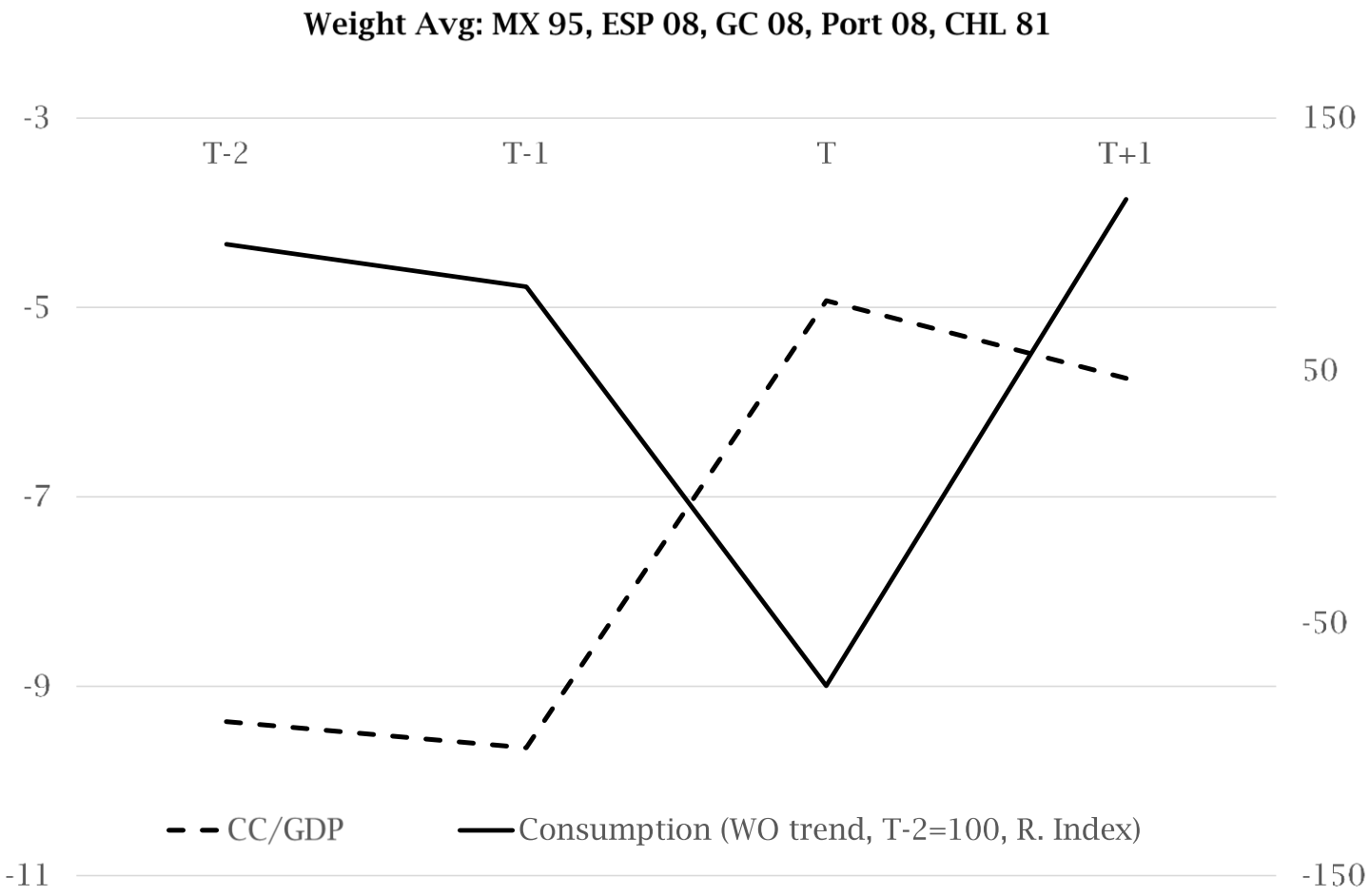
# Motivation

- How is the anatomy of a macroeconomic crises?
- Pre-phase, crises, recovery / spill over / stagnation
- Due to the possibly distinct behavior in the short and long run (Mendoza 2010), we need stationarity and ergodicity to test this difference
- Due to the disruptive nature of a crises, we may loose smoothness
- 1) Is the long run affected by the anatomy of the crises?

# Anatomy of the Crises



# Anatomy of the crises

- Pierri, Montes Rojas, Mira (2019): Pre-phase
    - Emerging economies can accumulate up to 25% of the GDP in external debt in 3-5

- Seoane and Yurdagul (2018): Post crises phase:
    - Heterogeneity after the crash

- **Evidence suggests that the time spell of the crises is more than 3 periods / years**

- Canonical models are silent wrt these facts.

- It is standard to identify the crises with shocks to the parameters or GPD; both exogenous.

- 2) Are global models capable to replicate an EMC?

# What is new?

- *We provide a positive answer to both questions using the workhorse model in the literature (Bianchi, 2011).*

- *1) We propose a Markov equilibrium notion, Generalized Markov Equilibria (GME), and prove that it has an ergodic steady state. Thus, we have a LLN to test the effects of the crises in the long run.*

- *2) we show that*

    - *The GME is capable of capturing the anatomy of the crises without changing neither the parameters nor support of exogenous shocks*
    - *A Minimal State Space (MSS) recursive equilibria replicates 1 of the 3 phases (crises)*

# How is this possible?

- *Crises?*
- *Macro models incorporate a **price dependent** collateral in order to capture the **spillover effects**.*
- There is a connection between the occasionally binding nature of the constraint, a crises and multiple equilibria; even in a MSS

- *Memory?*
- *The sequential equilibria incorporate memory requiring progressive measurability of equilibrium functions.*
- We can also think of a link between memory, phases of a crises and multiplicity inherited from the sequential equilibrium.

# How is this possible?

- That is, there are 2 sources of multiplicity.
- *The first, is the **cost** of capturing the **spillover effects** of a typical emerging market crises (i.e. on RER)*
- *The second, is the **cost** of **memory** which is relevant if we think that the pre-pase of a crises can help to explain the order of magnitude of the collapse and, maybe, the recovery / stagnation / spillover.*
- *Thus, bringing memory and collateral to the scene **calls for a selection mechanism whichs match data and preserves stationarity (MSS) and Ergodicity (GME).***

# What is new? Problems and solutions

- *We propose a convergent method for the Minimal State Space (MSS) recursive equilibria.*

- *This procedure is robust to the presence of multiplicity; at least the one related with the collateral constrains. Thus is capable of generating the "crises" phase.*

- ***This result implies a convergence theorem for state of the art algorithms in the literature***

    - The method is based on policy iterations approximating the consumption function using the primal (not the dual as it is standard)
    - As the operator induce an order topology, it allows for robust comparative statics
    - Allows us to assess the effects of economic policies directly on the decentralized equilibrium (No Ramsey!)

# What is new? Problems and solutions

- *We propose a novel equilibrium notion (GME) which is robust to any source of multiplicity.*

- *Using a GME:*

    - *We can match the different phases of a crises (short run)*
    - *It is possible to construct an ergodic selection (long run)*

- ***This result implies a calibration technique to test the effects of recurrent crises on the long run***

- *Argentina: 1961-2018, 22 out of 58 years of recession!!! (40%)*

- ***Bottom line: multiplicity is far from being a curse!!***

# Literature Review

- Theory (*in non-optimal economies):*
    - Minimal state space recursive equilibria: Datta, et. al. (2017), Mirman, et. al (2008)

*Existence Results for MSSRE (not in Mehra and Prescott, 1980)*

- Generalized Markov Equilibria: Feng, et. al. (2015), Duffie, et. al. (1994)

*Existence Results for GME*

- Simulations: Santos and Peralta Alva (2005), Pierri (2018)

*Accuracy of simulations*

- Stochastich Process (atom): Meyn and Tweedie (1993)

*Existence of a non-stochastic steady state*

# Literature Review

- Macro : 

    1) SG-U (2003), Chamberlain and Wilson (2000)
    2) SG-U (2016)
    3) SG-U (2018), Bianchi (2011)

- Macro : 

    4) Mendoza (1991)
    5) Mendoza and Smith (06), Kiyotaki and Moore (1998)

- Models 1-3 fits in our framework

- Models 4 -5 are outside the scope of this paper

- Why?

# Model Selection

- The class of models 1 to 3 has
  - ***1 dynamic equation,***
  - inequality / equality constraints,
  - market clearing conditions
  - intra-temporal optimality conditions.

- The class of models 4 and 5 has
  - ***2 dynamic equations***

- *2 assets, heterogeneous agents connected by market clearing conditions.*
- *It is difficult to keep the structural properties inherited from the optimization problem in the GE*

# Model Selection

- The theoretical results are presented for a model of the class 3.
- Why? It is the most general under the following "structure":

$$Max_{\{k_t^1\}} E_0 \sum_{t=0}^{\infty} u([k_t, k_{t+1}])$$

Subject to

$$g([k_t, k_{t+1}]) \geq 0,\ F([k_t, k_{t+1}]) = 0$$

- $k = [d, p, z]$ , k=[endogenous states, prices, exogenous states]
- $g([k_t, k_{t+1}])$ is the collateral constraint
- $F([k_t, k_{t+1}])$ contains feasibility and intra-temporal optimality

- The structure fits types 1 to 3 and form a class of models

- *Besides, it is the workhorse of the literature.*

# Sequential Equilibrium

- Skeleton of a financial crises in a SOE: a sudden stop.
- Additional assumptions wrt Bianchi (2011) and SG-U (2018).
- 2 goods: tradable and non-tradable
- 1 non-contingent real asset with fixed price
- 1 price: RER. An increase implies appreciation
- Endowments: non-tradable, fixed. Tradable, markov / i.i.d

$$c_t^T + p_t c_t^N + d_t = y_t^T + p_t y_t^N + \frac{d_{t+1}}{R}$$

# Sequential Equilibrium

- Distinctive feature in the literature: collateral

$$d_{t+1} \leq \kappa(p_t y_t^N + y_t^T)$$

- Intuition: debt to GDP has an upper bound, $\kappa$
- Problem: $\kappa$ is not observed. Callibration? Does it change?
- Distinctive feature of this paper: on preferences
- Besides additive separability across time and states (not across goods), we add ***uniform marginal utility*** (Braido 2013).
- Coupled with collateral, this preferences insure ***compactness of the sequential equilibrium. Critical for both types of RE.***

# Sequential Equilibrium

- Sufficient conditions for existence? Yes. As in Duffie, et. al. (1994), follows from Radner (1973) and induction.

- Compactness? Yes, almost everywhere, never uniform. We need:

    - CES + Stone Geary (upper bound) + Quasi-linear (bounded away from zero) type on instantaneous return function
    - Markov / iid
    - $\beta R < 1$.

- Given this structure, existence is a very ***weak requirement.***

- For iid and $\beta R \geq 1$, the structure of the problem brings Doob's theorem into the picture (super-martingale). We need satiation as in Sargent and Hansen (2009)

# Minimal State Space Equilibrium

- If we have a recursive equilibrium notion that insures short and long run match (GME), why should I care about a MSSRE?
    - Curse of dimensionality
    - Existence of a dual, connection sequential - recursive (envelope theorem)
    - Qualitative properties of the policy function (in the unconstrained regime)
    - Convergent algorithm based on a constructive existence proof (different from standard practice)
    - Robust comparative statics (policy in a non-Ramsey problem)

# Generalized Markov Equilibria

- In MSSRE we are going ***from a recursive to a sequential*** representation.
- We are restricting the possibility of capturing some paths that belong to this last type of equilibrium.
- The GME ***goes from the sequential to the recursive equilibria***
- The flexibility in the MSSRE have to came from the set of deep parameters and exogenous states
- The GME captures a bigger subset of the sequential equilibrium.
- As the distinctive feature of this 2 types of equilibrium is the amount of memory in each of them, ***we say that the GME is able to bring more memory into a computable framework***.

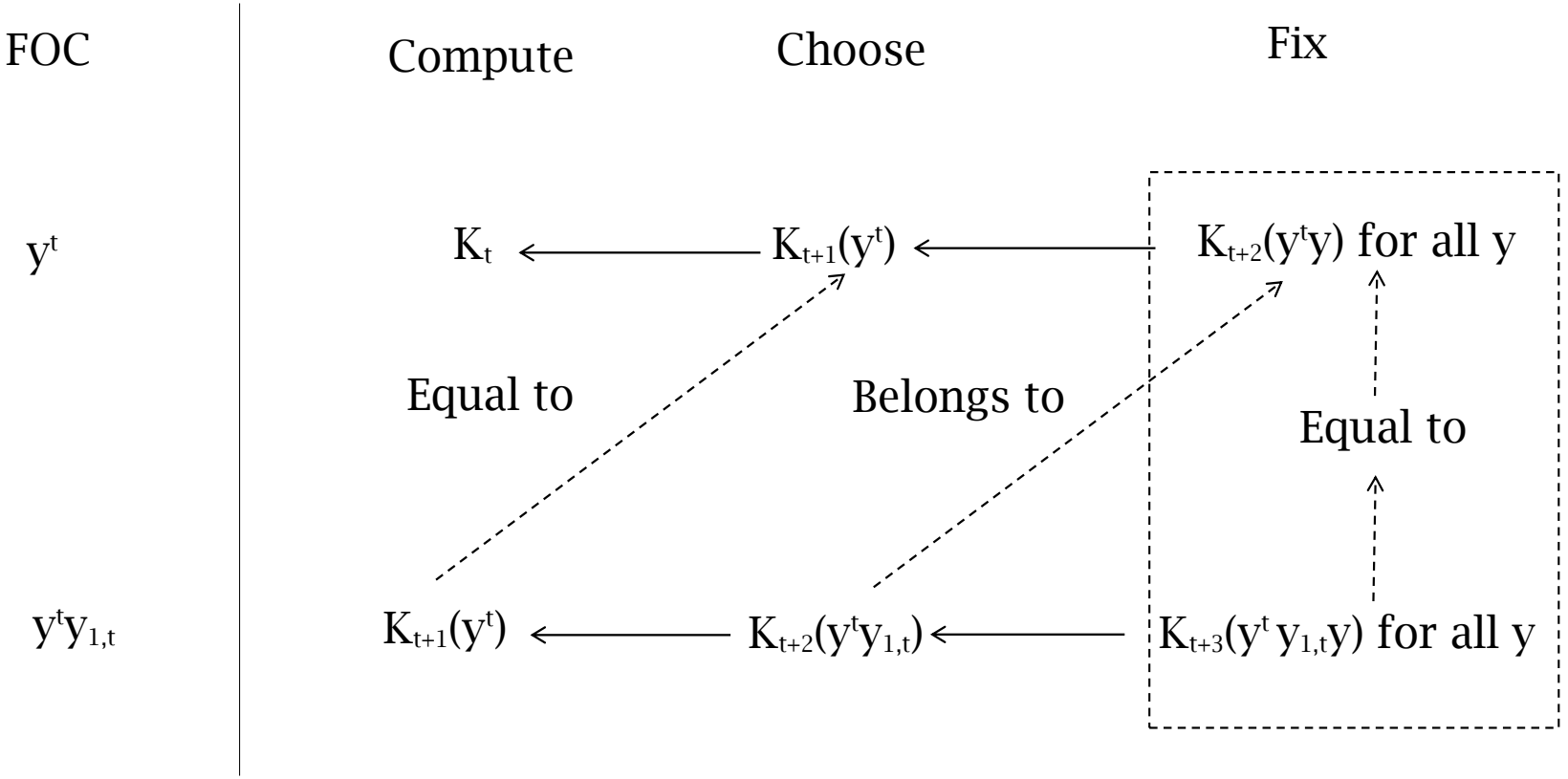
# Generalized Markov Equilibria

- The GME proceeds backwards.
- The 3 phases of the crises have t+2 periods with $\{\tilde{y}_0, \dots, \tilde{y}_{t+1}\} = \tilde{y}^{t+1}$.
- We define states as $k_s = [d_s, p_s, y_s]$, $s = 0, \dots, t+1$.
- Because the sequential equilibrium is compact, $k_s \in K$, any s.
- Euler equation + intra-temporal + feasibility implies restrictions on ***three states simultaneously***.
- Stating from t+1, we can:

  A) Fix $k_{t+3}(y_{t+2})$ in $K$ for each possible $y_{t+2} \in Y$ (defines the selection and the thus the transition function)
  B) For $\tilde{y}_{t+1}$ choose $k_{t+2}(\tilde{y}_{t+1})$ (typically binding collateral)
  C) Compute $k_{t+1}$

- Continue until t=0. If necessary $\infty > \#Y > t+1$ (flexibility!!)

# GME: Consistency



# GME, What do we gain?

- In a MSS given $(\tilde{y}_0, \tilde{d}_0)$, $k_{t+3}$ is pinned down by compositions of the policy function. ***In a GME, this is not determined yet !!***
- We can choose $k_{t+2}(\tilde{y}_{t+1})$ wisely. This is the "day" of the crises.
- The crises is identified with a hit of the collateral constraint.
- This permit us to pick $k_{t+1}(\tilde{y}_t)$ ***from a huge set*** formed by:
    - the intersection of any root in the collateral constraint (as in SG-U 2018)
    - the left had side of the Euler equation that holds with "≥"
- The paper shows that the 3 phases of a crises can be matched.

# GME, Stationarity and Ergodicity

- Restrictions coming from the picture above are weak (measurability is automatic, given compactness).
- We need to discipline the produce. We need a refinement.
- In order to capture the anatomy of the crises, we pick $k_{t+3}(y_{t+2})$ in $K$. This gives us ***stationarity (time independence).***
- We can define a stationary Markov process with state space given by $K$, which represent the sequential equilibrium.
- As we have a bigger state space, $[d, p, y]$ vs $[d, y]$ and the selection can be changed according to $\tilde{y}^{t+1}$, we have a bigger subset of the sequential equilibrium, thus ***more memory***.
- More important: ***we have used the right amount of memory !!***

# GME, Stationarity and Ergodicity

- We need an appropriate s.s.: an ***invariant measure***.
- The discontinuity of the selection is a challenge.
- There is a hope: an ***atom***. A point (a scalar) that is hit in finite time stating from any initial condition.
- What satisfies these properties? The collateral constraint.
- We have a big net: " $\geq$ " in the Euler equation.
- The paper shows that this point exists and is identified with a low but positive consumption level: ***a crises.***
- ***Bottom line: the crises generates a recurrence point and the process orbits around it.***
- ***The crises affects the long run behavior!!***

# Conclusions

- We claim that we can **trade** variability of the exogenous process (see for instance Seoane and Yurdagul, 2018) and of the parameter structure (see for instance Mendoza and Smith, 2006) for multiplicity and memory.

- A refinement mechanism disciplined by data (as for instance in Bhandari and McGrattan, 2019).

- Thus, we are constructing a parsimonious theoretical structure that represent the observed behavior without pressuring the deep parameters of the model.

- We are increasing the quality of the match!!

- We also have options and trade-offs for you:

    - Crises and numerical efficiency? MSSRE
    - Long run and short run? GME

Thank you!!

# Appendix and Back up Slides

# What is new? Problems and solutions

- Most papers in the “crises” literature have a quantitative flavor (Bianchi, (2011)).

- The results in this paper have a numerical / empirical implication:

  *by increasing the flexibility of the recursive equilibrium it is possible to borrow enough structure from the sequential equilibrium to help a parsimonious model to match the behavior observed in a macroeconomic crises and at the same time keep track of the long run.*

- Multiplicity permit us to "pick" the appropriate selection in order to match the observed paths without forcing neither the parameter nor the stochastic structure of the model.

  *we present a "flexible yet computable" recursive equilibrium notion based on the sequential equilibrium*

# To Do List: 60 minutes

- Motivation (10 minutes).
- Relation with the literature (5 minutes)
- Model selection (5 minutes)
- Model description (SCE) (10 minutes)
- MSSRE and implications (10 minutes)
- GME and implications (15 minutes)
- Concluding remarks (5 minutes)

# Model Selection

- We do not know if the theorems presented in this paper hold mutatis mutandis on all the models in "the suitable class" (i.e. in types 1,2,3).

- but they all have the adequate structure..

- The ***critical facts about these type of models*** *are 2*

    - It is easy to isolate the multiplicity associated with the collateral constraint and with that preserve some qualititative properties of the equilibrium which are essential to prove the existence of a MSSRE

    - Provided compacteness, the qualitative dynamics of the free problem are straigforward. If the collateral “hits frequently”, we can construct paths which are essential to insure ergodicity while keeping track of the crises.

# MSSRE - Optimization

- Frequently we observe papers using sequential and recursive dual optimization problems interchangeably (SG-U 2018, Bianchi 2011).

- We show that, provided compactness, they are all well-defined.

- The connection between ***primal and dual*** are guaranteed by Rincon Zapareto and Santos (2009).
    - Required because of inequality constraints.
    - Multipliers bounded and unique
    - Zero duality gap (value functions are well defined)

- The connection between ***recursive and sequential*** are guaranteed by envelope theorems for concave problems.

# MSSRE - Properties

- The structure of the problem imposes properties on the recursive equilibrium, some of them useful for the GME.
    - Curvature and smoothness of the return func. + continuity of the correspondence imply continuity of the PF over individual states.
    - In the unconstrained regime, because of market clearing, the intertemporal problem is independent of the RER
    - In the unconstrained regime, the RER is determined by intra-temporal optimality. It is increasing in tradable consumption.
    - It is a “partial” equilibrium problem with convex preferences as in Chamberlain and Wilson (1984) but with debt.
- These properties are essential to characterize the phases of the crises and to construct an ergodic markov equilibria.
- What happened with consumption, RER and debt in a recession before the crises?

# MSSRE - Algorithm

- The primal recursive MSS problem has the form:

$$V^*(d, y, Y, D; C^T) = Max_{c^T, c^N, d' \in G(s; C^T)} u(c^T, c^N) + \beta \int V^*(d', y', Y', \Phi(Y, D; C^T); C^T) \mu(dy')$$

  - $G(s; C^T)$ is the budget equation + collateral
  - $s = [d, y, Y, D]$
  - $d \ / \ D$ is the individual / aggregate endogenous state
  - $y \ / \ Y$ is the individual / aggregate exogenous state
  - $C^T$ is aggregate consumption which determine the RER
  - $\Phi$ is the law of motion of aggregate endogenous states

# MSSRE - Algorithm

- Due to the collateral, we have
  - Multiplicity
  - Different dynamics depending if we are constrained or not

- Multiplicity implies lack of smoothness.

- We need a non-smooth fixed point theorem. We draw from Tarski.

- Thus, we need monotonicity.

- Not possible in debt because of the 2 regimes: more debt implies less disposable income which implies more debt *only if not constrained*.

# MSSRE - Algorithm

- We ***construct a monotone 2 step operator in $C^T$ based on the Euler equation*** derived from the primal recursive problem.
- In order to insure monotonicity in $C^T$ the envelope is critical.
- Tarski typically have trivial fixed points.
- Due to bounded MU, it is problem.
- Thus, we build a 2 step method where the first step computes prices along the diagonal (i.e. $d = D$).
- ***The method is different from*** Rios Rull (1996), which the standard practice based on 1 step operator on $D$ using the primal recursive problem.
- Constructive existence implies convergence of the algorithm which avoids possible biases (see Frevenza, Martinez and Pierri, 2019).

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# MSSRE – Robust Comparative Statics

- We proved the existence of at least 1 non-trivial MSSRE.
- It may not be continuous. How do we assess the effects of alternative economic policies?
- Robust Comprative statics (Acemoglu and Jensen, 2015)
- We borrow from the Topkis family. We proved that in equilibrium consumption is:
    - Increasing in $\kappa$
    - Decreasing in $\beta$
- We can then model the effects of an ad-valorem tax on debt by modifying $\beta$ directly from the de-centralized equilibrium as it is done in practice.
- NO RAMSEY

# Sequential Equilibrium

- Result, $\beta R < 1$: ***if*** SCE exist and MU is bounded, then compactness.
- Bounds on $y^T$ and on MU "breaks" the curse of Chamberlain and Wilson (2000).
- Result, $\beta R \geq 1$: we need a different structure. We have to impose ***satiation***.
- Sargent and Hansen (2009). The long run "dances" around the satiation or "bliss" point.
- To our knowledge, this connection has never been explored
- ***We are pinning down debt in a stochastic steady state. We are going beyond SG-U (03) !!***