Persistent current account deficits and balance of payments crises*

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Abstract

Persistent current account deficits are common among low- and middle-income countries. We evaluate when this situation triggers a crisis and characterize it. Using a non-parametric estimator, we find a critical value for the yearly current account deficit just before the crisis sets off. These findings give rise to a different type of crises: countries that have increased their external indebtedness by an accumulated amount of at least 26%-31% of the GDP in a time span of 3 to 5 years, when they are hit by a crisis, they suffer a more severe recession than an average sudden stop. We call these events persistent balance of payment crises. This generates a consumption fall of 3.1% and a current account reversal 4.9 percentage points. We also contribute to the structural characterization of balance of payment crises. Using a canonical model augmented with an endogenous interest rate, given enough persistence of the GDP, we can replicate these stylized facts. In this sense, the variability of national income in countries which are external net debtors is critical.

Keywords: persistence, current account, debt, balance of payment crisis.

JEL codes: F32, F41

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1 Introduction

Persistent current account (CA) deficits are a recurrent phenomena in a large fraction of low- and middle-income economies. In a panel of 34 countries between 1970 and 2016 almost half of the sample contains observations with a deficit of at least 2% of the Gross Domestic Product (GDP) during five consecutive years. That is, CA deficits are persistent. Since Calvo (1998) we know that emerging markets are frequently affected by balance of payments crises. Thus, it is natural to ask: When is this situation not sustainable? Heuristically, we can say it happens when these deficits are followed by a crisis. But, *is there a critical value for the yearly CA deficit observed before it?* What are the determinants of this threshold? Are crises preceded by persistent CA deficit different with respect to an average sudden stop as defined for instance in Calvo et al. (2008)? These questions are policy relevant as they may help to design macro-prudential policies based on a rule that is simple to communicate and easy to detect.

The purpose of this paper is to provide positive answers to these questions. We find that there is a monotonic relationship between the yearly CA deficit (in terms of GDP) that precedes *some* balance of payment crises and its duration (3 to 5 years). These variables give rise to *new stylized facts* which are characterized by a threshold for the CA deficit, its persistence level, the level of accumulated debt and the type of crises which revert, at least partially, the observed deficit. That is, given the spell of the deficit, "crossing" the identified threshold may induce a balance of payment crisis. Moreover, the critical CA deficit is decreasing in its persistence and it ranges between 6.2% and 8.8% of the GDP. We also investigate the effects of accumulated borrowing on several economies. We found that there is a *fragile* set of countries, those that have increased their external indebtedness by at least 26% to 31% of the GDP in 3 to 5 years. This is the first stylized fact: *decreasing threshold, stable borrowing.* When these countries are hit by a systemic shock, they suffer a balance of payment crisis which is more severe than the average sudden stop according to Calvo et al. (2008); a fact that gives rise to the second stylized fact: *severity of the crises.* In particular, we found that the identified events have a contraction in consumption and a CA reversal that is at least twice the size of an average sudden stop. The distinctive characteristic of these events is the size of the accumulated deficit for different persistence levels. In this sense, the results in this paper provide a measure of external fragility in terms of yearly current account deficit and its persistence. To investigate the determinants of this type of events we derive a general equilibrium model subject to an endogenous interest rate and calibrate it using a new technique. We show that the persistence of tradable and non-tradable GDP together with the sensibility of interest rate spreads to debt levels can generate the new stylized facts presented in this paper.

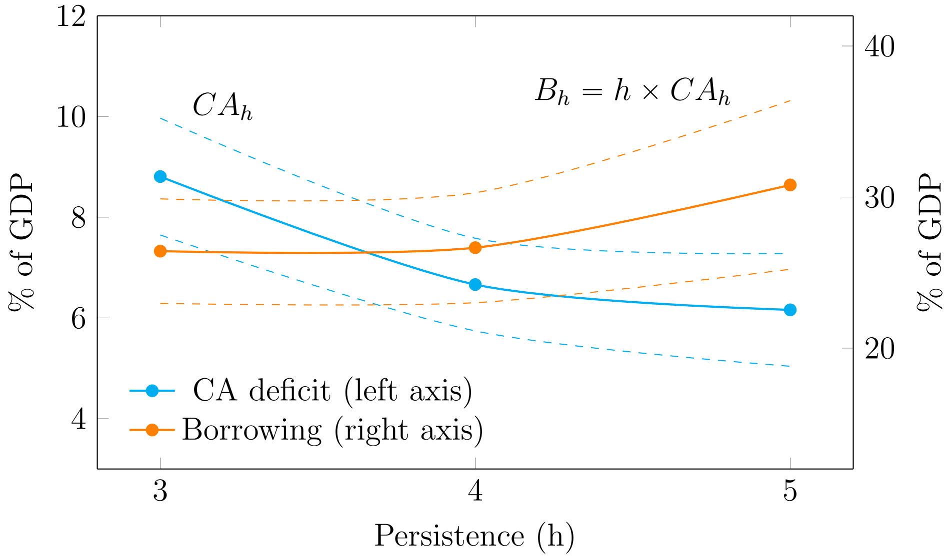
Table 1: Different type of Balance of Payment Crises

	$Acc.CA_2$	$Acc.CA_3$	$Acc.CA_4$	$Acc.CA_5$	ΔCA	Δc
This paper	15.5	26.4	26.6	30.8	4.9	−3.1%
Calvo et al. (2008)	7.2	11.0	14.3	17.5	1.8	0.5%

Notes: The values in Calvo et al. (2008) are constructed using their sample, ranging from 1990 to 2004, but restricting the countries to match the ones in our data base, ranging from 1960 to 2017. We then compute the simple average across the selected countries in both cases.

Table 1 summarizes the two stylized facts in our paper. Let $Acc.CA_h$ be the accumulated current account deficit in terms of the GDP during h consecutive years, ΔCA is the reversal of the CA to GDP ratio in terms of percentage points, and Δc is the growth rate of consumption at constant prices. The first type of variables define the pre-phase of a crisis and the last two variables the crisis itself. It is clear that a higher accumulated deficit previous to the crisis is associated with a more severe event, measured in terms of consumption and CA reversion. The table summarizes part of the first stylized fact (i.e., stable borrowing) and the second one (i.e., the severity of crises) but does not contain information about persistence and thresholds. To complement it, we present figure 1, which depicts a way of seeing the remaining part of the first stylized fact (i.e., decreasing thresholds). It depicts the monotonic relationship between the yearly CA deficit (in terms of GDP, CA in the left vertical axis) observed before a balance of payments crisis and its persistence (P, i.e. the spell of the CA deficit). In the right vertical axis, we plot the product of these two variables as a measure of accumulated external borrowing ($B(P) = P \times CA(P)$), also as a function of persistence. The depicted CA and B curves characterize the pre-pahse of a particular type of crises. There is a stable relationship between the rates at which the CA deficit decreases and its persistence increases, giving rise to a stable level of accumulated external indebtedness, between 26% and 31% of the GDP, which precedes the crises. Figure 1 together with table 1 allows us to characterize a special type of crises, which is preceded by persistent CA deficit and is more severe than an average sudden stop. We call this type of event *Persistent Balance of Payment Crises* (PBC).

Figure 1: Current account thresholds



Notes: Left vertical axis corresponds to yearly current account deficits. Right axis is accumulated current account deficits (borrowing). Both as percentage of GDP. The dashed lines represent ±1.96 standard error deviations with respect to the point estimates (calculated using wild bootstrap, see below).

The contribution of this paper to the literature is three-fold. First, we empirically identify and define a type of balance of payment crises using the persistence of large CA deficits (a generalization of one condition in the definition of sudden stop provided by Edwards, 2004). When a crisis hits the economies that satisfy these conditions (the treatment group), it produces a large drop in consumption, as compared with countries not having that condition (the control group) but also with other type of crises (i.e., an average sudden stops) which were identified using a different criteria, in particular not relying on the persistence of the CA deficit. Moreover, countries selected in the treatment group were all affected by major crises, including the global oil shocks in the 1970s, the Latin American debt crises of the 1980s, the Russian default in the 1990s and the subprime crisis in the 2000s. In this sense, this paper proposes novel stylized facts, that can be seen as refinement with respect to the systemic sudden stop literature as in Calvo et al. (2008). Contrarily to Forbes and Warnock (2012), Fratzscher (2012) or Cavallo et al. (2015), instead of looking at the capital account, we focus on the effects of the current account of the balance of payments. The observed drop in consumption and a CA reversal are also present in some definitions of a sudden stop (as in for instance Mendoza, 2010, and Bianchi, 2011). As a robustness check, we look for CA thresholds in the systemic sudden stop literature (see Calvo et al., 2008). We found that these thresholds are on average at a 1.0 standard deviation away from our estimated values.

Second, we contribute to the structural characterization of balance of payment crises. We propose a new calibration method based on the concept of "hitting times" (see e.g., Durret, 1996, p. 176). We study the behavior of accumulated CA deficits between crises. To match a persistent balance of payment crises we modify the canonical set-up due to Bianchi (2011) and add an endogenous interest rate, as in Garcia-Cicco et al. (2010), to it. That is, we incorporate an additional financial friction, reflecting the tension between debt levels and interest rates, to the standard collateral requirements in the literature which generates deleverage through the effects of exchange rates on tradable GDP. We identify these crises using different hitting times or, equivalently, by allowing a flexible number of periods without a crisis. As the model has a block recursive structure in the system of equations characterizing the sequential equilibrium between crises, we can easily identify the distinctive characteristic of those countries that have a positive match:

a highly persistent GDP, negative net external assets and enough variability is the market value of these assets. In particular, the norm of the matrix of coefficients in a standard VAR can be used to identify a positive match. Moreover, equipped with these results we can use the matched hitting time to calibrate the frequency of crises and consequently the tightness of borrowing limits using an endogenous interest rate. In this sense, this paper contributes decisively by improving the match of standard models to data; a quest that has been elusive in the literature (see Seoane and Yurdagul, 2019). We found that, if GDP is sufficiently persistent, the canonical model, without an endogenous interest rate, can replicate the dynamics of an economy that has suffered a PBC only for some persistence levels. However, the standard model fails to match a PBC with 5 years of persistence and the frequency of crises is too small compared with the literature. In order to improve the match, we add an endogenous interest rate. The intuition is straightforward: as these countries are net debtors and the interest rate is increasing in external debt, a CA deficit generates more indebtedness, especially through a reduction in National Income. Thus, we obtain not only a persistent CA deficit but also a significant level of external debt accumulated during the pre-phase of the crisis, and this happens frequently enough according to the standards in the literature (i.e., 5% as in Bianchi (2011)).

Finally, our results provide a useful guidance to policymakers. Balance of payment crises have shaken many countries, particularly emerging ones, for the last 40 years. From a macro-prudential policy perspective, our findings can help to elaborate leading indicators to anticipate crises (see Bianchi and Mendoza, 2018). Our policy implications, then, are crystal clear: fast accumulation of external debt is associated with a severe balance of payment crises, and so authorities must avoid reaching the 26%-31% of GDP threshold in the time span of 3 to 5 years.

This paper is organized as follows. Section 2 presents definitions and stylized facts. Section 3 contains the empirical strategy for identification and section 4 the empirical results. Section 5 shows the structural model and its estimation. Section 6 concludes.

2 Definitions and stylized facts

Our broad definition of crises includes episodes that involve the following characteristics simultaneously:

Definition: Persistent Balance of Payment Crises (PBC)

- (i) *Recession*: a sharp drop in (private) real consumption.
- (ii) *Persistent Current account deficit*: several years of consecutive current account deficits.
- (iii) *Reversion*: an acute correction of the current account after the sudden stop.

A PBC is related with the concept of sudden stop, which is originally due to Calvo (1998). However, there are many alternative definitions of sudden stops in the literature. These definitions can be classified in two types as being found in a) empirical papers; and b) theoretical (or structural) papers. The former type involves a relatively abnormal behavior of a particular episode when compared with similar countries and a pre-determined cut-off threshold given by a drop in capital inflows, see Calvo et al. (2004,2008), Edwards (2004). The latter is qualitative in nature and it omits any reference to the pre-phase of a balance of payment crises, see, for instance, Mendoza (2010) and Mendoza and Smith (2006). The empirical definitions refer to the dynamic behavior of the capital account of the balance of payments, while the theoretical types are based on certain path in the current account (CA). This difference is because most known models are based on real variables and, thus, they do not consider the role of international reserves.

Our definition stands in the middle of both types. This is because we want to perform a structural test which allows the order of magnitude involved in the event to be endogenously determined. We wish to connect the pre-phase of an event with a certain amount of accumulated current account deficit. Clause (ii) of the PBC definition above will give rise to a threshold, which determines if a certain country is prone, according to our data set, to a crisis. To recover the episodes from data, we follow a difference-in-differences strategy, which borrows from the relative nature of the empirical definition, but we extract from it a stylized fact that can be tested using a model in the small open economy family. Thus, our definition involves the behavior of the CA and it is more restrictive than any definition in the empirical or the theoretical class as it is concerned with elements of both types of existing definitions. On one hand, our definition will 'pick' a smaller subset of events when compared with the empirical literature. On the other hand, we will test for more than one country, as it is frequent in the theoretical literature, but with a parsimonious set of parameters; a fact that allows us to structurally test the presence of a PBC.

Instead of imposing an *ex-ante* criterion for the pre-phase or the PBC, we allow for multiple thresholds - persistence pairs as measured by the combination of a minimum level of CA deficit and the number of consecutive years which the actual figure was at or above that mark. For each threshold, the countries matching these criteria are placed in the "treatment group"; those that do not are in the "control group". We then draw a curve by mapping the difference between the change in consumption in the treatment group relative to the change in the control group (i.e., a d-in-d approach) against the yearly CA deficit in the threshold. The intuition is that if the level of accumulated deficit is associated with the a crisis, then this should be empirically observed in terms of the consumption dynamics: a drastic reduction in consumption, after averaging out countries' idiosyncratic differences. We have a different curve for each persistence level. The PBC is then identified as the "biggest" recession, in relative terms, measured by the steepest tangent of this curve. This allows us to determine the borrowing amount that is associated with the maximum discrimination between the implied treatment and control groups. This procedure relates to a 'large volume of foreign capital' as it is mentioned in the definition of sudden stops of Edwards (2004). This empirical strategy is further described in the next Section 3 and gives rise to the following stylized facts.

Stylized Fact 1: Decreasing threshold, stable accumulated borrowing. The minimal CA deficit among the treated ($\gamma(h)$) is decreasing in the persistence level for h. The level of accumulated borrowing is stable around 26-31% of the GDP.

Stylized Fact 2: Severity of the crisis. A PBC is associated with a CA reversion of 3.5 percentage points and a consumption drop of 5.0% in the year after the crisis. A typical sudden stop generates a CA reversal of 2.2 percentage points and a consumption drop of 0.6%.

Our structural estimation method in Section 5 consists in applying a small open economy model similar to the one in Bianchi (2011), augmented by an endogenous interest rate, in order to replicate a PBC. A balance of payment crisis occurs with a certain frequency, typically set to 5% (see Bianchi (2011)). Thus, we cannot test the presence of a sufficient condition for the occurrence of this type of crises. However, once we have identified the treatment group, we can compute the "size" of accumulated borrowing using the threshold for the current account deficit and its persistence. As the economies in the treatment group are net borrowers, an endogenous interest rate generate the required level of accumulated external debt due to the presence of negative income effects.

3 Empirical identification strategy

Our goal is to identify a PBC by evaluating a measure of accumulated borrowing that is associated with the expected consequences of this type of event. The empirical strategy is to link a persistent CA deficit with a discrete and significant fall in consumption. More specifically, we are interested in finding a clear discontinuity in the relative dynamics of consumption growth depending on the CA deficit level for different persistence levels.

The identification strategy has two steps. In the first step, we consider different values of CA deficits and persistence. A threshold is defined as the level of CA deficit of maximal separation between countries with that level of CA deficit or more vs. those with less CA deficit, in terms of changes in consumption. The intuition is that this magnitude and persistence of CA deficits are associated with a crisis. This is in fact a difference-in-differences (d-in-d) estimator, where the treatment corresponds to the CA deficit accumulation criterion. In the second step, for the group of countries that are at or above the first step threshold amount, we evaluate the average performance of those countries that also satisfy with conditions (i) and (iii) of PBC given above (i.e., recession and CA reversal).

Key to our identification strategy is that we define the PBC event not by evaluating its consequences (recession and CA reversal) but by using the data to look for a clear separation between countries below and above the endogenous threshold.

In sum, the empirical identification strategy below follows these steps:

- *First step: Accumulated CA deficit.* We find the value of the threshold, γ below, that maximizes the differences in consumption fall between the two groups, where the groups are constructed according to the level of accumulated CA deficits for different persistence levels, h.

- *Second step: Crisis.* Among the treated, we study those countries that satisfy (i) and (iii) of the PBC definition above for the threshold γ defined in the first step.

3.1 First step: An empirical measure of accumulated borrowing

The hypothesis here is that countries face a PBC when persistent CA deficits cannot be sustained. The identification of an event would thus correspond to different combinations of persistence in CA deficits. We are aware that the persistent behavior of the CA could be associated with a rigid fiscal deficit, an investment boom, or even a combination of the two; just to mention a few examples. In any case, the duration, and the order of magnitude of the external imbalance would be the same, which suggest that the accumulation rate of external debt may be at the center of the scene.

Specifically, a *threshold* is determined by the level of CA deficit per year during all the time periods involved in the pre-phase of the PBC event, a time window that we call *persistence*. The hypothesis is that this threshold will be observed before a given drop in consumption and a certain CA reversal. The countries in the *treatment group* have at least the threshold CA deficit during the pre-phase of a PBC. The *control group* will be defined by the complement, that is, they have less CA deficit.

Let CA_{it} be the current account of country i at time t, as a percentage of GDP. We argue that the PBC is associated with the magnitude of CA_{it} and its persistence. We consider the treatment vs. control mean effect for a given combination of (γ, h), where γ represents the CA threshold, i.e. $CA_{it} \leqslant -\gamma$ and h is the number of periods for which the CA deficit was at or above that given value (i.e. the persistence of the deficit). Define $H_{it}(\gamma, h)$ as an indicator function that takes the value 1 if the following condition is satisfied:

country i had experienced $-\gamma$ CA deficit for h years prior and including year t (i.e., $t-h+1,\dots,t-1,t$); 0 otherwise.

Define thus:

$$\mu_1(\gamma,h) = \mathbb{E}\left[\Delta ln(c_{i,t+1}) \mid H_{it}(\gamma,h)=1\right] - \mathbb{E}\left[\Delta ln(c_{i,t+1}) \mid H_{it}(\gamma,h)=0\right], \tag{1}$$

as the average difference between the treatment and control groups in consumption growth $\Delta ln(c_{i,t+1})$, where c corresponds to total consumption in real terms. Here we use the notation $\mathbb{E}$ for the empirical expectation or average. Note that the expectation is taken with respect to all the countries without further conditioning. We are thus concerned with a significant drop in consumption that will be associated with this accumulated CA deficit. The above equation thus defines a non-parametric d-in-d estimator.

Note that this strategy is designed to capture the typical event for an "average" country. Thus, the threshold may not be accurate to describe specific cases. In other words, a PBC is also related to a myriad of country specific components but our identification procedure consists in averaging out this specificity, keeping instead the mean effect over those events that satisfy the required conditions in the treatment group.

The first step in our identification strategy relies on finding a discontinuity in the average change in relative consumption at a value of γ for different fixed values of h, years accumulated of CA deficits. We assume that $\gamma \in \Gamma$ where Γ is a compact set. To avoid being deceived by mere visual inspection, we identify it by looking at the maximum numerical derivatives. More formally define,

$$\gamma^*(h,\epsilon) = \underset{\gamma\in\Gamma}{\text{argmax}}\, \mu_1'(\gamma,h,\varepsilon), \tag{2}$$

where

$$\mu_1'(\gamma,h,\varepsilon) \equiv \frac{\mu_1(\gamma,h) - \mu_1(\gamma-\varepsilon,h)}{\varepsilon} \tag{3}$$

is the numerical derivative or slope of $\mu_1(\gamma,h,\varepsilon)$. $\gamma^*(h,\epsilon)$, the CA deficit level for h years, is thus the main estimator in the paper. Then, $\gamma^*(h,\epsilon)\times h$ is the *empirical measure of accumulated borrowing.*

3.2 Second step: Characterizing the PBC

For the measure of accumulated borrowing found in the first step, we evaluate countries according to conditions (i) and (iii) of the PBC definition above. If a PBC occurs, we expect that for those countries involved consumption drops and CA deficit reverses. Therefore, we further characterize PBC by its empirical effects:

$$\mu_2(\gamma,h) = \mathbb{E}\left[\Delta ln(c_{i,t+1}) \mid H_{it}(\gamma,h)=1, \Delta ln(c_{i,t+1})<0, \Delta(CA_{i,t+1})>0\right], \tag{4}$$

and

$$\mu_3(\gamma,h) = \mathbb{E}\left[\Delta(CA_{i,t+1}) \mid H_{it}(\gamma,h)=1, \Delta ln(c_{i,t+1})<0, \Delta(CA_{i,t+1})>0\right]. \tag{5}$$

These two are the main PBC characterizations. $\mu_2(\gamma^*(h,\epsilon),h)$ is a measure of PBC recession, a fall in consumption. $\mu_3(\gamma^*(h,\epsilon),h)$ is a measure of PBC CA reversal. The pairs country-year (i,t) that satisfy the criteria $\{H_{it}(\gamma^*(h,\epsilon),h) = 1 \;\&\; \Delta ln(c_{i,t+1}) < 0 \;\&\; \Delta(CA_{i,t+1}) > 0\}$ are defined as the PBC episodes. The episodes could be consecutive years of the same country, an issue that is discussed below to avoid countries double counting.

4 Empirical estimation

4.1 Data

We construct a panel of 34 countries for which reliable data are available for the period 1970-2016, with annual frequency. The variables collected include CA in (current) dollars and percentage of GDP and consumption, GDP, and investment in (2010 constant) dollars. We also performed an estimation of the tradable and non-tradable categorization by grouping the value added of agricultural and manufacturing sectors (tradables), and construction and service sectors (non-tradables).

The sources are the World Bank Database; the Economic Commission for Latin America and the Caribbean Statistics; the National Accounts Section of the United Nations Statistics Division. In order to complete the database, we also asked for specific data to several national statistical institutes.

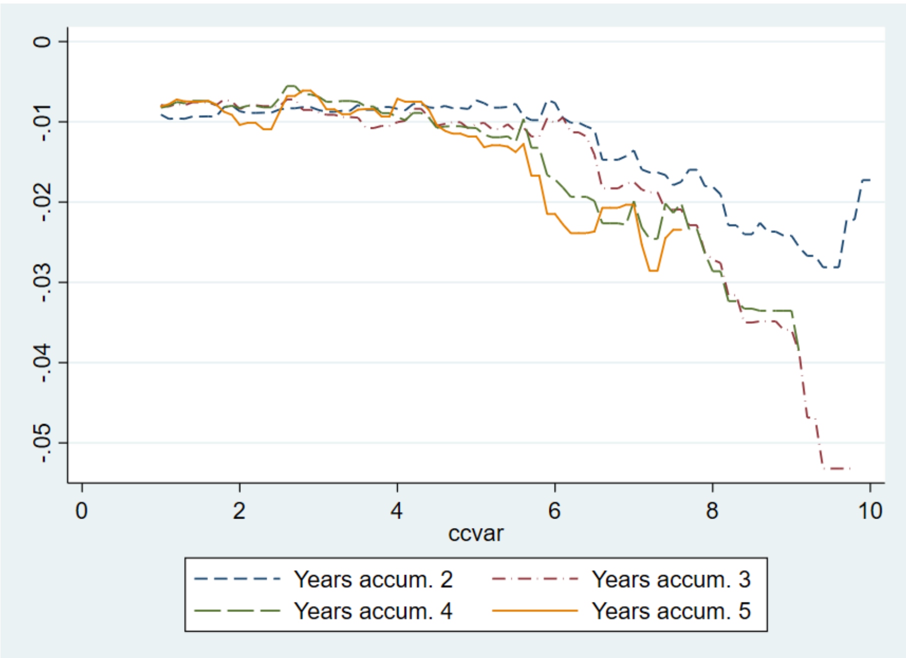
We cover fewer countries but for a longer lapse of time when compared with Edwards (2004) (i.e., 157 countries between 1970-2001) but more countries during a longer period when compared with Calvo et al. (2004) (i.e., 32 countries between 1990-2001).

The database is available from the authors upon request.

4.2 First step estimation

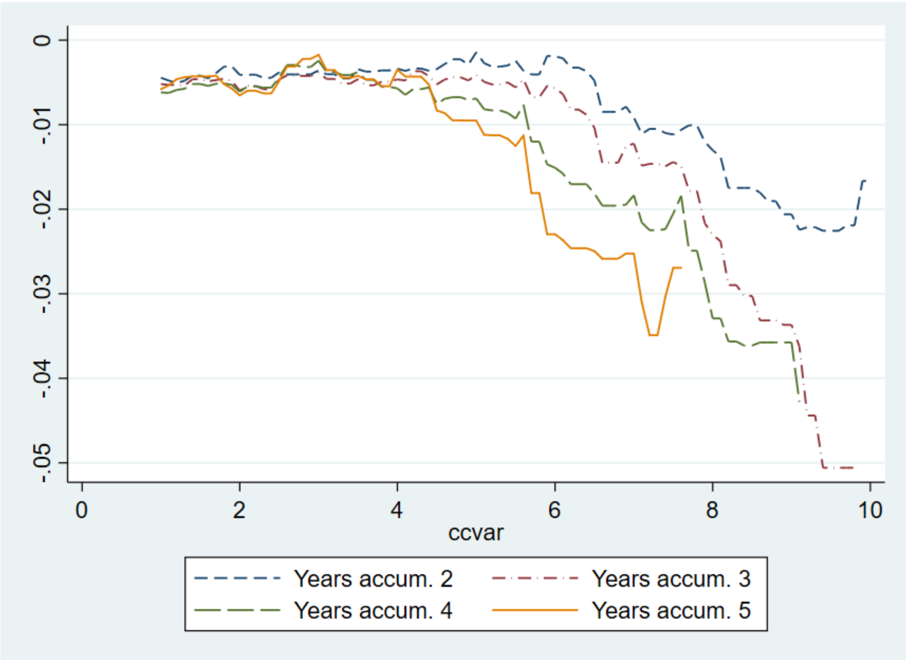
Figure 2 plots the d-in-d consumption value in eq. (1) for every pair of (γ, h), where we consider a grid of values with $h = 2, \ldots, 5$ and $\gamma \in \Gamma = \{1, 1.1, ..., 10\}$. The data in the figure is the main input for the subsequent analysis. By looking at the figure, we can informally spot the corresponding CA deficits associated with the PBC. For instance, if we take 5 years of persistence ($h = 5$, see the solid line), there is a jump in the figure at about 6% CA deficit. While we are specifically interested in consumption behavior to characterize the crisis, the PBC also affects the whole economy since the same patterns are also observed for GDP and investment, see Figures 3 and 4.

Figure 2: First step difference-in-differences estimator, consumption



Notes: The figure reports the $\mu_1(\gamma, h)$ estimates in (1) for $h = 2, 3, 4, 5$ and γ (ccvar) in $\Gamma = \{1, 1.1, ..., 10\}$.

Figure 3: First step difference-in-differences estimator, GDP

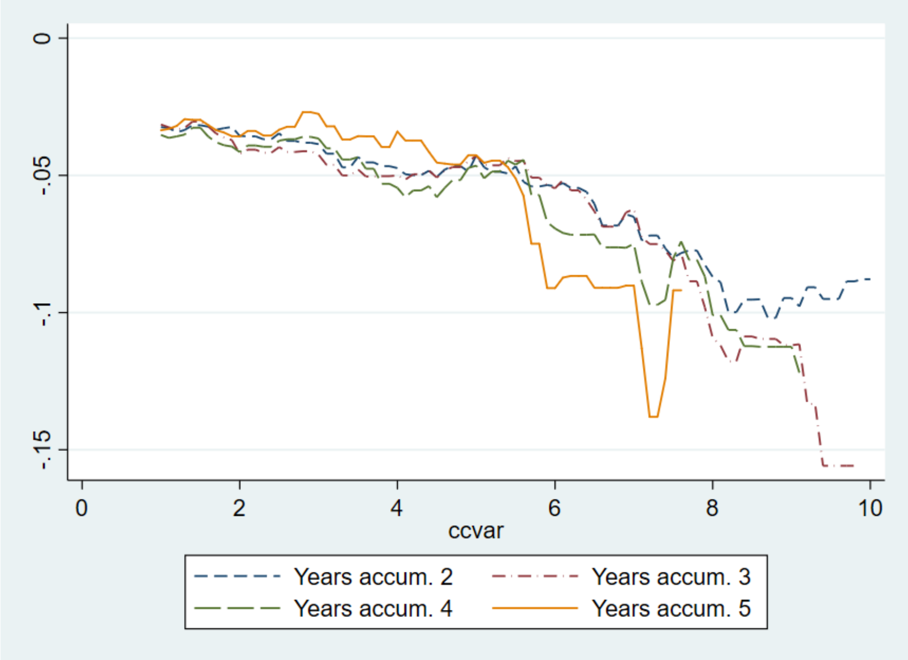


Notes: The figure reports the $\mu_1(\gamma, h)$ estimates (using GDP instead of consumption) in (1) for $h = 2, 3, 4, 5$ and γ (ccvar) in $\Gamma = \{1, 1.1, ..., 10\}$.

Consider now the estimation of the γ threshold as described in Section 3.1, eq. (2). That is, we will consider the CA deficits accumulated for up to 5 years, and we will consider CA levels each year with values in Γ, that contains up to 10% of GDP. Our interest lies in finding the threshold γ for which this difference in relative consumption has a clear discontinuity for every h. The assumption is that this discontinuity is only due to the PBC crisis. Although strong, this hypothesis allows us to keep the set of variables involved in the empirical identification strategy simple, a fact that will be crucial for the structural procedure in subsequent sections. We take $\varepsilon \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ for the numerical derivatives or slopes.

Tables 12-15 in the Appendix reports the estimated maximum slopes, together with the associated γ, for $h = 2, 3, 4, 5$, respectively. All reported

Figure 4: First step difference-in-differences estimator, investment



Notes: The figure reports the $\mu_1(\gamma, h)$ estimates (using investment instead of consumption) in (1) for $h = 2, 3, 4, 5$ and γ (ccvar) in $\Gamma = \{1, 1.1, ..., 10\}$.

slopes are negative, thus we refer to the maximum absolute value of slopes. To evaluate the robustness of our strategy we compute the three maximum slopes ordered by rank, i.e., the first, second and third biggest values of the slopes. We also compute the standard errors associated with the three values using wild bootstrap. For the bootstrap procedure we first construct a dataset containing for each country-year pair (i, t) the values of $\Delta ln(c_{i,t+1})$ together with all the information about $H_{it}(\gamma, h)$ for $h = 2, \ldots, 5$ and $\gamma \in \Gamma = \{1, 1.1, ..., 10\}$. Next, we perform wild bootstrap on this sample, and estimate γ using the same numerical slope estimator.

The PBC level of accumulated borrowing should be search for within those values. Table 2 contains the weighted average threshold given by

$$\sum_{\varepsilon, rank} \gamma^*(h, \varepsilon, rank)\omega(\varepsilon, rank), \tag{6}$$

where $\gamma^*(h, \varepsilon, rank)$ is the maximum empirical derivative, ordered by position or rank $= 1, 2, 3,$[1] for a given h and ε, and $w(\varepsilon, rank)$ weights accordingly to the number of episodes (which will depend on the second step estimation that follows). In particular, the weight $\omega(\varepsilon, rank)$ is constructed as the number of cases in the treatment group for a particular pair $(\varepsilon, rank)$, that also satisfy the second step criteria below (fall in consumption, CA reversal) over the total number of events for a given persistence level. The table finds levels of accumulated borrowing that are in line with the informal visual inspection in Figure 2. For instance, a 4-year persistence ($h = 4$) has an estimate of γ value of 6.7, such that $6.7 \times 4 = 26.6$. This can be interpreted as the fact that a country that borrowed an accumulated 26.6% of GDP in 4 years is a typical candidate for an PBC event. If we use a 5-year persistence ($h = 5$), it is associated with a γ value of 6.2, such that $6.2 \times 5 = 30.8$ is the associated threshold measure of accumulated borrowing.

[1] The order implies that $\mu_1'(\gamma^*(h, \varepsilon, 1), h, \varepsilon) < \mu_1'(\gamma^*(h, \varepsilon, 2), h, \varepsilon) < \mu_1'(\gamma^*(h, \varepsilon, 3), h, \varepsilon)$.

Table 2: Estimation results: Weighted average

h	γ	Borrowing ($\gamma \times h$)	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
2	7.74	15.49	-0.0516	3.88	15.12
	(0.75)	(1.45)	(0.00057)	(0.252)	(3.24)
3	8.81	26.42	-.05049	3.53	8.12
	(0.58)	(1.73)	(0.0006)	(0.123)	(1.34)
4	6.66	26.65	-0.0473	3.51	10.30
	(0.46)	(1.82)	(0.0014)	(0.173)	(1.55)
5	6.16	30.80	-0.0500	3.06	7.88
	(0.56)	(2.79)	(0.0036)	(0.126)	(3.97)

Notes: Estimation of average values given by eq. (6) using the results in Tables 12-15. Standard errors in parenthesis are calculated using wild bootstrap.

Table 5 contains the main empirical findings of the paper and gives rise to one of the mentioned stylized facts. Namely, the pre-phase of a balance of payment crises is not only relevant, but also has a stable structure captured by the amount of accumulated borrowing, which averages up to 30% of the GDP when we restrict the persistence levels to 3 to 5 years.

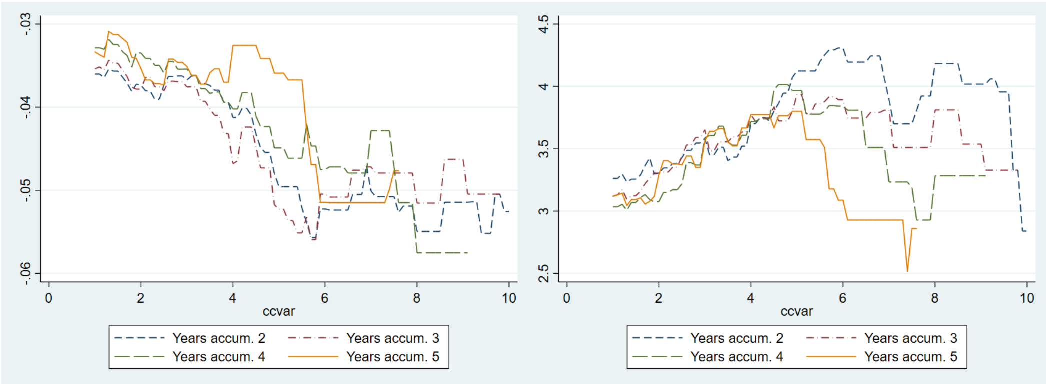
4.3 Second step estimation

Having identified the critical measure of accumulated borrowing, we consider now the second step estimation. Our interest lies in characterizing countries that experience PBC by finding the change in consumption and CA $\mu_2(\gamma^*, h)$ and $\mu_3(\gamma^*, h)$, from eqs. (4) and (5), respectively, after the γ^* estimation in the first step.

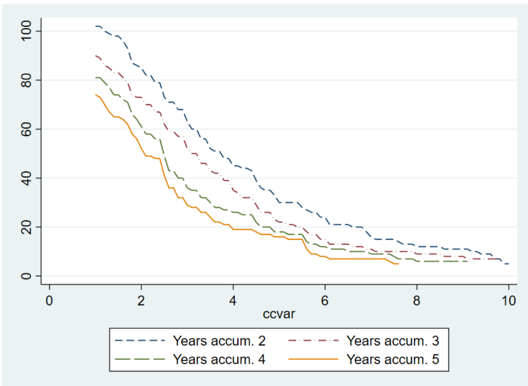
Figure 5 shows these results for every pair (γ, h). The figure also reports the number of episodes (country, year) that satisfy the corresponding criterion. As expected, as the magnitude and persistence of CA deficit increases the number of episodes decreases.

Figure 5: Second step: crises

(a) Fall in consumption (b) Current account reversal



(c) Number of episodes



Notes: The figures report the $\mu_2(\gamma, h)$ (fall in consumption), $\mu_3(\gamma, h)$ (current account reveral) and number of episodes estimates described in eqs. (4) and (5) for $h = 2, 3, 4, 5$ and γ (ccvar) in $\Gamma = \{1, 1.1, ..., 10\}$.

Tables 12-15 report the average values of $\mu_2(\gamma, h)$ and $\mu_3(\gamma, h)$ for all the potential episodes in terms of the maximum empirical derivative. These are given by columns Δlnc_{t+1} and ΔCA_{t+1}, respectively. The calculated weighted average for those parameters using the same formula as in eq. (6)

appears in Table 2. In general, most values coincide in terms of the magnitude of the fall in consumption and CA reversal. As noted in stylized fact 2, presented in section 2, for consumption fall, most estimates coincide in a 4-5% drop. For CA reversal (as percentage of GDP), the values are framed within 2.5-4.5%.

To further characterize a PBC, we consider the following. As noted, events can be overlapped with each other for the same country. That is, it is possible to observe an event in the treatment group for $h = 2, 3, 4, 5$, provided that consumption falls, and the CA deficit reduces. Thus, we keep the longest duration spell and compute the performance of those episodes.

Table 3 reports the characterization of typical PBC events, together with the average values. Note that there are no events for $h = 3$ that are not also encountered with $h = 4$, thus we do not report the value for this persistence value. We can identify 2 major debt crises that affected all the countries involved: the first one occurred in the late 70s-early 80s and the second one is due to the sub-prime crisis. Thus, in this sense, *persistent CA deficits* and *systemic sudden stops*, as in Calvo et al. (2008), are connected. We explore this connection in the next subsection.

Table 3: Second Step: PBC characterization

γ	h	$\Delta ln(c_{t+1})$	ΔCA_{t+1}	$\Delta ln(GDP_{t+1})$	$\Delta ln(I_{t+1})$	(i,t)
9.6	2	-1.0	7.4	0.3	-10.5	Israel 75
6.7	4	-3.9	6.9	5.3/-4.5*	-27.8	Panama 81
6.7	4	-8.9	6.0	-2.6	-8.9	Peru 77
6.7	4	-1.7	5.1	-3.6	-16.9	Spain 08
6.2	5	-5.9	5.4	-7.3	-27.8	Costa Rica 81
6.2	5	-0.8	3.6	-4.3	-13.9	Greece 08
6.2	5	1.1/-1.5*	2.9	-1.4	-12.1	Honduras 81
6.2	5	-1.2	1.7	-3.0	-7.6	Portugal 08
9.6	2	-1.0	7.4	0.3	-10.5	Average
6.7	4	-4.8	6.0	-3.6	-17.9	Average
6.2	5	-2.4	3.4	-4.0	-15.4	Average

Notes: * growth rate for period t instead of $t+1$.

4.4 Relationship with other events

PBCs can be linked to sudden stops. To serve this purpose, we will use the list of these last type of events contained in Calvo et al. (2004) and Calvo et al. (2008). According to our definition, PBCs require the presence of accumulated CA deficit with a different threshold (condition (ii)), followed by a consumption drop and a current account reversal in the year of the crisis (conditions (i) and (iii), respectively). Table 4 compares these conditions, represented by the variables $\gamma(h) \times h$ for $h = 2, 3, 4, 5$, ΔC and ΔCA, for the average event according to PBCs and SS. As can be seen, PBCs shows some differences with respect to sudden stops, including larger pre-crisis accumulated CA deficits, and larger post-crisis adjustments in consumption and current account.

Table 4: Persistent Balance of Payments Crises and Sudden Stops[1]

Event	N	$\gamma(2) \times h$	$\gamma(3) \times h$	$\gamma(4) \times h$	$\gamma(5) \times h$	ΔC	ΔCA
PBCs	8	-15.5	-26.4	-26.6	-30.8	-3.1	4.9
SSs(04)	13	-6.7	-10.5	-13.0	-15.6	-1.6	2.5
SSs(08)	23	-7.2	-11.0	-14.3	-17.5	0.5	1.8

1: $\gamma(h) \times h$ and ΔCA are expressed in percentage points with respect to GDP, ΔC in year over year growth rates. In the case of sudden stops, N refers to the quantity of events present in our database. The averages for the growth rate of consumption and current account reversals are computed for $\gamma = 2, 4, 5$ based on the events listed in table 3.

Table 5 reports selected cases from major systemic events. We apply to them our definition for the sake of comparison. The list of events includes a few examples from the Latin American Debt Crisis (early 80s), the Tequila Crisis (1994/5), and the Asian/Rusia Crisis (1998). The crises in 1994 and 1998 observed in Mexico and Ecuador respectively barely fail to meet the pre-phase criteria. Most of the events observed during the South American Debt Crisis did not show a CA reversal.

Table 5: Systemic Events: Selected Cases[1]

Event	$\gamma(2) \times h$	$\gamma(3) \times h$	$\gamma(4) \times h$	$\gamma(5) \times h$	ΔC	ΔCA	$\frac{\gamma-\gamma_i}{\sigma_e}$
Arg 81	-6.3	-6.6	-4.3	-2.5	-4.6	-0.9	1.7
Bra 81	-9.9	-14.6	-18.1	-20.9	4.2	-1.3	0.7
Chi 82	-22.8	-29.6	-35.1	-41.9	-5.1	3.6	0.1
Mex 94	-10.3	-17.0	-21.7	-24.6	-5.0	5.2	0.7
Arg 94	-7.7	-10.3	-10.7	-7.2	-3.6	2.3	2.0
Chi 98	-9.1	-13.1	-14.9	-17.7	7.1	-0.4	0.9
Ecu 98	-11.0	-11.2	-16.2	-21.0	-10.8	14.5	1.2
Arg 99	-8.7	-12.7	-15.1	-17.1	-0.5	1.0	0.9

1: $\gamma(h) \times h$ and ΔCA expressed in percentage points with respect to GDP, ΔC in year over year growth rates.

The last column provides a measure, $(\gamma - \gamma_i)/\sigma_e$, to gauge how far is each case from our threshold γ. The quantity is computed by finding the minimal yearly CA deficit for $h = 2, 3, 4, 5$ and then comparing this threshold γ_i to our estimated values γ. As in all these cases $\gamma_i < \gamma$, $(\gamma - \gamma_i)$ is positive. We choose the smallest value for this difference. We then adjust it by the standard deviation of the CA deficit to GDP (for the whole sample), σ_e. For instance, when $h = 5$ we identified a threshold of the CA of 6.2% of GDP (see Table 2). In the case of Argentina 1999, the minimal yearly CA deficit for this persistence is 2.0%. Adjusting this difference by the standard deviation (4.85%), we conclude that the threshold for that event lies 0.9 standard deviations away from our estimated threshold. The closest event to our estimations is Chile 1982.

4.5 Robustness checks

The empirical identification above relied on finding a clear discontinuity in terms of consumption drop, based on the level of accumulated CA deficits. This subsection explores alternative specifications to evaluate the robustness of the previous findings.

First, we compute the $\mu_1(\gamma, h)$ functions in eq. (1) using medians instead of averages,

$$\tilde{\mu}_1(\gamma, h) = \mathbb{M}\left[\Delta ln(c_{i,t+1}) \mid H_{it}(\gamma, h) = 1\right] - \mathbb{M}\left[\Delta ln(c_{i,t+1}) \mid H_{it}(\gamma, h) = 0\right], \tag{7}$$

where $\mathbb{M}$ denotes the median. The idea is that the results above could be driven by extreme values, thus when searching for a discontinuity this could be affected by outliers. The results in Table 6 are computed in the same way as those in Table 2 but using medians in the first step. The results are qualitatively very similar to those found for the difference in means. That is, they identify qualitatively similar levels of accumulated borrowing, and they lie within two standard errors of each other.

Table 6: Estimation results: Weighted average using medians in the first step

h	γ	Borrowing ($\gamma \times h$)	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
2	6.70	13.39	-0.0516	4.09	20.63
	(0.60)	(1.19)	(0.0013)	(0.174)	(4.64)
3	8.55	25.65	-.0492	3.48	8.87
	(0.57)	(1.71)	(0.0010)	(0.076)	(2.03)
4	6.81	27.26	-0.0497	3.59	11.20
	(0.71)	(1.82)	(0.0028)	(0.125)	(2.02)
5	5.06	25.32	-0.0403	3.42	15.29
	(0.55)	(2.77)	(0.0039)	(0.160)	(4.13)

Notes: Estimation of average values given by eq. (7). Standard errors in parenthesis are calculated using wild bootstrap.

Second, the first step strategy for identification described above used a criterion of maximum empirical derivatives or slopes. As an alternative measure, we consider formal econometric tests based on the theory of structural breaks. Consider a regression model of $\mu_1(\gamma, h)$ (calculated using averages) on a constant. Thus the idea is to search for an structural break in the intercept that occurs at a given value of γ, by considering the *sup* $Wald$ test of Hansen (1997).[2] In turn this corresponds to the maximum discrimination between the treatment and control group that defines $\mu_1(\gamma, h)$.

Table 7 presents the results of this second robustness exercise. We present the threshold corresponding the maximum Wald statistic value, and the two consecutive ordered values. Overall, the results show again a similar picture to that described in the empirical section. The findings suggest an accumulated borrowing measure of about 26-31% (i.e., $\gamma \times h$) that corresponds to a consumption drop of about 5% and a CA adjustment of 3-4%.

[2]Hansen (1997) tests is designed for a time-series model where the structural break occurs at a certain time. We are replacing time with γ.

Table 7: Sup Wald Tests

h	Rank^a	Wald	γ	Borrowing ($\gamma \times h$)	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
2	1	386.6	7.1	14.2	-0.0508	3.70	15
	2	376.8	7.2	14.4	-0.0508	3.70	15
	3	363.9	7.3	14.6	-0.0508	3.70	15
3	1	352.2	7.9	23.7	-0.0479	3.51	10
	2	347.0	8	24.0	-0.0515	3.81	9
	3	338.5	8.1	24.3	-0.0515	3.81	9
4	1	320.3	6.1	24.4	-0.0472	3.81	11
	2	317.1	6.0	24.0	-0.0475	3.84	12
	3	316.0	6.2	24.8	-0.04729	3.81	11
5	1	416.0	5.7	28.5	-0.0469	3.18	9
	2	394.4	5.8	29.1	-0.0469	3.18	9
	3	381.2	5.9	29.0	-0.0514	3.09	8

Notes: Hansen (1997) sup Wald test for structural breaks using a regression of $\mu_1(\gamma, h)$ on a constant and γ. ^{a}: rank corresponds to the ordered values of the Wald statistics.

5 Structural model estimation

This section describes a canonical and parsimonious structural model to test the presence of accumulated borrowing. We use the workhorse in the literature, a recursive version of the model in Bianchi (2011). The theoretical structure focuses only on the behavior of consumption and CA. Financial frictions are introduced by assuming incomplete capital markets and the presence of a collateral constraint. A crisis occurs when the level of debt hits the collateral constraint.[3]

[3]We are aware of the existence of more complex and realistic models to represent the findings in this paper. For instance, using a recursive equilibrium, Mendoza (2010) highlights the importance of investment. Along the same lines, Mendoza and Smith (2006) takes into account the interaction between two assets in order to match the order of magnitude of the recession generated by a sudden stop. On a more theoretical branch, Schmitt-Grohé and Uribe (2017) exploit the multiplicity of the sequential equilibria to generate more "variability" in the time series behavior of the simulated paths. However,

5.1 The canonical model

The model is the same as in Bianchi (2011), except in the characterization of the equilibrium, which is adjusted to improve the description of the calibration process. We will structurally test the presence of accumulated borrowing as characterized in condition (ii) of the definition of PBC in Section 2. Note that this type of events may involve more than 2 consecutive periods, as persistence can vary between 2 and 5 years. Thus, we will have to consider a history of shocks and not only the last of them as it is frequently done in the recursive literature.

We assume a small open endowment economy populated by an infinitely lived representative agent who consumes tradable and non-tradable goods, c^T and c^N, respectively. The relative price of non-tradables is denoted by p. The agent can also borrow/save using an internationally traded asset b (if $b > 0$ we say that the agent holds net foreign assets), paying a constant interest rate, $r > 0$. This type of trade is restricted by a borrowing constraint. It is required that the ratio of debt to current income, measured in tradable units, must lie above $-\kappa < 0$ with $\kappa > 0$. The borrowing constraint is interpreted to be a collateral constraint and the pledgeable object is a fraction κ of income. Let $y^T + py^N$ be the total income in any given period, then the principal paid by the agent is given by $\max\left\{b, -\kappa(y^T + py^N)\right\}$. We assume that y=(y^N, y^T) follows a finite state Markov process with transition matrix q constructed using an auxiliary regression. In particular, we use a 2-variables (tradable

considering the numerical burden implied by the test, we require a parsimonious model. The endogenous variables depicted in Bianchi (2011) are sufficient to verify the presence of accumulated borrowing as we only need to model the behavior of the CA and the possibility of a crisis.

and non-tradable sectors) VAR(1) model.

Finally, preferences are standard in the literature: the inter-temporal problem is characterized by a CRRA instantaneous utility function with parameter $\sigma > 0$, while the intra-temporal problem assumes CES preferences with parameters $\eta > -1, \omega \in (0, 1)$.

The agent solves the following maximization problem:

Problem 1

$$\max_{\{c_t^T, c_t^N, b_{t+1}\}} E_0 \sum \beta^t \frac{C_t^{1-\sigma} - 1}{1 - \sigma}, \tag{8}$$

subject to

$$C_t = \left[\omega \left(c_t^T\right)^{-\eta} + (1 - \omega) \left(c_t^N\right)^{-\eta}\right]^{-1/\eta}$$

$$b_{t+1} + c_t^T + p_t c_t^N = b_t(1 + r) + y_t^T + p_t y_t^N,$$

$$b_{t+1} \geqslant -\kappa(y_t^T + p_t y_t^N),$$

$$b_0 \in \mathbb{R}, y_0 \in \mathbb{R}_{++}^2,$$

where the second equation is the flow budget constraint and the third is the collateral requirement.

Now we can define a (sequential) competitive equilibrium. Randomness in this economy comes from the Markovian process that drives the endowments.[4]

[4]This guarantees the existence of an infinite horizon process for $\{y_t\}$, $(\Omega, \mathcal{F}, \mu_{y_0})$, where Ω is the sample space which lies in the space of infinite bounded sequences, $\mathcal{F}$ is the associated sigma algebra with filtration $\mathcal{F}_t$ and μ_{y_0} is the associated probability measure for a given initial condition, y_0 (see Stokey, Lucas and Prescott, 1989, ch.7).

Definition: Sequential Competitive Equilibrium (SCE)

A SCE for this economy is composed by 4 progressively $\mathcal{F}_t$-measurable functions $p, (c^T, c^N, b)$ such that:

- (a) Given p, (c^T, c^N, b) solves Problem 1.
- (b) For each $t, \omega_t \in \mathcal{F}_t$; $c^N(\omega_t) = y^N(\omega_t)$

We assume that the SCE is compact. Under this assumption, we can assure that for each $t, \omega_t \in \mathcal{F}_t$, $\left[p, c^T, c^N, b\right](\omega_t) \in K$ with $K \subset \mathbb{R}^4$ compact[5]. In contrart with Bianchi (2011), we use the primal characterization of the sequential maximization problem in Definition 1 as it is easier to structurally define the moment to match, the hitting time of the collateral constraint. The remaining aspects of the model are as in Bianchi (2011) and, thus, we defer the details to the Appendix.

Let Θ be the set of structural parameters in the system of equations. Specifically, Θ contians any possible value of $\kappa, \eta, \omega, \sigma, \beta$. Our empirical identification strategy involves three conditions. The first one, which involve the definition of a threshold γ, considers the relative behavior of the treated. As the model described above is related with an emerging economy for each set of structural parameters $\theta \in \Theta$, we can only test the presence of those conditions that affect a treated country. However, we can test the presence of a PBC as its definition can be applied individually to each economy. We

[5]For a set of sufficient conditions that ensure the compactness of the SCE see Pierri and Reffett (2021).

choose to start with condition (ii) in the definition of a PBC, persistent CA deficits. We estimate a VAR for the tradable and non-tradable components of the GDP. Using the covariance and auto-regressive coefficients matrixes, we derive a 2 state discrete markov process for endowments [6]. We then look for $\theta_i \in \Theta$ which satisfies:

$$CA_{i,s}(\theta_i) \equiv \frac{b_{s+1} - b_s}{p_s y_s^N + y_s^T} \leqslant -\gamma, \quad s = 1, ..., h, \tag{9}$$

where $\gamma > 0$, $y_s = (y_s^N, y_s^T)$ are drawn from the mentioned markov process and we have omitted the subscripts i in the RHS of eq. (9). Note that the above equation concerns with the realization of several random variables in the model. However, as suggested by the definition of SCE, these functions are generated by the underlying stochastic process $(\Omega, \mathcal{F}, \mu_{y_0})$ which in turn imply that we must look for a sequence of shocks $\{y_s\}_1^h$ capable of matching eq. (9). As we have to focus on the unconstrained regime, using the system of equations which characterize it (formed by eqs. (A1),(A3) and (A4) in the Appendix), we can figure out the type shocks y that may generate (9). The "block recursive" nature of the unconstrained sequential equilibria, which is described in the Appendix, allows us to match separately the numerator and the denominator of eq. (9).

Finally, we now turn to the relevance of *hitting times* in the identification process. It is usual in the literature (see Bianchi, 2011, p. 3411) to say that a "crisis has occurred" if the collateral constraint binds (i.e. $b_{t+1} \leqslant -\kappa(y_t^T + p_t y_t^N)$). However, the literature is silent with respect to the pre-phase of a

[6] The results of these estimations are available under request.

balance of payment crisis. By looking at the definition of the SCE, it is easy to see that we can define an event as the number of time periods τ between crises (i.e. the $t, \omega_t \in \mathcal{F}_t$ that generates $b_{t+1}(\omega_t) > -\kappa(y_t^T + p_t y_t^N)(\omega_t)$ with $t = 1, ..., \tau$) and in $\tau + 1$ the collateral constraint is binding. This sequence of shocks may generate different accumulated deficits, between periods τ and $\tau - h + 1$ if we are interested in a persistence level of h years, depending on τ. We are interested in finding a CA deficit that generates an accumulation of external debt of at least $\gamma \times h$ in periods $\tau - h + 1, ..., \tau$. Note that it is possible to observe a level of borrowing above the threshold but not a PBC (i.e., the collateral constraint is not binding in period $\tau + 1$). We will say that there is a crisis for a time spell τ and persistence level h if we observe an accumulated CA deficit with respect to GDP of at least $\gamma(h) \times h$ in the last h periods of a τ spell.

5.2 Structural estimation procedure

To test the results in the empirical section, we build a random variable: the number of periods without crisis, that is, the time between hits to the collateral constraint. This variable contains our structural measure of persistence (h). As we cannot replicate the behavior of the control group (as per the d-in-d estimation above) in this class of models, we try to match the CA path for an arbitrary country in the treatment group. According to Table 3, we found PBC with persistence levels of $h = 2, 4, 5$. However, we will only structurally test the presence of spells 4 and 5 as we only found 1 case with $h = 2$

We describe here the procedure that will be used to verify the presence of a PBC for some empirically meaningful set of parameters $\theta \in \Theta$. We must first take care of a numerical procedure. The first choice, following the structural literature (see Bianchi, 2011; Mendoza and Smith, 2006), is to compute the model using recursive minimal state space representation. As we are dealing with a decentralized equilibrium with incomplete markets, we do not have welfare theorems. Thus, we must take care of the evolution of prices, in particular the real exchange rate, p.

As we have assumed the compactness of the SCE, we can choose a set K which contains all possible values for the endogenous variables in the model. In particular, for each $t, \omega_t \in \mathcal{F}_t$, $\left[p, c^T, c^N, b\right](\omega_t) \in K \equiv P \times K_1 \times K_2 \times B$. Let p_t^0 be an initial guess for the price function with $p : B \times Y \to P$. Note that given p_t^0, the system of equations that characterize the equilibrium, (A1)-(A4) in the Appendix, can be solved for any possible values of $(b_t, y_t) \in B \times Y$, giving rise to a function $b_{t+1}^0(b_t, y_t; p_t^0)$, taking values in B. We call this object *Policy Function*. Moreover, after obtaining b_{t+1}^0, we can update p using the intra-temporal optimality condition, eq. (A2) in the Appendix. Thus, we can define the following procedure:

Computable Recursive Equilibrium

- 1) Set B, Y, $\delta > 0$ and p_t^0.
- 2) Compute b_{t+1}^0 using equations (A1)-(A4).
- 3) Update p_t^0 using equation (A2).
- 4) continue until $\sup \left\| b_{t+1}^{n+1} - b_{t+1}^{n} \right\| < \delta$.

Step 4) implies the existence of number $N(\delta)$ such that $n \geqslant N(\delta)$ implies:

$$\sup_{(b_t,y_t)\in B\times Y} \left\| b_{t+1}^{n+1}(b_t, y_t; p_t^{n+1}) - b_{t+1}^{n}(b_t, y_t; p_t^{n}) \right\| < \delta. \tag{10}$$

After achieving convergence in the above procedure, equipped with a policy and a price function, we can try to verify condition (9) which is the structural analogous of of point (ii) in the definition of PBC, associated with persistence. Then we verify conditions (i) and (iii) (i.e., recession and CA reversal respectively) to complete the structural test of a PBC.

We must test the presence of a spell h of length 4 and 5 years in a selected group of countries. For these countries i we will look for the set of parameters $\theta_i \in \Theta$ such that condition (ii) (i.e. $CA_{i,t-s} < -\gamma(h)$ for $s = 1, ..., h$) is satisfied. We proceed in several steps for each persistence level h:

Structural Estimation Procedure

- i) Find the values $\theta_i \in \Theta$ that has the maximum level of accumulated borrowing in any pre-phase of length h for several simulations of length T.

- ii) For these values of θ_i, or any vector of parameters nearby, find if there is a longer pre-phase spell τ which contains the desired level of accumulated borrowing in the last h years.

5.3 Results using the canonical model

In this section we present the results of the structural estimation procedure. We take one case based on our empirical findings, from Table 3, which is Peru 1977, and one from the systemic sudden stop literature adjusted to our framework, presented in Table 5, i.e. Argentina 1998. This procedure is designed to match the level of accumulated borrowing in the definition of PBC, condition (ii) for a persistence level of 4 (Peru in 1977) and 5 years (Argentina 1998).[7] In both cases the frequency of crises is significantly below the one observed in the literature (5%, see Bianchi (2011)). These fact will lead us to move away from the canonical model and incorporate an endogenous interest rate to the analysis.[8]

Argentina 98, $h = 5$

The table below contains the parameters obtained using our structural estimation procedure. In the appendix we show that our results not only replicate Bianchi's procedure, but also our data generates a closer match.

[7]We are choosing Argentina to verify the performance of the structural estimation procedure. As we are modeling the balance of payment crises in these country in 1998, based on the results in Calvo et al. (2008), we can also use this event to test the performance of the model in Bianchi (2011). The author also calibrates the model to Argentinean data but solved the model using a different procedure with respect to ours. In particular, contrarily to what is done frequently in the literature, we are using hitting times to match accumulated borrowing. After checking if our simulations can replicate the results in Bianchi (2011) using the parameter structure in that paper, we can try to verify if there is a PBC hit Argentina in 1998.

[8]We select the events that has a stationary bivariate $AR(1)$ process for (y^T, y^N). Some countries, even after de-trending the series, have an autoregressive coefficient above 1, a fact that does not allow us to discretize the process using the Tauchen (1986) procedure. Even if we can discretize a non-stationary process, using the results in Fella et al. (2019), for instance, the required compactness of the SCE will make simulations unsuitable.

Table 8: Estimated parameters

Argentina 98

β	κ	η	ω	σ
0.91	0.32	0.21	0.31	2.0

We will test the presence of persistence in the CA. We present the level of accumulated borrowing achieved by the simulations (B_i), the empirical threshold for this event (γ_i, see section 4.4), the persistence level associated with this threshold (h), the empirical minimal borrowing to be matched ($\gamma_i \times h$) and the distance with respect to our findings using the d-in-d approach ($(\gamma - \gamma_i)/\sigma_e$). Finally, we evaluate if the simulations generate the reported level of borrowing B_i followed by a crisis, where we are looking for a binding collateral constraint in the period that follows the last unconstrained year.

The results of the structural estimation are presented in the table below. The first step is borrowed from the empirical section in this paper. The thresholds γ and γ_i, for the persistence same $h = 5$, were presented in Tables 3 and 5 respectively. We are reporting the empirical threshold ($\gamma_i = -2.0$) according to our data base and the associated borrowing (10.0) to be matched. Finally, we present the scaled deviation from the d-in-d approach for this event.

Notice that Argentina 1998 is a PBC but with milder characterises when we compare it with the results in table 2. For instance, the empirical threshold (γ_i, 2.0) is below the estimated one (γ, 6.2). Similarly, the event has a milder CA reversion (0.7 percentage points vs. 3.1 in table 2) and consumption drop (1.3 percentage points vs. 5.0 in table 2). Argentina 1998 shows

Table 9: Simulations

Argentina 98

B_i	γ_i	h	$\gamma_i \times h$	$(\gamma - \gamma_i)/\sigma$	Step	Crisis?
6.7	2.0	5	10.0	1.2	1	YES
14.4	2.0	5	10.0	1.2	2	NO
12.3	2.0	5	10.0	1.2	2	NO

that sometimes a sudden stop, as defined by Calvo (2008), is a milder PBC.

Regarding the quality of the match, the results of the table indicate that transitions to a crisis are in some cases "too fast" and in others "too slow". In particular, the first fact can be verified by looking at the maximal level of borrowing for a pre-phase of 5 years followed by a crisis (6.7% of the GDP, in the second row of the table). The empirical evidence suggests that this value must have been larger that that (i.e., 10%), even if we allow for some statistical error (i.e., 1.2%). The second fact can be verified by looking at the remaining rows: when we replicate the level of borrowing and the persistence level (in the second and third rows), we cannot generate any crisis. That is, convergence to the collateral constraint is "too slow".

Peru 77, $h = 4$

The table below describes the results for Peru 1977. We found that the model can replicate the dynamics of the pre-phase of the PBC. This happens because GDP is 15% more persistent than in Argentina, as measure by the norm of the matrix of autoregressive coefficients of a standard VAR [9]. However-

[9] We split the GDP intro its tradable and non-tradable components and estimate a first order VAR, which is available under request. The eigenvalues of the matrix of autoregressive coefficients for Perú are both equal to 0.84 and 0.74 for Argentina.

ever, the probability of observing this type of crisis is lower when compared with the literature (i.e., 5% in Bianchi (2011)): we simulate 5000 spells of 20 periods each (i.e., $h = 4, \tau = 20$) and we only found 1 match. That is, for a persistence of 4 years, the probability is 0.02%. This result is of course due to the restrictiveness of the conditions imposed to replicate the presence of accumulated borrowing in Peru and is captured by a bigger value of κ (i.e., 0.96 for Peru and 0.32 for Argentina).

Table 10: Estimated parameters and simulations

Peru 77

Step	β	κ	η	ω	σ	B_i	γ	h	$\gamma \times h$
1	0.965	0.96	0.32	0.21	2.00	9.7	6.7	4	26.8
2	0.965	0.96	0.32	0.21	2.00	36.7	6.7	4	26.8

5.4 The model with endogenous interest rates

In this section we show that a PBC can be generated by a canonical small open economy model augmented with endogenous interest rates. In this model, the system of equations has a "block recursive structure" generated by the market clearing condition for non-tradables (see the definition of the SCE). In particular, the CA deficit, measured in tradables, must be generated by tradable income shocks (see equation (A5) in the Appendix). That is, the persistence of the CA is independent of non-tradable shocks y^N and prices p. These last two variables affect only the level of the CA to GDP ratio (see equation (A6) in the Appendix). It follows then that the persistence of tradable income is critical to generate a PBC. During the pre-phase the collateral constraint is not binding, thus consumption is smoother than income which implies that the only way to generate 4 or 5 years of excessive borrowing is due to a strong and persistent recession. This does not happen frequently. Thus, the role of and endogenous interest rate is critical as it will allow us to generate significant income effects because the two selected countries were net borrowers at the time of the crisis.

We borrow the theoretical structure from García-Cicco et al. (2010). We now assume that interest rate is increasing in the level of aggregate debt, $D \equiv -B$. That is, there is an individual endogenous state variable, external assets b, that is under the control of the representative agent. In equilibrium, we have $b = B$. Thus, the agent fails to internalize the effects of excessive borrowing at the optimization or individual level. Following Garcia-Cicco et al. (2010), the interest rate is now:

$$r = r^* + \psi(e^{(D-\bar{D})})$$

To interpret (5.4), as the market rate is increasing in the level of debt, we can imagine that there is default risk and $e^{(D-\bar{D})}$ is the premium with respect to the risk free rate r^*. This last variable is fixed. Thus, in contrast to García-Cicco et al. (2010), we do not include interest rate shocks (i.e. a stochastic r^*) but an additional friction to complement the presence of a collateral constraint and jointly achieve the frequency of crises and accumulated borrowing which are observed in the data. Moreover, ψ and $\bar{D}$ are simply scale factors ii order to calibrate the sensitivity of the market rate with respect to business cycle fluctuations. The table below contain the results of the calibration process.

Table 11: Simulation results for the model with an endogenous interest rate

Country / Variable	ψ	$\bar{D}$	τ	Frequency	B_i	$\gamma_i \times h$	Crisis?
Argentina 98	0.10	-0.41	10	4.8%	8.1%	10.0%	YES
Peru 77	0.002	-0.31	12	5.1%	26.7%	26.8%	YES

The grid for debt is $[-031, -1.22]$ so $\bar{D}$ is close to the upper bound of the set in both cases. This fact, together with the values of ψ, guarantees a reasonable order of magnitude for the interest rates as $\psi(e^{(D-\bar{D})})$ will be typically between 0 and 1. For Peru we reduced κ from 0.96 to 0.96/4 to increase the frequency of crises. Moreover, also for this country, we changed β from 0.965 to 0.976. Note that now the equilibrium interest rate is endogenous and for Argentina we have a match with $\beta = 0.91$. This implies that balanced growth path interest rate satisfies: $1/\beta = R = 1.0989$. In order to respect this steady state, we set $1/\beta = 1/0.976 = R = 1.0989/4$. Thus, we get an equivalent rate but with quarterly basis; in line with the adjustment of κ. The values reported in the table above, for the case of Peru, are then annualized from row simulations [10]. The sensitivity of spreads to debt levels, captured by ψ, is at least twice as in García-Cicco et al. (2010). These authors want to replicate long run regularities and thus their estimates are not intended to capture the frequency of crises.

The model with an endogenous interest rate not only matches the empirical frequency of crises, around 5% (see Bianchi (2011)), but also at least

[10] We would like to thank to J. García-Cicco for pointing out this to us.

the 80% of the empirical accumulated borrowing. The intuition goes as follows: an increase in the interest rate may generate a reduction in consumption due to a negative income effect. This is so as these countries are net debtors. Given everything else, this may imply a reduction in debt. However, as total income is composed by 2 components, tradable income and debt services, debt may still increase. Formally, equilibrium consumption satisfy $b_{t+1} = y_t + r_t b_t - c_t$. Thus, the effect in the last 2 terms of the right hand side of the budget equation are not ex-ante defined. Our simulation results suggest that the reduction in $r_t b_t$ more than compensates the drop in consumption. Thus, as we were not able to match the occurrence of a PBC without an endogenous interest rate, fluctuations in tradable National Income $y_t + r_t b_t$, as opposed to tradable GDP y_t, are critical to generate the type of crises described in this paper.

6 Conclusions

Persistent current account deficits are a recurrent phenomenon in a large fraction of low- and middle-income economies. Since these dynamics are usually followed by balance of payments crises, we answer the questions of when does a country have effectively accumulated a critical level of debt and if there is a cut-off value for the CA deficit just before the crisis sets off. These issues are key, as they may help to design macro-prudential policies by applying a rule that are easy to follow and to communicate.

Our paper is a contribution to the characterization of balance of payment crises. First, our empirical analysis shows that there is a robust measure of accumulated borrowing indicating that external crises can be associated with persistent current account deficits that accumulate about 26%-31% of GDP in a time span of 3 to 5 years. We call these events persistent balance of payment crises (PBC). Second, these crisis are more severe than an average sudden stop, producing a drop in real consumption of about 3.5% and a current account reversal of 5.0 percentage points of the GDP. This finding is robust to different estimates of the threshold. Our results provide a potential measure of external fragility.

We test these empirical findings by structurally estimating a standard model due to Bianchi (2011) augmented by an endogenous interest rate. The model allow us to replicate the pre-phase of a PBC and match the observe frequency of these events. We contribute by proposing a novel moment to match, hitting times, and highlight the relevance of national income and the persistence of the GDP to generate severe crises.

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Appendix

Table 12: Maximum numerical derivatives 2 years accumulated of CA deficit

ε	Rank[a]	Slope	γ	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
0.1	1	-0.039	8.2	-0.0550	4.2	12
		(0.0060)	(0.72)	(0.0020)	(0.154)	(3.43)
	2	-0.038	6.6	-0.0505	4.2	20
		(0.0082)	(0.77)	(0.0019)	(0.213)	(3.80)
	3	-0.024	7.1	-0.0508	3.7	15
		(0.0086)	(0.86)	(0.0015)	(0.226)	(4.20)
0.2	1	-0.030	9.8	-0.0505	3.3	7
		(0.0047)	(1.10)	(0.0009)	(0.385)	(4.48)
	2	-0.025	10.0	-0.0525	2.8	5
		(0.0058)	(1.55)	(0.0010)	(0.547)	(6.46)
	3	-0.017	6.6	-0.0505	4.2	20
		(0.0052)	(1.54)	(0.0013)	(0.579)	(6.72)
0.3	1	-0.040	9.8	-0.0505	3.3	7
		(0.0078)	(1.16)	(0.0011)	(0.418)	(5.30)
	2	-0.033	10.0	-0.0525	2.8	5
		(0.0102)	(1.52)	(0.0014)	(0.591)	(6.99)
	3	-0.018	6.0	-0.0522	4.3	24
		(0.0094)	(1.56)	(0.0014)	(0.605)	(7.24)
0.4	1	-0.052	10.0	-0.0525	2.8	5
		(0.0088)	(0.80)	(0.0013)	(0.476)	(3.68)
	2	-0.044	9.8	-0.0505	3.3	7
		(0.0118)	(1.32)	(0.0017)	(0.555)	(5.84)
	3	-0.029	8.4	-0.0550	4.2	12
		(0.0117)	(1.36)	(0.0019)	(0.480)	(6.04)
0.5	1	-0.087	10.0	-0.0525	2.8	5
		(0.0203)	(0.94)	(0.0011)	(0.472)	(4.35)
	2	-0.050	9.8	-0.0505	3.3	7
		(0.0194)	(1.30)	(0.0015)	(0.521)	(5.58)
	3	-0.043	6.8	-0.0505	4.2	20
		(0.0152)	(1.28)	(0.0016)	(0.459)	(5.48)

Notes: Identification of PBC using maximum derivatives or slopes of $\mu_1(\gamma, h)$ in the direction of γ. [a]: rank corresponds to the ordered absolute values of the slopes. Δlnc_{t+1}, ΔCA_{t+1} and # Episodes correspond to the characterization in eqs. (4), (5) and the number of episodes satisfying the PBC criterion, respectively, defined in Section 3.2. Wild bootstrap standard errors in parenthesis.

Table 13: Maximum numerical derivatives 3 years accumulated of CA deficit

ε	Rank[a]	Slope	γ	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
0.1	1	-0.080	9.2	-0.0505	3.3	7
		(0.0116)	(0.73)	(0.0008)	(0.130)	(1.36)
	2	-0.063	9.4	-0.0505	3.3	7
		(0.0162)	(1.12)	(0.0014)	(0.226)	(2.07)
	3	-0.042	6.6	-0.0476	3.8	12
		(0.0148)	(1.13)	(0.0018)	(0.221)	(3.05)
0.2	1	-0.032	9.4	-0.0505	3.3	7
		(0.0046)	(0.38)	(0.0003)	(0.129)	(0.78)
	2	-0.025	9.2	-0.0505	3.3	7
		(0.0057)	(0.56)	(0.0008)	(0.222)	(1.16)
	3	-0.018	8.2	-0.0515	3.8	9
		(0.0047)	(0.76)	(0.0012)	(0.230)	(1.80)
0.3	1	-0.048	9.4	-0.0505	3.3	7
		(0.0117)	(0.58)	(0.0007)	(0.206)	(1.21)
	2	-0.025	8.4	-0.0515	3.8	9
		(0.0115)	(0.95)	(0.0011)	(0.235)	(2.06)
	3	-0.021	9.6	-0.0505	3.3	7
		(0.0073)	(1.18)	(0.0015)	(0.224)	(2.93)
0.4	1	-0.068	9.4	-0.0505	3.3	7
		(0.0133)	(0.45)	(0.0008)	(0.147)	(0.85)
	2	-0.048	9.6	-0.0505	3.3	7
		(0.0153)	(0.96)	(0.0014)	(0.220)	(1.75)
	3	-0.038	8.4	-0.0515	3.8	9
		(0.0124)	(1.06)	(0.0019)	(0.208)	(1.95)
0.5	1	-0.092	9.6	-0.0505	3.3	7
		(0.0143)	(0.49)	(0.0007)	(0.151)	(0.84)
	2	-0.084	9.4	-0.0505	3.3	7
		(0.0203)	(0.79)	(0.0014)	(0.217)	(1.41)
	3	-0.053	8.4	-0.0515	3.8	9
		(0.0199)	(1.00)	(0.0019)	(0.206)	(1.91)

Notes: See notes to Table 12.

Table 14: Maximum numerical derivatives 4 years accumulated of CA deficit

ε	Rank[a]	Slope	γ	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
0.1	1	-0.053	9.1	-0.0575	3.3	6
		(0.0081)	(1.00)	(0.0038)	(0.170)	(1.98)
	2	-0.037	8.2	-0.0575	3.3	6
		(0.0081)	(1.39)	(0.0062)	(0.276)	(3.17)
	3	-0.036	5.7	-0.0447	3.8	13
		(0.0065)	(1.20)	(0.0059)	(0.316)	(3.04)
0.2	1	-0.032	5.7	-0.0447	3.8	13
		(0.0017)	(0.93)	(0.0030)	(0.302)	(2.33)
	2	-0.031	7.1	-0.0428	3.2	9
		(0.0031)	(1.02)	(0.0048)	(0.289)	(2.37)
	3	-0.028	7.5	-0.0472	3.2	8
		(0.0051)	(1.10)	(0.0060)	(0.213)	(4.64)
0.3	1	-0.033	5.7	-0.0447	3.8	13
		(0.0022)	(0.82)	(0.0017)	(0.310)	(2.28)
	2	-0.033	7.5	-0.0472	3.2	8
		(0.0042)	(0.86)	(0.0030)	(0.322)	(2.27)
	3	-0.031	7.1	-0.0428	3.2	9
		(0.0055)	(0.81)	(0.0037)	(0.325)	(2.02)
0.4	1	-0.042	5.9	-0.0475	3.8	12
		(0.0033)	(0.82)	(0.0020)	(0.406)	(2.27)
	2	-0.037	7.7	-0.0515	2.9	7
		(0.0038)	(0.96)	(0.0030)	(0.440)	(2.68)
	3	-0.036	5.7	-0.0447	3.8	13
		(0.0036)	(0.95)	(0.0035)	(0.409)	(2.61)
0.5	1	-0.074	5.9	-0.0475	3.8	12
		(0.0066)	(0.87)	(0.0023)	(0.418)	(2.32)
	2	-0.065	7.7	-0.0515	2.9	7
		(0.0089)	(0.88)	(0.0031)	(0.409)	(2.28)
	3	-0.057	7.9	-0.0515	2.9	7
		(0.0091)	(0.76)	(0.0043)	(0.332)	(1.94)

Notes: See notes to Table 12.

Table 15: Maximum numerical derivatives 5 years accumulated of CA deficit

ε	Rank[a]	Slope	γ	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
0.1	1	-0.050	7.1	-0.0515	2.9	7
		(0.0059)	(0.72)	(0.0027)	(0.129)	(2.39)
	2	-0.048	5.9	-0.0514	3.1	8
		(0.0096)	(0.89)	(0.0051)	(0.200)	(3.77)
	3	-0.040	5.7	-0.0469	3.2	9
		(0.0124)	(1.23)	(0.0071)	(0.284)	(7.08)
0.2	1	-0.025	7.1	-0.0515	2.9	7
		(0.0024)	(0.71)	(0.0023)	(0.117)	(2.25)
	2	-0.025	5.7	-0.0469	3.2	9
		(0.0047)	(0.90)	(0.0037)	(0.176)	(4.03)
	3	-0.024	5.9	-0.0514	3.1	8
		(0.0055)	(1.00)	(0.0056)	(0.272)	(4.27)
0.3	1	-0.029	5.9	-0.0514	3.1	8
		(0.0035)	(0.78)	(0.0025)	(0.124)	(2.60)
	2	-0.024	7.5	-0.0476	2.9	5
		(0.0043)	(1.00)	(0.0039)	(0.183)	(4.70)
	3	-0.022	5.7	-0.0469	3.2	9
		(0.0045)	(1.07)	(0.0047)	(0.225)	(5.36)
0.4	1	-0.044	5.9	-0.0514	3.1	8
		(0.0084)	(0.35)	(0.0021)	(0.105)	(1.18)
	2	-0.039	6.1	-0.0515	2.9	7
		(0.0107)	(0.66)	(0.0040)	(0.174)	(3.30)
	3	-0.022	5.7	-0.0469	3.2	9
		(0.0102)	(1.01)	(0.0049)	(0.218)	(6.65)
0.5	1	-0.068	6.1	-0.0515	2.9	7
		(0.0145)	(0.46)	(0.0018)	(0.091)	(1.90)
	2	-0.066	5.9	-0.0514	3.1	8
		(0.0198)	(0.71)	(0.0035)	(0.152)	(3.07)
	3	-0.033	7.3	-0.0515	2.9	7
		(0.0193)	(0.93)	(0.0055)	(0.236)	(4.00)

Notes: See notes to Table 12.

Characterization of the model

We are allowed to characterize a (compact) SCE by means of a sequence of FOCs. We choose a primal version of the Karush-Kuhn-Tucker conditions because of their simplicity. We look for a system of equations that characterize definition 1. Thus, for each $t, \omega_t \in \mathcal{F}_t$, $[p(\omega_t), y(\omega_t), b(\omega_t)]$ must satisfy:

$$\left[\left[X_t(p_t, b_{t+1}) - \beta(1+r)E_t(X_{t+1}(b_{t+1})) \right] \left[b_{t+1} + \kappa(p_t y_t^N + y_t^T) \right] \right] (\omega_t) = 0 \tag{A1}$$

$$p_t = \left(\frac{1-\omega}{\omega} \right) \left[\frac{(1+r)b_t + y_t^T - max\left\{ b_{t+1}, -\kappa(y_t^T + p_t y_t^N) \right\}}{y_t^N} \right]^{(1+\eta)} \tag{A2}$$

Eq. (A1) characterizes the inter-temporal behavior and it is composed by 2 brackets, the Euler equation and the collateral constraint, both greater or equal than zero. Eq. (A2) controls the inter-temporal behavior. The preference structure implies that $X_t(\omega_t)$ for each $t, \omega_t \in \mathcal{F}_t$ must satisfy:

$$X_t(\omega_t) = \left[\left[\omega \left(c_t^T \right)^{-\eta} + (1-\omega) \left(y_t^N \right)^{-\eta} \right]^{\frac{-1+\sigma-\eta}{\eta}} \left(c_t^T \right)^{-\eta-1} \right] (\omega_t) \tag{A3}$$

$$c_t^T(\omega_t) = \left[(1+r)b_t + y_t^T - \max\left\{ b_{t+1}, -\kappa(y_t^T + p_t y_t^N) \right\} \right] (\omega_t) \tag{A4}$$

Eq. (A1) gives rise to 2 different regimes: one characterized by a constrained level of debt, which is related to the occurrence of a balance of payment crises, and one that is unconstrained. As we are interested in the pre-phase of the crises, we must focus on the former. Thus, in order to match the facts found in the empirical section, we must concentrate on the non-binding version of the system of equations above. In this sense, it is convenient to characterize the SCE using the primal version of optimality because of at least two reasons: i) the Lagrange multiplier associated with the collateral constraint will not be needed, 2) the system formed by equations (A1) to (A4) can be decomposed in two parts, where the first (eqs. (A1),(A3),(A4)) determines the level of assets. Thus, the equations above can be written in a "block recursive" style where the first part determines b_{t+1} and the second p_t. As our definition of PBC only concerns the behavior of consumption and CA, we focus on the first part.

Characterization of the unconstrained regime

Formally, if the collateral constraint does not bind, we have:

$$X_t(y_t^T, b_{t+1}) = \beta(1+r)E_t(X_{t+1}(b_{t+1})) \tag{A5}$$

$$p_t = \left(\frac{1-\omega}{\omega}\right)\left[\frac{(1+r)b_t + y_t^T - b_{t+1}}{y_t^N}\right]^{(1+\eta)} \tag{A6}$$

Note that eqs. (A1),(A3),(A4) imply that the Euler equation, (A5), is independent of y_t^N and p_t. Thus, we have highlighted the dependence on y_t^T

and b_{t+1} in equation (A5). This equation will allow us to match the numerator of (9). In particular, the convexity of preferences implies that any *decreasing* sequence $\{y_s^T\}_1^h$ will generate a *decreasing* sequence $\{b_{s+1} - b_s\}_1^h$. That is, a recession caused by tradable output implies a decline in net external assets, which in the model is associated unequivocally with a downturn in the CA as we are not considering valuation effects. Taking into account the extensive numerical burden of the structural test, we must discretize the state space of Y. Thus, the number of possible elements in $\{\Delta y_s\}_1^{h-1}$ is limited by the cardinality of Y. Fortunately, the bounded nature of the SCE can be used to solve this problem. Suppose that the optimal choice in time $s-1$ is $\hat{b}_s$ and that $y_s^T = y_{LB}^T$, where y_{LB}^T is the lower bound in Y^T with $Y = Y^T \times Y^N$. Thus, we have:

$$X_s(y_{LB}^T, \hat{b}_s) > \beta(1+r)E_t(X_{s+1}(\hat{b}_s))$$

The above equation implies $\hat{b}_{s+1} < \hat{b}_s$. Thus, we look for a *weakly decreasing sequence* $\{y_s^T\}_1^h$ to replicate an adequate behavior in the numerator of (9). Of course, we will have $\hat{b}_{s+1} < \hat{b}_s$ until we hit the collateral constraint in period, say, τ.

As regards the denominator, note that: i) typically, (y^T, y^N) will be positively correlated (this will be shown in the next subsection), ii) p is *increasing* in c^T. Thus, *even for the same value of* $b_{s+1} - b_s < 0$, the ratio will be *decreasing* as GDP measured in tradables will go down if $y_{s+1}^T \leqslant y_s^T$.

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Standard deviations estimates: model and data

Model	Model in Bianchi (2011)	Data	Data in Bianchi (2011)
2.5	2.8	2.7	3.6

Accuracy test of the algorithm

The Table above presents the standard deviation of the CA to GDP ratio computed using simulations, in the first 2 columns, and the different data sets in our paper and in Bianchi (2011), the last 2 columns. The purpose of this table is to show that the numerical methods used in this paper are even more accurate than the ones in Bianchi (2011).