The empirical dimension of overborrowing*

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Abstract

Persistent current account deficits are common among low- and middle-income countries. We evaluate when this situation triggers a sudden stop crisis. Using a non-parametric estimator, we find a critical value for the yearly current account deficit just before the crisis sets off and these findings give rise to an empirical measure of overborrowing: countries that have increased their external indebtedness by an accumulated amount of at least 26%-31% of the GDP in a time span of 3 to 5 years are more prone to be hit by a sudden stop crisis. The typical crisis produces a consumption drop of 4% and a current account reversal of 2.5-4.5% of GDP. We also contribute to the structural characterization of sudden stops. Using a canonical model, we can replicate these stylized facts. Moreover, we compute the corresponding ratio of net debt to GDP. This parameter is two or three times bigger than the benchmark value in the literature, a fact that improves the empirical performance of the model. From a policy perspective, our findings help to elaborate leading indicators to anticipate a sudden stop.

Keywords: sudden stops, current account deficits, debt, balance of payment crisis.

JEL codes: F32, F41

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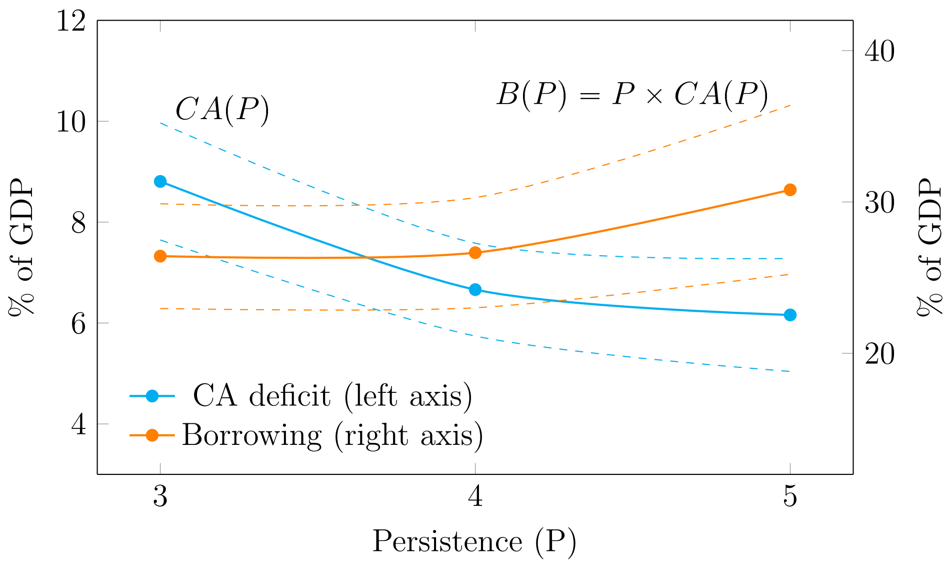
1 Introduction

Persistent current account (henceforth CA) deficits are a recurrent phenomenon in a large fraction of low- and middle-income economies. In a panel of 34 countries between 1970 and 2016 almost half of the sample contains observations with a deficit of at least 2% of Gross Domestic Product (GDP) during five consecutive years. Since in general these dynamics are followed by balance of payments crises, they beg important questions: When does a country have effectively overborrowed? Heuristically, we can say it happens when these deficits are followed by a crisis. But, *is there a critical value for the yearly CA deficit observed before the crisis?* This question is policy relevant as it may help to design macro-prudential policies based on a rule that is simple to communicate and easy to detect.

The purpose of this paper is to provide a positive answer to the last question. We find that there is a monotonic relationship between the yearly CA deficit (in terms of GDP) that precedes a sudden stop (henceforth SS) and its duration (3 to 5 years). These variables give rise to a threshold, as "crossing" it may induce a balance of payment crisis. Moreover, the CA deficit is decreasing in its persistence and it ranges between 6.2% and 8.8% of the GDP. We also investigate the effects of accumulated borrowing on several economies. We found that there is a *fragile* set of countries, those that have increased their external indebtedness by at least 26% to 31% of the GDP in 3 to 5 years. When these countries are hit by a systemic shock, they suffer a SS. In this sense, the results in this paper provide a measure of external fragility in terms of yearly current account deficit and its persistence, a measure that we call *the empirical dimension of overborrowing.*

Figure 1 summarizes the main findings and stylized facts in this paper. It depicts the monotonic relationship between the yearly CA deficit (in terms of GDP, CA in the figure) observed before a balance of payments crisis and its persistence (P, i.e. the spell of the CA deficit). In the right vertical axis, we plot the product of these two variables as a measure of accumulated external borrowing ($B(P) = P \times CA(P)$). The depicted CA and B curves indicate the threshold that characterize a SS crisis. There is a stable relationship between the rates at which the CA deficit decreases and its persistence increases, giving rise to the empirical dimension of overborrowing. That is, we found a stable accumulated increase in external indebtedness (i.e. B) between 26% and 31% of the GDP which precedes a balance of payment crisis.

Figure 1: The empirical dimension of overborrowing. Summary of the main findings



Notes: Left vertical axis corresponds to yearly current account deficits (CA). Right axis is accumulated current account deficits (borrowing). Both as percentage of GDP. The dashed lines represent 2 standard error deviations with respect to the point estimates.

To structurally test the presence of overborrowing we use a standard theoretical model in the small open economy literature, Bianchi (2011). We found that, if GDP is sufficiently persistent, the model can replicate the dynamics of an economy that has suffered a SS after 2 to 5 years of persistent CA deficits. We use these results to compute the maximal value of net external indebtedness before a balance of payments crisis. We found that this parameter is 2 or 3 times bigger than the benchmark value in the literature and it is increasing in the persistence of the yearly deficit. This fact allows the model to replicate the observed net external debt-to-GDP ratio, a frequently used measure of fragility, more accurately when compared with the state-of-the-art parameterization.

This paper contribution to the literature is three-fold.

First, we empirically identify and define SS episodes using the persistence of large CA deficits (a generalization of one condition in the definition of SS provided by Edwards, 2004). When the SS hits on the economies that satisfy these conditions (the treatment group), it produces a large drop in consumption, as compared with countries not having that condition (the control group). Countries selected in the treatment group were all affected by major crises, including the global oil shocks in the 1970s, the Latin American debt crises of the 1980s, the Russian default in the 1990s and the subprime crisis in the 2000s. In this sense, this paper proposes a novel definition of SS that can be used in empirical and theoretical papers.

This definition allows us to establish a threshold for external borrowing. We can compute it as the simple accumulation of yearly CA deficits, ranging between 6.2% and 8.8.% of the GDP, during 3 to 5 time periods in the pre-phase of the SS, adding up to 26%-31% of the GDP. When a particular country reaches this threshold, we observed a drop in consumption (averaging 4% of GDP) and a CA reversal (which ranged between 2.5-4.5% of GDP), two features that are also present in the definition of a SS (as in for instance Mendoza, 2010, and Bianchi, 2011). As a robustness check, we applied our definition to the systemic SS literature (see Calvo et al., 2008). We found that the CA values in those episodes are on average at a 0.7 standard deviation from our estimated thresholds. We also use alternative robustness estimators to compute the overborrowing measures with similar estimates.

It must be noted that our empirical definition of overborrowing contrasts with the one frequently used in the literature. In particular, since Bianchi (2011), overborrowing is defined as an excessive level of indebtedness with respect to a *normative benchmark* (i.e. the solution to a constrained Pareto efficient version of the model). Our definition can be identified in data in a simple and clear manner, without the necessity of structurally estimating a model.

Second, we contribute to the structural characterization of SSs. We propose a calibration method based on the concept of "hitting times" (see e.g., Durret, 1996, p. 176). We study the behavior of accumulated CA deficits between crises. Using the canonical set-up due to Bianchi (2011), we identify those countries that have overborrowed using different hitting times or, equivalently, by allowing a flexible number of periods without a crisis. As the model has a block recursive structure in the system of equations characterizing the sequential equilibrium between crises, we can easily identify the distinctive characteristic of those countries that have overborrowed: a highly persistent GDP. In particular, the norm of the matrix of coefficients in a standard VAR can be used to identify a positive match. Moreover, equipped with these results we can use the matched hitting time to calibrate the frequency of crises and consequently the tightness of borrowing limits. We found that the critical ratio of net debt to GDP is significantly bigger for those countries in the treatment group, closer to the observed level. Following Mendoza and Smith (2006), we interpret this result as suggesting that a SS preceded by overborrowing is an infrequent event, at least when compared to the benchmark parametrization used in the literature. For instance, we observe at most 2 positive matches out of 2000 cases (0.1%) when the "typical" event was assumed to have a frequency of 5% (see Bianchi, 2011, p. 3411). This change in the frequency of crises is a result of a more restrictive definition of SS and is the reason behind the improved match in terms of net debt to GDP. In this sense, this paper contributes decisively by improving the match of standard models to data; a quest that has been elusive in the literature (see Seoane and Yurdagul, 2019).

Finally, our results provide a useful guidance to policymakers. SSs episodes have shaken many countries, particularly emerging ones, for the last 40 years. From a macro-prudential policy perspective, our findings can help to elaborate leading indicators to anticipate a SS (see Bianchi and Mendoza, 2018). Our policy implications, then, are crystal clear: fast accumulation of CA deficits and/or significant external debt issuance increase the likelihood of a SS, and so authorities must be aware before reaching the 26%-31% of GDP threshold in the time span of 3 to 5 years.

This paper is organized as follows. Section 2 describes the canonical definitions of sudden stops. Section 3 presents the empirical strategy for identification and Section 4 presents the empirical results. Section 5 presents the structural model and its estimation. Section 6 concludes.

2 Definition of a Sudden Stop

Our broad definition of a sudden stop (SS) includes episodes that involve the following characteristics simultaneously:

Definition: Sudden Stop

- (i) *Recession*: a sharp drop in (total) real consumption.
- (ii) *Overborrowing*: several years of persistent accumulated current account deficits.
- (iii) *Reversion*: an acute correction of the current account after the sudden stop.

The concept of SS is originally due to Calvo (1998), but there are many alternative definitions in the literature. These can be classified in two types as being found in a) empirical papers; and b) theoretical (or structural) papers. The former type involves a relatively abnormal behavior of a particular episode when compared with similar countries and a pre-determined cut-off threshold given by a drop in capital inflows, see Calvo et al. (2004,2008), Edwards (2004). The latter is qualitative in nature and it omits any reference to the pre-phase of a SS, see, for instance, Mendoza (2010) and Mendoza and Smith (2006). The empirical definitions refer to the dynamic behavior of the capital account of the balance of payments, while the theoretical types are based on certain path in the current account (CA). This difference is because most known models are based on real variables and, thus, they do not consider the role of international reserves.

Our definition stands in the middle of both types. This is because we want to perform a structural test of our definition of SS, which allows the order of magnitude involved in the event to be endogenously determined. We wish to connect the pre-phase of an event with an empirical notion of overborrowing. Clause (ii) of the SS definition above will give rise to a threshold, which determines if a certain country has overborrowed and is thus prone to a SS. To recover the episodes from data, we follow a difference-in-differences strategy, which borrows from the relative nature of the empirical definition, but we extract from it a stylized fact that can be tested using a model in the small open economy family. Thus, our definition involves the behavior of the CA and it is more restrictive than any definition in the empirical or the theoretical class as it is concerned with elements of both types of existing definitions. On one hand, our definition will 'pick' a small subset of events when compared with the empirical literature. On the other hand, we will test for more than one country, as it is frequent in the theoretical literature, but with a parsimonious set of parameters; a fact that allows us to structurally test the presence of overborrowing.

Instead of imposing an *ex-ante* criterion for the pre-phase or the SS, we allow for multiple thresholds - persistence pairs as measured by the combination of a minimum level of CA deficit and the number of consecutive years which the actual figure was at or above that mark. For each threshold, the countries matching these criteria are placed in the "treatment group"; those that do not are in the "control group". We then draw a curve by mapping the difference between the change in consumption in the treatment group relative to the change in the control group (i.e., a d-in-d approach) against the CA deficit in the threshold. The intuition is that if this level of overborrowing is associated with the SS event, then this should be empirically observed in terms of the consumption dynamics: a drastic reduction in consumption, after averaging out countries' idiosyncratic differences. We have a different curve for each persistence level. The SS is then identified as the "biggest" reversion in capital flows, in relative terms, measured by the steepest tangent of this curve. This allows us to determine the most likely overborrowing measure in terms the maximum discrimination between the implied treatment and control groups. This procedure corresponds relates to a 'large volume of foreign capital' as it is mentioned in the definition of SS of Edwards (2004). This empirical strategy is further described in the next Section 3.

Our structural estimation method in Section 5 consists in applying a standard minimal state space recursive representation as in Bianchi (2011) in order to replicate condition (ii) of our definition of SS. Thus, this paper tests a sufficient condition for the presence of overborrowing in a small open economy. Typically, overborrowing is defined using a constrained Pareto efficient equilibrium as a benchmark. As the purpose of this paper is to describe an empirical measure of this phenomenon, we must avoid using the normative definition of it. Because the model refers to a small open economy, it is not possible to test for condition (i) as it requires to model a control group. Thus, we cannot test the presence of a sufficient condition for the occurrence of a SS. However, once we have identified the treatment group, we can compute the "size" of overborrowing using the threshold for the current account deficit and its persistence.

3 Empirical identification strategy

As explained above, our goal is to identify the SS event by evaluating a measure of overborrowing that is associated with the expected consequences of a SS. The empirical strategy is to link the measure of overborrowing that is associated to a discrete and significant fall in consumption measured in (constant) dollar terms. More specifically, we are interested in finding a clear discontinuity in the relative dynamics of consumption growth depending on the CA deficit level for different persistence levels.

The identification strategy has two steps. In the first step, we consider different values of CA deficits given by magnitude and persistence, which corresponds to a measure of overborrowing. A threshold is defined as the level of CA deficit of maximal discrimination or separation between countries with that level of CA deficit vs. those with less CA deficit, in terms of consumption fall. The intuition is that this level of magnitude and persistence of CA deficit is associated with a crisis. This is in fact a difference-in-differences (d-in-d) estimator, where the treatment corresponds to the CA deficit accumulation criterion. In the second step, for the group of countries that are at or above the first step threshold amount, we evaluate the average performance of those countries that also satisfy with conditions (i) and (iii) of SS given above (i.e., recession and CA reversal).

In sum, the empirical identification strategy below follows these steps:

- *First step: Overborrowing.* We find the value of the threshold, γ below, that maximizes the differences in consumption fall between the two groups, where the groups are constructed according to the level of accumulated CA deficits for different persistence levels, h.

- *Second step: SS crisis.* Among the treated, we study those countries that satisfy (i) and (iii) of the SS definition above for the threshold γ defined in the first step.

3.1 First step: An empirical measure of overborrowing

The hypothesis is that countries face a SS when persistent CA deficits cannot be sustained anymore. The identification of an event would thus correspond to different combinations of persistence in CA deficits. We are aware that the persistent behavior of the CA could be associated with a rigid fiscal deficit, an investment boom, or even a combination of the two; just to mention a few examples. In any case, the duration, and the order of magnitude of the external imbalance would be the same, which suggest that the accumulation rate of external debt may be at the center of the scene. We will try to disentangle *ex-post* for every SS, the possible origin of the disequilibrium, as this information could be useful for future research.

Specifically, a *threshold* is determined by the level of CA deficit per year during all the time periods involved in the pre-phase of the SS event, a time window that we call *persistence*. The hypothesis is that this threshold will be observed before a given drop in consumption and a certain CA reversal. The countries in the *treatment group* have at least the threshold CA deficit during the pre-phase of a SS. The *control group* will be defined by the complement, that is, they have less CA deficit.

Let CA_{it} be the current account of country i at time t, as a percentage of GDP. We argue that the SS is associated with the magnitude of CA_{it} and its persistence. We consider the treatment vs. control mean effect for a given combination of (γ, h), where γ represents the CA threshold, i.e. $CA_{it} \leqslant -\gamma$ and h is the number of periods for which the CA deficit was at or above that given value (i.e. the persistence of the deficit). Define $H_{it}(\gamma, h)$ as an indicator function that takes the value 1 if the following condition is satisfied: country i had experienced $-\gamma$ CA deficit for h years prior and including year t (i.e., $t-h, t-h+1m, \ldots, t$); 0 otherwise.

Define thus:

$$\mu_1(\gamma, h) = \mathbb{E}\left[\Delta ln(c_{i,t+1}) \mid H_{it}(\gamma, h) = 1\right] - \mathbb{E}\left[\Delta ln(c_{i,t+1}) \mid H_{it}(\gamma, h) = 0\right], \tag{1}$$

as the average difference between the treatment and control groups in consumption growth $\Delta ln(c_{i,t+1})$, where c corresponds to total consumption in real terms. Here we use the notation $\mathbb{E}$ for the empirical expectation or average. Note that the expectation is taken with respect to all the countries without further conditioning. We are thus concerned with a significant drop in consumption that will be associated with this accumulated CA deficit. The above equation thus defines a non-parametric d-in-d estimator.

Note that this strategy is designed to capture the typical event for an "average" country. Thus, the threshold may not be accurate to describe specific cases. In other words, a SS is also related to a myriad of country specific components. Our identification procedure consists in averaging out this specificity in each country, keeping instead the mean effect over those events that satisfy the required conditions in the treatment group.

The first step in our identification strategy relies on finding a discontinuity in the average change in relative consumption at a value of γ for different fixed values of h, years accumulated of CA deficits. We assume that $\gamma \in \Gamma$ where Γ is a compact set. To avoid being deceived by mere visual inspection, we identify it by looking at the maximum numerical derivatives. More formally define,

$$\gamma^*(h, \epsilon) = \underset{\gamma \in \Gamma}{\operatorname{argmax}} \, \mu_1'(\gamma, h, \varepsilon), \tag{2}$$

where

$$\mu_1'(\gamma, h, \varepsilon) \equiv \frac{\mu_1(\gamma, h) - \mu_1(\gamma - \varepsilon, h)}{\varepsilon} \tag{3}$$

is the numerical derivative or slope of $\mu_1(\gamma, h, \varepsilon)$. $\gamma^*(h, \epsilon)$, CA deficit level for h years, is thus the main estimator in the paper. Then, $\gamma^*(h, \epsilon) \times h$ is the *empirical measure of overborrowing.*

3.2 Second step: Characterizing the SS crisis

For the overborrowing measure found in the first step, we evaluate countries according to conditions (i) and (iii) of the SS definition above. If a SS occurs, we expect that for those countries involved consumption drops and CA deficit reverses. Therefore, we further characterize SS by its empirical effects:

$$\mu_2(\gamma, h) = \mathbb{E}\left[\Delta ln(c_{i,t+1}) \mid H_{it}(\gamma, h) = 1, \Delta ln(c_{i,t+1}) < 0, \Delta(CA_{i,t+1}) > 0\right], \tag{4}$$

and

$$\mu_3(\gamma, h) = \mathbb{E}\left[\Delta(CA_{i,t+1}) \mid H_{it}(\gamma, h) = 1, \Delta ln(c_{i,t+1}) < 0, \Delta(CA_{i,t+1}) > 0\right]. \tag{5}$$

These two are the main SS characterizations. $\mu_2(\gamma^*(h, \epsilon), h)$ is a measure of SS recession, a fall in consumption. $\mu_3(\gamma^*(h, \epsilon), h)$ is a measure of SS CA reversal. The pairs country-year (i, t) that satisfy the criteria $\{H_{it}(\gamma^*(h, \epsilon), h) = 1 \ \& \ \Delta ln(c_{i,t+1}) < 0 \ \& \ \Delta(CA_{i,t+1}) > 0\}$ are defined as the SS episodes. The episodes could be consecutive years of the same country, an issue that is discussed below to avoid countries double counting.

4 Empirical estimation

4.1 Data

We construct a panel of 34 countries for which reliable data are available for the period 1970-2016, with annual frequency. The variables collected include CA in (current) dollars and percentage of GDP and consumption, GDP, and investment in (2010 constant) dollars. We also performed an estimation of the tradable and non-tradable categorization by grouping the value added of agricultural and manufacturing sectors (tradables), and construction and service sectors (non-tradables).

The sources are the World Bank Database; the Economic Commission for Latin America and the Caribbean Statistics; the National Accounts Section of the United Nations Statistics Division. In order to complete the database, we also asked for specific data to several national statistical institutes.

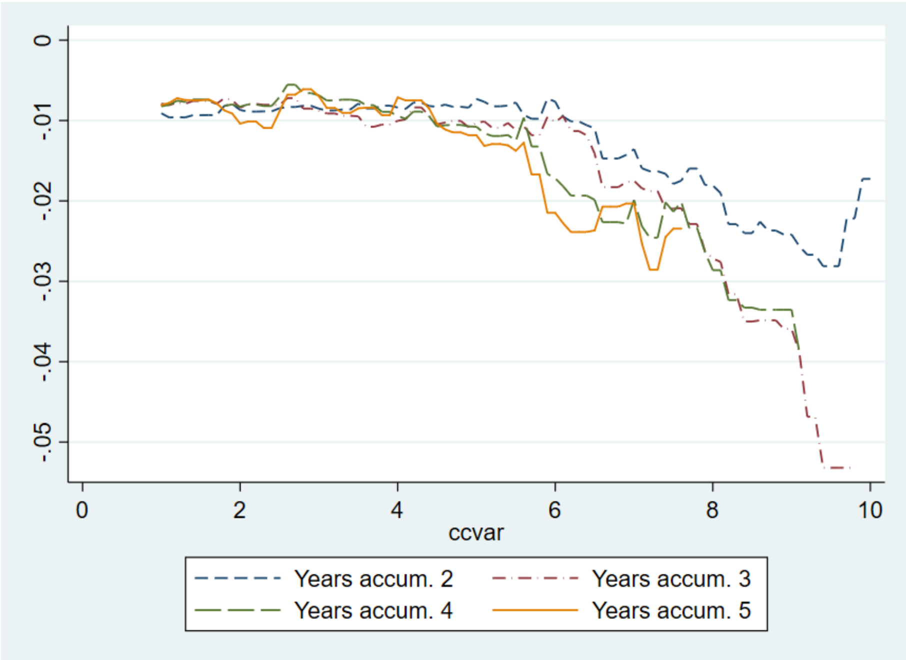
We cover fewer countries but for a longer lapse of time when compared with Edwards (2004) (i.e., 157 countries between 1970-2001) but more countries during a longer period when compared with Calvo et al. (2004) (i.e., 32 countries between 1990-2001).

The database is available from the authors upon request.

4.2 First step estimation

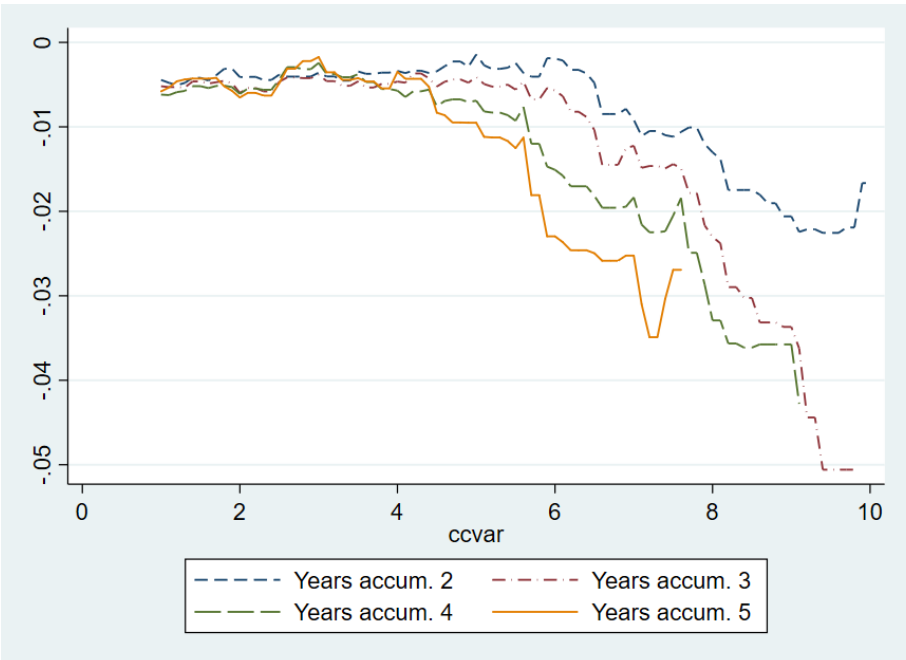
Figure 2 plots the d-in-d consumption value in eq. (1) for every pair of (γ, h), where we consider a grid of values with $h = 2, \ldots, 5$ and $\gamma \in \Gamma = \{1, 1.1, ..., 10\}$. The data in the figure is the main input for the subsequent analysis. By looking at the figure, we can informally spot the corresponding CA deficits associated with the SS. For instance, if we take 5 years of persistence ($h = 5$, see the solid line), there is a jump in the figure at about 6% CA deficit. While we are specifically interested in consumption behavior to characterize the SS crisis, the SS also affects the whole economy since the same patterns are also observed for GDP and investment, see Figures 3 and 4.

Figure 2: First step difference-in-differences estimator, consumption



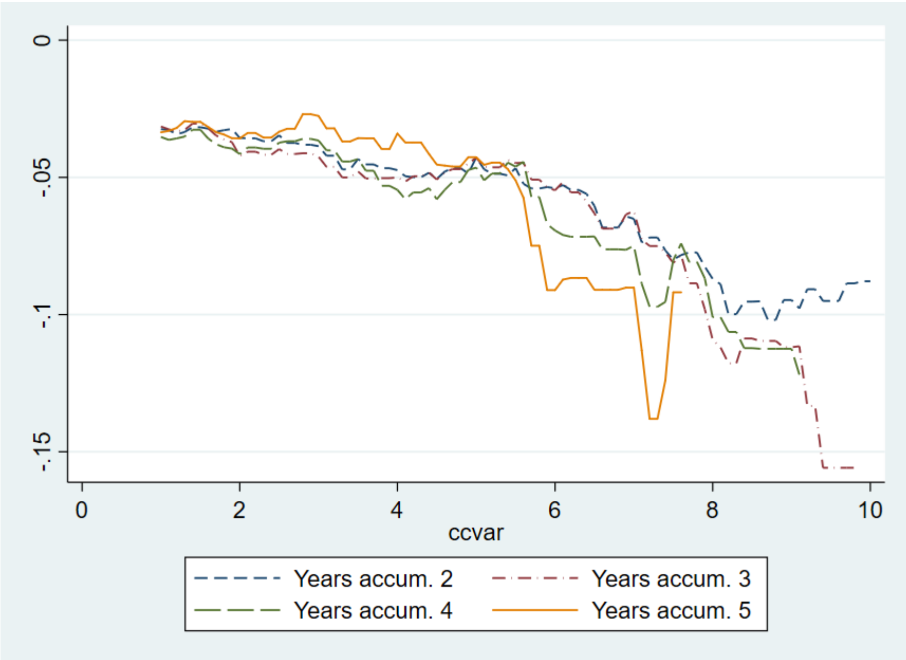
Notes: The figure reports the $\mu_1(\gamma, h)$ estimates in (1) for $h = 2, 3, 4, 5$ and γ (ccvar) in $\Gamma = \{1, 1.1, ..., 10\}$.

Figure 3: First step difference-in-differences estimator, GDP



Notes: The figure reports the $\mu_1(\gamma, h)$ estimates (using GDP instead of consumption) in (1) for $h = 2, 3, 4, 5$ and γ (ccvar) in $\Gamma = \{1, 1.1, ..., 10\}$.

Figure 4: First step difference-in-differences estimator, investment



Notes: The figure reports the $\mu_1(\gamma, h)$ estimates (using investment instead of consumption) in (1) for $h = 2, 3, 4, 5$ and γ (ccvar) in $\Gamma = \{1, 1.1, ..., 10\}$.

Consider now the estimation of the γ threshold as described in Section 3.1, eq. (2). That is, we will consider the CA deficits accumulated for up to 5 years, and we will consider CA levels each year with values in Γ, that contains up to 10% of GDP. Our interest lies in finding the threshold γ for which this difference in relative consumption has a clear discontinuity for every h. The assumption is that this discontinuity is only due to the SS crisis. Although strong, this hypothesis allows us to keep the set of variables involved in the empirical identification strategy simple, a fact that will be crucial for the structural procedure below. We take $\varepsilon \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$ for the numerical derivatives or slopes.

Tables 1-4 reports the estimated maximum slopes, together with the associated γ, for $h = 2, 3, 4, 5$, respectively. To evaluate the robustness of our strategy we compute the three maximum slopes ordered by rank, i.e., the first, second and third biggest values of the slopes. We also compute the standard errors associated with the three values using wild bootstrap. For the bootstrap procedure we first construct a dataset containing for each country-year pair (i, t) the values of $\Delta ln(c_{i,t+1})$ together with all the information about $H_{it}(\gamma, h)$ for $h = 2, \ldots, 5$ and $\gamma \in \Gamma = \{1, 1.1, ..., 10\}$. Next, we perform wild bootstrap on this sample, and estimate γ using the same numerical slope estimator.

Table 1: Maximum numerical derivatives 2 years accumulated of CA deficit

ε	Rank[a]	Slope	γ	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
0.1	1	-0.039	8.2	-0.0550	4.2	12
		(0.0060)	(0.72)	(0.0020)	(0.154)	(3.43)
	2	-0.038	6.6	-0.0505	4.2	20
		(0.0082)	(0.77)	(0.0019)	(0.213)	(3.80)
	3	-0.024	7.1	-0.0508	3.7	15
		(0.0086)	(0.86)	(0.0015)	(0.226)	(4.20)
0.2	1	-0.030	9.8	-0.0505	3.3	7
		(0.0047)	(1.10)	(0.0009)	(0.385)	(4.48)
	2	-0.025	10.0	-0.0525	2.8	5
		(0.0058)	(1.55)	(0.0010)	(0.547)	(6.46)
	3	-0.017	6.6	-0.0505	4.2	20
		(0.0052)	(1.54)	(0.0013)	(0.579)	(6.72)
0.3	1	-0.040	9.8	-0.0505	3.3	7
		(0.0078)	(1.16)	(0.0011)	(0.418)	(5.30)
	2	-0.033	10.0	-0.0525	2.8	5
		(0.0102)	(1.52)	(0.0014)	(0.591)	(6.99)
	3	-0.018	6.0	-0.0522	4.3	24
		(0.0094)	(1.56)	(0.0014)	(0.605)	(7.24)
0.4	1	-0.052	10.0	-0.0525	2.8	5
		(0.0088)	(0.80)	(0.0013)	(0.476)	(3.68)
	2	-0.044	9.8	-0.0505	3.3	7
		(0.0118)	(1.32)	(0.0017)	(0.555)	(5.84)
	3	-0.029	8.4	-0.0550	4.2	12
		(0.0117)	(1.36)	(0.0019)	(0.480)	(6.04)
0.5	1	-0.087	10.0	-0.0525	2.8	5
		(0.0203)	(0.94)	(0.0011)	(0.472)	(4.35)
	2	-0.050	9.8	-0.0505	3.3	7
		(0.0194)	(1.30)	(0.0015)	(0.521)	(5.58)
	3	-0.043	6.8	-0.0505	4.2	20
		(0.0152)	(1.28)	(0.0016)	(0.459)	(5.48)

Notes: Identification of SS using maximum derivatives or slopes of $\mu_1(\gamma, h)$ in the direction of γ. [a]: rank corresponds to the ordered absolute values of the slopes. Δlnc_{t+1}, ΔCA_{t+1} and # Episodes correspond to the characterization in eqs. (4), (5) and the number of episodes satisfying the SS criterion, respectively, defined in Section 3.2. Wild bootstrap standard errors in parenthesis.

Table 2: Maximum numerical derivatives 3 years accumulated of CA deficit

ε	Rank[a]	Slope	γ	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
0.1	1	-0.080	9.2	-0.0505	3.3	7
		(0.0116)	(0.73)	(0.0008)	(0.130)	(1.36)
	2	-0.063	9.4	-0.0505	3.3	7
		(0.0162)	(1.12)	(0.0014)	(0.226)	(2.07)
	3	-0.042	6.6	-0.0476	3.8	12
		(0.0148)	(1.13)	(0.0018)	(0.221)	(3.05)
0.2	1	-0.032	9.4	-0.0505	3.3	7
		(0.0046)	(0.38)	(0.0003)	(0.129)	(0.78)
	2	-0.025	9.2	-0.0505	3.3	7
		(0.0057)	(0.56)	(0.0008)	(0.222)	(1.16)
	3	-0.018	8.2	-0.0515	3.8	9
		(0.0047)	(0.76)	(0.0012)	(0.230)	(1.80)
0.3	1	-0.048	9.4	-0.0505	3.3	7
		(0.0117)	(0.58)	(0.0007)	(0.206)	(1.21)
	2	-0.025	8.4	-0.0515	3.8	9
		(0.0115)	(0.95)	(0.0011)	(0.235)	(2.06)
	3	-0.021	9.6	-0.0505	3.3	7
		(0.0073)	(1.18)	(0.0015)	(0.224)	(2.93)
0.4	1	-0.068	9.4	-0.0505	3.3	7
		(0.0133)	(0.45)	(0.0008)	(0.147)	(0.85)
	2	-0.048	9.6	-0.0505	3.3	7
		(0.0153)	(0.96)	(0.0014)	(0.220)	(1.75)
	3	-0.038	8.4	-0.0515	3.8	9
		(0.0124)	(1.06)	(0.0019)	(0.208)	(1.95)
0.5	1	-0.092	9.6	-0.0505	3.3	7
		(0.0143)	(0.49)	(0.0007)	(0.151)	(0.84)
	2	-0.084	9.4	-0.0505	3.3	7
		(0.0203)	(0.79)	(0.0014)	(0.217)	(1.41)
	3	-0.053	8.4	-0.0515	3.8	9
		(0.0199)	(1.00)	(0.0019)	(0.206)	(1.91)

Notes: See notes to Table 1.

Table 3: Maximum numerical derivatives 4 years accumulated of CA deficit

ε	Rank[a]	Slope	γ	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
0.1	1	-0.053	9.1	-0.0575	3.3	6
		(0.0081)	(1.00)	(0.0038)	(0.170)	(1.98)
	2	-0.037	8.2	-0.0575	3.3	6
		(0.0081)	(1.39)	(0.0062)	(0.276)	(3.17)
	3	-0.036	5.7	-0.0447	3.8	13
		(0.0065)	(1.20)	(0.0059)	(0.316)	(3.04)
0.2	1	-0.032	5.7	-0.0447	3.8	13
		(0.0017)	(0.93)	(0.0030)	(0.302)	(2.33)
	2	-0.031	7.1	-0.0428	3.2	9
		(0.0031)	(1.02)	(0.0048)	(0.289)	(2.37)
	3	-0.028	7.5	-0.0472	3.2	8
		(0.0051)	(1.10)	(0.0060)	(0.213)	(4.64)
0.3	1	-0.033	5.7	-0.0447	3.8	13
		(0.0022)	(0.82)	(0.0017)	(0.310)	(2.28)
	2	-0.033	7.5	-0.0472	3.2	8
		(0.0042)	(0.86)	(0.0030)	(0.322)	(2.27)
	3	-0.031	7.1	-0.0428	3.2	9
		(0.0055)	(0.81)	(0.0037)	(0.325)	(2.02)
0.4	1	-0.042	5.9	-0.0475	3.8	12
		(0.0033)	(0.82)	(0.0020)	(0.406)	(2.27)
	2	-0.037	7.7	-0.0515	2.9	7
		(0.0038)	(0.96)	(0.0030)	(0.440)	(2.68)
	3	-0.036	5.7	-0.0447	3.8	13
		(0.0036)	(0.95)	(0.0035)	(0.409)	(2.61)
0.5	1	-0.074	5.9	-0.0475	3.8	12
		(0.0066)	(0.87)	(0.0023)	(0.418)	(2.32)
	2	-0.065	7.7	-0.0515	2.9	7
		(0.0089)	(0.88)	(0.0031)	(0.409)	(2.28)
	3	-0.057	7.9	-0.0515	2.9	7
		(0.0091)	(0.76)	(0.0043)	(0.332)	(1.94)

Notes: See notes to Table 1.

Table 4: Maximum numerical derivatives 5 years accumulated of CA deficit

ε	Rank[a]	Slope	γ	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
0.1	1	-0.050	7.1	-0.0515	2.9	7
		(0.0059)	(0.72)	(0.0027)	(0.129)	(2.39)
	2	-0.048	5.9	-0.0514	3.1	8
		(0.0096)	(0.89)	(0.0051)	(0.200)	(3.77)
	3	-0.040	5.7	-0.0469	3.2	9
		(0.0124)	(1.23)	(0.0071)	(0.284)	(7.08)
0.2	1	-0.025	7.1	-0.0515	2.9	7
		(0.0024)	(0.71)	(0.0023)	(0.117)	(2.25)
	2	-0.025	5.7	-0.0469	3.2	9
		(0.0047)	(0.90)	(0.0037)	(0.176)	(4.03)
	3	-0.024	5.9	-0.0514	3.1	8
		(0.0055)	(1.00)	(0.0056)	(0.272)	(4.27)
0.3	1	-0.029	5.9	-0.0514	3.1	8
		(0.0035)	(0.78)	(0.0025)	(0.124)	(2.60)
	2	-0.024	7.5	-0.0476	2.9	5
		(0.0043)	(1.00)	(0.0039)	(0.183)	(4.70)
	3	-0.022	5.7	-0.0469	3.2	9
		(0.0045)	(1.07)	(0.0047)	(0.225)	(5.36)
0.4	1	-0.044	5.9	-0.0514	3.1	8
		(0.0084)	(0.35)	(0.0021)	(0.105)	(1.18)
	2	-0.039	6.1	-0.0515	2.9	7
		(0.0107)	(0.66)	(0.0040)	(0.174)	(3.30)
	3	-0.022	5.7	-0.0469	3.2	9
		(0.0102)	(1.01)	(0.0049)	(0.218)	(6.65)
0.5	1	-0.068	6.1	-0.0515	2.9	7
		(0.0145)	(0.46)	(0.0018)	(0.091)	(1.90)
	2	-0.066	5.9	-0.0514	3.1	8
		(0.0198)	(0.71)	(0.0035)	(0.152)	(3.07)
	3	-0.033	7.3	-0.0515	2.9	7
		(0.0193)	(0.93)	(0.0055)	(0.236)	(4.00)

Notes: See notes to Table 1.

The SS level of overborrowing should be search for within those values. Table 5 contains the weighted average threshold given by

$$\sum_{\varepsilon,rank} \gamma^*(h,\varepsilon,rank)\omega(\varepsilon,rank), \tag{6}$$

where $\gamma^*(h,\varepsilon,rank)$ is the maximum empirical derivative, ordered by position or rank $= 1,2,3,$[1] for a given h and ε, and $w(\varepsilon,rank)$ weights accordingly to the number of episodes (which will depend on the second step estimation that follows). In particular, the weight $\omega(\varepsilon,rank)$ is constructed as the number of cases in the treatment group for a particular pair $(\varepsilon,rank)$, that also satisfy the second step criteria below (fall in consumption, CA reversal) over the total number of events for a given persistence level. The table finds overborrowing levels that are in line with the informal visual inspection in Figure 2. For instance, a 4-year persistence ($h = 4$) has an estimate of γ value of 6.7, such that $6.7 \times 4 = 26.6$. This can be interpreted as the fact that a country that borrowed an accumulated 26.6% of GDP in 4 years is a typical candidate for an SS event. If we use a 5-year persistence ($h = 5$), it is associated with a γ value of 6.2, such that $6.2 \times 5 = 30.8$ is the associated threshold measure of overborrowing.

[1] The order implies that $\mu_1'(\gamma^*(h,\varepsilon,1),h,\varepsilon) < \mu_1'(\gamma^*(h,\varepsilon,2),h,\varepsilon) < \mu_1'(\gamma^*(h,\varepsilon,3),h,\varepsilon)$.

Table 5: Estimation results: Weighted average

h	γ	Borrowing ($\gamma \times h$)	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
2	7.74	15.49	-0.0516	3.88	15.12
	(0.75)	(1.45)	(0.00057)	(0.252)	(3.24)
3	8.81	26.42	-.05049	3.53	8.12
	(0.58)	(1.73)	(0.0006)	(0.123)	(1.34)
4	6.66	26.65	-0.0473	3.51	10.30
	(0.46)	(1.82)	(0.0014)	(0.173)	(1.55)
5	6.16	30.80	-0.0500	3.06	7.88
	(0.56)	(2.79)	(0.0036)	(0.126)	(3.97)

Notes: Estimation of average values given by eq. (6) using the results in Tables 1-4. Standard errors in parenthesis are calculated using wild bootstrap.

From the first step we can obtain the following stylized fact.

Stylized Fact 1: Decreasing threshold, stable overborrowing. The minimal CA deficit among the treated ($\gamma(h)$) is decreasing in the persistence level for h. The level of overborrowing is stable around 26-31% of the GDP.

We interpret this fact as evidence against the "deep pockets" assumption. Following Calvo (1999) and Cerruti et al. (2015), we argue that the assets under lenders' management are critical to understand an emerging market crisis. In particular, note that a borrower can accumulate a well-defined amount of debt at markets rates (i.e. between 26 and 31% of GDP, depending on the level of persistence). Our findings suggest that the level of wealth of the brokers trading this type of securities could be a determinant of the SS as the fraction of the total portfolio allocated to emerging economies could be constant all along the duration spell of the threshold. As Table 5 shows, these borrowing levels cannot be sustained for more than h years, thus we say that the listed countries have actually *overborrowed*.

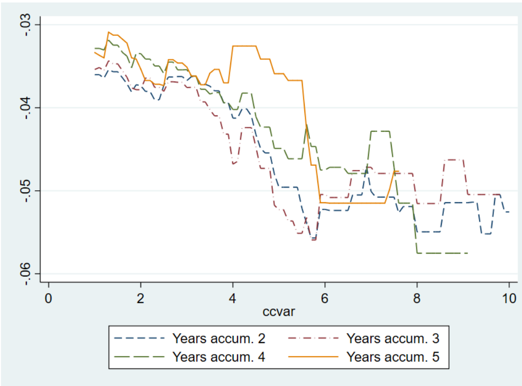
4.3 Second step estimation

Having identified the measure of overborrowing, we consider now the second step estimation, which characterizes SS. Our interest lies in characterizing countries that experience SS by finding $\mu_2(\gamma^*, h)$ and $\mu_3(\gamma^*, h)$, from eqs. (4) and (5), respectively, from the γ^* estimation in the first step.

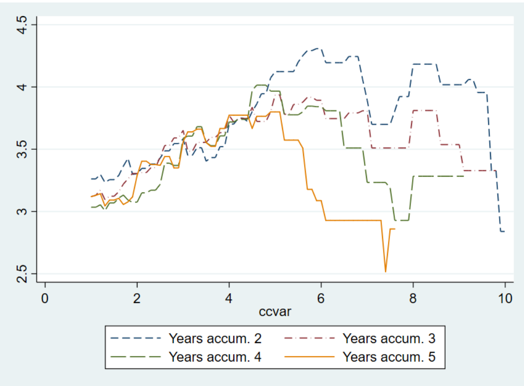
Figure 5 shows these results for every pair (γ, h). The figure also reports the number of episodes (country, year) that satisfy the corresponding criterion. As expected, as the magnitude and persistence of CA deficit increases the number of episodes decreases.

Figure 5: Second step: SS crises

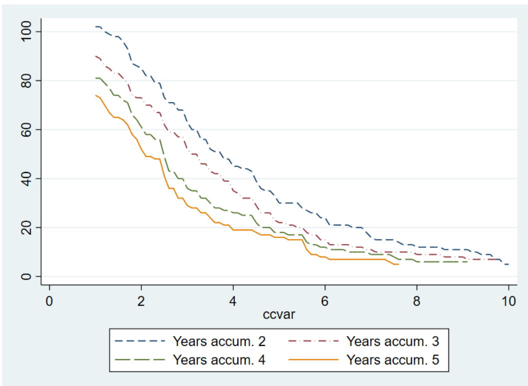
(a) Fall in consumption



(b) Current account reversal



(c) Number of episodes



Notes: The figures report the $\mu_2(\gamma, h)$ (fall in consumption), $\mu_3(\gamma, h)$ (current account reveral) and number of episodes estimates described in eqs. (4) and (5) for $h = 2, 3, 4, 5$ and γ (ccvar) in $\Gamma = \{1, 1.1, ..., 10\}$.

Tables 1-4 report the average values of $\mu_2(\gamma, h)$ and $\mu_3(\gamma, h)$ for all the potential episodes in terms of the maximum empirical derivative. These are given by columns Δlnc_{t+1} and ΔCA_{t+1}, respectively. The calculated weighted average for those parameters using the same formula as in eq. (6) appears in Table 5. In general, most values coincide in terms of the magnitude of the fall in consumption and CA reversal. For consumption fall, most estimates coincide in a 4-5% drop. For CA reversal (as percentage of GDP), the values are framed within 2.5-4.5%.

To further characterize the SS crises, we consider the following. As noted, events can be overlapped with each other for the same country. That is, it is possible to observe an event in the treatment group for $h = 2, 3, 4, 5$, provided that consumption falls, and the CA deficit reduces. Thus, we keep the longest duration spell and compute the performance of those episodes.

Table 6 reports the characterization of typical SS events, together with the average values. Note that there are no events for $h = 3$ that are not also encountered with $h = 4$, thus we do not report the value for this persistence value. In the Appendix A we qualitatively describe the 8 episodes presented in Table 6. We can identify 2 major debt crises that affected all the countries involved: the first one occurred in the late 70s-early 80s and the second one is due to the sub-prime crisis. Thus, in this sense, *overborrowing* and *systemic sudden stops*, as in Calvo et al. (2008), are connected. We explore this connection in the next subsection.

Table 6: Second Step: SS characterization

γ	h	$\Delta ln(c_{t+1})$	ΔCA_{t+1}	$\Delta ln(GDP_{t+1})$	$\Delta ln(I_{t+1})$	(i,t)
9.6	2	-1.0	7.4	0.3	-10.5	Israel 75
6.7	4	-3.9	6.9	5.3/-4.5*	-27.8	Panama 81
6.7	4	-8.9	6.0	-2.6	-8.9	Peru 77
6.7	4	-1.7	5.1	-3.6	-16.9	Spain 08
6.2	5	-5.9	5.4	-7.3	-27.8	Costa Rica 81
6.2	5	-0.8	3.6	-4.3	-13.9	Greece 08
6.2	5	1.1/-1.5*	2.9	-1.4	-12.1	Honduras 81
6.2	5	-1.2	1.7	-3.0	-7.6	Portugal 08
9.6	2	-1.0	7.4	0.3	-10.5	Average
6.7	4	-4.8	6.0	-3.6	-17.9	Average
6.2	5	-2.4	3.4	-4.0	-15.4	Average

Notes: * growth rate for period t instead of $t+1$.

The following stylized fact emerges.

Stylized Fact 2: Decreasing crises, persistent deficit. The reversal in the current account (i.e. $\Delta ln(CA_{i,t+1})(h) > 0$) is decreasing in the persistence of overborrowing h. However, none of the treated registered a surplus after the crisis.

This behavior could have been caused by the presence of a lender of last resource as there seems to be a minimum level of credit (around 2% of the GDP) that is granted despite the SS. In addition, there could have been heterogeneity among lenders.

4.4 Relationship with other events

We verify the presence of overborrowing in an extensive list of SSs as computed in Calvo et al. (2008). Considering that all events in the treatment group can be associated with two major international crises, we use the results in the systemic SSs literature to check the accuracy of our estimates.

Table 7 contains all the events classified as a SS by Calvo et al. (2008) which at the same time satisfy the definition of SS used in this paper. To make those results comparable with ours, we: (a) restrict attention to the countries listed in our database, (b) filtered the events with conditions (i), (ii) and (iii) of our empirical definition. Namely, we require a consumption drop and a current account reversal in the year of the SS (conditions (i) and (iii), respectively) and the presence of overborrowing with a different threshold (condition (ii)).

Table 7: Accuracy of the estimated thresholds

$(\gamma - \gamma_i)/\sigma$	h	$\mu_2, \Delta ln(c_{i,t+1})$	$\mu_3, \Delta(CA_{i,t+1})$	(i,t)
0.62	5	-1.3	0.6	Argentina 98
1.2	5	-11.4	14.5	Ecuador 98
0.2	5	-0.7	3.2	Peru 98
0.68	5	-4.5	6.1	Average

Calvo et al. (2008) reported 23 episodes with different thresholds. However, after imposing our conditions we can only keep 3 of them. The standard deviation of $\Delta(CA_{i,t+1})$ is 4.8. Thus, we confirm that our estimates are accurate as we can find an average threshold less than 0.7 standard deviation away from our results. To estimate the threshold in Calvo et al. (2008), we took the identified SS in that paper and computed the minimal yearly CA deficit for all possible persistence levels (i.e., $h = 2, 3, 4, 5$). We compare these thresholds $\gamma_i(h)$ with our values $\gamma(h)$ and kept the closest value in terms of the standard deviation of CA in the whole sample. For instance, in the case of Argentina 1998, we identify a threshold of 3% of the GDP and a persistence level of 5 years. The critical value is at 0.62 standard deviations away from our estimated threshold for that duration (i.e., 6.2%).

4.5 Robustness checks

The empirical identification above relied on finding a clear discontinuity in terms of consumption drop, based on the level of accumulated CA deficits. This subsection explores alternative specifications to evaluate the robustness of the previous findings.

First, we compute the $\mu_1(\gamma, h)$ functions in eq. 1 using medians instead of averages,

$$\tilde{\mu}_1(\gamma, h) = \mathbb{M}\left[\Delta ln(c_{i,t+1}) \mid H_{it}(\gamma, h) = 1\right] - \mathbb{M}\left[\Delta ln(c_{i,t+1}) \mid H_{it}(\gamma, h) = 0\right], \tag{7}$$

where $\mathbb{M}$ denotes the sample median. The idea is that the results above could be driven by extreme values, thus when searching for a discontinuity this could be affected by outliers. The results in Table 8 are computed in the same way as those in Table 5 but using medians in the first step. The results are qualitatively very similar to those found for the difference in means. That is, they identify qualitatively similar levels of overborrowing, and they lie within two standard errors of each other.

Table 8: Estimation results: Weighted average using medians in the first step

h	γ	Borrowing ($\gamma \times h$)	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
2	6.70	13.39	-0.0516	4.09	20.63
	(0.60)	(1.19)	(0.0013)	(0.174)	(4.64)
3	8.55	25.65	-.0492	3.48	8.87
	(0.57)	(1.71)	(0.0010)	(0.076)	(2.03)
4	6.81	27.26	-0.0497	3.59	11.20
	(0.71)	(1.82)	(0.0028)	(0.125)	(2.02)
5	5.06	25.32	-0.0403	3.42	15.29
	(0.55)	(2.77)	(0.0039)	(0.160)	(4.13)

Notes: Estimation of average values given by eq. (7). Standard errors in parenthesis are calculated using wild bootstrap.

Second, the first step strategy for identification described above used a criterion of maximum empirical derivatives or slopes. As an alternative measure, we consider formal econometric tests based on the theory of structural breaks. Consider a regression model of $\mu_1(\gamma, h)$ (calculated using averages) on a constant. Thus the idea is to search for an structural break in the intercept that occurs at a given value of γ, by considering the *sup* $Wald$ test of Hansen (1997).[2] In turn this corresponds to the maximum discrimination between the treatment and control group that defines $\mu_1(\gamma, h)$.

Table 9 presents the results of this second robustness exercise. We present the threshold corresponding the maximum Wald statistic value, and the two consecutive ordered values. Overall, the results show again a similar picture to that described in the empirical section. The findings suggest an overborrowing measure of about 26-31% (i.e., $\gamma \times h$) that corresponds to a consumption drop of about 5% and a CA adjustment of 3-4%.

[2]Hansen (1997) tests is designed for a time-series model where the structural break occurs at a certain time. We are replacing time with γ.

Table 9: Sup Wald Tests

h	Rank[a]	Wald	γ	Borrowing ($\gamma \times h$)	Δlnc_{t+1}	ΔCA_{t+1}	# Episodes
2	1	386.6	7.1	14.2	-0.0508	3.70	15
	2	376.8	7.2	14.4	-0.0508	3.70	15
	3	363.9	7.3	14.6	-0.0508	3.70	15
3	1	352.2	7.9	23.7	-0.0479	3.51	10
	2	347.0	8	24.0	-0.0515	3.81	9
	3	338.5	8.1	24.3	-0.0515	3.81	9
4	1	320.3	6.1	24.4	-0.0472	3.81	11
	2	317.1	6.0	24.0	-0.0475	3.84	12
	3	316.0	6.2	24.8	-0.04729	3.81	11
5	1	416.0	5.7	28.5	-0.0469	3.18	9
	2	394.4	5.8	29.1	-0.0469	3.18	9
	3	381.2	5.9	29.0	-0.0514	3.09	8

Notes: Hansen (1997) sup Wald test for structural breaks using a regression of $\mu_1(\gamma, h)$ on a constant and γ. [a]: rank corresponds to the ordered values of the Wald statistics.

5 Structural model estimation

This section describes a canonical and parsimonious structural model to test the presence of overborrowing. We use the workhorse in the literature, a recursive version of the model in Bianchi (2011). The theoretical structure focuses only on the behavior of consumption and CA. Financial frictions are introduced by assuming incomplete capital markets and the presence of a collateral constraint. A crisis occurs when the level of debt hits the collateral constraint. We are aware of the existence of more complex and realistic models to represent the findings in this paper. For instance, using a recursive equilibrium, Mendoza (2010) highlights the importance of investment. Along the same lines, Mendoza and Smith (2006) takes into account the interaction between two assets in order to match the order of magnitude of the recession generated by a SS. On a more theoretical branch, Schmitt-Grohé and Uribe (2017) exploit the multiplicity of the sequential equilibria to generate more “variability” in the time series behavior of the simulated paths. However, considering the numerical burden implied by the test, we require a parsimonious model. The endogenous variables depicted in Bianchi (2011) are sufficient to verify the presence of overborrowing as we only need to model the behavior of the CA and the possibility of a crisis.

5.1 The model

The model is similar to Bianchi (2011), except in the characterization of the equilibrium, which is adjusted to improve the description of the calibration process. We will structurally test the presence of overborrowing as characterized in condition (ii) of the definition of SS in Section 2. Note that overborrowing may involve more than 2 consecutive periods, as persistence can vary between 2 and 5 years. Thus, we need to characterize the equilibrium sequentially, and not recursively as it is usually done in the literature.

We assume a small open endowment economy populated by an infinitely lived representative agent who consumes tradable and non-tradable goods, respectively c^T and c^N. The relative price of non-tradables is denoted by p. The agent can also borrow/save using an internationally traded asset b (if $b > 0$ we say that the agent holds net foreign assets), paying a constant interest rate, $r > 0$. This type of trade is restricted by a borrowing constraint. It is required that the ratio of debt to current income, measured in tradable units, must lie above $-\kappa < 0$ with $\kappa > 0$. The borrowing constraint is interpreted to be a collateral constraint and the pledgeable object is a fraction κ of income. Let $y^T + py^N$ be the total income in any given period, then the principal paid by the agent is given by $\max\left\{b, -\kappa(y^T + py^N)\right\}$. We assume that y=(y^N, y^T) follows a finite state Markov process with transition matrix q constructed using an auxiliary regression for each country in Table 6. In particular, we use a 2-variables (tradable and non-tradable sectors) VAR(1) model. The estimated coefficients appear in Appendix B.

Finally, preferences are standard in the literature: the inter-temporal problem is characterized by a CRRA instantaneous utility function with parameter $\sigma > 0$, while the intra-temporal problem assumes CES preferences with parameters $\eta > -1, \omega \in (0, 1)$.

The agent solves the following maximization problem:

Problem 1

$$\max_{\left\{c_t^T, c_t^N, b_{t+1}\right\}} E_0 \sum \frac{C_t^{1-\sigma} - 1}{1 - \sigma}, \tag{8}$$

subject to

$$C_t = \left[\omega \left(c_t^T\right)^{-\eta} + (1 - \omega)\left(c_t^N\right)^{-\eta}\right]^{-1/\eta}$$

$$b_{t+1} + c_t^T + p_t c_t^N = b_t(1 + r) + y_t^T + p_t y_t^N,$$

$$b_{t+1} \geqslant -\kappa(y_t^T + p_t y_t^N),$$

$$b_0 \in \mathbb{R}, y_0 \in \mathbb{R}_{++}^2,$$

where the second equation is the flow budget constraint and the third is the collateral requirement.

Now we can define a (sequential) competitive equilibrium. Randomness in this economy comes from the Markovian process that drives the endowments.[3]

Definition 1: Sequential Competitive Equilibrium (SCE)

A SCE for this economy is composed by 4 progressively $\mathcal{F}_t$-measurable functions $p, (c^T, c^N, b)$ such that:

- (a) Given p, (c^T, c^N, b) solves Problem 1.
- (b) For each $t, \omega_t \in \mathcal{F}_t$; $c^N(\omega_t) = y^N(\omega_t)$

To guarantee the compactness of the SCE, we must impose the existence of an upper bound on debt, $-D > 0$, which is never binding. Under this assumption, we can assure that for each $t, \omega_t \in \mathcal{F}_t$, $\left[p, c^T, c^N, b\right](\omega_t) \in K$ with $K \subset \mathbb{R}^4$ compact. In contrart with Bianchi (2011), we use the primal characterization of the sequential maximization problem in Definition 1 as it is easier to structurally define the moment to match, the hitting time of the collateral constraint. The remaining aspects of the model are as in Bianchi (2011) and, thus, we defer the details to the Appendix C.

[3]This guarantees the existence of an infinite horizon process for $\{y_t\}$, $(\Omega, \mathcal{F}, \mu_{y_0})$, where Ω is the sample space which lies in the space of infinite bounded sequences, $\mathcal{F}$ is the associated sigma algebra with filtration $\mathcal{F}_t$ and μ_{y_0} is the associated probability measure for a given initial condition, y_0 (see Stokey, Lucas and Prescott, 1989, ch.7).

Let Θ be the set of structural parameters in the system of equations. Our empirical identification strategy involves three conditions, listed in the definition of SS in Section 2. The first one, (a), considers the relative behavior of the treated. As the model described above is related with an emerging economy for each set of structural parameters $\theta \in \Theta$, we can only test the presence of those conditions that affect a treated country. Thus, it is only possible to test a *necessary condition* for the occurrence of a SS and a *sufficient condition* for overborrowing. We choose to test condition (b). Formally, we look for $\theta_i \in \Theta$ which satisfies:

$$CA_{i,s}(\theta_i) \equiv \frac{b_{s+1} - b_s}{p_s y_s^N + y_s^T} \leqslant -\gamma, \quad s = 1, ..., h, \tag{9}$$

where $\gamma > 0$ and we have omitted the subscripts in the RHS of eq. (9) for exposition purposes. Note that the above equation concerns with the realization of several random variables in the model. However, as suggested by the definition of SCE, these functions are generated by the underlying stochastic process $(\Omega, \mathcal{F}, \mu_{y_0})$ which in turn imply that we must look for a sequence of shocks $\{y_s\}_1^h$ capable of matching eq. (9). As we have to focus on the un-constrained regime, using the system of equations which characterize it (formed by (11),(13),(14) in the Appendix C), we can figure out the type shocks y that may generate (9). More to the point, the "block recursive" nature of the unconstrained sequential equilibria, which is described in the Appendix C, allows us to match separately the numerator and the denominator of eq. (9).

Finally, we now turn to the relevance of *hitting times* in the identification process. It is usual in the literature (see Bianchi, 2011, p. 3411) to say that a "crisis has occurred" if the collateral constraint binds (i.e. $b_{t+1} \leqslant -\kappa(y_t^T + p_t y_t^N)$). However, the literature is silent with respect to the pre-phase of a SS. By looking at the definition of the SCE, it is easy to see that we can define an event as the number of time periods τ between crises (i.e. the $t, \omega_t \in \mathcal{F}_t$ that generates $b_{t+1}(\omega_t) \geqslant -\kappa(y_t^T + p_t y_t^N)(\omega_t)$ with $t = 1, ..., \tau$). This sequence of shocks may generate different accumulated deficits depending on τ. We are interested in finding a CA deficit above the threshold γ for each possible persistence level h with $h \leqslant \tau$. Note that it is possible to observe a level of borrowing above the threshold but not a SS (i.e., the collateral constraint is not binding in the period which follows the last overborrowing period). We will say that there is a crisis for a time spell τ and persistence level h if we observe a CA deficit which is above $\gamma(h)$ in the last h periods of a τ spell.

5.2 Structural estimation procedure

To test the results in the empirical section, we build a random variable: the number of periods without crisis, that is, the time between hits to the collateral constraint. This variable contains our structural measure of persistence (h). As we cannot replicate the behavior of the control group (as per the d-in-d estimation above) in this class of models, we try to match the CA path for an arbitrary country in the treatment group. According to Table 6, we replicate the behavior of economies with $h = 2, 4, 5$ as these are the persistence levels found after applying conditions (i) to (iii) of our definition of SS.

We describe here the procedure that will be used to verify the presence of overborrowing for some empirically meaningful set of parameters $\theta \in \Theta$. We must first take care of a numerical procedure. The first choice, following the structural literature (see Bianchi, 2011; Mendoza and Smith, 2006), is to compute the model using recursive minimal state space representation. As we are dealing with a decentralized equilibrium with incomplete markets, we do not have welfare theorems. Thus, we must take care of the evolution of prices, in particular the real exchange rate, p.

As we have insured the compactness of the SCE, we can choose a set K which contains all possible values for the endogenous variables in the model. In particular, for each $t, \omega_t \in \mathcal{F}_t$, $\left[p, c^T, c^N, b\right](\omega_t) \in K \equiv P \times K_1 \times K_2 \times B$. Let p_t^0 be an initial guess for the price function with $p : B \times Y \to P$. Note that given p_t^0, the system of equations (11)-(14) can be solved for any possible values of $(b_t, y_t) \in B \times Y$, giving rise to a function $b_{t+1}^0(b_t, y_t; p_t^0)$, taking values in B. We call this object *Policy Function*. Moreover, after obtaining b_{t+1}^0, we can update p using eq. (12). Thus, we can define the following procedure:

Computable Recursive Equilibrium

- 1) Set B, Y, $\delta > 0$ and p_t^0.
- 2) Compute b_{t+1}^0 using equations (11)-(14).
- 3) Update p_t^0 using equation (12).
- 4) continue until $\sup \left\| b_{t+1}^{n+1} - b_{t+1}^n \right\| < \delta$.

Step 4) implies the existence of number $N(\delta)$ such that $n \geqslant N(\delta)$ implies:

$$\sup_{(b_t, y_t) \in B \times Y} \left\| b_{t+1}^{n+1}(b_t, y_t; p_t^{n+1}) - b_{t+1}^{n}(b_t, y_t; p_t^{n}) \right\| < \delta. \tag{10}$$

After achieving convergence in the above procedure, equipped with a policy and a price function, we can try to verify condition (9). In order to test it, we need to refine the definition of SS so it is possible to identify the requirements concerning each specific country. Those are the structurally testable conditions as we can only model the behavior within the treatment group. Let CA_t be the current account at time t, as a percentage of GDP, Y_t. We guess that the SS is associated with the magnitude of CA_t and its persistence. To verify this hypothesis, we split condition (i) in our definition in two: the first one will test the relative behavior of the treated and the second one will keep track of the time series performance of the affected countries. Thus, we have evaluated $E(\Delta ln(C_{t+1}))$, which is the average variation in logs of the relative consumption C, in the empirical section. This variable determines the threshold together with the treatment group. We take the expectations for a given combination of (γ, h), where γ represents the current account threshold, i.e. $CA_t < -\gamma$ and h is the number of periods for which the current account deficit was at or above that given value (i.e. the persistence of the deficit). Note, further, that the expectation is taken with respect to all the countries in the treatment group (i) and in the control (j) group. Finally, C_t is the ratio between the consumption in the treatment and in the control group.

That is, we look for events at time t satisfying:

Empirical Identification Strategy: Conditions

- (1.1) $E(\Delta ln(C_{t+1})) < 0$;
- (2.2) $\Delta ln(c_{i,t+1}) < 0$ for all i in the treatment group;
- (2) $CA_{i,t} < -\gamma, CA_{i,t-1} < -\gamma, ..., CA_{i,t-h+1} < -\gamma$ for all i in the treatment group;
- (3) $\Delta(CA_{i,t+1}) > 0$ for all i in the treatment group.

Table 6 contains the empirical results. For each of the countries i in this table we will look for the set of parameters $\theta_i \in \Theta$ such that condition (ii) in the empirical identification strategy (i.e. $CA_{i,t-s} < -\gamma(h)$ for $s = 1, ..., h$) is satisfied. We proceed in several steps for each persistence level h:

Structural Estimation Procedure

- i) Find the values $\theta_i \in \Theta$ that has the maximum level of accumulated borrowing in any pre-phase of length h for several simulations of length T.
- ii) For these values of θ_i, or any vector of parameters nearby, find if there is a longer pre-phase spell τ which contains the desired level of accumulated borrowing in the last h years.
- iii) For these values of θ_i, or any vector of parameters nearby, find if there is a sequence of shocks which contains the desired level of accumulated borrowing in h years.

In step iii), we are looking for a decreasing sequence of shocks and possibly a stagnation phase (see the characterization of the unconstrained regime in the Appendix C). However, Claessens and Ghosh (2016) found that in emerging economies recessions last 4 or eventually 8 quarters. These facts imply that we must restrict the sequence of decreasing shocks to a maximum of 3 periods as we are calibrating the model with yearly data. More to the point, some authors do not find a persistent stagnation process. Thus, we have to restrict simulations to $\{\Delta y_s < 0\}_{t+1}^{t+2}$ and $y_{LB} = y_{t+3}$, where s is any time period within a pre-phase of a SS of length $\tau \geqslant h$.

5.3 Results

In this section we present the results of the structural estimation procedure. We take 3 cases based on our empirical findings, from Table 6, and 1 from the systemic SS literature adjusted to our framework, presented in Table 7, i.e. Argentina 1998. As mentioned before, this procedure is designed to match only the overborrowing clause in the definition of SS, condition ii). We have a positive match for 2 out of 4 cases: Israel, with a persistence of 2 years, and Peru, with 4. However, there is no match for Honduras and Argentina. The difference between these groups of countries can be found in the income shocks persistence as estimated in the Appendix B. The positive results are observed in those countries with a high level of persistence in income.

Note that the results in Table 7 can be used to verify the accuracy of the numerical procedure. As we are modeling the SS in Argentina 1998, based on the results in Calvo et al. (2008), we can use this event to test the performance of the model in Bianchi (2011), who calibrates the model to Argentinean data. In particular, after checking if our simulations can replicate the results in Bianchi (2011) using the parameter structure in this paper, we can try to match the SS that hit Argentina in 1998 using the parameters set in that paper.

In order to select the countries in Table 6, we choose to keep 1 country for each persistence level h. Moreover, among those countries, we select the events that has a stationary bivariate $AR(1)$ process for (y^T, y^N) according to the results in the appendix. Some countries, even after de-trending the series, have an autoregressive coefficient above 1, a fact that does not allow us to discretize the process using the Tauchen (1986) procedure. Even if we can discretize a non-stationary process, using the results in Fella et al. (2019), for instance, the required compactness of the SCE will make simulations unsuitable.

Argentina 98, $h = 5$

The table below shows the parameters used in Bianchi (2011), the standard deviation (STD) of the ratio CA to GDP obtained using the simulations in the original article and using our results. Moreover, we present the STD computed from data in Bianchi (2011) and using our database.

Our results not only replicate Bianchi's procedure, but also our data generates a closer match.

We now proceed to test the presence of overborrowing in the SS that affected Argentina in 1998 as reported in the empirical section. We present

Table 10: Argentina 98, Accuracy of simulations

Standard deviations estimates: model and data

Model	Model in Bianchi (2011)	Data	Data in Bianchi (2011)
2.5	2.8	2.7	3.6

Common parameter values

β	κ	η	ω
0.91	0.32	0.21	0.31

the level of accumulated borrowing achieved by the simulations (B_i), the empirical threshold for this event (γ_i), the persistence level associated with this threshold (h), the empirical minimal borrowing to be matched ($\gamma_i \times h$) and the distance with respect to our findings using the d-in-d approach ($(\gamma - \gamma_i)/\sigma$). Finally, we evaluate if the simulations generate the reported level of borrowing B_i followed by a crisis, where we are looking for a binding collateral constraint in the period that follows the last overborrowing year.

The first step of our structural procedure is borrwed from the empirical section in this paper. The threshold γ and the persistence h were presented in Table 7, except for the last row, which contains the largest accumulated borrowing (12.2) and the associated threshold (0.8) that could be obtained using a "tailor made" simulated path according to step iii) in our structural procedure. In particular, we simulate a severe recession that lasted 2 years, a stagnation period and later on we took a path from the $AR(1)$ process calibrated to Argentinean data using the autoregressive coefficients and errors reported in the Appendix B. We found an overborrowing spell of 4 years, which was followed by a crisis (i.e. the collateral constraint was binding in

Table 11: Argentina 98, Estimation results

B_i	γ_i	h	$\gamma_i \times h$	$(\gamma - \gamma_i)/\sigma$	Step	Crisis?
6.7	3.0	5	15.0	0.7	1	YES
14.4	3.0	5	15.0	0.7	2	NO
12.3	3.0	5	15.0	0.7	3	NO
12.2	0.8/2.0	4	8.0	1.2	3	YES

the fifth period). Moreover, we are reporting the empirical threshold (−2.0) according to our data base and the associated borrowing (8.0) to be matched. Finally, we present the scaled deviation from the d-in-d approach for this event.

The results of the table indicate that: i) the model cannot generate an empirically meaningful CA deficit, ii) transitions to a crisis are in some cases "too fast" and in others "too slow". In particular, the first fact can be verified by looking at the maximal level of borrowing for a pre-phase of 5 years followed by a crisis (6.7% of the GDP, in the second row of the table). The empirical evidence suggests that this value must have been more than twice the reported figure (15%), even if we allow for some statistical error (i.e., 0.7%). The second fact can verify by looking at the remaining rows: when we are close to replicate the level of borrowing and the persistence level (third and fourth rows), we cannot generate any crisis. That is, convergence to the collateral constraint is "too slow". In the last row, we observe the opposite situation: the simulated threshold (0.8) is below the empirical one (2.0) which is not only quite distant to our findings (1.2 STD away from it) but also has lower persistence ($h = 4$ vs. $h = 5$).

To solve these problems, we: i) increase the value of the subjective discount factor β, ii) the fraction of pledgeable GDP κ will be calibrated. A higher β implies a steeper policy function for debt and thus bigger CA deficits in a recessionary path. Moreover, a "flexible" κ will allow us balance the trade-off between the frequency of the crises and the level of borrowing in the pre-phase of the SS. Thus, in the following 3 cases, those obtained using the d-in-d approach, we set $\beta = 0.965$ and let κ to adjust.

Israel 75, $h = 2$

The table below presents the results for Israel 1975. We show the selected parameters as well as the simulation results. In step i) we are reporting the average parameters for 5 ordered maximal values in terms of borrowing (i.e., B_i). We simulated a time series of 21,000 periods and kept only a spell of 2 periods without crisis.

We used the parameters in step i) as a starting point for the next strategy. In step ii) we are allowing the pre-phase τ to be equal to 50 (years) in a time series of length 100,000. As we are discarding the first 1000 simulations (the length of the simulated path is 101,000), we are assuming that the path has reached a stable equilibrium (i.e., it is ergodic).

Assuming ergodicity, we can interpret the simulated path as containing 2000 replicas of a spell of 50 periods without crisis. We found two SSs preceded by a similar level of overborrowing (16.1%). Both spells have a yearly deficit over the required level implied by the presence of a persistent threshold, 7.7% per year. Thus, the probability of observing this event is 0.1% (i.e., 2 over 2000 cases). That is, as it is known in the literature since

Table 12: Israel 1975, Estimation results

Step	κ	η	ω	σ	B_i	γ	h	$\gamma \times h$
1	1.00	0.97	0.31	1.89	9.3	7.7	2	15.4
2	0.71	1.34	0.37	1.89	16.1	7.7	2	15.4

Mendoza and Smith (2006), SS are rare events.[4] The value of κ, among others, balanced the trade-off between overborrowing and the occurrence of crises efficiently. Bianchi (2011) found a lower value for this parameter simply because the author wanted to match a more recurrent type of crises. In fact, the targeted frequency of a crisis in that paper was 5.5%.

These findings are not surprising as the type of event we are trying to match is constructed using the intersection of several definitions of a SS. As we are looking to match a considerable number of events using a parsimonious and tractable model, the scope of the definition must be reduced. However, the estimated model allows us to replicate more closely the observed ratio of external debt to GDP in Israel during that time. According to our estimates, this ratio, after netting out only international reserves, reached almost 200% of the GDP. Our estimate for κ, suggest that the level of net external debt oscillated around 70%; which seems more reasonable than the original 30% proposed by Bianchi (2011).

[4]These authors claimed that: "Precautionary saving (...) works to minimize the likelihood that these (SS) states are reached, since agents dislike large drops in consumption, and hence they wish to save and manage their portfolios so as to avoid frequent Sudden Stops. As a result, long run business cycle moments for economies with and without financial frictions display minimal differences and the long-run probability of observing binding margin requirements is relatively low, consistent with the fact that Sudden Stops are rare events". The preferences used in this paper, as it is usual in the literature, generates a precautionary savings motive.

Table 13: Peru 1977, Estimation results

Step	κ	η	ω	σ	B_i	γ	h	$\gamma \times h$
1	0.96	0.32	0.21	2.00	9.7	6.7	4	26.8
2	0.96	0.32	0.21	2.00	36.7	6.7	4	26.8

Note how the parameters change from step i) to step ii): basically, we only need a reduction in the elasticity of substitution (i.e., a hike in η), with respect to step i), generating a CA that is more responsive to price and income fluctuations. This parameter value coupled with a higher β, with respect to Bianchi (2011), allows us to capture the required level of borrowing. Moreover, κ is well above the estimated value in Bianchi (2011) (i.e., 0.32) as step ii) required 50 years without a crisis to accumulate a sufficiently high CA deficit (i.e., above 7.7% of the GDP).

Peru 77, $h = 4$

The table below describes the results for Peru 1977. We found not only that the model can replicate the dynamics of the pre-phase of the SS but also that the probability of observing this type of crisis is lower when compared to the case of Israel 1975: we simulate 5000 spells of 20 periods each and we only found 1 match. That is, for a persistence of 4 years, the probability is reduced to 0.02% (from 0.1% for a persistence of 2 years). This result is of course due to the restrictiveness of the conditions imposed to replicate the presence of overborrowing in Peru and is captured by a bigger value of κ (i.e., 0.96 for Peru and 0.71 for Israel).

Honduras 81, $h = 5$

The results for Honduras highlight the limitations of the canonical model to capture the presence of overborrowing. Even if we allow for a bigger pledgeable fraction of GDP, to generate a larger current account spell in the pre-phase, we could not match the stylized facts. By looking at the table below it is easy to see that: i) borrowing is insufficient, ii) the threshold found in the simulated current account spell is well below its empirical counterpart for countries with a similar persistence level (for $h = 5$, γ_i is on average of 0.68 STDs away from our estimated results as can be seen in Table 7).

Table 14: Honduras 81, Estimation results

Step	κ	η	ω	σ	B_i	γ	h	$\gamma \times h$	$(\gamma - \gamma_i)/\sigma$	Crisis?
1	0.92	0.69	0.26	2.00	7.1	6.2	5	31.0	NA	YES
2	1.00	0.69	0.26	2.00	26.4	6.2	5	31.0	1.26	YES
3	1.00	0.69	0.26	2.00	11.4	6.2	5	31.0	1.05	NO

In step 2 we have used a 30 years spell and the selected path in step 3 is a 2 years recession followed by stagnation.

5.4 Take away points

We claimed that the pre-phase of a SS must be generated by a canonical savings model. In this model, the system of equations has a “block recursive structure” because of the market clearing condition for non-tradables (see the definition of the SCE). In particular, the CA deficit, measured in tradables, must be generated by readable income shocks (see equation (15)) while the effect of consumption on prices (see equation (16)) feedback only on the CA to GDP ratio. Thus, the persistence of tradable income is critical to generate the overborrowing spell. More to the point, as during the pre-phase the collateral constraint is not binding, consumption is smoother than income which implies that the only way to generate 5 years of excessive borrowing is due to a strong and persistent recession.

Note that we have a positive match for Israel, with a persistence of 2 years and Peru, with 4. However, we fail to replicate a similar type of event in Honduras. The difference between these countries is in the matrix of auto-regressive coefficients (see Appendix B). While the norm for this matrix in Israel is 0.77, for Peru is 1.03 but for Honduras is 0.59. Thus, the persistence level of income is Israel is sufficient to match a spell of 2 years. The positive result in Peru is explained by the hike in persistence of income. However, the lack of inertia, especially in tradable income, in Honduras could not be compensated by any change in the parameter structure. This fact seems reasonable as only κ is related to the duration of the spell as it affects the frequency of crises. However, if κ is "too" high, we may have substantial accumulated borrowing not followed by a crisis as in Honduras.

We believe that it is matter of future research to provide a solution to the excessive dependence of the model on income shocks.

6 Conclusions

Persistent current account deficits are a recurrent phenomenon in a large fraction of low- and middle-income economies. Since these dynamics are usually followed by balance of payments crises, we answer the questions of when does a country have effectively overborrowed and if there is a cut-off value for an accumulated deficit just before the crisis sets off. These issues are key, as they may help to design macro-prudential policies by applying a rule that are easy to follow and to communicate.

Our paper is a contribution to the characterization of sudden stops. First, our empirical analysis shows that there is a robust overborrowing measure indicating that sudden stops can be associated with persistent current account deficits that accumulate about 26%-31% of GDP in a time span of 3 to 5 years. Second, the typical sudden stop event produces a drop in real consumption of about 4% of GDP and a current account reversal of 2.5%-4.5% of GDP. This finding is robust to different estimates of the threshold. Our results provide a potential measure of external fragility that we dubbed as the empirical dimension of *overborrowing*.

To check the conceptual features of overborrowing, we test these empirical findings in a standard structural model due to Bianchi (2011). The model results allow us to compute the maximal value of net external indebtedness that a country can reach before being hit by a balance of payments crisis. According to our calculations, this parameter is two or three times bigger than the benchmark value normally employed in the literature. Considering the restrictive nature of our definition, a sudden stop preceded by overborrowing is a rare event, which implies a somewhat “loose” borrowing limit. This last fact implies a better match of the model taking into account the observed level of net external indebtedness.

Our results may give useful advice to policy makers. Many “stress” indicators displaying debt and financial conditions has been developed to anticipate a potential hit of a threshold that would trigger a sudden stop. However, all these indicators are usually empirically weak and/or rarely based on formal theoretical ground. We obtained a simpler measure, which also happens to be easy to understand. Our findings can help elaborate better leading indicators to anticipate a sudden stop and eventually avoiding the awful consequences of a crisis. In practice, after significant external debt issuance, local authorities may consider taking measures to tackle persistent current account deficits that are near to the estimated threshold. Specifically, early policies intended to adjust the current account before the threshold has been reached can help prevent a much more drastic and painful adjustment later. Sudden stops, it must be recalled, do not just bring a deep drop of real consumption, but also make future borrowing less likely. Fast accumulation of current account deficits or debt issuance severely increase the likelihood of a sudden stop, and so authorities must be aware of reaching the 26%-31% of GDP threshold in the time span of 3 to 5 years.

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Appendix A: Characterizing selected SS episodes

Here we describe the 8 episodes presented in Table 6.

Israel 1975, $h = 2$

In close relationship to a huge and systematic US capital inflow, Israel started in the late 60s to experience many years of CA deficits. This deficit was particularly high in 1975, when it reached almost 12% of GDP. This path suffered a drastic change in 1976, when the current account had to adjust by almost 10 percentage points and consumption fell by 3% in the following two years.

Even though the Yom Kippur War was in part responsible for this crash, the main shock was due to the hike in oil prices back in 1973. After those years, inflation accelerated in the 1970s, rising steadily from 13% in 1971 to 111% in 1979. The economy experienced "stagflation" and the government was forced to postpone its full-employment policy until the 1980s. But inflation kept gathering pace and explode into an hyperinflation at mid-80s. From the sudden stop of 1975 up to 1985, nominal and real instability ruled the economy.

Panama 1981, $h = 4$

Panama experienced permanent CA deficits since 1970 averaging 7.7% of GDP. In 1981 the external deficit soared to 10.2% and accumulated 25 percentage points in just 3 years. One year later, consumption adjusted downwards by almost 4%. There were at least three reasons for this hard landing.

First, the extreme vulnerability in external and fiscal accounts observed in Central America in general, and Mexico in particular (a key trade partner). Second, the oil crisis that triggered a brutal upwards adjustment in international interest rates. Third, the unexpected death of president Torrijos that unraveled a severe political crisis.

In 1981 the external sector suffered a drastic reversal, accumulating a contraction in the current account deficit of 13% of GDP in the two following years. Consumption growth returned in 1983 and 1984, but Panama could not fully recover from the crisis until the 90s.

Peru 1977, $h = 4$

The government of President Velasco inherited recessionary conditions in 1968, and soon started to boost aggregate demand. Thus, there was an increasing CA deficit reaching a peak of 12% of GDP by 1975. The following two years continued with this trend: 10% more in 1976 and another 8.5% in 1977. Due to a fixed exchange rate regime, in an increasingly inflationary context, the real exchange rate fell steadily up to 1975. By 1975, external creditors had lost confidence in Peru's ability to repay its debts and capital inflows slowed down.

Whether because of such external pressure or because of growing internal opposition to the increasingly arbitrary decisions of the government, the Peruvian military government decided to replace Velasco in 1975. The Government kept the deficit running, but in 1978 capital inflows stopped suddenly and the external sector had to adjust by 6% in that year, and by an additional

7.5% the following year. The corresponding adjustment in consumption was by -9% and -2% each year.

Costa Rica 1981, $h = 5$

The economic crisis in Costa Rica between 1980 and 1982 was triggered by a serious financial collapse that hit the Central American country during the administration of Rodrigo Carazo Odio. This event is considered one of the worst economic crises in the country's history. The Costa Rica economy, which at that time depended almost exclusively on coffee and banana exports, faced a sudden drop in international export prices. The conflict with Nicaragua, at that time involved in a civil war and during which the Costa Rican government supported the Sandinista rebels, caused further tensions between countries, in particular with the Somoza regime and United States.

The oil crises added problems to energy imports. Therefore, the Costa Rican fiscal deficit rose to 11% and forced the country to default loans granted by international organizations, an event that quickly cut off access to financing. During this period unemployment doubled from 4.4% in 1979 to 9.4% in 1982, and the crisis brought also shortages of basic products, hyperinflation and high popular discontent.

The Costa Rican external sector showed permanent CA deficits since the 60s (or more accurately, since data can be gathered). But the deficit increased quickly since 1978, accumulating more that 50% of GDP in just four years. In 1981, the crisis exploded and consumption started to fall. In that year, private expenditure fell by almost 13%, and another 6% in 1982.

Spain 2008, $h = 4$

During the subprime crisis in 2007 and the bankruptcy of Lehman Brothers a year later, the "Great Recession" began. Inside the Eurozone, the crisis hit harder on economies with real state bubbles and a large financial sector, as was the case of Spain. Although 2008 began with a low public sector debt (about 36% of GDP), this figure was the result of an artificially high tax revenues resulting from the housing bubble. Spain was a "peculiar" case in Europe because of the significant social costs associated with the collapse: total unemployment peaked almost 27% in the first quarter of 2013, and youth unemployment was reported to be 50%.

As the time of the crash, the economy as a whole was experiencing a huge CA deficit, around 10% of GDP. Moreover, in the previous 4 years Spain had been accumulating 35 percentage points of external deficits in terms of GDP. Without the possibility of adjusting the real exchange to restore competitiveness, Spain suffered not just a SS, but also a painful adjustment in the four years that followed. That is, Spain suffered in 2009 a contraction of real consumption of 1.7%, contrasting with an average growth of 4% since the creation of the Euro until 2007. The CA deficit experienced a sharp reduction of 5 percentage points of GDP in 2008, but the process continued until 2012, when the CA finally balanced and Spain restored growth at a slow rate.

Portugal 2008, $h = 5$

The case of Portugal is somewhat different from Spain. Although the crisis hit both economies hard, Portugal could not take advantage from the Euro arrangement in the years following 2000, as Spain clearly did. From 2001 to 2007 Portugal grew at a rate of barely 1.1% a year on average. Even that, the economy was not immune to the 2008 events that worsened the situation for the Portuguese economy. The failure to exploit the advantages of the Euro was due primarily to a competitiveness problem: when entering the Eurozone Portugal had already a 10% CA deficit, a fact that could not be corrected along the Euro-partnership decade. The adjustment finally started in 2008 and intensified severely since 2011.

In 2008 Portugal suffered its first shock, and the following year consumption growth in real terms was -1.2% of GDP. This was the first negative figure 1993, the year when the Exchange Rate Mechanism collapsed. After a slight rebound in 2009, consumption fell again for three consecutive years at a -3.3% pace. The CA began its adjustment in 2009 by almost 2 percentage points of GDP, but the process was much more severe since 2011, with the external gap closing in just two years, starting from a 10% deficit.

Greece 2008, $h = 5$

The case of Greece was dramatic, mainly because of what happened since 2009 on. During the golden years of the Euro, the country grew at an impressive rate of 4% on average, which allowed an average of 3.8 annual percent of consumption progress. However, between 2000 and 2009 Greece showed a

CA deficit of 8.8% of GDP on average, with a 2008 peak of 14.5%. Although it was clear that there was a problem with the official fiscal figures, which were found to be false, the external gap was clear to everyone.

In 2009 the SS hit the country and Greece started a painful adjustment that keeps going on today. The Greek crisis was triggered by the sovereign debt crisis. Without the possibility of an exchange rate adjustment, European authorities and the IMF came to the rescue and urged strong austerity measures. Far from recovering confidence, this agenda produced a huge downturn. In 2009 the CA deficit had to adjust by almost 5 percentage points of GDP and the following year consumption experienced a contraction of almost 6% in real terms. The process did not stop there, and the country continued a recessive path until very recently. Just to illustrate the point, in 2016 GDP was 25% below the last peak, that was attained in 2007. Social and political disturbances followed. The seasonally adjusted unemployment rate grew from 7.5% in September 2008 to a then record high of 23.1% in May 2012, while the youth unemployment rate time rose from 22.0% to 54.9%.

Honduras 1981, $h = 5$

In late 1979, as insurgency spread in neighboring countries, and Honduran military leaders support the United States policies in the region, a fact that implied a decisive financial support. Honduran defense spending rose throughout the 1980s until it consumed 20 to 30% of the national budget and the country soon became the tenth largest recipient of United States assistance aid.

The increasing dependence of the Honduran economy on foreign aid was aggravated by a severe, region wide economic decline during the 1980s. As a consequence, private investment plummeted and capital flight soared. To make matters worse, coffee prices plunged on the international market and remained low throughout the decade. The CA went from an average deficit of 9.4% between 1974 and 1981 to -3.3% between 1986 and 1990. Contrarily to the remaining countries in the treatment group, consumption in Honduras did not experienced a drastic reduction. However, there was a severe drop in investment (12% in 1982) that affected GDP (-1.5%).

Appendix B: Correlation Matrices

The estimates below correspond to a 2-variables (tradable and non-tradable, logarithm of real product) VAR(1) model. Matrix ρ contains the autoregressive parameters and matrix V the variance-covariance matrix of reduced form residuals.

$$\rho_{Israel\,75'} = \begin{bmatrix} 0.393 & 0.114 \\ -0.077 & 0.758 \end{bmatrix} \qquad V_{Israel\ 75'} = \begin{bmatrix} 0.191 & 0.020 \\ 0.020 & 0.094 \end{bmatrix}$$

$$\rho_{Panama\ 81'} = \begin{bmatrix} 0.536 & -0.002 \\ -0.106 & 0.819 \end{bmatrix} \qquad V_{Panama\ 81'} = \begin{bmatrix} 0.115 & 0.064 \\ 0.064 & 0.190 \end{bmatrix}$$

$$\rho_{Peru\ 77'} = \begin{bmatrix} 1.001 & -0.234 \\ 0.143 & 0.687 \end{bmatrix} \qquad V_{Peru\ 77'} = \begin{bmatrix} 0.213 & 0.173 \\ 0.173 & 0.178 \end{bmatrix}$$

$$\rho_{Spain\ 08'} = \begin{bmatrix} 1.038 & -0.176 \\ 0.271 & 0.668 \end{bmatrix} \qquad V_{Spain\ 08'} = \begin{bmatrix} 0.059 & 0.059 \\ 0.059 & 0.046 \end{bmatrix}$$

$$\rho_{CostaRica\ 81'} = \begin{bmatrix} 1.055 & -0.358 \\ 0.253 & 0.569 \end{bmatrix} \qquad V_{CostaRica\ 81'} = \begin{bmatrix} 0.148 & 0.075 \\ 0.075 & 0.066 \end{bmatrix}$$

$$\rho_{Greece\ 08'} = \begin{bmatrix} 0.806 & -0.004 \\ 0.250 & 0.740 \end{bmatrix} \qquad V_{Greece\ 08'} = \begin{bmatrix} 0.110 & 0.052 \\ 0.051 & 0.086 \end{bmatrix}$$

$$\rho_{Honduras\ 81'} = \begin{bmatrix} 0.574 & 0.120 \\ 0.050 & 0.155 \end{bmatrix} \qquad V_{Honduras\ 81'} = \begin{bmatrix} 0.133 & 0.035 \\ 0.035 & 0.067 \end{bmatrix}$$

$$\rho_{\,Portugal\ 08'} = \begin{bmatrix} 0.808 & -0.208 \\ 0.191 & 0.623 \end{bmatrix} \qquad V_{Portugal\ 08'} = \begin{bmatrix} 0.115 & 0.033 \\ 0.033 & 0.090 \end{bmatrix}$$

Appendix C: Characterization of the theoretical model

Characterization of the theoretical model, Definition 1

We are allowed to characterize a (compact) SCE by means of a sequence of FOCs. We choose a primal version of the Karush-Kuhn-Tucker conditions because of their simplicity. We look for a system of equations that characterize definition 1. Thus, for each $t, \omega_t \in \mathcal{F}_t$, $[p(\omega_t), y(\omega_t), b(\omega_t)]$ must satisfy:

$$\left[\left[X_t(p_t, b_{t+1}) - \beta(1+r)E_t(X_{t+1}(b_{t+1}))\right]\left[b_{t+1} + \kappa(p_t y_t^N + y_t^T)\right]\right](\omega_t) = 0 \tag{11}$$

$$p_t = \left(\frac{1-\omega}{\omega}\right)\left[\frac{(1+r)b_t + y_t^T - max\left\{b_{t+1}, -\kappa(y_t^T + p_t y_t^N)\right\}}{y_t^N}\right]^{(1+\eta)} \tag{12}$$

Eq. (11) characterizes the inter-temporal behavior and it is composed by 2 brackets, the Euler equation and the collateral constraint, both greater or equal than zero. Eq. (12) controls the inter-temporal behavior. The preference structure implies that $X_t(\omega_t)$ for each $t, \omega_t \in \mathcal{F}_t$ must satisfy:

$$X_t(\omega_t) = \left[\left[\omega\left(c_t^T\right)^{-\eta} + (1-\omega)\left(y_t^N\right)^{-\eta}\right]^{\frac{-1+\sigma-\eta}{\eta}}\left(c_t^T\right)^{-\eta-1}\right](\omega_t) \tag{13}$$

$$c_t^T\left(\omega_t\right) = \left[(1+r)b_t + y_t^T - \max\left\{b_{t+1}, -\kappa(y_t^T + p_t y_t^N)\right\}\right](\omega_t) \qquad (14)$$

Eq. (11) gives rise to 2 different regimes: one characterized by a constrained level of debt, which is related to the occurrence of a balance of payment crises, and one that is unconstrained. As we are interested in the pre-phase of the crises, we must focus on the former. Thus, in order to match the facts found in the empirical section, we must concentrate on the non-binding version of the system of equations above. In this sense, it is convenient to characterize the SCE using the primal version of optimality because of at least two reasons: i) the Lagrange multiplier associated with the collateral constraint will not be needed, 2) the system formed by equations (11) to (14) can be decomposed in two parts, where the first (eqs. (11),(13),(14)) determines the level of assets. Thus, the equations above can be written in a "block recursive" style where the first part determines b_{t+1} and the second p_t. As our definition of overborrowing only concerns the behavior of consumption and CA, we focus on the first part.

Characterization of the unconstrained regime of the model

Formally, if the collateral constraint does not bind, we have:

$$X_t(y_t^T, b_{t+1}) = \beta(1+r)E_t(X_{t+1}(b_{t+1})) \qquad (15)$$

$$p_t = \left(\frac{1-\omega}{\omega}\right)\left[\frac{(1+r)b_t + y_t^T - b_{t+1}}{y_t^N}\right]^{(1+\eta)} \qquad (16)$$

Note that eqs. (11),(13),(14) imply that the Euler equation, (15), is independent of y_t^N and p_t. Thus, we have highlighted the dependence on y_t^T and b_{t+1} in equation (15). This equation will allow us to match the numerator of (9). In particular, the convexity of preferences implies that any *decreasing* sequence $\{y_s^T\}_1^h$ will generate a *decreasing* sequence $\{b_{s+1} - b_s\}_1^h$. That is, a recession caused by tradable output implies a decline in net external assets, which in the model is associated unequivocally with a downturn in the CA as we are not considering valuation effects. Taking into account the extensive numerical burden of the structural test, we must discretize the state space of Y. Thus, the number of possible elements in $\{\Delta y_s\}_1^{h-1}$ is limited by the cardinality of Y. Fortunately, the bounded nature of the SCE can be used to solve this problem. Suppose that the optimal choice in time $s-1$ is $\hat{b}_s$ and that $y_s^T = y_{LB}^T$, where y_{LB}^T is the lower bound in Y^T with $Y = Y^T \times Y^N$. Thus, we have:

$$X_s(y_{LB}^T, \hat{b}_s) > \beta(1+r)E_t(X_{s+1}(\hat{b}_s))$$

The above equation implies $\hat{b}_{s+1} < \hat{b}_s$. Thus, we look for a *weakly decreasing sequence* $\{y_s^T\}_1^h$ to replicate an adequate behavior in the numerator of (9). Of course, we will have $\hat{b}_{s+1} < \hat{b}_s$ until we hit the collateral constraint in period, say, τ.

As regards the denominator, note that: i) typically, (y^T, y^N) will be positively correlated (this will be shown in the next subsection), ii) p is *increasing* in c^T. Thus, *even for the same value of* $b_{s+1} - b_s < 0$, the ratio will be *decreasing* as GDP measured in tradables will go down if $y_{s+1}^T \leqslant y_s^T$.