An Ergodic Theory of Sovereign Default

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Research Questions

- What are the effects of private debt in an economy subject to default risk?
  - *Due to pecuniary externalities,* in a small open economy, private external debt can destabilize the decentralize economy. Private external debt is unstable or unsustainable.
- What is the role of the Government?
  - It can *globally* stabilize an inherently unstable economy using default as an instrument, affecting it *permanently*. With capital controls, authorities act exante. With default, they act expost.
- We show that default can be used as an stabilization policy. By ruling out explosive / unstable paths, the Government increases welfare.

Outline

- Motivation
  - Relevance of private external debt
  - Local vs Global moments
- Preview of the main results
- Model
- Results
  - Theoretical
  - Calibration-Numerical
- Conclusions

Preliminary and distinctive elements

- Terminology
  - Constructive existence: proof + algorithm
  - Stationarity: time-independence (state space and policy functions)
  - Ergodicity: LLN + global stability of the stochastic steady state.
- Distinctive elements
  - Non-uniform convergence in the initial condition of the operator ($\neq$ CMT): order theory, default punishment
  - Discontinuities ($\neq$ SLP (1989)): atoms
  - Centralized default-decentralized debt ($\neq$ Arellano (2008)): two-step operators, pecuniary externalities, b and B.
  - Stochastic discrete time phase diagrams: qualitative characterization of global dynamics.

Motivation

Relevance of private external debt

Table 1: External debt in Argentina and Chile (2021)

bln of Dollars	$Public_{PV}$ (I)	$Public_{IO}$ (II)	I+II	*Private* (III)	I+III	III/(I+III)
Argentina	87	74	161	73	160	46%
Chile	46	0	46	192	238	81%

Note: "$Public_{PV}$" refers to public external debt with private lenders, "*IO*" stands for international organisations, "I+ II" is total public debt. *Private* is private external debt. "I+III" is the total debt subject to default, which excludes debt with IO. The last column computes the share of private debt to total defaultable debt. Source: Central Bank of Chile and National Institute of Public Statistics (INDEC) for Argentina.

Motivation

Local vs Global moments

Table 2: Global and local moments for Argentina

CA/GDP		External Assets	
1983-2001	1960-2017	1983-2001	1960-2017
-2.4%	-0.7%	-36.3%	-25.7%

Note: CA/GDP stands for current account to GDP. Net external assets refer to the overall figure, which includes total public and private debt, including international organisms. In the body of the paper we present evidence in favor of these differences for private and defaultable external debt.

Relationship with the literature

- Main contribution: derive an ergodicity result in discontinuous small open economies with financial frictions in the sense of Kim and Zhang (12). As ergodicity is connected with global stochastic stability, we also provide a novel rational behind default: it stabilizes an inherently unstable economy.
  - Ergodicity: SLP (89) continuity, DGMM (94) sunspots, Pierri and Reffett (2023) expanded state space. *We don't need sunspots or enlarged state space. Constructiveness (accuracy) and efficiency.*
  - Interpretation: Zame (93) completes markets. *We prove that default is an stabilization policy. Permanent-long run implications of default risk.*
  - Constructive existence: Aguiar and Amador (19), Feng and Santos (21) but *no stability or ergodicity. Local vs global stylized facts.*
  - Financial Frictions, Bianchi(11): deleverage is induced by a crisis generated by an OBIC (collateral). As default the threshold is endogenous, *default risk has an important role in BOP crises.*

Theory

Basics of the model

- Small open economy subject to (endogenous) default risk.
- Risk averse households:
  - issue one-period real foreign non-contingent net external assets,
  - consume, and
  - receive an endowment.
- There is a unique good (no real exchange rate fluctuations).
- Households take prices as given. They *do not internalize the consequences on interest rates of a change in debt levels. This generates unstable / unsustainable paths in the absence of default*.

Theory

Basics of the model

- International investors are deep-pockets, risk neutral agents.
- Benevolent government that every period decides to allow the private sector to repay the foreign debt or forces a default.
- The Government:
  - eliminate excessive borrowing by forcing the private sector to default,
  - optimize *centralized default with decentralized borrowing*,
  - defaults through a drastic change in the economic environment surrounding private indebtedness (i.e., a depreciation).
- Decentralized borrowing and pecuniary externalities gives a rational for Government intervention.

Theory

Equations

- Private optimization: choose b_+, given b, B, y and h

$$V^c(b, B, y; h) = \max_{b_+ \geq -\bar{b}} u(F(b, B, y) - b_+) + \beta \mathbb{E}[V(b_+, h(B, y), y'; h)]. \tag{1}$$

$F(b, B, y) = y + bR(B)$, h is the aggregate law of motion for assets B_+.

- Default decision.

$$\textit{Default if} \quad V^c(B, y) \leq V^{def}(y). \tag{2}$$

Theory

Equilibrium Equations

- $b = B$,
- h determines B_+, $b_+ = B_+$ and both depend on B, y
- Repay

$$V^c(B, y) = u(y - B_+ + BR(B)) + \beta \mathbb{E} \max\{V^c(B_+, y'), V^{def}(y')\} \quad (3)$$

- Default - Exclusion

$$V^{def}(y) = u(y^{def}(y)) + \beta \mathbb{E}\{\theta V^c(0, y') + (1 - \theta) V^{def}(y')\} \quad (4)$$

Where θ is the probability of gaining access to the market.

Theory

Pricing Equation

- Interest rate (i.i.d. shocks)

$$R(B') = R^* \left[\sum_{y' \in [Y_{LB}, Y_{UB}]} \pi(y') \mathbb{I} \left\{ V^c(B', y') \geq V^{def}(y') \right\} \right]^{-1} \tag{5}$$

$\mathbb{I}$ is an indicator function and R^* is the gross international risk free rate.

- Default sets

$$\begin{Bmatrix} \text{Repay} & \textit{if } B \geq \bar{B}(y) & \text{as this implies} & V^c(B, y) \geq V^{def}(y) \\ \text{Default} & \textit{if } B < \bar{B}(y) & \text{as this implies} & V^c(B, y) < V^{def}(y) \end{Bmatrix} \tag{6}$$

Results

Theory - Operator

Definition [Nested two-steps fixed Point Operator]

We prove the existence of a stationary equilibrium using the operator:

- Coleman-Reffett. Given R, The Euler equation generates an operator A with $c_n \to Ac_n = c_{n+1}$.
- Aguiar-Amador.
  - Equations (3) and (4) induce an operator $\mathbb{T}$ with: $\left[V_j^c, V_j^{def}\right](c_n) \to \mathbb{T}\left[V_j^c, V_j^{def}\right](c_n) = \left[V_{j+1}^c, V_{j+1}^{def}\right](c_n)$.
  - This operator has a fixed point $\left[V_*^c, V_*^{def}\right](c_n)$.
- Equations (2) and (5) update $R\left(\left[V_*^c, V_*^{def}\right](c_n)\right)$.
- The Coleman-Reffett operator updates c using $R\left(\left[V_*^c, V_*^{def}\right](c_n)\right)$.
- Continue until convergence with $R\left(\left[V_*^c, V_*^{def}\right](c_*)\right) \equiv R_*$ and c_* is a fixed point of A.

Results

Theory - Existence Theorem

Theorem 1 [Existence of stationary computable equilibria (SCE)]

Assuming:

- finite i.i.d shocks (we only need this for ergodicity),
- u is increasing, concave and satisfy Inada,
- $\beta R < 1$,
- a restriction of the default penalty to be presented below,

there exist at least 2 SCE, $H(\bar{c}_0)$ and $H(\underline{c}_0)$, with $c_*(\underline{c}_0)(B, y) \leq c_*(\bar{c}_0)(B, y)$ for all $B \in \mathbb{B}$ given y.

Results

Theory - Existence Theorem

- As $\beta R < 1$, $c_*(\bar{c}_0)(B, y)$ is a strictly positive fixed point.
- The private economy is continuous. Thus, adding sufficiently curvature to u, such that A maps up from $\underline{c}_0$, we can prove uniqueness $c_*(\underline{c}_0)(B, y) = c_*(\bar{c}_0)(B, y)$.
- Equation (5) may generate multiple ordered and possibly discontinuous equilibria for the Government. We assume that the Government picks the highest in terms of welfare. This is implemented every time the Government updates R after solving the second step fixed point problem, before starting again the first step.
- Thus, we prove that the SCE is *bounded a.e., constructive and unique*.

Results

Theory - Endogenous default penalty

- Constructiveness is not uniform (as in the CMT). Thus, we need to carefully pick the initial condition of the operator $R_0, c_0, V_0^c, V_0^{def}$.
- This condition must be consistent (i.e., $R_0 = R^*$, $V_0^c \geq V_0^{def}$, and c_0 is an extreme element in the space of candidate consumption functions). Thus, we need an additional endogenous variable, y^{def}.

Let $\mathbb{C}$ be the space of consumption functions and $\mathbb{B}$ the compact set containing any B. Let $c_0 \in \mathbb{C}$. Then $y^{def}(y)$ with $y \in Y$ satisfies:

1. $V_0^c(B, y) = u(c_0(B, y)) + \beta \mathbb{E}\left\{V_0^c(y + R^* B - c_0(B, y), y')\right\}$
2. $V_0^c(B, y) \geq V_0^{def}(y) =$ $u(y^{def}(y)) + \beta \mathbb{E}\left\{(1-\theta) V_0^{def}(y') + (\theta) V_0^c(0, y')\right\}$ for all $y, B \in Y \times \mathbb{B}$.
3. c_0 satisfies $\bar{c}_0 = SUP(\mathbb{C})$ or $\underline{c}_0 = INF(\mathbb{C})$.
4. $y \geq y^{def}(y)$ for all $y \in Y$

Results

Theory - Implications of y^{def}-OBIC

- The definition of defaults sets, equation (6), implies that default is triggered by an occasionally binding price dependent inequality constraint (OBIC).
- We can connect the theory of default with the theory of BOP crises as (6) triggers as a sudden stop, deleverage and Fisherian deflation happens instantaneously.
- Default risk and pecuniary externalities generate BOP crises due to the inherent instability of private external debt.
- For certain levels of GDP or debt, autarky is welfare improving as it prevents explosive debt paths.
- Novel rationale behind BOP crises - Sudden Stops

Results

Theory Ergodicity

If we assume that:

- there is at least 1 non-stochastic steady state (i.e., $B^N = b_+(B^N, y_{LB})$),
- and that y^{def} is independent of y,

we can prove ergodicity.

Theorem 2 [Ergodicity and Global Stability]

Assume that there exist a unique compact SCE and there exist y^{def} such that $B^N(y_{LB}) < \bar{B}(y^{def}) < 0$. Then the SCE is globally stable with *a unique ergodic probability measure* μ_*^{def}.

Results

Theory - Main

- This paper is the first one to propose a global constructive ergodic theory of default risk.
- In the calibrated model we found that there are 3 type of paths:

  – non-sustainable. They are observed when the economy has both low assets and low GDP.

  – sustainable and stable.

  – sustainable and unstable. They are observed when there is a low level of external assets but a high level of GDP. Instability is connected concave policy function and net external debt.

Results

Stability and Ergodicity- Phase diagram

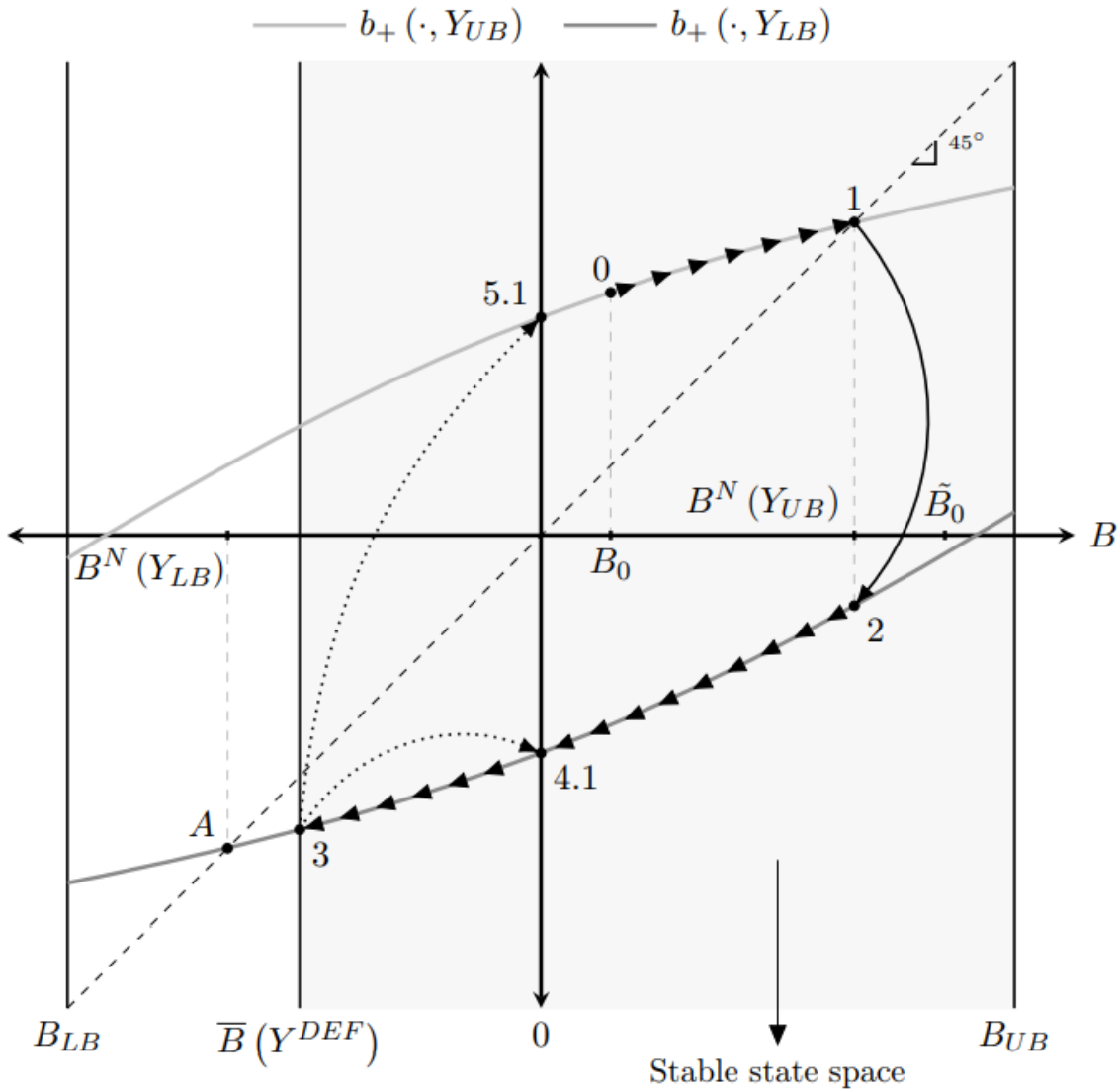

Figure 1: Non-empty and (saddle path) stable default set

Results

Calibrated Phase diagram

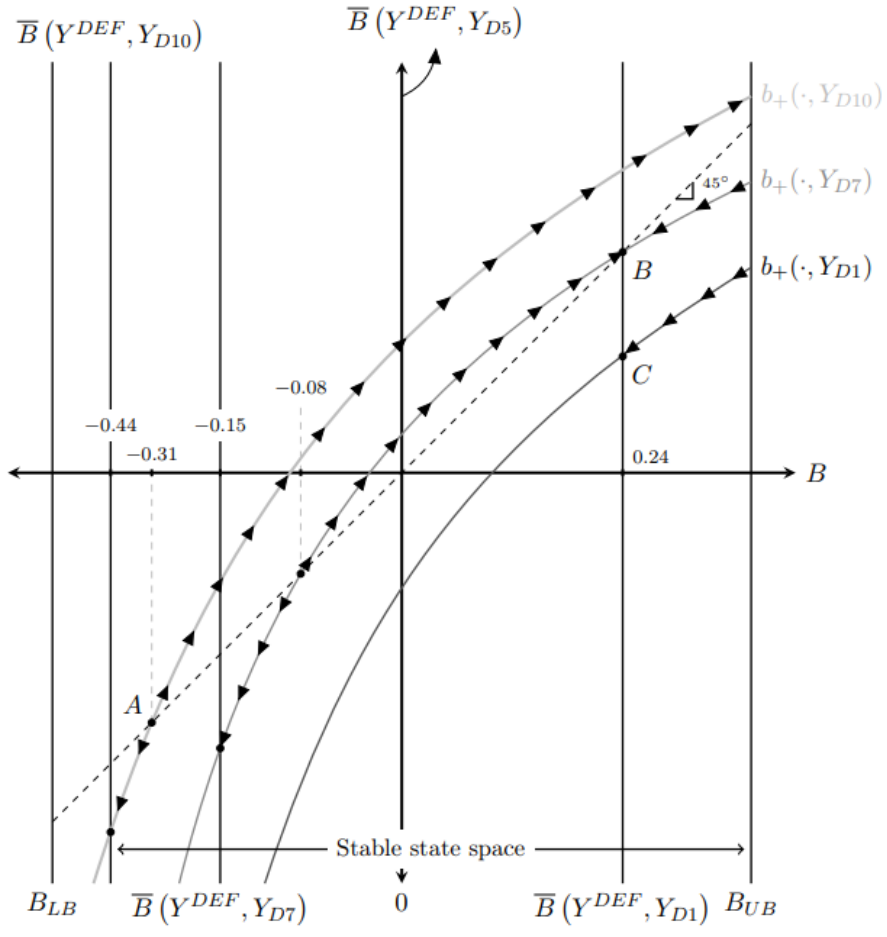

Figure 4: Calibrated phase diagram ($\mu = -2.0\%$, $STD = 0.08$) part A

Note: The phase diagram in this picture follows from the model calibrated to Argentina with the calibration described in Section 4.

Results

Calibration

Table 3: Results

Variable	$(R(B)B)/Y^*$	Def. freq.*	B/Y	CA/Y	$C.V.(CA/Y)$
Data	-0.6%	3.0%	-1.4%	-0.8%	3.6
Model	-0.6%	2.4%	-2.0%	-1.3%	4.2

Note: * denotes moments that are matched using the simulated method of moments. The rest of the statistics are non-targeted moments. $(R(B)B)/Y$ are (yearly) interest payments of private external debt with respect to GDP. "Def. freq." is the frequency of default for events that were preceded by 19 years (between 1983 and 2001) of open access to the international credit markets. B/Y are yearly capital payments (i.e., amortizations) of foreign private debt over GDP. CA/Y is the current account to GDP and C. V. is the coefficient of variation of CA/Y, its standard deviation divided by its mean.

Summary of the main results

- Stationarity (Markov exogenous shocks).
  - New 2 step operator (Coleman-Reffett + Aguiar-Amador).
  - Default costs are endogenous and act as a selection mechanism.
  - Provide sufficient conditions to prove uniqueness and compactness.
- Ergodicity (i.i.d. exogenous shocks)
  - It can be at most 1 default, identified with the value of GDP at the event, per ergodic distribution. Once and for all decisions have permanent effects.
  - The unique ergodic distribution is at the same time a globally stable stochastic steady state.
- Computation and simulation based estimation
  - Constructive proofs in minimal SS generate an efficient algorithm.
  - In the ergodic model, accuracy of simulations and global stochastic dynamics go hand-in-hand.

Final slide

Conclusions

- We present the conditions under which the dynamics of a sovereign default model of private external debt are stationary, ergodic and globally stable.
- Constructive results: the model can be used for the accurate computation of global long run stylized facts.
- Default can be used to derive a stable unconditional distribution, one for each possible event, which in turn allows us to characterize globally positive probability paths.
- We show that the stable (stochastic) steady state and the ergodic distribution are actually the same object.
- We found 3 type of paths: non-sustainable and sustainable; among this last category trajectories can be either stable or unstable.
- In the absence of default, non-sustainable and unstable paths generate explosive trajectories for debt. The government can use the default of private external debt as a stabilization policy.

Final slide

THANKS!

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Back up slides

APPENDIX

Results

Theory - Main

- There are sustainable but unstable paths associated with a negative net external position and rapid changes in this position (non-concavity). We numerically found that even if GDP is above the median, paths can be unstable even though they are sustainable.
- The distinctive characteristic of unsustainable paths is the absence of a non-stochastic steady state.
- In the absence of default, non-sustainable and unstable paths generate explosive trajectories for debt. *By deriving the notion of stable state space*, a subset of the compact state space, we show that the Government can use default as a stabilization policy.

Results

Theory - Main

- We show that the presence of default can be used to derive a stable unconditional distribution, *one for each possible event*, which in turn allows us to *characterize globally positive probability paths*.
- We show that the stable and the ergodic distribution are actually the same object. Thus, we suggest a potential answer for the role of default in open economies: we show that private external debt generates *unstable and unsustainable* debt paths, even for high levels of GDP. In this context, default can be used by a benevolent Government to stabilize the economy.
- As our results are constructive, the model can be used for the accurate computation of global long run stylized facts as well as local short run moments. *We derive a 2 step convergent operator to show the existence of a unique statationary equilibria*.

Results

Implications of stationarity, ergodicity and global stability

- (1) Every time a country defaults, *it generates a new ergodic and stable distribution*. Thus, *every default, even transitory, has a permanent effect on the economy*. The size of the default, as measured by the haircut, matters for unconditional moments of key macroeconomic variables.
- (2) A recession generates default and re-entry to international capital markets depends on the level of output. If GDP is below an endogenous threshold, then net external debt cannot be traded.
- (3) Precautionary savings play a crucial role. Contrarily to the results in the literature with public debt, households and firms accumulate assets in good times.
- (4) For our calibrated version of the model, 90% of time, net external assets fluctuates at +/- 1 standard deviation away from the mean (as against 68% in the standard normal distribution). Thus, *default has a drastic permanent effect as it severely affects financial development as measured by the level of net external assets*.

Results

Implications of stationarity, ergodicity and global stability

- (5) Unstable paths are not only associated with low assets and high output, but also with the curvature of the consumption function: in the unstable region, this function is convex (with respect to endogenous states), which implies that private agents change their asset position at a fast pace, destabilizing the economy.
- (6) In our model, the default cost (the penalty) is essential to achieve stationarity and ergodicity. In the first type of equilibrium, as there can be multiplicity, the penalty is used as a selection mechanism. After imposing a strengthening on preferences, we show uniqueness. Then the default cost can be used to determine the stable state space (i.e., the ergodic support of the steady state distribution).
- (7) If paths are unsustainable, the model would be stabilized by a repudiation even when the economy has a positive net asset position.
- (8) By defaulting, policy makers ensure that when the economy returns to asset markets, with substantial probability the private sector will save and the economy will have a high level of output; that is, unstable and unsustainable paths will not happen with high probability.