

# External Debt Dynamics in an Endogenous Growth Model

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# Motivation-Research Questions

## Cyclical behavior of private external debt

- ▶ Emerging markets engage in pro-cyclical external borrowing (PCB, Mendoza (1991), etc.).
- ▶ PCB is typically characterized by a negative correlation between TB and GDP. We revisit this fact from a new perspective.
- ▶ Does PCB lead to sustainable external debt?
- ▶ In terms of Debt-to-GDP ratio, recessions may lead to:
  - an increase in Debt-to-GDP if the recession is strong enough.
  - a reduction in Debt-to-GDP if the recession is mild enough.
- ▶ Same thing happens in an expansion.
- ▶ Due to the long-run nature of the question and the relevance of GDP growth, we need *endogenous growth* to fully capture the essence of the interaction between PCB and external debt sustainability.

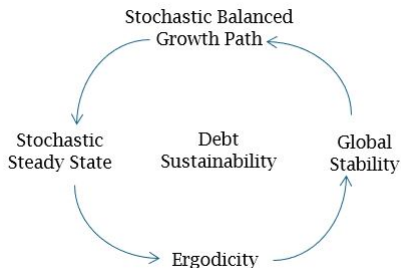
# Motivation-Research Questions

## Cyclical behavior of private external debt

- ▶ Finding conditions for debt sustainability in emerging PCB markets may lead to explain why these economies are far more volatile when compared with advanced economies.
- ▶ Surprisingly, the literature does not provide a unified framework to analyze (external) debt sustainability in a stochastic environment, let alone the effects of PCB on the long-run behavior of debt.
- ▶ From D'erasmo and Mendoza (2016), state-of-the-art debt sustainability analysis based on Debt-to-GDP ratios, even in a non-stochastic environment, does not:
  - guarantee the converge to a finite S-S from the current level of debt,
  - characterizes finite dynamic paths of endogenous variables.
- ▶ The literature focuses on the inter-temporal budget constraint and relationship between exogenous growth rates and the interest rate.

# Preview of the results

- ▶ We claim that the problems associated with sustainability analysis are generated by a missing ingredient that connects RBC with endogenous growth (trend and cycle): the notion of a *Dynamically Stable Stochastic Balanced Growth Path*.



- ▶ We derive a **testable theory of external debt dynamics in a endogenous growth environment**.

## Preview of the results

- ▶ Middle income countries **spent 50% of time engaging in PCB.**
- ▶ We derive an optimal **stochastic endogenous growth model** for open economies. A constrained planner determines the optimal timing of PBC, which is tied to the distribution of stationary TFP shocks.
- ▶ By proving the global stochastic stability of the balanced growth path and the sustainability of external debt, we show that the model matches the stylized fact.
- ▶ We derive simple and testable rules that characterize sustainable paths, allowing for persistent current account deficits.
- ▶ **External debt sustainability, cyclical borrowing and endogenous growth are essential to replicate BC facts in emerging economies.**

# Stylized fact

Unhedged debt  $D/K$  vs growth

*Argentina Private Leverage  $D/K$  (X) vs Growth (Y) 1950 - 2019*

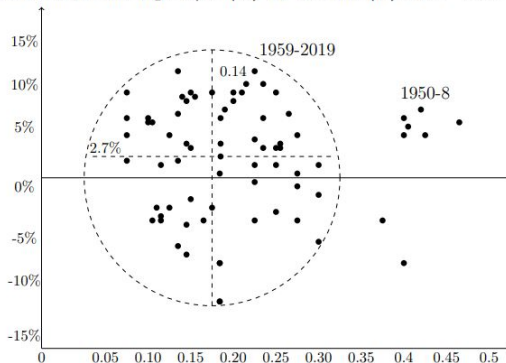


Figure 2: Argentina 1950-2019

In 1958 Argentina suffered a structural change that increased capital stock, lowering unhedged debt.

- ▶  $D/K$  is independent of the size of the country. CCB/PCB is defined WRT to the medians of  $D/K$  and of the growth rate of GDP.

# Stylized fact

## Results

Table 1: Pro-cyclical borrowing according to different measures

Middle Income Countries	Pro-cyclicality (figure 1)	$Corr(TB, \hat{y})$
MEX, ESP, BRL, ITA, ARG, PRT, PRY, URU	49.83%	-0.179
BOL, ECU, CHL, PER, COL, BEL	51.02%	0.057

Pro-cyclicality is defined as the fraction of time that a given country spends in the north-east and south-west quadrants of figure 1.  $TB$  stands for trade balance with respect to GDP. We report the median across countries of the pro-cyclicality and of the correlation between  $TB$  and  $\hat{y}$  for each row. Countries are defined as middle income based on the position in the 2019, the last year in the sample, ranking of PPP GDP per capita restricted by the availability of data. Countries with a negative correlation between  $TB$  and  $\hat{y}$ : MEX (Mexico), ESP (Spain), BRL (Brazil), ITA (Italy), ARG (Argentina), PRT (Portugal), PRY (Paraguay), URU (Uruguay). Countries with a positive correlation: BOL (Bolivia), ECU (Ecuador), CHL (Chile), PER (Peru), COL (Colombia), BEL (Belgium). Details are in the appendix.

- ▶ The frequency of PCB is similar for countries with a positive or negative correlation between  $TB$  and growth, which is the standard measure of PCB in the literature.
- ▶ Thus our stylized fact uses a broader notion of PCB.



# Theory

## Centralized Economy

- ▶ Small open economy with capital  $K$  and net external assets  $B$ .
- ▶ To generate endogenous growth, we assume, as in the AK model, that technology is linear and production depends only on capital.
- ▶ We assume that preferences are homothetic. Following Stokey (1994), this structure generates a well defined *homogeneous* dynamic programming program even though  $K$  and  $B$  maybe unbounded. Formally, preferences and technology are given by:

$$U(C) = \frac{(C)^\theta}{\theta} \quad (1)$$

$$Y = AK \quad (2)$$

- ▶  $A$  is total factor productivity. The agent is also endowed with an initial level of debt  $B$ . She can trade assets internationally at the (gross) risk-free rate  $R$ .

### Stochastic centralized optimization problem

$$V(K, B, A) = \text{Max}_{K', B'} \quad u(AK + RB - K' - B') + \beta E_A V(K', B', A')$$

Subject to

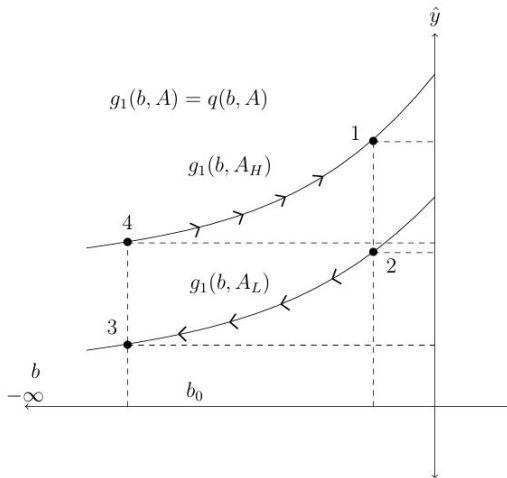
$$(i) -B' \leq -RB, (ii) K' \leq AK, (iii) K' + B' \leq AK + RB.$$

- ▶ Restriction (i) is stronger than a standard transversality condition and implies a simple rule: *you are not allowed to pay debt with more debt*, restriction (ii) bounds the growth rate of capital as  $u$  is unbounded above, (iii) is a non-negative restriction for consumption.
- ▶ Let  $b \equiv B/K$ ,  $\hat{y} \equiv K'/K$  (in the AK model, for the same shock, GDP growth equals capital stock growth).

# Results

## Phase diagram-Relationship between growth and external assets

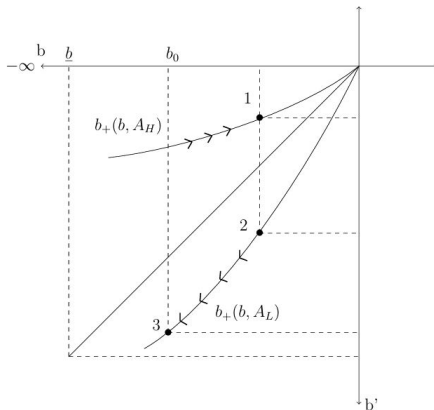
- $g_1(b, A) = K'/K$  is  $\uparrow$  in  $b$ . Indebted economies grow less.



# Results

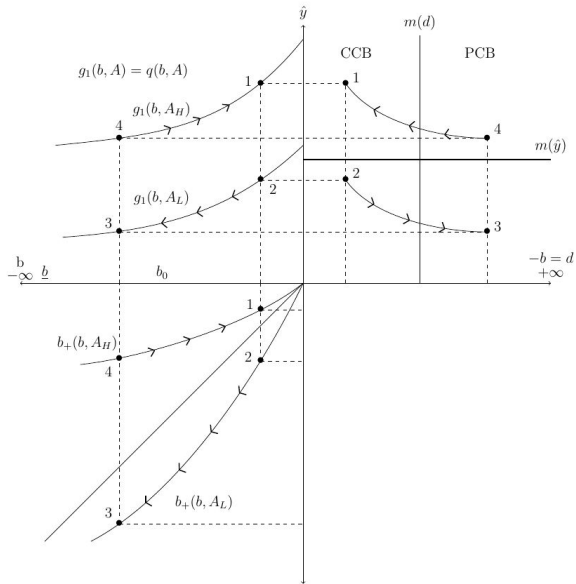
## Phase diagram-Dynamic of the balanced growth path- Dynamics of $b$

- $\downarrow b' = (\downarrow g_2(\downarrow b, A))/(\downarrow g_1(\downarrow b, A))$ ,  $g_2(b, A) = B'/K < 0$ . Indebted economies needs to borrow more.



# Results

Stability - Phase diagram- All the pieces together



# Results

## Trajectories

- ▶ Start at point 3 (high debt, low growth, CCB region).
- ▶ A positive shock moves the system to point 4 in the PCB region. The economy grows at rate  $\hat{y}(4)$ , but the growth rate of consumption is lower than  $\hat{y}(4)$  because  $A_L < R < A_H$ .  
This is the *the consumption smoothing effect*.
- ▶ The economy is increasing total net wealth by accumulating capital and reducing the stock of debt.  $K$  is growing faster than  $B$  (we are in  $b_+(\cdot, A_H)$ ). Both the aggregate growth rate  $\hat{y}$  (given by  $g_1(b, A_H)$ ) and  $b$  (given by  $b_+(\cdot, A_H)$ ) increase, moving to point 1.
- ▶ The economy enters the CCB. When the economy reaches point 1 there is a shock to productivity which decreases from  $A_H$  to  $A_L$ .

# Results

## Trajectories

- ▶ The economy moves from point 1 to 2 (PCB region) and the dynamics of debt is now described by  $b_+(\cdot, A_L)$ .
- ▶ When productivity is  $A_L$ , the expected productivity of the economy is higher than the actual level. It is then optimal to increase borrowing and reduce capital accumulation in order to smooth consumption.
- ▶  $B$  grows faster than  $K$ . The growth rate of the economy and  $b$  decrease (from point 2 to 3). At point 3 (CCB region) there is a positive shock (from  $A_L$  to  $A_H$ ), the debt dynamics is again driven by  $b_+(\cdot, A_H)$ , generating *recurrence and global stability*.

# Results

## Global Stability-Sustainability

- ▶ (1) If productivity is volatile, the economy will display large variations in growth rates. If productivity is persistent, we should observe wider variations in the stock of normalized debt.
- ▶ (2) With a low probability of  $A_H$  or low values of  $A_H$ ,  $b_+(\cdot, A_H)$  is close to the  $45^\circ$  line and  $b_+(\cdot, A_L)$  far away from it. Thus, *the distribution of Debt-to-GDP is tilted to the left.*
- ▶ Conditions for debt sustainability are given by the support and probability distribution of TFP shocks, Debt-to-GDP is not constant and there is no threshold value for the growth rate of GDP (WRT  $R$ ).
- ▶ (3) With a high  $R$ , the probability of high-productivity events (relative to  $R$ ) is lower and the distribution of Debt-to-GDP moves to the left.
- ▶ *For high levels of  $A_H$  or if it has a high probability, debt is sustainable: it is bounded by a finite upper bound with probability one.*



# Results

## Calibration

- ▶ The table below contains the results of the calibration

Table 4: Simulation results

Statistic	$PC(-b, \hat{y})$	$AVG(\hat{y})$
Data	57.1%	2.5%
Model	56.1%	3.0%
Garcia Cicco, et. al	96.0%	-3.3%

PC stands for pro-cyclicality (i.e., the frequency at which simulations or data stays above or below the median in both variables),  $-b$  is unhedged debt,  $\hat{y}$  is the net growth of GDP (due to the AK technology is equal to  $((A_+K_+/AK) - 1)\%$ ), and AVG stands for average. The row labeled "Garcia Cicco, et. al. (2010)" solves the model presented in this paper but using the parameters borrowed from that paper, which we present in table 2.

- ▶ We match pro-cyclicality (57.1% vs 56.1%) and mean GDP growth (2.5% vs 3.0%).<sup>1</sup> The transition matrix for  $A$  satisfies  $\pi(A_i, A_i) = 0.58$ .
- ▶ Interest rate ( $R = 1.01$ ) is below the growth rate ( $AVG(\hat{y}) = 0.03$ ).

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<sup>1</sup>To construct asset prices we take the reciprocal of the country risk premium. The implicit assumption is that the only risk associated with debt is the sustainability risk.

# Final slide

## Conclusions

- ▶ We prove the existence and global stability of the balanced growth path in a stochastic endogenous AK growth model for open economies.
- ▶ Global stability of the balanced growth path is the mirror image of external debt sustainability.
- ▶ The frequency of PCB is consistent with a sustainable economy.
- ▶ Sustainable debt (and thus cyclical borrowing) is consistent with persistent Current Account Deficits.

Final slide

THANKS!

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Back up slides

APPENDIX

# Theory

## Centralized Economy

- ▶ The budget constraint is given by:

$$C = AK + RB - K' - B' \quad (3)$$

- ▶  $A$  includes depreciation and  $R$  could be smaller than 1 (i.e., negative real rate).

**Assumption 1:** (a)  $X \equiv [K, B] \in [0, +\infty) \times (-\infty, 0]$ , (c)  $\theta \in (0, 1)$

**Assumption 1':** b)  $A_i > 0$ , with  $A_i \in \mathbb{A}$ . **Assumption 2':**  $\beta\alpha^\theta < 1$  with  $\alpha \equiv \text{Max}\{R, A_i\}$  with  $A_i \in \mathbb{A}$ . **Assumption 3':**  $A_j < R < A_i$ , with  $A_j, A_i \in \mathbb{A}$ .

# Results

## Calibration

- ▶ Calibrated parameters  $\beta = 0.95, \theta = 0.7, A_{LB} = 0.94, A_{UB} = 1.07, R = 1.01, \pi(A_i, A_i) = 0.58$
- ▶ These values can be used to verify assumptions 1', 2', 3'. The first one (i.e.,  $A_i > 0$  for all  $i$ ) can be seen immediately as  $A_{LB} > 0$ . The second one (i.e.,  $\beta\alpha^\theta < 1$ , with  $\alpha \equiv \max\{A_i, R\}$  for all  $i$ ) requires some computations:  $\beta\alpha^\theta = 0.95(1.07)^{0.7} = 0.996 < 1$ . Finally, the third one (i.e.,  $A_{LB} < R < A_{UB}$ ) follows from the listed values.
- ▶ We also include the values for the diagonal of the transition matrix  $\pi(A_i, A_i)$  for all  $A_i$ , which is critical to understand the connection between the global stochastic dynamics and the pro-cyclicality found in Argentina.