Mathematical Model of Flow in a Meandering Channel

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Abstract

The present study presents a mathematical model of flow in a deep, weakly meandering open channel. A complete mathematical model for the tangential velocity, radial velocity, vertical velocity and pressure is developed for the entire the flow domain. Experimental data have been used to develop empirical equations for a few parameters; parameters which are embedded in the mathematical model. The mathematical model is in agreement with mathematical models developed by other investigators. A fundamental strength of the present mathematical model is that it is valid for the entire flow domain, both near the walls as well as in the core region.

Introduction

An open channel is a conduit in which a liquid flows with a free surface. The free surface is actually an interface between the moving liquid and an overlying fluid medium and is subject to constant pressure. In engineering practice, activities for utilization of water resources involve open channel of varying magnitude in one way or other. Flows in natural rivers, streams and rivulets, artificial, that is, man-made canal for transmitting water from a source to a place of need, such as for irrigation water supply and hydro power generation. It is evident that the size, shape and roughness of open channel vary over a sizable range, covering a few order of magnitudes. Basically, all open channel channels have a bottom slope and the mechanism of flow is akin to the movement down an inclined plane due to gravity. The component of the weight of the liquid along the slope acts as driving force.

The boundary resistance at the perimeter acts as the resisting force. Water flows in open channel is largely in the turbulent regime with negligible surface tension effects. In addition, the fact that water behaves as an incompressible fluid leads one to appropriate the importance of the force due to gravity as the major force and the Froude Number as the prime non-dimensional number governing the flow phenomenon in open channels.

Natural channels include all watercourses that exist naturally on the earth, varying in size from tiny hillside rivulets, through brooks, streams, small and large rivers, to tidal estuaries. The hydraulic properties of the natural channels are generally very irregular. In some cases empirical assumption reasonably consistent with actual observations and experience may be made such that the conditions of flows in these channels become amenable to the analytical treatment of theoretical hydraulics.

The artificial channels are those constructed or developed by human effort; navigation channels, power canals, irrigation canals and flumes, floodways etc. as well as model channels that are built at the laboratory for testing purposes. The hydraulic properties of such channels can be either controlled to the extent as desired or designed to meet given requirement. The application of hydraulic theories to artificial channels will, therefore, produce results fairly close to the actual condition and hence, are reasonably accurate for practical design purposes.

There have been a number of attempts to mathematically model the flow in meandering channels (Engelund(1977); Ikeda, Parker and Sawai(1981); Odgaard(1986a,b); Ikeda and Nishimura(1986); Odgaard(1989a,b)). Most of these are essentially models describing flow in shallow, weakly meandering erodible bed streams. A feature of such streams is that the flow may be assumed to be two-dimensional. Although most meandering river stretches probably fall in the shallow, weakly-meandering category, one can find cases where the river channel is somewhat deeper. In such stretches, one cannot apply the shallow meander models.

The ratio mean depth divided by the width (or *d/b*), is a convenient parameter to use when talking about shallow and deep meanders. In shallow meanders, i.e. where *d/b* value is small, the influence of the wall is confined to a small zone near the wall which may be called the “wall zone”. The central portion, which may be called the “core zone”, is essentially free of wall effects. In deep meanders, i.e. where the *d/b* value is larger, such a core zone does not exist and the influence of the wall is felt throughout the flow.

Similarly the ratio width divided by least centreline radius of curvature (or b/r_{cm}) is a useful parameter in connection with the sinuosity of the meandering channel. The larger the value of this parameter, the more strongly meandering a channel is.

These two parameters, considered together, can be used to classify meandering channels into different categories. For example, in Ikeda and Nishimura's (1986) channel, *d/b* equalled 0.18 and b/r_{cm} equalled 0.44. So this channel fell in the shallow and weakly meandering category. In Kar's (1977) experiments, *d/b* varied from 0.453 to 0.985 while b/r_{cm} equalled 1.06. These experiments, therefore, can be said to have been conducted on a deep, strongly meandering channel flow.

In the present study conducted at IIT Kharagpur (Bhattacharya (1995)), a mathematical study is made of, and experiments have been conducted on, a mild meander which falls into the so-called deep category. A rigid bed is considered as it is felt that in the present state of knowledge of deep, mild meanders, obtaining a picture of the basic nature of flow in such channels is more appropriate.

The meander model formulated in the present work satisfies all boundary conditions – at the bed and at the side walls. In shallow, weak meanders, because of order of magnitude considerations, considerable simplifications can be made in the governing equations but for deep meanders such simplifications cannot be made in the governing equations. Therefore for deep meanders, one has to solve the full equations which are quite complex. So, in these circumstances, in place of a direct solution of the governing equations, a different approach is adopted in the present study. Because the model formulated in the present study is not restricted to shallow meanders, it is valid from wall to wall whereas the shallow meander models are for the core zone.

Review of Literature

Meandering of river is one of the natural processes. Leliavsky (1955) in his renowned book says that "The centrifugal effect (which causes the superelevation) may possibly be visualized as the fundamental principle of the meandering theory, for it represents the main cause of the helicoidal cross-currents which remove the soil from the concave banks, transported the eroded material across the channel, and deposit in on the convex banks, thus intensifying the tendency towards meandering. It follows therefore that the slightest accidental irregularity in channel formation turning as it does, the stream lines from their straight course may under certain circumstances constitute their focal point for the erosion process which leads to ultimately to meander".

According to Inglis (1947), "Where however, banks are not tough enough to withstand the excess turbulent energy developed during floods, the banks erode and the river widen and shoals…. In channels with widely fluctuating discharges and silt charges, there is a tendency for silt to deposit at one bank and for the river to move to the other bank. This is the origin of meandering….".

A meander is called shallow if its depth is small compared to its width and it is called mild-meandering if its width is small compared to its least centerline radius of curvature. These ratios are considered to be very small in most analysis as in most natural channels. The effect of considering these ratios to be very small is that the governing equation of the flow (three momentum and one continuity) is much simplified from order of magnitude consideration. However, not all-natural meanders are shallow.

Investigators have studied meanders and compound channel flows for a long time. Thomson (1876) was probably the first to point out the existence of spiral motion in a curved open channel.

Rozovskii (1957) used order of magnitude consideration and assumptions of eddy viscosity, vertical distribution of tangential (streamwise) velocity components and zero net lateral discharge and he found an approximate solution for the radial velocity component from the equation of motion. He also tried to solve for the radial distribution of the tangential velocity component and growth and decay of the spiral motion. He concluded that the solution for the vertical distribution of the radial velocity based on the logarithmic distribution of the vertical of the tangential velocity is acceptable. It should be noted that the Reynold's number for the experiments were generally low.

Kikkawa, Ikeda and Kitagawa (1976) reported a study on the flow and bed topography in a constant curvature bend. In their flow model, they assumed that the radius of curvature is large and that the width is sufficiently large compared with the depth. They considered the equation of motion governing the secondary flow and obtain an equation for the radial velocity distribution.

Zimmermann and Kennedy (1978) made a formulation for the transverse bed slope in a constant curvature bend. In the process of doing so, they assumed that a power law could express the vertical distribution of the tangential velocity.

Meandering channel flow is considerably more complexes than constant curvature bend flow. Unlike a constant curvature bend, where one finds that in the initial portion of the bends, the flow is in the developing stage and thereafter it is in the developed stage, in meanders due to the continuous streamwise variation of the radius of curvature, no developed region is reached and the flow parameters vary continuously in the streamwise direction.

De Vriend and Geldof (1983) compared the results of a mathematical model for the depth-averaged main flow velocity in shallow river bends with measurements in two consecutive sharply curved short bends in the river Dommel.
They found that,

a) The velocity distribution in the Dommel showed essentially the same features as observed in other channels of similar geometry.

b) A mathematical model derived from the "Kalkwijk/De Vriend model" worked rather well in the greater part of the two bends if secondary flow convection is ignored.
c) The inward skewing tendency of the velocity distribution near the entrance of the bend was due to the main flow inertia combined with the longitudinal pressure gradients arising from the growth of the transverse surface slope.
d) In the river section under consideration, the gradual outward shift of the main velocity maximum further downstream in the bend was a matter of retard adaptation of the flow to the bed configuration, rather than of secondary flow convection.
e) Secondary flow convection was only important in the last part of each of the surveyed beds; there it seemed to hamper the outward skewing of the flow instead of enhancing it.
f) As the flow stage became higher, the main velocity maximum tended to shift towards the inner bank; this was attained to the reduced a symmetry of the channel, rather than to the slower development of the secondary flow.

Ikeda and Nishimura (1986) presented an analytical model for describing three-dimensional flow. This was presented as an extension of the depth-averaged two-dimensional flow model of Engelund (1974) which had not treated the secondary flow in sinuous channels. To describe the flow they split the flow into the depth-averaged two-dimensional flow component and secondary flow component. The depth-averaged flow was expressed as the sum of unperturbed reach-averaged hydraulic variables and the perturbed quantities denoting perturbations induced by curvature.
The reach-averaged tangential velocity, the perturbed component of tangential velocity and the perturbed component of radial velocity were evaluated (the reach-averaged radial velocity is zero). Next, an expression was formulated for the zero depth-average pure helical component of the radial velocity. These results were incorporated in a bed topography model and the bed topography model was validated thus indirectly validating the flow model also.

Johannession and Parker (1989a) studied the secondary flow in a mildly sinuous channel. They obtained a derivation of the phase lag (the lag between the secondary flow and the channel curvature). They found that the predicted lag was small in natural channels. However, the predicted lag was much higher in many experimental channels, in agreement with data.

Johannesson and Parker (1989b) presented an analytical model for calculating the lateral distribution of the depth-averaged primary flow velocity in meandering rivers. The method they used took into account the convective transport of primary flow momentum by secondary flow.

Zhou, Chang and Stow (1993) reported a study on the phase lag of secondary flow in meanders.

Bhattacharya (1995) conducted a mathematical study and experiments on a deep mild meander without and with adjacent flood plains.

Geometric Elements of Channel Section

Geometric elements are properties of a channel section that can be defined entirely by the geometry of the section and the depth of flow. These elements are very important and area used extensively in flow computations.
For simple rectangular channel sections, the geometric element can be expressed mathematically in terms of the depth of flow and other dimensions of the section.

The definition of several geometric elements of channel cross-section are given below

1. **Depth of flow:** The depth of flow is the vertical distance of the lowest point of a channel section from the free surface. Strictly speaking, the depth of flow section is the depth of flow normal to the direction of flow or the height of the channel section containing the water.
2. **Stage:** It is the elevation or vertical distance of the free surface above a datum. If a lowest point of the channel section is chosen as the datum, the stage is identical with the depth of flow.

3. **Top width (T):** It is the width of the channel section at the free surface.

4. **Water area (A):** It is the cross sectional area of the flow normal to the direction of flow.

5. **Wetted perimeter (P):** It is the length of the line of intersection of the channel-wetted surface with a cross sectional plane normal to the direction of flow.

6. **Hydraulic Radius (R):** It is the ratio of the water area to its wetted perimeter i.e

$$R=A/P$$

7. **Hydraulic Depth (D):** It is the ratio of water area to the width of the top

$$D=A/T$$

8. **Sinuosity (S_r):** It is defined as the ratio of one wave length of the main channel in the down-valley direction (l_s) to the one wave length of the main channel along its center line (l_c)

$$S_r = l_s / l_c$$

Distribution of tangential and radial velocity in meandering channel

There are a number of mathematical models of shallow meanders in the literature (Odgaard 1986a, 1986b, 1989a, 1989b; Ikeda Nishimura 1986: Smith and McLane1984) but to the knowledge of writer, there exists no model capable of describing the flow in deep meandering channels, unlike shallow menders, in deep menders there does not exist a central zone of flow where effect of the side-walls is insignificant. Kar (1977) and Das (1984) carried out experiment on flow in a sinuous channel as part of their overall experiment works Theoretical studies on meandering channels has been reported by Bhattacharya and Kar (1992,1993) and an experimental work has been reported in Kar and Bhattacharya (1995).

A curvilinear system as shown in Fig 5.1 is used. The s axis is along the centreline, positive in the downstream direction: and the z axis is perpendicular to the s and n axes the points normally upward from the channel bed. For very small slope, the z axis can be assumed to coincide with the vertical axis. Fig 5.1 which is a plan view is also the definition sketch. The channel bed is assumed to have no lateral slope and the depth is assume to have the same order of magnitude as the width .the channel centreline is given by a cosine curve and the width of the channel is constant. The channel boundaries are non-erodible.

For deep weak menders, v (the radial velocity) is an order less than u (the tangential velocity) and w (the vertical velocity) is of the order v. there is no wall-zone and the wall effect can be felt throughout the flow. In shallow menders, the wall effect is limited to the wall zone. The core zone is substantially free of wall effect and in the zone the flow is conventionally analysed by assuming it to be two dimensional neglecting variation the pressure from hydrostatic condition. In deep mender vertical velocity must be retained in the governing equation throughout the flow and the pressure must be taken to the flow are of complex nature.

Ikeda, Parker and Sawai (1981), analysing shallow, weak meanders, considered the following equation for $\acute{u}$ and $\acute{v}$ (the depth-averaged tangential and radial velocities)

$$\acute{u}\frac{\partial \acute{u}}{\partial s}+\acute{v}\frac{\partial \acute{u}}{\partial n}+\frac{\acute{u}\acute{v}}{rc}=-g\frac{\partial \xi}{\partial s}-\frac{\zeta_s}{\rho d}(1)$$

$$\acute{u}\frac{\partial \acute{v}}{\partial s}+\acute{v}\frac{\partial \acute{v}}{\partial n}-\frac{\acute{u}^2}{rc}=-g\frac{\partial \xi}{\partial n}-\frac{\zeta_n}{\rho d}(2)$$

$$\frac{\acute{v}}{r_c}+\frac{\partial(\acute{v}d)}{\partial n}+\frac{\partial(\acute{u}d)}{\partial s}=0(3)$$

where

ξ is the water-surface elevation
τ_s and τ_n are the bed stresses; and
d is the local depth.

The above are the St. Venant equations with the metric coefficients neglected. τ_s and τ_n are evaluated using a constant friction factor C_f as

$$\zeta_s=\rho C_f \acute{u}\sqrt{\acute{u}^2+\acute{v}^2}(4)$$

$$\zeta_n=\rho C_f \acute{v}\sqrt{\acute{u}^2+\acute{v}^2}(5)$$

After decomposing $\acute{u}$ in the following manner

$$\acute{u}=U+u'(6)$$ □

and following Ikeda, Parker and Sawai (1981), Ikeda and Nishimura (1986) obtained for u□ (U is the zeroth-order value that is the reach-averaged tangential velocity) as follows:

(ks)
$a\sin(ks)+b\cos$ ¿
¿
$\frac{u'}{U}=\frac{n}{r_{cm}}$ ¿

So,

$$(ks)$$
$$a \sin(ks) + b \cos ¿$$
$$¿$$
$$\acute{u} = U + U \frac{n}{r_{cm}} ¿$$

where a and b are coefficients.

It may be noted that this equation does not satisfy the no-slip condition at the sidewalls.
In the present analysis, the equation has been modified into the following form:

$$\acute{u} = \acute{u}_0 + \acute{u}_1 (9)$$

$$\left(\frac{2\pi}{L} s\right)$$
$$A_1 \sin\left(\frac{2\pi}{L} s\right) + A_2 \cos ¿$$
$$¿$$
$$¿\acute{u}_0 + \acute{u}_0 \left(\frac{n}{r_{cm}}\right) ¿$$

with

$$1 - \left(\frac{2n}{b} ¿^{\hat{m}}\right] (11)$$
$$\acute{u}_0 = \left(\frac{\hat{m}+1}{\hat{m}}\right) U ¿$$

where

$\acute{u}_0$ = depth-averaged zeroth order tangential velocity, the zeroth order tangential velocity, the zeroth order state being a straight channel whose width is the same as that of the curved channel and whose depth of flow is the reach-averaged of the reach-averaged depth of the curved channel.

r_{cm} is the centreline radius of curvature, at the apex of the cosine curve

L = wavelength (measured along the centreline)

s and n are the stream wise and the cross-stream co-ordinates respectively; and

A and A_2 are coefficients.

This form of equation is for application in deep channels. It is the evident that this equation satisfies the no-slip condition at the sidewalls. $\hat{m}$ is an even integer used in the power law describing the

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lateral variation of the zeroth order tangential velocity. A and A_2 are different form Ikeda and Nishimura's (1986) *a* and *b* are to be evaluated later. *U* may be evaluated by Manning's equation as

$$U = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} (12)$$ where

n = Manning's coefficient of roughness
R = reach-averaged hydraulic radius and
S = reach-averaged slope.

The equation

$$1 - ¿(13)$$

$$\acute{u}_0 = \left(\frac{\hat{m}+1}{\hat{m}}\right) U ¿$$ is equivalent to the equation

$$\frac{2n}{b} ¿^{\hat{m}} (14)$$

$$\frac{\acute{u}_0}{\acute{u}_{0c}} = 1 - ¿$$ where

$\acute{u}_{0c}$ = depth-averaged zeroth order tangential velocity at the centreline. (This is incidentally, equivalent to u because the depth-averaged tangential velocity at the centreline has no first order component).

The tangential velocity in a straight channel is zero at the side-walls (to satisfy the no slip condition) and then increases very rapidly to almost its centreline value and remains almost constant over the central part of the cross-section.

To find the depth averaged radial velocity, the depth averaged continuity equation

$$\frac{\partial \acute{u}}{\partial s} + \frac{1}{r_c} \frac{\partial}{\partial n} (\acute{v}\, r) = 0 (15)$$ where

$\acute{v}$ is the depth –averaged radial velocity
r is the radius of curvature and
r_c is the radius of curvature at centreline,

is used with the equation for u and the equation

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$$(\frac{2\pi}{L}s)$$

$$A \sin\left(\frac{2\pi}{L}s\right) - A_2 \cos ¿$$

$$¿$$

$$\acute{v} = \frac{r_c}{r}\left(\frac{\hat{m}+1}{\hat{m}}\right) U \frac{2\pi}{L} ¿$$

$$[\frac{1}{2r_{cm}}\left\{\left(\frac{b}{2}\right)^2 - n^2\right\} - \left(\frac{2}{b}\right)^{\hat{m}} \frac{1}{r_{cm}(\hat{m}+2)}\left\{\left(\frac{b}{2}\right)^{\hat{m}+2} - n^{\hat{m}+2}\right\}](17)$$

results.

From the above equation, the following equation for $\acute{v}_c$, the value of $\acute{v}$ at the centerline, can be obtained and it is as follows

$$(\frac{2\pi}{L}s)$$

$$A \cos\left(\frac{2\pi}{L}s\right) - A_2 \sin ¿$$

$$\acute{v}_c = \left(\frac{\hat{m}+1}{\hat{m}}\right) U \frac{2\pi}{L} ¿$$

$$[\frac{1}{2r_{cm}}\left(\frac{b}{2}\right)^2 - \left(\frac{2}{b}\right)^{\hat{m}} \frac{1}{r_{cm}(\hat{m}+2)}\left(\frac{b}{2}\right)^{\hat{m}+2}](18)$$

The above equation can be simplified to

$$(\frac{2\pi}{L}s)$$

$$A \cos\left(\frac{2\pi}{L}s\right) - A_2 \sin ¿$$

$$¿$$

$$\acute{v}_c = \left(\frac{\hat{m}+1}{\hat{m}+2}\right) \frac{U}{r_{cm}} \frac{\pi}{L} \left(\frac{b}{2}\right)^2 ¿$$

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At those point where $\acute{v_c} = 0$,

$$\left(\frac{2\pi}{L}s\right)$$

$$A\cos\left(\frac{2\pi}{L}s\right) - A_2\sin ¿$$

$$¿$$

$$\left(\frac{\hat{m}+1}{\hat{m}+2}\right)\frac{U}{r_{cm}}\frac{\pi}{L}\left(\frac{b}{2}\right)^2 ¿$$

At these points therefore, since $\left(\frac{\hat{m}+1}{\hat{m}+2}\right)\frac{U}{r_{cm}}\frac{\pi}{L}\left(\frac{b}{2}\right)^2 \neq 0$,

$$A\cos\left(\frac{2\pi}{L}s\right) - A_2\sin\left(\frac{2\pi}{L}s\right) = 0 \quad (21)$$

Now, as per the current modelling, $\acute{v_c} = 0$ at the value of s given by the following equation

$$s = \frac{L}{4} + \frac{L\sigma}{2\pi} + i\frac{L}{2} \quad (22)$$

Where $i = 0,1,2,3\ldots\ldots$
and σ is the phase angle by which the point where $\acute{v_c} = 0$ lags the geometrical point of inflection.

So,

$$A\cos\left(\frac{2\pi}{L}\left(\frac{L}{4} + \frac{L\sigma}{2\pi} + i\frac{L}{2}\right)\right) - A_2\sin\left(\frac{2\pi}{L}\left(\frac{L}{4} + \frac{L\sigma}{2\pi} + i\frac{L}{2}\right)\right) = 0 \quad (23)$$ or

$$\tan\left(\frac{2\pi}{L}\left(\frac{L}{4} + \frac{L\sigma}{2\pi} + i\frac{L}{2}\right)\right) = \frac{A}{A_2} \quad (24)$$ or

$$\tan\left[\left(\frac{\pi}{2}+\sigma\right)+i\pi\right]=\frac{\Lambda}{A_2}(25)$$ or

$$\sigma=¿\frac{\Lambda}{A_2}(26)$$ or
$$\tan ¿$$

$$A_2=\Lambda\tan\sigma(27)$$

Therefore, the equation for $\acute{u}$ is rewritten as follows:

$$\left(\frac{2\pi}{L}s\right)$$
$$\sin\left(\frac{2\pi}{L}s\right)+\tan\sigma\cos ¿$$
$$¿$$
$$\acute{u}=\acute{u}_0+\acute{u}_0\left(\frac{n}{r_{cm}}\right)\Lambda ¿$$

The equation for $\acute{v}$ is rewritten as follow

$$\left(\frac{2\pi}{L}s\right)$$
$$\cos\left(\frac{2\pi}{L}s\right)-\tan\sigma\sin ¿$$
$$\acute{v}=\frac{r_c}{r}\left(\frac{\hat{m}+1}{\hat{m}}\right)U\frac{2\pi}{L}\Lambda ¿$$

$$\left[\frac{1}{2r_{cm}}\left\{\left(\frac{b}{2}\right)^2-n^2\right\}-\left(\frac{2}{b}\right)^{\hat{m}}\frac{1}{r_{cm}(\hat{m}+2)}\left\{\left(\frac{b}{2}\right)^{\hat{m}+2}-n^{\hat{m}+2}\right\}\right](29)$$

These equations contain some undetermined coefficients ($\hat{m}$, Λ *and* σ) which are to be obtained from experimental data.

It was found difficult to formulate an analytic expression for the phase lag, σ, in the deep mender so it was decide to modify a shallow mender phase lag equation, by adding a correction coefficient, for use in deep menders. The correction coefficient is found by matching the equation with the experimental data of present study Bhattacharya (1995).

The shallow mender phase lag equation chosen (Bhattacharya,1995) is that proposed by Zhou, Chang and Stow (1993) (some of the notation has been recast into the terminology of the present analysis)

$$\sigma = \tan^{-1}\left\{\frac{\pi}{\kappa}\left[\frac{1}{\kappa}+\left(\frac{8}{f}\right)^{\frac{1}{2}}\right]\frac{d_0}{L}\right\} \quad (30)$$ where

κ= von Karman's coefficient,

d_0 = reach-averaged flow depth,

and f = friction factor = $\frac{8k^2}{v^2}$ where

v is the "power –law" coefficient appearing in the vertical distribution of the tangential velocity. κ has a value of 0.4. v has a value of 7 in straight channels and lower values in curved channels.

The phase lag equation proposed by Zhou Chang and Stow (1993) is chosen because it was shown by Zhou Chang and Stow (1993) that the phase lag obtained by the equations proposed by them agreed better with experimentally measured phase lags than the phase lags obtained by the equations proposed by Kitanidis and Kennedy (1984), Ikeda and Nishimura (1986) and Johannesson and Parker (1989a).

Zhou Chang and Stow's (1993) equation is modified in the present analysis to the equation

$$\sigma = \hat{k}\tan^{-1}\left\{\frac{\pi}{\kappa}\left[\frac{1}{\kappa}+\left(\frac{8}{f}\right)^{\frac{1}{2}}\right]\frac{d_0}{L}\right\} \quad (31)$$

where, $\hat{k}$ is a coefficient which is to be obtained by matching the equation with the experimentally obtained phase lag for deep meander.

By taking the values of k obtained by Bhattacharya (1995) from his experimental data and by curve fitting we get an empirical equation for k . The data used are given in Table 1,

Table 1

d/b	$\hat{k}$
0.363	0.425

0.233	0.375
0.190	0.295

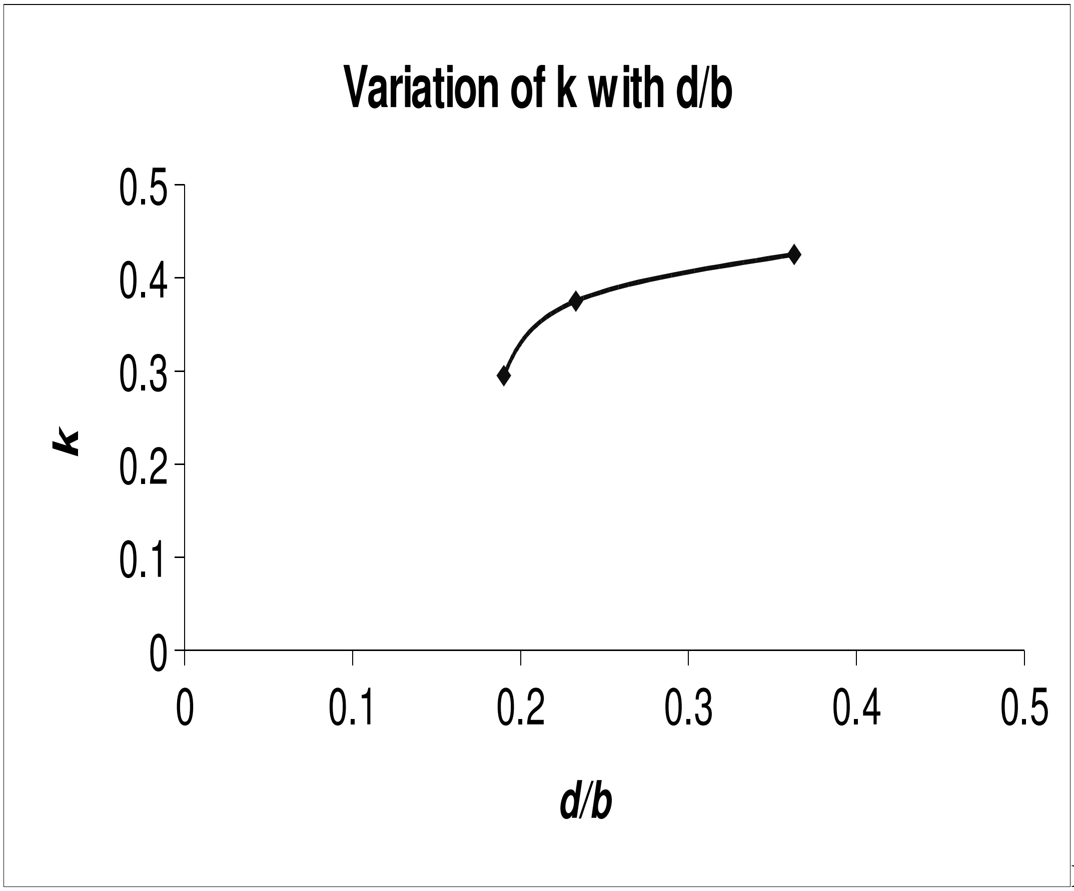

Figure 1

By using a parabolic curve fitting, an equation for *k* in terms of *d/b* is obtained as given below

$$k = 4.634 \pm \sqrt{3.463(d/b) - 11.44} \quad (32)$$

The procedure adopted for finding out Λ involves finding it from the depth averaged tangential velocity, $\bar{u}$,(Bhattacharya,1995)

$$\bar{u} = \bar{u}_0 + \bar{u}_0 \left[\frac{n}{r_{cm}}\right] \Lambda \left[\sin\frac{2\pi}{L}s + \tan\sigma\cos\frac{2\pi}{L}s\right] \quad (33)$$

$$\frac{\bar{u}}{\bar{u}_0} = 1 + \Lambda \left[\frac{n}{r_{cm}}\right]\left[\sin\frac{2\pi}{L}s + \tan\sigma\cos\frac{2\pi}{L}s\right] \tag{34}$$

or,

$$\Lambda = \frac{\frac{\bar{u}}{\bar{u}_0} - 1}{\left[\frac{n}{r_{cm}}\right]\left[\sin\frac{2\pi}{L}s + \tan\sigma\cos\frac{2\pi}{L}s\right]} \tag{35}$$

where,

$$\bar{u}_0 = \left(\frac{m+1}{m}\right)U\left[1 - \left(\frac{2n}{b}\right)^m\right]$$

$$\bar{u}_c = \left[\frac{m+1}{m}\right]U$$

Again

From tis equation we can get the different m at different sections (except those at which U ¿ $\bar{u}_c$ ¿ ¿ where it is not possible to find m) and averaged to get the value of m to be used. Table3.2 gives the values of m

Table 4

Serial No.	m
1	26
2	58
3	158

And finally taking the data which is given below (Bhattacharya,1995)

Table 5

Λ	*d/b*
0.385	0.195
0.263	0.235
0.150	0.365

An equation for Λ is obtained by curve fitting.

The equation for Λ is,

$$\Lambda = 0.369 \pm \sqrt{0.136 - (0.677 - (d/b)}$$ …….. (36)

where,

$$v_{\max,s=\sigma} = \frac{r_c}{r}\left(\frac{m+1}{m}\right) U \frac{2\pi}{L}\left(0.369 \pm \sqrt{0.136\text{-}0.283(0.677\text{-d/b})}\right)\left[\cos\frac{2\pi}{L}\sigma - \tan\sigma\sin\frac{2\pi}{L}\sigma\right]$$

$$1-\left(\frac{2n}{b}\right)^m]+$$

¿
¿

$$U\frac{r_c}{r}\left(\frac{\overline{u}}{U}\right)^2\left[\cos\frac{2\pi}{L}\sigma - \tan\sigma\sin\frac{2\pi}{L}s\right]f\left(\frac{z}{d}\right)$$

$$\left[\frac{1}{2r_{cm}}\left\{\left(\frac{b}{2}\right)^2 - n^2\right\} - \frac{1}{mr_{cm}}\right.$$ ¿

¿.

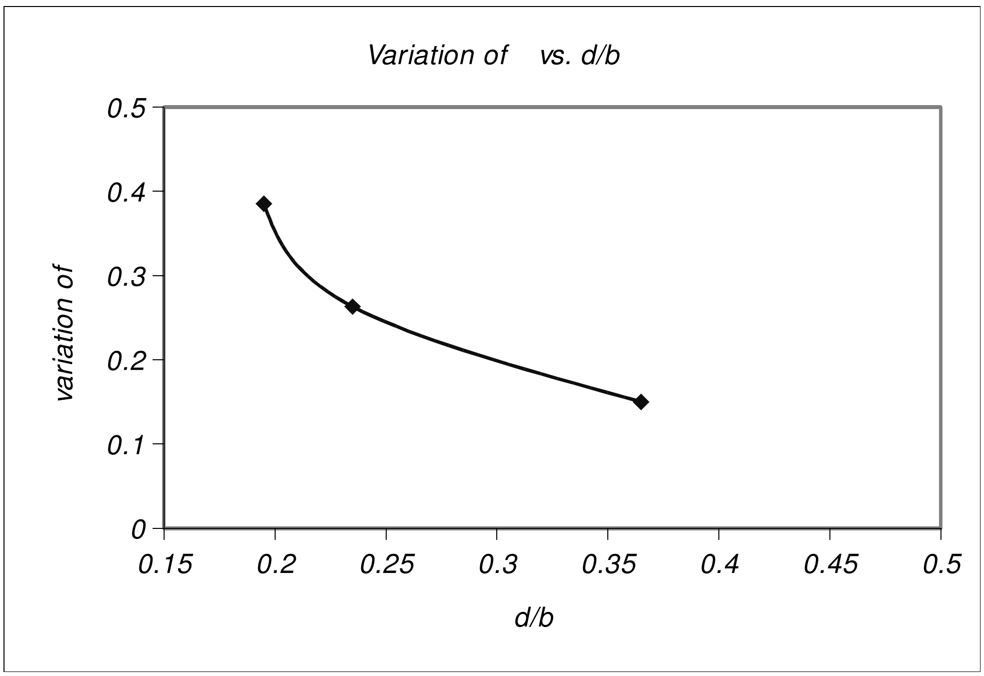

Figure 2

and $$\sigma = \left(4.634 \pm \sqrt{3.4363(d/b - 11.44)}\right)\tan^{-1}\left[\frac{\pi}{\kappa}\left\{\frac{1}{\kappa} + \left(\frac{8}{f}\right)^{1/2}\right\}\frac{d_0}{L}\right]$$

As an extension of the model for □u, if it is described to model the point tangential velocity, u_0 it can be modeled by the equation

$$u = u_0 + u_1 \quad (37)$$

$$¿\,\acute{u}_0\left(\frac{n_1+1}{n_1}\right)\left(\frac{z}{d}\right)^{\frac{1}{n_1}} + \acute{u}_1\left(\frac{n_2+1}{n_2}\right)\left(\frac{z}{d}\right)^{\frac{1}{n_2}} \quad (38)$$

Where $\acute{u}_0$, $\acute{u}_1$ are as have already been modeled. Different power law coefficients (n_1 , n_2) may be used in the zeroth order tangential velocity formulation and the first order tangential velocity formulation as has been done above if it is desired to stimulate experimentally any observed irregularities in the vertical distribution of the tangential velocity. However, using $n_1 = n_2$ is sufficiently accurate for the most purposes.

It may be noted that if $n_1 = n_2 = v$, then the equation for u may be written as

$$u = \acute{u}_0\left(\frac{n_1+1}{n_1}\right)\left(\frac{z}{d}\right)^{\frac{1}{v}} + \acute{u}_1\left(\frac{n_2+1}{n_2}\right)\left(\frac{z}{d}\right)^{\frac{1}{v}} \quad (39)$$

$$u = \left(\acute{u}_0 + \acute{u}_1\right)\left(\frac{n_1+1}{n_1}\right)\left(\frac{z}{d}\right)^{\frac{1}{v}} \quad (40)$$

$$¿\,\acute{u}\left(\frac{n_1+1}{n_1}\right)\left(\frac{z}{d}\right)^{\frac{1}{v}} \quad (41)$$

Which is the model of the vertical distribution of the tangential velocities used by Odgaard (1986a, 1989a) and is also the well-known “power law” for the vertical distribution of the stream wise velocity. Since the power law is not done in the present study.

The point radial velocity is modelled as the summation of the depth-average radial velocity and a zero depth- average “residual” component of the radial velocity includes an assumption of its vertical distribution.

Support for the assumption of a vertical distribution of the radial velocity is provided by Ikeda and Nishimura (1986) who assume that the radial velocity retains a certain shape in the vertical throughout the flow domain inn a shallow meander with changes only in magnitude and phase. This assumption has been shown to be approximately corrected by Johannesson and Parker (1989a).

Any modelling of the radial velocity must incorporate in itself the net lateral mass flux which is present in the sinuous channel flow (but absent in fully developed constant-radius bend flow). Odgaard’s (1989a) vertical profile of the radial velocity is linear and his equation for the radial velocity is given by the following equation:

$$v = \acute{v} + 2 v'_s \left(\frac{z}{d} - \frac{1}{2} \right) (42)$$

Where v = radial velocity,

$\acute{v}$ = depth-averaged radial velocity,

v'_s = $v_s - \acute{v}$ where v_s = value of v at any water surface,

z = vertical coordinate,

d = local depth,

The radial velocity distribution used by Kitanidis and Kennedy (1984) does not take into account the net lateral mass flux. The radial velocity distribution used by them has a linear vertical profile with the inward flux in balance.

It seems that the vertical distribution of the radial velocities in a deep channel differs from that in a shallow channel in one significant point. In the shallow channels, a linear or roughly linear variation is present in the central portion of the vertical profile both for the meander and for fully developed constant- radius bend flows (Kikkawa, Ikeda and Kitagawa (1976), Falcon Ascanio and Kennedy (1983) but in a deep channel the vertical profile of the radial velocity does not show that characteristics and must be considered to be non-linear throughout.

Experimental data from the present experiment is taken and by curve-fitting it is found to be the cubic parabola is the most suitable representation of the vertical distribution of the radial velocity.

The residual component of the radial velocity, v_R is formulated as follows:

$$v_R = U \frac{r_0}{r} \left(\frac{\acute{u}}{U} \right)^2 \left[\cos \frac{2\pi}{L} s - tan\sigma \sin \frac{2\pi}{L} s \right] f \left(\frac{z}{d} \right) (43)$$

Where $f\left(\frac{z}{d}\right)$ is a function of $\frac{z}{d}$.

The most appropriate form of $f\left(\frac{z}{d}\right)$ is a cubic parabola and then Eqn. (5.35) may be written as follows:

$$v_R = U \frac{r_0}{r} \left(\frac{\acute{u}}{U} \right)^2 \left[\cos \frac{2\pi}{L} s - tan\sigma \sin \frac{2\pi}{L} s \right] [C_0 + C_1 \left(\frac{z}{d} \right) + C_2 \left(\frac{z}{d} \right)^2 + C_3 \left(\frac{z}{d} \right)^3] (44)$$

Where C_0 , C_1 , C_2 and C_3 are coefficients, all of which lie between -1 and 1.

The function $\left[\cos\frac{2\pi}{L}s - tan\sigma \sin\frac{2\pi}{L}s\right]$ has been selected to be the same function that appears in the formulation of $\acute{v}$. It is to be noted that v_R vanishes at the side walls because $\acute{u}$ vanishes at the sidewalls. The terms $\left(\frac{\acute{u}}{U}\right)^2$ is modelled after Ikeda and Nishimura's (1986) term $\hat{f}^2\left(\frac{\acute{u}}{U}\right)^2$ which appears in their formulation of v_R . This artificial function, $\hat{f}$, was used by Ikeda and Nishimura's (1986) to make their formulation for v_R match at the sidewalls as their $\acute{u}$ does not satisfy the sidewall matching requirement. Such an artificial function has not been needed in the present study as here the formulation for $\acute{u}$ has been made in a way that ensures vanishing at the sidewalls.

The four unknown coefficients appearing in the cubic parabola describing the vertical distribution of the residual component of the radial velocity are evaluated from experimental data.

Therefore,

$$v_{\max,s=\sigma} = (\bar{v} + v_R)_{\max,\, s=\sigma} \qquad (45)$$

$$v_{\max,s=\sigma} = \frac{r_c}{r}\left(\frac{m+1}{m}\right)U\frac{2\pi}{L}\Lambda\left[\cos\frac{2\pi}{L}\sigma - \tan\sigma\sin\frac{2\pi}{L}\sigma\right]$$
$$\left[\frac{1}{2r_{cm}}\left\{\left(\frac{b}{2}\right)^2 - n^2\right\} - \left(\frac{2}{b}\right)^m\frac{1}{r_{cm}(m+2)}\left\{\left(\frac{b}{2}\right)^{m+2} - n^{m+2)}\right\}\right] +$$
$$U\frac{r_c}{r}\left(\frac{\bar{u}}{U}\right)^2\left[\cos\frac{2\pi}{L}\sigma - \tan\sigma\sin\frac{2\pi}{L}s\right]f\left(\frac{z}{d}\right) \qquad (46)$$

Now it is assumed that *m>>2* (which is justified by the experimental result of Bhattacharya, 1995),

So,

$$v_{\max, s=\sigma}=\frac{r_c}{r}\left(\frac{m+1}{m}\right)U\frac{2\pi}{L}A\left[\cos\frac{2\pi}{L}\sigma-\tan\sigma\sin\frac{2\pi}{L}\sigma\right]$$
$$1-\left(\frac{2n}{b}\right)^m]+$$
¿
¿
$$U\frac{r_c}{r}\left(\frac{\overline{u}}{U}\right)^2\left[\cos\frac{2\pi}{L}\sigma-\tan\sigma\sin\frac{2\pi}{L}s\right]f\left(\frac{z}{d}\right)$$
$$\left[\frac{1}{2r_{cm}}\left\{\left(\frac{b}{2}\right)^2-n^2\right\}-\frac{1}{mr_{cm}}\text{¿}\right.$$
¿. (47)

$$v_{\max, s=\sigma}=\frac{r_c}{r}\left(\frac{m+1}{m}\right)U\frac{2\pi}{L}\left(0.369\pm\sqrt{0.136\text{-}0.283(0.677\text{-d/b})}\right)\left[\cos\frac{2\pi}{L}\sigma-\tan\sigma\sin\frac{2\pi}{L}\sigma\right]$$
$$1-\left(\frac{2n}{b}\right)^m]+$$
¿
¿
$$U\frac{r_c}{r}\left(\frac{\overline{u}}{U}\right)^2\left[\cos\frac{2\pi}{L}\sigma-\tan\sigma\sin\frac{2\pi}{L}s\right]f\left(\frac{z}{d}\right)\ldots\ldots\ldots(48)$$
$$\left[\frac{1}{2r_{cm}}\left\{\left(\frac{b}{2}\right)^2-n^2\right\}-\frac{1}{mr_{cm}}\text{¿}\right.$$
¿.

Comparison of tangential and radial velocity distribution with those obtained by other investigators

Odgaard (1989a) present the present formula for the depth averaged radial velocity at the centreline, $\acute{v}_c$:

$$\acute{v}_c=\frac{C}{8}\acute{u}_c\frac{b^2}{d_c}\frac{d}{ds}\left(S_{TC}+U_{TC}\right)(49)$$ where

C = coefficient

$\acute{u}_c$ = depth-averaged tangential velocity at the centerline

d_c = depth of the centreline

S_{TC} = transverse bed slope at centerline, and

U_{TC} = normalized transverse velocity gradient at the centreline

$$\text{¿}d_c\left[\frac{\partial}{\partial n}\left(\frac{\acute{u}}{\acute{u}}\right)\right]_c$$

For a right bed, S_{TC} = 0 and Odgaard's equation for $\acute{v}_c$ can be written in following form

$$\acute{v}_c = ¿$$

$$sinusoidal\,function\,having\,dimension\,of\ \frac{1}{length^2}$$

$$\left(non-dimensional\,constant\right) X\ U\ b^2\ X\ ¿$$

In the present modelling, $\acute{v}_c$ can be written as

$$\left(\frac{2\pi}{L}s\right)$$

$$\frac{\Lambda}{L\,r_{cm}}\cos\left(\frac{2\pi}{L}s\right) - \frac{\Lambda\tan\sigma}{L\,r_{cm}}\sin ¿$$

$$¿$$

$$\acute{v}_c = \left(\frac{\hat{m}+1}{\hat{m}+2}\right)\frac{\pi}{4}U\,b^2 ¿$$

which is also of the form

$$\acute{v}_c = ¿$$

$$sinusoidal\,function\,having\,dimension\,of\ \frac{1}{length^2}$$

$$\left(non-dimensional\,constant\right) X\ U\ b^2\ X\ ¿$$

The laboratory and field data of DeVriend and Geldof (1983), Kikkawa et al. (1976), Odgaard (1984), Zimmermann and Kenney (1978) and Throne et al. (1983) show that u is essentially constant along the channel centreline and that its variation in the transvers direction is nearly linear over the central portion of the cross section (Odgaard 1989a)

In the general modelling, the expression for u_c, the tangential velocity at the centerline can be written as follows

$$u_c = u_0\, ¿_{n=0} + u_1\, ¿_{n=0}$$

$$¿u_0\, ¿_{n=0}\left(\frac{n_1+1}{n_1}\right)\left(\frac{z}{d}\right)^{\frac{1}{n_1}} + u_1\, ¿_{n=0}\left(\frac{n_2+1}{n_2}\right)\left(\frac{z}{d}\right)^{\frac{1}{n_2}}$$

$$(\frac{2\pi}{L}s)$$

Now, $\sin(\frac{2\pi}{L}s)+\tan\sigma\cos¿$

$$\acute{u}_1=\acute{u}_0(\frac{n}{r_{cm}})\Lambda ¿$$

and so $\acute{u}_1=0$ at the centerline

So, $u_c=u_0 ¿_{n=0}\left(\frac{n_1+1}{n_1}\right)\left(\frac{z}{d}\right)^{\frac{1}{n_1}}=\left(\frac{\hat{m}+1}{\hat{m}}\right)U\left(\frac{n_1+1}{n_1}\right)\left(\frac{z}{d}\right)^{\frac{1}{n_1}}$

Therefore u_c is constant.

In the present modelling, the derivative of the tangential velocity in the transverse direction can be written as

$$(\frac{2\pi}{L}s)$$

$$\sin\left(\frac{2\pi}{L}s\right)+\tan\sigma\cos¿$$

$$¿$$

$$\frac{\partial u}{\partial n}=\left(\frac{\hat{m}+1}{\hat{m}}\right)U\left[-\left(\frac{2}{b}\right)^{\hat{m}}\hat{m}n^{\hat{m}-1}\right]\left(\frac{n_1+1}{n_1}\right)\left(\frac{z}{d}\right)^{\frac{1}{n_1}}+\left(\frac{\hat{m}+1}{\hat{m}}\right)U\left[\frac{1}{r_{cm}}\frac{-1}{r_{cm}}\left(\frac{2}{b}\right)^{\hat{m}}(\hat{m}+1)n^{\hat{m}}\right]\Lambda ¿$$

which is constant only at the centreline. So, in the present modelling the transverse variation of the tangential velocity is linear at the centreline.

Based on previous work where the velocity field in a constant radius band with d/b << 1 and d/ r_c <<1 was solved by the perturbation method resulting in the finding that $\acute{v}$ had a parabolic distribution in the n direction (this solution had a good agreement with experiments), Yen and Ho (1990) adopted the following distribution of $\acute{v}$ for their work on bed evolution in (constant-radius) channel bends:

$$\acute{v}=\acute{v}_c\left[1-4\left(\frac{n}{b}\right)^2\right](52)$$

In the present modelling $\acute{v}$ and $\acute{v}_c$ are as follows:

$$\left(\frac{2\pi}{L}s\right)$$

$$\cos\left(\frac{2\pi}{L}s\right)-\tan\sigma\sin ¿$$

$$¿$$

$$\acute{v}=\frac{r_c}{r}\left(\frac{\hat{m}+1}{\hat{m}}\right)U\frac{2\pi}{L}A ¿$$

$$\left(\frac{2\pi}{L}s\right)$$

$$\cos\left(\frac{2\pi}{L}s\right)-\tan\sigma\sin ¿$$

$$¿$$

$$\acute{v}_c=\left(\frac{\hat{m}+1}{\hat{m}}\right)U\frac{2\pi}{L}A ¿$$

Therefore, $\acute{v}$ can be written as

$$\acute{v}=\frac{r_c}{r}\acute{v}_c\left(\frac{\hat{m}+2}{\hat{m}}\right)\left(\frac{2}{b}\right)^2\left[2\left(\frac{\hat{m}+1}{\hat{m}}\right)\right]\left[\frac{1}{2}\left\{\left(\frac{b}{2}\right)^2-n^2\right\}-\left(\frac{2}{b}\right)^{\hat{m}}\frac{1}{(\hat{m}+2)}\left\{\left(\frac{b}{2}\right)^{\hat{m}+2}-n^{\hat{m}+2}\right\}\right](55)$$

which can be written as

$$\acute{v}=\frac{r_c}{r}\acute{v}_c\left(\frac{\hat{m}+2}{\hat{m}}\right)\left[\left\{1-\left(\frac{2n}{b}\right)^2\right\}-\frac{2}{(\hat{m}+2)}\left\{1-\left(\frac{2n}{b}\right)^{\hat{m}+2}\right\}\right](56)$$

If the metric coefficient is ignored $\frac{r_c}{r}=1$ .Further, if d/ r_c << 1, that is if a shallow channel is considered $\hat{m}$ is very large or

$$\frac{\hat{m}+2}{\hat{m}}=1(57)$$

And

$\frac{2}{(\hat{m}+2)} = 0 \quad (58)$ It is noticed that $\left\{1-\left(\frac{2n}{b}\right)^{\hat{m}+2}\right\}$ can never be more than 1. So ignoring the metric coefficient and for shallow flow,

$$\acute{v} = \acute{v}_c\left[1-4\left(\frac{n}{b}\right)^2\right] \quad (59)$$

Which is identical to the expression for $\acute{v}$ adopted by Yen and Ho (1990).

Distribution of vertical velocity and pressure in the meandering channel

An approximate formulation has been made for the distribution of vertical velocity and pressure in the meandering channel. The distribution of vertical velocity may have considerable implication for sediment transport, particularly for suspended sediment transport.

In a constant curvature bend, where a single spiral system of circulation exists, the vertical velocity is directed upwards near the inner walls and downwards towards the outer wall. The vertical velocity is zero at both the inner and the outer wall to satisfy the “no-slip” condition. Also it is clearly zero at both the bed and the free surface.

In the present study, it has been experimentally found that a single spiral system of circulation exists in a mildly meandering channel at flows that are not shallow.

This observation, made on the basis of radial velocity measurements, enables one to state that the vertical velocity must be directed upwards near the inner wall and downwards near the outer wall. Thus it may be stated that along any vertical line, the velocity is the same sign throughout and it is possible to conveniently express the vertical distribution of the vertical velocity in terms of depth-averaged vertical velocity.

$\acute{w}_s = 0, n = \frac{b}{4}$

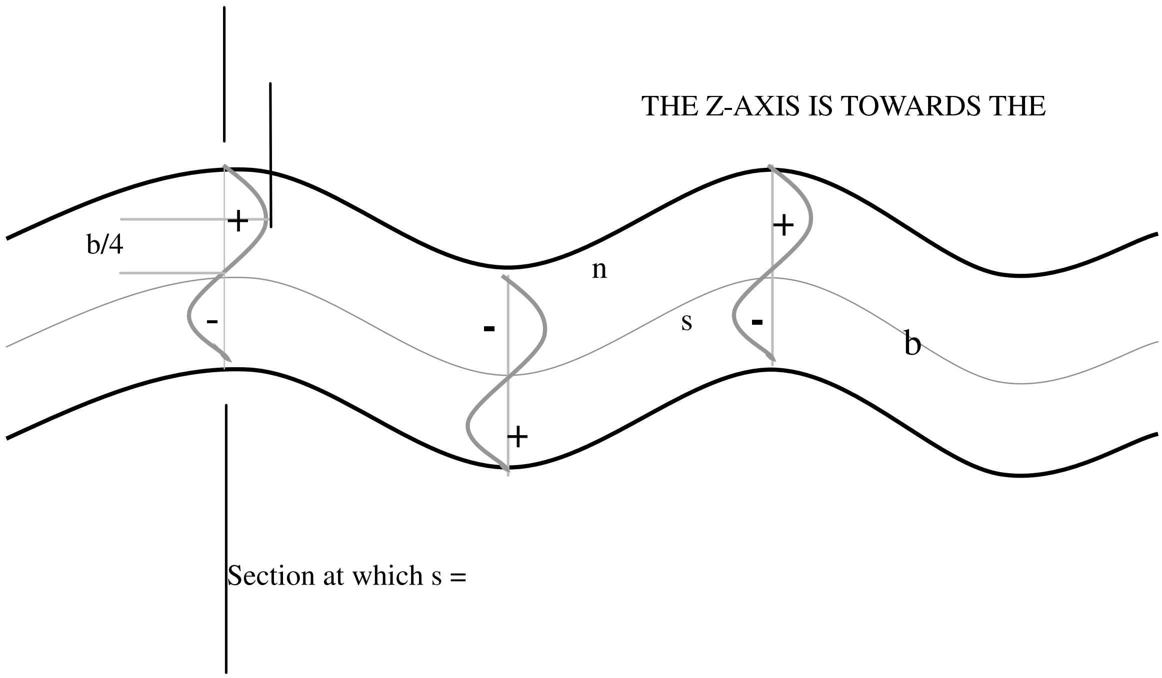

Figure 3: Distribution of depth-averaged vertical velocity

Since an approximate formulation is sought, the vertical distribution of the vertical velocity is considered to be parabolic and is considered to be given by an equation of the form

$$w = \acute{w}\left[c_1 + c_2\left(\frac{z}{d}\right) + c_3\left(\frac{z}{d}\right)^2\right] \quad (60)$$

subject to the following conditions

i. At z = 0, w = 0
ii. At z = d, w = 0
iii. At z = $\frac{d}{2}$, w = w_{max}

Where w_{max} is the maximum vertical velocity on that vertical. For a parabolic distribution

$$w_{max} = \frac{3}{2}\acute{w} \quad .$$

Using the above conditions, the following equation is obtained:

$$w = 6\acute{w}\left[\left(\frac{z}{d}\right) - \left(\frac{z}{d}\right)^2\right] (61)$$

With the reference to the Fig.1 which shows the distribution of $\acute{w}$ in plan, the following for the transverse distribution of $\acute{w}$ at the section where s = 0 can be down

$$\acute{w}_{s=0} = \acute{w}_{s=0, n=\frac{b}{4}} \sin\frac{2\pi}{b} n (62)$$

And adding a streamwise variation to the above distribution, the following equation is obtained

$$\acute{w} = \acute{w}_{s=0, n=\frac{b}{4}} \sin\frac{2\pi}{b} n cos\frac{2\pi}{L} s (63)$$

$\acute{w}_{s=0, n=\frac{b}{4}}$ must now be estimated. As per the current modelling, at any section the maximum value of point vertical velocity occurs at n = $\frac{b}{4}$, z = $\frac{d}{2}$, If it is now assumed from dimensional considerations that

$$\acute{w}_{s=0, n=\frac{b}{4} z=\frac{d}{2}} = \frac{b}{d} v_{max, s=0} (64)$$

Where $v_{max, s=0}$ is the maximum point radial velocity at the section s = 0, then since

$$\acute{w}_{s=0, n=\frac{b}{4}} = \frac{2}{3} \acute{w}_{s=0, n=\frac{b}{4} z=\frac{d}{2}} (65)$$

$$\acute{w}_{s=0, n=\frac{b}{4}} = \frac{2}{3}\frac{b}{d} v_{max, s=0} (66)$$

So, $\acute{w} = \frac{2}{3}\frac{b}{d} v_{max, s=0} \sin\frac{2\pi}{b} n cos\frac{2\pi}{L} s (67)$

and $w = 4\frac{b}{d} v_{max, s=0} \sin\frac{2\pi}{b} n cos\frac{2\pi}{L} s\left[\left(\frac{z}{d}\right) - \left(\frac{z}{d}\right)^2\right] (68)$

The above equation gives a rough estimate of the vertical velocity. The above equation can be further improved by considering that since the radial velocity and vertical velocity are both curvature-driven and are both components of the circulation, they are in phase with each other and their development lags behind the channel platform by the same amount. So, the maximum vertical velocity is found not at the section s = 0 but at section s = σ, where σ is the phase lag. Then the equation point vertical velocity becomes

$$w=4\frac{b}{d}v_{max,\,s=\sigma}\sin\frac{2\pi}{b}ncos\frac{2\pi}{L}(s-\sigma)[\left(\frac{z}{d}\right)-\left(\frac{z}{d}\right)^2](69)$$

The formulation for the point vertical velocity does not satisfy the continuity equation. However, since an idea of the overall nature of the flow is desired rather than an exact description of the flow is desired rather than an exact description of the flow details, this is not considered to be a serious defect. In Kitanidis and Kennedy (1984), the assumed distribution of the radial and vertical velocities violate the continuity equations locally and also violate the boundary conditions but are retained because , in context of their analysis, the details of distributions are not very important.

For steady-state flow, using the following very simplified equation

$$\frac{-\partial(p+\gamma z)}{\partial z}=\rho w\frac{\partial w}{\partial z}(70)$$

Where

p = pressure
γ = unit weight of water
ρ = density of water

The expression of w in terms of $\acute{w}$ and the boundary condition at the water surface, the following very approximate equation is obtained for the pressure

$$\rho=\gamma(d-z)-36\rho\acute{w}^2\left[\frac{1}{2}\left(\frac{z}{d}\right)^2-\left(\frac{z}{d}\right)^3+\frac{1}{2}\left(\frac{z}{d}\right)^4\right](71)$$

The equation reduce to the hydrostatic equation when $\acute{w}$ = 0 because pressure has not been measured in the experiments, it has not been possible to compare the above equation with the actual pressure distribution.

This equation is further expanded in the present study thereby getting,

$$p=\gamma(d-z)-16\rho[\frac{1}{2}(z/d)^2-(z/d)^3+\frac{1}{2}(z/d)^4](\frac{d}{b})^2V^2_{\max,s=0}\sin^2\frac{2\pi}{b}n\cos^2\frac{2\pi}{L}(s-\sigma)\ldots\ldots\ldots(72)$$

So one can rewrite above equation as,

$$p=\gamma(d-z)-16\rho[\frac{1}{2}(\frac{z}{d})^2-(\frac{z}{d})^3+\frac{1}{2}(\frac{z}{d})^4](\frac{d}{b})^2\sin^2\frac{2\pi}{b}n\cos^2\frac{2\pi}{L}(s-\sigma)[\varphi]^2$$

(73)

Now the equation (73) can be rewritten as,

$$p = \gamma(d-z) - 16\rho[\frac{1}{2}(\frac{z}{d})^2 - (\frac{z}{d})^3 + \frac{1}{2}(\frac{z}{d})^4](\frac{d}{b})^2 \sin^2\frac{2\pi}{b} n \cos^2\frac{2\pi}{L}(s-\sigma)[\varphi]^2 \ldots\ldots\ldots\ldots(74)$$

where, ϕ *is defined as*

$$\phi = v_{max,s=\sigma}$$

From the equation (74) it can be concluded that if

$$\cos\frac{2\pi}{L}(s-\sigma) = 0$$

then

$$p = \gamma(d-z)$$

i.e pressure is hydrostatic.

Now, $s = \sigma + \frac{L}{4} + i\frac{L}{2}$

and we can say that at the sections $s = \sigma + \frac{L}{4} + i\frac{L}{2}$, the pressure is hydrostatic.

So one can rewrite equation (74) as,

$$p = \gamma(d-z) - 16\rho[\frac{1}{2}(\frac{z}{d})^2 - (\frac{z}{d})^3 + \frac{1}{2}(\frac{z}{d})^4](\frac{d}{b})^2 \sin^2\frac{2\pi}{b} n \cos^2\frac{2\pi}{L}(s-\sigma)[\varphi]^2 \quad (75)$$

It has not been attempted to obtain a more accurate expression for the pressure because of the peripheral nature of the pressure to the main purpose of the current work.Also, a boundary layer can be visualised very close to the channel bed where the pressure distribution is linearly varying with depth. If it is required to model the bed load transport by modeling the rolling motion or saltation of individual grains first, and then aggregating to find the total bed load, then the static pressure distribution around the grain can be considered as linearly increasing with the depth.

Conclusion

A single spiral system of circulation exists for flow in the meandering channel used in the present experiments. In contrast to the flow in the much more strongly meandering channel of Kar (1977) where a double spiral system of circulation exists. This implies that in meandering channels with relatively less sinuosity a single spiral system of circulation exists and with increase in sinuosity, the single spiral system of circulation changes into a double spiral system of circulation.

For the meandering channel of the present study, the thread of maximum tangential velocity lies close to the inner wall. This has also been found in the experimental work of Kar (1977) and is in contrast with the findings, present in the literature, for shallow meanders particularly with erodible beds. This indicates that whereas for shallow meanders the tangential velocity maximum is shifted outward by the secondary flow, this redistribution process is much less effective for larger depths of flow.

It is experimentally observed in the present meandering channel that there is not much irregularity in the vertical distribution of the tangential velocity and so the well-established power law may be assumed to be valid. This is in contrast to the findings of Kar (1977) who found, for the much more strongly meandering channel used in his experiments, that the vertical distribution of the tangential velocity changes its nature and becomes highly irregular and the power law or logarithmic law do not hold good for the vertical distribution of the tangential velocity in meandering channels of high sinuosity.

Even for a simple meander, Manning's n is found to increase with increasing depth of flow. Manning's n increases monotonically from 0.55 at the lowest depth to 0.85 at the highest depth. Therefore, the use of an average value of n does not result in an accurate prediction of the mean velocity of the simple meander. Thus it follows that Manning's n is dependent on flow stage and field engineers who calculate the mean velocity in meanders by Manning's formula assuming a fixed value of Manning's n incur a significant error in their calculations.

For a simple meander, the experimentally obtained phase lags are much less than the phase lags calculated using Zhou, Chang and Stow's (1993) formula which was developed for use in shallow meanders. The coefficient k lies between 0 and 1 in all cases.

The coefficient Λ is also found to lie between 0 and 1 in all cases.

The tangential velocity model for the meander shows a fairly good agreement with the observed tangential velocity where the calculated tangential velocity is more towards the outer bank whereas the observed tangential velocity is greater towards the inner bank.

For the meandering channel, the vertical distribution of the radial velocity may be modelled in a cubic parabolic form.

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