Analytical Derivation of Diffusio-osmosis Electric Potential and Velocity Distribution of an Electrolyte in a Fine Capillary Slit

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Abstract-- **This paper critically reviews electrokinetic flow mechanism (diffusio-osmosis) of electrolyte solutions in micro/nano channels. We have attempted to show the complete analytical derivation and the missing steps from previous literature to obtain diffusio-osmosis electric potential and velocity distribution of an electrolyte in a fine capillary slit. Diffusio-osmotic transport occurs due to the presence of a charged surface (electrostatic interactions between surface and electrolyte) which is induced by the imposed electrolyte concentration gradient. Electrostatic and induced potential distribution and velocity distribution were determined from using Poisson, linearized Poisson–Boltzmann and modified Navier–Stokes equations, respectively. Various assumptions such as having a constant concentration gradient imposed along the axial direction have been considered to ease our calculations. It was found that normalized induced electric field increases with an increase in the electo-kinetic distance from the center to a maximum value at the slit wall; and normalized velocity decreases monotonically with the increase in the electro-kinetic distance. The effect of the induced electric field was found to be of dominant significance on the diffusio-osmotic flow.**

Index Term-- **Diffusio-osmosis; transport; electrolyte; porous media; charged capillary wall**

1. INTRODUCTION

Electrokinetic flows of electrolyte solutions in charged micro/nano channels has gained much interest in many areas of science and engineering. In electrokinetics, electro-osmosis has been the most known transport mechanism used to manipulate the fluid flow in microfluidic and nanofluidic systems. Electro-osmosis transport occurs due to the interaction between an external, tangential, electric field with the electric double layer (EDL) adjacent to a charged wall [1].

Principally, transport of fluids in micropore systems (porous media) involves three different driving forces: pressure difference between the two ends of a capillary pore (convection), concentration gradient of the impermeable solute between the two bulk solutions surrounding the pore on both ends (osmosis), and tangential electrical fields affecting the electrical double layer (EDL) near a charged pore wall (electro-osmosis). Transport problems related to the three mentioned driving forces has been studied extensively in the past [2,3,4].

Conversely, it has been found that there is another driving force for electrolyte solutions in a capillary pore which is known as diffusioosmosis and is associated with the presence of a concentration gradient of a permeable solute along the capillary pore. Diffusio-osmosis is an important electrokinetic flow mechanism in microchannels and nanochannels for understanding transport of fluids in porous media [2,4]. In an electrolyte system, diffusio-osmosis transport results from the electrostatic interaction between a tangential concentration gradient of the electrolyte and a charged wall. Dissociation of electrolytes along the charged solid surface would result in a fluid flow by two mechanisms: (1) chemi-osmotic effect due to stresses developed by tangential gradient of excess pressure within the EDL (2) electro-osmotic effect due to the inequality of the tangential diffusive and convective fluxes of the two electrolyte ions which is related to the macroscopic electric field [1,2].

Understanding the mechanism of diffusio-osmotic flow of electrolyte solutions in porous media would make it possible to improve and develop much more advanced applications related to separation technologies. Permeable membranes for molecular separation, nanofluidic devices for biological and chemical analysis and polymer electrolyte fuel cells for energy storage are some important applications of diffusio-osmotic flow [1].

The diffusio-osmotic flow of an electrolyte solution in a slit (capillary channel) which is induced by the presence of a charged surface induced by the imposed electrolyte concentration gradient has been discussed and investigated theoretically by Ma and Keh (2006) [2].

Analytical solutions of the diffusio-osmotic transport of a permeable solute (electrolyte solution) in the tangential direction of a capillary channel (slit) between two identical parallel and charged plates has been derived and identified. The use of the well-known Poisson and linearized Poisson–Boltzmann equations, which apply to the case of low potentials or low fixed-charge densities, with the basics of transport phenomena in electrolyte systems allowed us to obtain the electrostatic potential distribution of an electrolyte solution inside a capillary pore. Electrostatic potential distribution would help us in calculating thickness of EDL adjacent to the charged walls which is an arbitrary thickness relative to the

capillary pore size. Moreover, fluid velocity profile along the tangential direction (the horizontal position) was obtained semi-analytically from solving the modified Navier–Stokes equation with neglecting electric current arising from the cocurrent diffusion, electric migration, and diffusio-osmotic convection of the electrolyte ions. It should be noted that any charged surface may have either a constant surface potential or a constant surface charge density with an arbitrary quantity, which would play a key role in simplifying our transport problem analysis [2,4,5]

2. PROBLEM FORMULATION & MATHEMATICAL MODELING

The steady diffusio-osmosis flow of a symmetric electrolyte solution in a capillary channel between two identical parallel charged plates is investigated to obtain both electric potential and velocity distributions within the slit which has a separation distance of $2h$. For simplicity, we need to only consider the half region $0 \leq y \leq h$ as shown in Fig. 1 where further analysis is similar to the case of the charged plate [1,2].
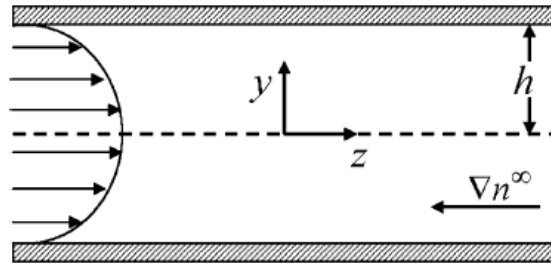

Fig. 1. Diffusion-osmosis flow in a capillary slit due to the presence of an applied concentration gradient of an electrolyte.

(a) Electrostatic potential distribution

Considering the given assumptions with the utilization of the well-known Poisson equation in microscopic models as shown in Eq. (1) and Eq. (2); substitution of Eq. (2) into Eq. (1) gives us Eq. (3) [1-3,6].

$$\nabla^2 \psi = -\frac{\rho_e}{\varepsilon} \tag{1}$$

$$\rho_e = F \sum_i z_i n_i \tag{2}$$

where ψ is the electric potential, ρ_e is the charge density, $\varepsilon = 4\pi\varepsilon_0\varepsilon_r$, ε_r is the relative permittivity of the electrolyte solution and ε_0 is the permittivity of a vacuum, F is Faraday's constant, z_i is a valence (a positive or negative integer), Z is a valence (a positive integer), and n_i represents local concentrations of both cations, $n_+(y,0)$, and anions, $n_-(y,0)$ in the electrolyte solution [1,2].

$$\frac{d^2\psi}{dy^2} = -\frac{4\pi Ze}{\varepsilon}[n_+(y,0) - n_-(y,0)] \tag{3}$$

The Boltzmann distribution equation, Eq. (4), relates between the local ionic concentrations to the electrostatic potential as shown in Eq. (5) [1,2].

$$n_i = n^\infty\, e^{\pm\bar{\psi}_i} \tag{4}$$

$$n_\pm = n^\infty\, e^{-\bar{\psi}} \quad ; \quad n_- = n^\infty\, e^{+\bar{\psi}} \tag{5}$$

where $\bar{\psi} = (z_i F\psi/RT) = (Ze\psi/kT) = $ is the dimensionless electrostatic potential at a position y from the wall relative to that in the bulk solution, R is universal gas constant, k is Boltzmann constant, T is the absolute temperature. Substituting Eq. (5) into Eq. (3) results in the well-known Poisson–Boltzmann equation, Eq. (6) [1-3,6];

$$\frac{d^2\bar{\psi}}{dy^2} = \mathcal{K}^2 \sinh \bar{\psi} \tag{6}$$

where $\mathcal{K} = [4\pi(Ze)^2 n^\infty(z=0)/\varepsilon kT]^{1/2}$ is the Debye screening parameter. Eq. (7) and Eq. (8) show the boundary conditions within the slit; which were utilized to solve Eq. (6) [1,2].

$$y = h \,;\; \bar{\psi} = \zeta \tag{7}$$

$$y = 0 \,;\; \frac{d\bar{\psi}}{dy} = 0 \tag{8}$$

$$\int \left(\frac{d^2\bar{\psi}}{dy^2}\right) d\bar{\psi} = \frac{1}{2}\left(\frac{d\bar{\psi}}{dy}\right)^2 + C \tag{9}$$

$$\cosh^2 \bar{\psi} = \frac{1}{2}[\cosh(2\bar{\psi}) + 1] \tag{10}$$

To solve Eq. (6), we employ the mathematical identities in Eq. (9) and Eq. (10) and integrate from $y = 0$ to $y = y$ using the boundary condition in Eq. (8). Note that $\bar{\psi}_c = \bar{\psi}(y = 0)$ is the potential at the median plane between the slit walls [1,2,5-7].

$$\left(\frac{d\bar{\psi}}{dy}\right)^2 = 2\int_{\bar{\psi}_c}^{\bar{\psi}} \left(\frac{d^2\bar{\psi}}{dy^2}\right) d\bar{\psi} - C \tag{11}$$

$$\left(\frac{d\bar{\psi}}{dy}\right)^2 = 2\mathcal{K}^2[\cosh \bar{\psi} - \cosh \bar{\psi}_c] \tag{12}$$

$$\left(\frac{d\bar{\psi}}{dy}\right)^2 = 2\mathcal{K}^2\left[2\left(\cosh^2\frac{\bar{\psi}}{2} - 1\right) - 2\left(\cosh^2\frac{\bar{\psi}_c}{2} - 1\right)\right] \tag{13}$$

$$\left(\frac{d\bar{\psi}}{dy}\right)^2 = 4\mathcal{K}^2\left[\cosh^2\left(\frac{\bar{\psi}}{2}\right) - \cosh^2\left(\frac{\bar{\psi}_c}{2}\right)\right] \tag{14}$$

$$\frac{d\bar{\psi}}{dy} = 2\mathcal{K}\left[\cosh^2\left(\frac{\bar{\psi}}{2}\right) - \cosh^2\left(\frac{\bar{\psi}_c}{2}\right)\right]^{1/2} \tag{15}$$

When we have a surface potential of ζ, the potential profile in a capillary slit can be determined numerically from Eq. (15). However, if there is a surface charge density of ρ_e, the boundary condition in Eq. (7) should be replaced by the Gauss condition as [1,2,5-7];

$$y = h \,;\; \frac{d\bar{\psi}}{dy} = \frac{4\pi\rho_e}{\varepsilon} \tag{16}$$

Applying the new boundary condition in Eq. (15) results in:

$$\frac{1}{2}\left(\frac{4\pi\rho_e}{\varepsilon\mathcal{K}}\right)^2 = \cosh\left(\frac{Ze\zeta}{kT}\right) - \cosh\left(\frac{Ze\bar{\psi}_c}{kT}\right) \tag{17}$$

Dimensionless and scaling analysis for Eq. (17) [1-3,6];

LHS: $\frac{4\pi\rho_e}{\varepsilon\mathcal{K}}[=]\frac{\rho_e}{\varepsilon\mathcal{K}}[=]\frac{C/m^2}{(C/Vm)*(1/m)}[=]V$

RHS: $\frac{Ze\zeta}{kT}[=]\frac{Ze\bar{\psi}_c}{kT}[=]\frac{C\;V}{(J/K)*K}[=]\frac{C\;V}{(J/K)*K}[=]\frac{V}{V}[=]1$

where C is Coulomb (the SI unit of electric charge), m is meter (the length unit), V is volt (the SI unit of electromotive force), J is joule (a derived unit of energy; $J = kg\;m^2\;s^{-2} = Newton \times m = Watt \times s$), and K is Kelvin (the SI unit of thermodynamic temperature). Thus, we need to make the LHS dimensionless by multiplying LHS with the scaling factor $(Ze/kT)^2$, then LHS becomes,

LHS: $\frac{4\pi\rho_e}{\varepsilon\mathcal{K}}*\frac{Ze}{kT}[=]\frac{\rho_e}{\varepsilon\mathcal{K}}*\frac{Ze}{kT}[=]\frac{C/m^2}{(C/Vm)*(1/m)}*\frac{C}{(J/K)*K}[=]\frac{V}{V}[=]1$

$$\frac{1}{2}\left(\frac{4\pi Ze\rho_e}{\varepsilon\mathcal{K}kT}\right)^2 = \cosh\left(\frac{Ze\zeta}{kT}\right) - \cosh\left(\frac{Ze\bar{\psi}_c}{kT}\right) \tag{18}$$

Eq. (18) provides the relation between ζ and ρ_e for an arbitrary value of $\mathcal{K}h$.

(b) Induced electric potential

The total flux can be expressed by using Nernst-Einstein equation as in Eq. (19) [1-3,6];

$$J_\pm = -D_\pm\left(\nabla n_\pm + \frac{z_i F}{RT}n_\pm\nabla\psi\right) + n_\pm u \tag{19}$$

where $u = u(y)$ is the fluid velocity relative to the slit walls in the direction of decreasing electrolyte concentration, $D_+ \; and \; D_-$ are the diffusion coefficients of the cations and anions, respectively, $E = E(y)$ is the macroscopic electric field induced by the concentration gradient of the electrolyte. Substitution of $(z_i F\psi/RT) = (Ze\psi/kT)$ into Eq. (19) and applying the principle of superposition for the electric potential gives [1-3,6];

$$J_\pm = -D_\pm\left[\nabla n_\pm + \frac{Ze}{kT}n_\pm(\nabla\psi - E)\right] + n_\pm u \tag{20}$$

Assuming that $J_+ = J_- = J$ is possible if we have no net electric current arising from the transport of cations and anions by cocurrent diffusion, electric migration, and diffusio-osmotic convection; which will yield to have an analytical expression for the induced electric field distribution as obtained in Eq. (21) [1,2,7];

$$E = \frac{kT\nabla n^\infty}{Zen^\infty(z=0)}\left[\frac{(1+\beta)e^{-\bar{\psi}} - (1-\beta)e^{\bar{\psi}}}{(1+\beta)e^{-\bar{\psi}} + (1-\beta)e^{\bar{\psi}}} + \frac{Pe\sinh\bar{\psi}}{(1+\beta)e^{-\bar{\psi}} + (1-\beta)e^{\bar{\psi}}}\frac{u}{U^*}\right] \tag{21}$$

Where $U^* = 2kT|\nabla n^\infty|/\eta\mathcal{K}^2$ is a characteristic value of the diffusio-osmotic velocity, $\beta = (D_+ - D_-)/(D_+ + D_-)$ is the dimensionless effective diffusivity, $Pe = [4n^\infty(z=0)U^*/(D_+ + D_-)|\nabla n^\infty|] = [8n^\infty(z=0)kT/(D_+ + D_-)\eta\mathcal{K}^2]$, and η is the

fluid viscosity. Eq. (21) shows that $E(y)$ is strongly dependent on the local electrostatic potential ψ and fluid velocity u. To estimate the induced electric field E numerically, we need to solve for u as shown in part (c) [1,2,5].

(c) Fluid velocity distribution

The momentum balance from the modified Navier-Stoke equations can be applied for the steady diffusio-osmotic flow in a capillary slit to determine pressure and velocity distributions. Assuming the fluid is Newtonian and incompressible, y and z direction equations, respectively, gives [1-3,6];

$$\frac{\partial p}{\partial y} + Ze(n_+ - n_-)\frac{d\psi}{dy} = 0 \tag{22}$$

$$\eta\frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial z} - Ze(n_+ - n_-)E = 0 \tag{23}$$

Boundary conditions for velocity profile [1-3,6];

$$y = h \, ; u = 0 \tag{24}$$

$$y = 0 \, ; \frac{du}{dy} = 0 \tag{25}$$

Solving for pressure distribution from Eq. (5) and Eq. (22) and integrate from $\psi(y = 0) = \psi_c$ to $\psi(y = h) = \psi$ gives [1-3,6];

$$\int_p^{p_0} \frac{\partial p}{\partial y} dy + Ze\left(n^\infty e^{-\bar\psi} - n^\infty e^{+\bar\psi}\right)\int_{\psi_c}^{\psi} \frac{d\psi}{dy} dy = 0 \tag{26}$$

$$p_0 - p + n^\infty Ze\left(e^{-\bar\psi} - e^{+\bar\psi}\right)(\psi - \psi_c) = 0 \tag{27}$$

$$p = p_0 + n^\infty kT\left(e^{-\bar\psi} - e^{+\bar\psi}\right)(\bar\psi - \bar\psi_c) \tag{28}$$

$$p = p_0 + 2n^\infty kT(-\sinh\bar\psi - \sinh\bar\psi_c) \tag{29}$$

$$\sinh\bar\psi + \cosh\bar\psi = ? \tag{30}$$

Evaluating Eq. (30)

$$\frac{1}{2}e^{\bar\psi} + \frac{1}{2}e^{-\bar\psi} + \frac{1}{2}e^{\bar\psi} - \frac{1}{2}e^{-\bar\psi} = e^{\bar\psi} \tag{31}$$

for $\mathcal{K} \to \infty$; $(Ze \to \infty$ or $kT \to 0)$, thus $\bar\psi \to 0$; $e^0 = 1$, and then Eq. (27) becomes

$$\sinh\bar\psi + \cosh\bar\psi = 1 \tag{32}$$

Substitution of Eq. (32) into Eq. (29) results in

$$p = p_0 + 2n^\infty kT(\cosh\bar{\psi} - \cosh\bar{\psi}_c) \tag{33}$$

where p_0 is the pressure on the mid-plane between slit walls and at $y = 0$. Substituting Eq. (5) and Eq. (33) into Eq. (23) and integrating with respect to y-direction twice subject to the given boundary condition in Eq. (24) and Eq. (25) gives [1,2,7];

$$\eta\frac{\partial^2 u}{\partial y^2} = \frac{\partial p}{\partial z} - Ze(n_+ - n_-)E = 0 \tag{34}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\eta}\left[2|\nabla n^\infty|kT(\cosh\bar{\psi} - \cosh\bar{\psi}_c) - Zen^\infty E\left(e^{-\bar{\psi}} - e^{\bar{\psi}}\right)\right] \tag{35}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\eta}[2|\nabla n^\infty|kT(\cosh\bar{\psi} - \cosh\bar{\psi}_c) + 2Zen^\infty E \sinh\bar{\psi}] \tag{36}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{2|\nabla n^\infty|kT}{\eta}\left[(\cosh\bar{\psi} - \cosh\bar{\psi}_c) + \frac{Zen^\infty E}{kT|\nabla n^\infty|} \sinh\bar{\psi}\right] \tag{37}$$

$$u = \frac{2|\nabla n^\infty|kT}{\eta\mathcal{K}^2}\int_{\mathcal{K}h}^{\mathcal{K}y}\int_0^{\mathcal{K}y}\left[(\cosh\bar{\psi} - \cosh\bar{\psi}_c) + \frac{Zen^\infty E}{kT|\nabla n^\infty|} \sinh\bar{\psi}\right]d(\mathcal{K}y)d(\mathcal{K}y) \tag{38}$$

$$\frac{u}{U^*} = \int_{\mathcal{K}h}^{\mathcal{K}y}\int_0^{\mathcal{K}y}\left[(\cosh\bar{\psi} - \cosh\bar{\psi}_c) + \frac{Zen^\infty E}{kT|\nabla n^\infty|} \sinh\bar{\psi}\right]d(\mathcal{K}y)d(\mathcal{K}y) \tag{39}$$

To estimate the induced electric field E numerically, we solve for u from Eq. (39) and $\bar{\psi}$ from Eq. (15); and plug both of them into Eq. (21) [1,2].

3.   RESULTS AND DISCUSSION

From Fig. 2 and for specific values of the dimensionless parameters, $E(\mathcal{K}y)/E(0)$ is positive and increases with an increase in the electo-kinetic distance (y/h) from unity at the symmetrical position at the center $(y = 0)$ to a maximum value at the slit wall $(y = h)$ when Pe = 0. Normalized electric field decreases with an increase in $Ze|\zeta|/kT$ and increases with an increase in $\zeta/|\zeta|\beta$ if it is not too close to unity. It was found that the effect of the electrolyte convection on the local induced electric field in the slit (when Pe = 1) could be quite significant even if there is a low zeta potential at the walls [1,2].
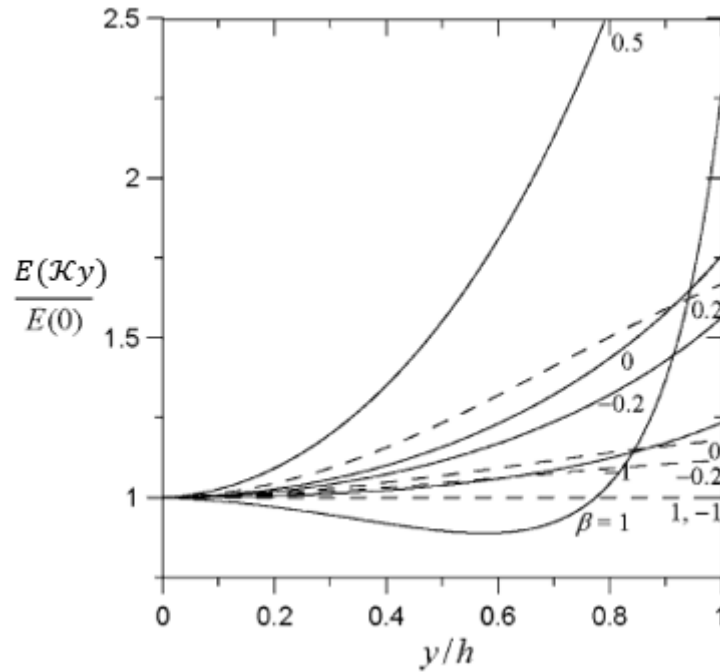
Fig. 2. Normalized electric field induced by an electrolyte gradient parallel to the walls of a slit versus the dimensionless coordinate y/h for the case of $\mathcal{K}y = 1$ with various values of the parameter β with $Ze\zeta/kT = 2$; solid curves: Pe = 1, dashed curves: Pe = 0.

From Fig. 3 and for specific values of the dimensionless parameters, the dimensionless velocity u/U^* decreases monotonically with the increase in the electro-kinetic distance (y/h). In other words, one can also understand that u/U^* increases monotonically and remarkably with an increase in the value of $\mathcal{K}h$. Also, it was determined that when $\beta = 0$, u/U^* increases with an increase in Pe and as Pe $\to \infty$, u/U^* becomes finite, Fig. 4.

When $Pe = \beta = 0$, the normalized velocity u/U^* is negative and the fluid flows toward the higher electrolyte concentration due to the chemi-osmotic contribution only. Similarly, when $Pe = 0$ and $\zeta/|\zeta|\beta \neq 1$, u is negative; however, if $Pe = 0$ and $\zeta/|\zeta|\beta \approx 1$ the fluid velocity u is positive, meaning that the fluid flows against the electrolyte concentration gradient. The fluid velocity u/U^* monotonically increases as $Ze|\zeta|/kT$ increase, as concluded from Fig. 3 and Fig. 4.

Comparing the analysis results in a fine capillary slit with the previous identified results in a fine capillary tube may help us to understand the diffusio-osmotic transport behavior. In both cases, in general, the effect of the induced electric field in the EDL is found to be of dominant significance on the diffusio-osmotic flow. However, for the tube case, the fluid velocity may have more than one reversal in direction over a small surface potential range and the flow rate does not always increase with an increase in the electro-kinetic radius of the capillary [8,9,10].
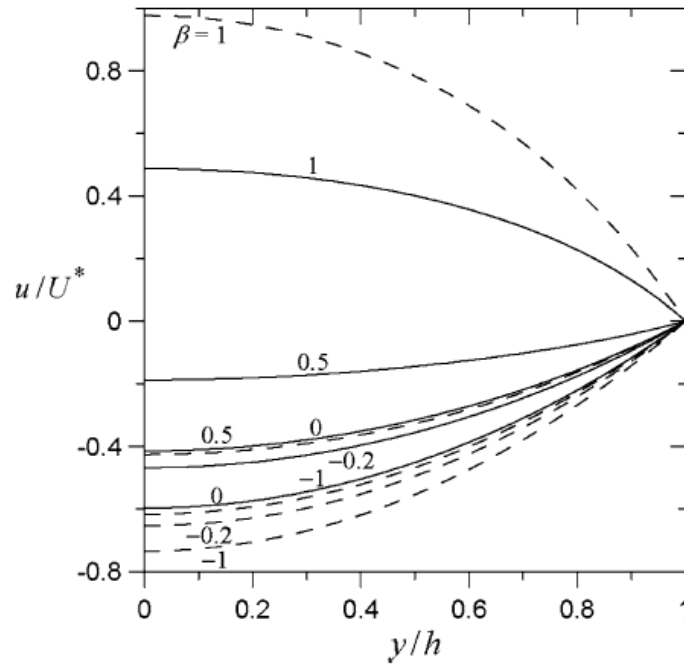
Fig. 3. Normalized diffusio-osmotic velocity in a slit versus the dimensionless coordinate y/h for the case of $\mathcal{K}h = 1$ with various values of the parameter β with $Ze\zeta/kT = 2$; solid curves: Pe = 1, dashed curves: Pe = 0.
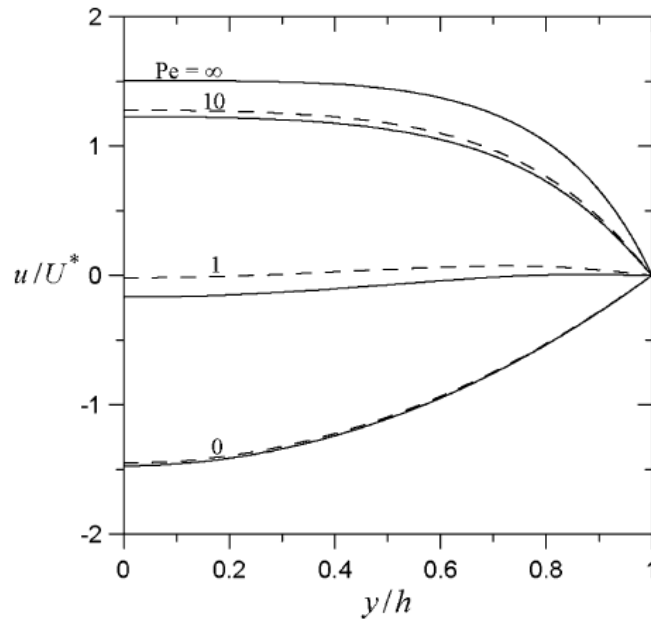

Fig. 4. Normalized diffusio-osmotic velocity in a slit versus the dimensionless coordinate y/h for the case of $Ze\zeta/kT = 4$ with $\mathcal{K}h = 1$; solid curves: β =−0.2, dashed curves: β = 0.

4. THEORY SHORTCOMINGS AND LIMITATIONS

Diffusio-osmotic flow within a capillary slit was derived with various assumptions such that the fluid is Newtonian, steady state, incompressible, y-direction dependent only, only slightly nonuniform in the electrolyte concentration and there is only a small external gradient of the electrolyte concentration along the plane wall and near the parallel plates in a slit [1,5]. The consideration of many assumptions in solving the distribution profile of both induced electric potential and fluid velocity may result in some kind of errors and deviation from a real case scenario. It is suggested that numerical analysis should be compared with experimental results to have a better judgment on the validity of the theory.

Moreover, it was assumed that the applied electrolyte concentration gradient ∇n^∞ is constant along the tangential direction since there is only a small external gradient; where n^∞ is the linear electrolyte concentration in the bulk phase and far away from the surface at y→∞ [1,2].

The theoretical and numerical investigations were carried out for electrolyte systems with bare walls. In contrast to this, the existence of a surface charge layer can result in a quite different diffusio-osmotic flow relative to a charged plane wall and/or relative to a capillary slit with bare walls. Earlier studies found that the structure of the surface charge layer have a tremendous effect on the flow velocity depending on the capillary characteristics and the electrolyte solution [9]. For example, a slit with a surface charge layer can lead to an augmented or a diminished electro-kinetic flow and the flow may also be reversed relative to that in a capillary with bare walls [4].

Linearized Poisson-Boltzmann equation have only a few terms analyzed (after applying Taylor's series expansion to the non-linear form) and this may result in a huge error in calculations if and only if the further terms have large magnitudes. Non-linear form of Poisson-Boltzmann will obviously result in having a much better prediction, but the problem will be more difficult to solve analytically.

X. Xing (2011) applied the non-linear Poisson-Boltzmann equation for two parallel uniformly charged plates and obtained new exact asymptotic results in various regime, but it turned out that the theory has some limitations and assumptions that might affect exact solutions which include the following [10,11]: (1) crystallization; high surface charge density may crystallize the counter-ion density on the plate surface (2) Neutral bound pairs; the response of ions form neutral bound pairs and ions with multiple valence to the external charged objects is a serious ununderstood issue, and (3) chemical interactions; there must be short range chemical interactions between ions and charged surfaces in real situations. Taking previous issues into consideration is critical in analyzing transport of electrolytes. Even though crystallization possibility has been extensively explored by many authors, the problem has not been completely understood. Also, the latter two issues must be investigated thoroughly and should be major missions of future works [10].

5. CONCLUSION

Electrostatic and induced potential distribution and velocity distribution of an electrolyte solution inside a capillary pore were investigated. In general, the normalized induced electric field is positive and increases in the electo-kinetic distance from the center $(y = 0)$ and normalized velocity decreases monotonically with the increase in the electro-kinetic distance.

However, an increase in Pe results in a monotonic increase in u/U^* and as Pe $\to \infty$, u/U^* becomes finite. When Pe $= 0$ and $\beta = 0$, E vanishes everywhere and u is negative. When u is negative, it means the fluid flows toward the higher electrolyte concentration; but, having a positive u indicates that the fluid flows against the electrolyte concentration gradient. The effect of the induced electric field was found to be of dominant significance on the diffusio-osmotic flow. Thus, it is important to understand the mechanism of diffusio-osmotic flow of electrolyte solutions in porous media to improve and develop related separation technologies.

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