The H-Hierarchy

Mathematical Backbone of the SFVFS™ Programme

H_0 through H_{12} · Six Kimi-Confirmed Upgrades · Dimensional Taxonomy · Master Equation

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SFVFS™ Positioning System · Segment 9 of 12 · Framework
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CF CONSISTENT not PASS

Fourteen equations trace the full $Seed_1 \rightarrow Form_1 \rightarrow Void \rightarrow Form_2 \rightarrow Seed_2$ cycle. Six notational upgrades, confirmed by Kimi, make the cycle structure visible in the equations themselves. The master equation H_C is the cycle written as a contour integral. The dimensional taxonomy maps the same progression onto physical space. The programme named itself in the last line.

The hierarchy is the formal skeleton that connects the abstract mathematics of Segments 1–3 (The Pinch, The Needle's Eye, The Cartographer) to the physical exemplars of Segments 5–7 (Saturn, AMOC, Tokamak) and the atlas of Segment 8. It is the map's internal geometry.

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1. Plain English

The H-Hierarchy is a sequence of fourteen mathematical objects, each one built from the previous, that traces the complete SFVFS™ cycle from its most primitive form to its closure.

Start with H_0: the rawest material. A contour integral of the logarithmic derivative of the Riemann zeta function — the most fundamental object you can build from the prime numbers and the zeta zeros together. No structure imposed. Just the raw signal.

H_1 squares it to get an energy density. H_2 strips the main term to isolate the oscillatory residue — the interesting part. H_3 takes the limit and extracts the invariant: the H3 Invariant, the seed that crystallises. This is the $Seed_1$ phase.

H_4 fires the bilateral operator $B[H_3](\sigma) = H_3(\sigma) + H_3(1-\sigma)$. This is the hourglass made algebraic: the function and its reflection across the critical line $\sigma = 1/2$, summed. The $FORM_1$ phase activates. H_5 adds nonlinear self-interaction. H_6 decomposes into flow components (H_6a, H_6b-i) that close, and the terminal barrier H_6b-ii that does not.

Then comes the void. Ψ_void is not a number or a limit. It is the threshold between the analytical world of $Form_1$ (where σ is a complex variable) and the geometric world of $Form_2$ (where the structure lives on the circle S^1 in operator space). Crossing the void means changing mathematical language entirely.

H_7 through H_9 reconstruct the geometry in the new language: S^1 operator space, metric structure, optimal configuration. H_9 is the DN analog — the natural resting configuration, the Needle's Eye in pure mathematics.

H_{10} through H_{12} are the $Seed_2$ phase. But here is the key Kimi confirmation from 23 March 2026: these three rungs are epiphenomenal. They are not traversed. They are produced by stopping at H_9. The arc parks at the optimal configuration and the $Seed_2$ emerges as a consequence — the geometry re-establishes itself without being explicitly constructed. The cycle implied its own closure.

H_C is the master equation: the entire cycle written as a single contour integral along the SFVFS™ path. The last line of the master equation reads: flow(H_0 → H_∞). The programme named itself.

And the fold: $H_0 = H_\infty$. The pre-seed and the quantum limit are the same object. −1D = 4D. Source = Identification. The cycle closes on itself.

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2. Hierarchy Overview

Object	Phase	Role
H_0	Pre-seed	The primal integral. Raw material before the cycle begins.
H_1	$Seed_1$	Energy density. Seed activation.
H_2	$Seed_1$	Oscillatory residue. Main term stripped.
H_3 · S_seed	$Seed_1$	**H3 Invariant. The seed crystallises.**
H_4 · $B[H_3]$	$Form_1$	Bilateral hourglass. FORM activated.
H_5	$Form_1$	Nonlinear activation on bilateral structure.
H_6a	$Form_1$ (flow)	**Flow component. Closes.**
H_6b-i	$Form_1$ (flow)	**Structural inheritance. Closes.**
H_6b-ii	**$Form_1$**	**Terminal barrier. RH-strength. Does not close.**

	(STATIC)	Stable under subdivision at depth 3.
Ψ_void	**VOID**	**Transition point. Not a limit. Not a value. The needle's eye.**
H_7	$Form_2$	Geometric emergence. S^1 operator space.
H_8	$Form_2$	Metric structure. Distance + asymmetry.
H_9	**$Form_2$**	**Optimal geometric configuration. DN analog. Arc parks here.**
H_{10}	*$Seed_2$*	*Epiphenomenal. Global integration over moduli space. Not traversed — produced by parking at H_9.*
H_{11}	*$Seed_2$*	*Epiphenomenal. Return to critical line. Geometric meets analytical.*
H_{12}	*$Seed_2$*	*Epiphenomenal. Weighted synthesis. Seed remembers void.*
H_Conductor	Closure	Modular symmetry acting on weighted hierarchy.
H_∞	Closure	Quantum limit. Cycle at infinite refinement.
H_C	**Master**	**The cycle as contour integral. The programme named itself.**

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3. The Hierarchy

Pre-Seed H_0 · *The primal integral*

H_0 is constructed from the logarithmic derivative of the completed zeta function ξ(s), which incorporates both the trivial and non-trivial zeros of the Riemann zeta function. Define the correction term:

$$c(\sigma) := \log(\pi) + \tfrac{1}{2}\sigma + \tfrac{1}{2}(1-\sigma) + \Gamma(\sigma/2) + \Gamma((1-\sigma)/2)$$

where Γ is the Euler gamma function. The primal integral is the principal value contour integral of the logarithmic derivative of ξ along a vertical line segment:

> **$H_0(\sigma,T)$** $P.V. \int_{-T}^{T} \xi'/\xi\,(\sigma+it)\,dt$ — *Primal integral. Raw material. Encodes zeros and poles of ζ.*

H_0 captures all the information in the zero distribution: each non-trivial zero ρ = β + iγ contributes a pole to ξ′/ξ at s = ρ. The principal value regularises the conditional convergence. At σ = 1/2 (the critical line), $H_0(1/2, T)$ measures the accumulated zero density up to height T. This is the starting material. No structure has been imposed.

Seed₁ H₁ → H₃ · *Invariant crystallisation*

> **H₁(σ,T)** $|H_0(\sigma,T)|^2$ — *Energy density. Seed activation. Positive definite.*

H_1 converts the signed contour integral to an energy density by squaring the modulus. This is the first irreversible step: phase information is partially lost, but the magnitude of the zero contribution is preserved. H_1 grows like T log T on average (by the explicit formula), providing the main term that H_2 will strip.

> **H₂(σ,T)** $|H_0(\sigma,T) - (2T \log T - c(\sigma)T)|^2$ — *Oscillatory residue. Main term subtracted.*

H_2 subtracts the leading-order main term $2T \log T - c(\sigma)T$, which accounts for the smooth background distribution of zeros. What remains is the oscillatory residue: the deviation of the actual zero distribution from its smooth average. This is the interesting part — the signature of the prime distribution encoded in the zeros. $H_2 \to 0$ if the zeros are distributed exactly as the smooth average predicts; any deviation shows up here.

> **H₃(σ)** $\limsup_{T\to\infty} [\min_{\Phi} H_2(\sigma,T) + \|Tr\, U_{\Phi}(\sigma)\|^2]$ — *H3 Invariant. The seed crystallises.*

H_3 takes the lim sup over T of the minimum over an admissible family of operators U_Φ. The minimisation selects the operator that best accounts for the oscillatory residue; the lim sup extracts the irreducible component that no operator can eliminate.

The U_Φ operator family. U_Φ is a family of bounded linear operators on the Hardy space $H^2(H_{1/2})$ parametrised by admissible phase functions $\Phi: \mathbb{R} \to \mathbb{R}$. The precise functional-analytic specification — exact conditions on Φ, domain and range of U_Φ, continuity properties — requires a dedicated Kimi session post-exhibition. For exhibition purposes: U_Φ represents the best available analytic approximation to the zero structure; H_3 measures what survives after optimal approximation. If $H_3 > 0$, the invariant is non-trivial and the seed has crystallised.

> **★ Upgrade 1 S_seed(σ) := H₃(σ)** — *Seed crystallisation named as explicit output*

Form₁ H₄ → H₆ · *Bilateral hourglass → terminal barrier*

> **★ Upgrade 2 B[f](σ) := f(σ) + f(1−σ)** — *The bilateral hourglass operator*

The bilateral operator B reflects a function across the critical line $\sigma = 1/2$ and sums. It is self-adjoint on $L^2((1/2-\varepsilon, 1/2+\varepsilon))$ and commutes with the functional equation of ζ. The fixed points of B are precisely the functions symmetric about $\sigma = 1/2$. RH is the statement that the zeros of ζ lie at the fixed point set of B.

> **H₄(σ)** $B[H_3](\sigma) := H_3(\sigma) + H_3(1-\sigma)$ — *FORM activated. Bilateral structure fires.*

H_4 applies the bilateral operator to the seed H_3. The result is symmetric: $H_4(\sigma) = H_4(1-\sigma)$ by construction. This is the algebraic hourglass: the functional equation made explicit in the hierarchy. $H_4 > 0$ requires the seed to be non-trivial on at least one side of the critical line.

> **$H_5(\sigma)$** $H_4(\sigma) + \lambda|H_4(\sigma)|^2$ — *Nonlinear activation. $\lambda > 0$ is a coupling constant.*

H_5 adds a quadratic self-interaction term controlled by the coupling constant $\lambda > 0$. This models the self-reinforcing structure of the bilateral form: where H_4 is large, the nonlinearity amplifies it further. The coupling λ sets the scale of the Form_1 activation and connects to the renormalisation-group structure of the hierarchy near the void.

> **★ Upgrade 3 $H_6(\sigma) = H_6a(\sigma) + H_6b(\sigma)$** — *Terminal Form decomposes into flow + barrier*
>
> **$H_6a(\sigma)$** $H_5(\sigma)$ — *Flow component. Closes by Montel + Fatou–Riesz.*
>
> **$H_6b(\sigma)$** $\mu|H_4(\sigma)| = H_6b\text{-}i(\sigma) + H_6b\text{-}ii(\sigma)$ — *Barrier. $\mu > 0$ measures barrier intensity.*

H_6a is the flow component. It inherits the BMOA bounds from the infrastructure (Theorems 2.1–2.3 in The Pinch) and closes by standard limit arguments. H_6b-i is the structural inheritance component; it closes by symmetry preservation. H_6b-ii is the terminal static barrier: the statement that $G_0 = E \cdot \xi$ in $H^2(H_{1/2})$. It does not close. It is RH-strength. Stable under subdivision at depth 3.

> **FLAG: H_6b-i empty/full determination** [PENDING — post-exhibition]
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> Whether a BKM-closable component exists within $\mu|H_4(\sigma)|$ remains to be determined by dedicated Kimi session. Must match the split-to-stable analysis in The Pinch. If H_6b-i is empty, $H_6b = H_6b$-ii directly — the barrier is the terminal static wall with no flow sub-component. This does not change the FSC classification: all flow collapses to the static core regardless.

Void Ψ_{void} · *The needle's eye*

> **★ Upgrade 4 $\Psi_{void} :=$ threshold($H_6 \to H_7$)** — *Named transition point, not a limit*

The void threshold Ψ_{void} is the infimum of σ where $H_6(\sigma) \neq H_7(\sigma)$ in operator norm:

$$\Psi_{void} = \inf\{ \sigma > 0 : H_6(\sigma) \neq H_7(\sigma) \text{ in operator norm } \}$$

Three properties define Ψ_{void} precisely:

Not a limit. The system does not approach Ψ_{void} from below along a convergent sequence. It is a threshold of divergence — the greatest lower bound where the two regimes first differ.

Not a value. H_6 and H_7 are not defined at the same point; Ψ_{void} is the boundary between their domains. The void cannot be occupied.

A dimensional transition. To the left of Ψ_void: analytical σ-space (the complex variable world of Form₁). To the right: geometric S^1 operator space (the circle bundle world of Form₂). The void is where the mathematical language changes.

> **Kimi CF CONSISTENT.** *The Pinch is the void. H₆b-ii is the void's face in the RH scheme. Ψ_void is the same object as the void threshold in The Cartographer — inf{ η > 0 : G_η ≠ E·ξ in operator norm }.*

Form₂ H₇ → H₉ · *Geometric reconstruction*

Form₂ operates in a completely different mathematical language from Form₁. The analytical σ-variable is replaced by a geometric S^1 structure: the unit circle in the operator space associated with the Hilbert bundle over the spectral data. The Form₂ objects are no longer functions of a complex variable; they are functionals on geometric spaces.

> **H₇** $(1/2\pi) \int_{S^1} Tr_{H^- \otimes H^+} [(1 \otimes (\bigstar \circ M_r^*)) P_0(\tau) (\rho^- \otimes \rho^+)] \, d\tau$ — *Geometric emergence. S^1 operator space.*

H₇ is a trace functional over the circle S^1, acting on the tensor product of positive (H^+) and negative (H^-) energy Hilbert spaces. The operator $P_0(\tau)$ is the spectral projector at angle τ; M_r^* is the radial multiplication operator; ★ is the Hodge star on the operator algebra; ρ^-, ρ^+ are the density matrices for the negative and positive energy sectors. This expression is presently schematic — it indicates the correct functional structure but the full specification of the Hilbert bundle and trace-class conditions awaits a dedicated functional analytic embedding (flagged post-exhibition). The geometric content is correct: H₇ measures the asymmetry between positive and negative energy contributions over S^1.

> **H₈(K,ω)** $H_7 + \alpha(Tr\rho^- - Tr\rho^+)^2 + \beta \, dist_\chi(K, K_0)^2$ — *Metric structure. Distance + asymmetry. α, β > 0.*

H₈ adds two penalty terms to H₇. The first, $\alpha(Tr\rho^- - Tr\rho^+)^2$, penalises asymmetry between the positive and negative energy sectors — it is zero precisely when the sectors are balanced, which is the condition at the DN attractor. The second, $\beta \, dist_\chi(K, K_0)^2$, is the squared distance in the geometric class χ(ω) from a reference configuration K_0. Together they define a metric on the space of geometric configurations.

> **H₉(ω)** $\inf_{K \in \chi(\omega)} H_8(K, \omega)$ — *Optimal geometric configuration. DN analog. Arc parks here.*

H₉ minimises H₈ over all admissible geometric configurations K in the class χ(ω). This is the DN analog: the natural resting configuration of the geometric structure, the configuration that minimises the combined asymmetry and distance penalty. At H₉, the energy asymmetry $Tr\rho^- = Tr\rho^+$ (balanced sectors) and the configuration is at the optimal point in χ(ω). This is the Needle's Eye of the hierarchy in pure mathematics.

Hook Topology — Kimi CF CONSISTENT, 23 March 2026. *H_9 is the arc parking point. The hierarchy stops here. H_{10} through H_{12} are not traversed. They are epiphenomenal: geometrically implied by the arc parking at H_9, not produced by continuing through them. They exist in the hierarchy as limit objects — what the $Seed_2$ phase looks like when you inspect H_9 from the outside — but the programme does not need to pass through them to close. SFVFS™ is its own 13th example of this Kimi-documented phenomenon, alongside Aharonov–Bohm, Jordan's Lemma, and ten others.*

$Seed_2$ $H_{10} \to H_{12}$ · *Epiphenomenal — implied by parking at H_9*

These three rungs exist as limit objects. They are not traversed. They are the shadow cast by H_9 onto the full cycle. Cf. Hook Topology, Kimi CF CONSISTENT 23 March 2026.

> **H_{10}** $\int_{\mathscr{M}_g} H_9(\omega)\, d\mu_{WP}(\omega)$ — *Global integration over moduli space $\mathscr{M}_g$ with Weil-Petersson measure.*
>
> **H_{11}** $\sup_{f\in\mathscr{F}} |P.V. \int f(t)\, \xi'/\xi\,(\tfrac{1}{2}+it)\, dt|^2$ — *Return to critical line. Geometric meets analytical.*
>
> **★ Upgrade 5** $H_{12}(\sigma) = \Sigma_{k=0}^{11} w_k(\tfrac{1}{2})\, \dot{M}_k(\sigma)$ — *Weights from k=0..11 only. H_{12} excluded. No circularity.*

The weights are conditioned on prior critical-line values: $w_k(1/2) := H_k(1/2) / \Sigma_{j=0}^{11} H_j(1/2)$. The overline $\dot{M}_k$ denotes complex conjugate. The sum runs only to k = 11 to avoid circular self-weighting of H_{12}. The weighted synthesis H_{12} remembers the void: it encodes the critical-line values of all previous rungs, carrying the geometric memory of the passage through Ψ_{void} forward into the reconstituted seed.

Closure H_Conductor · H_∞ · H_C · *Cycle closure*

> **H_Conductor** $\inf_{\gamma \in PSL(2,\mathbb{Z})} \Sigma\, w_k(\tfrac{1}{2})\, H_k(\gamma(k)\cdot\tfrac{1}{2})$ — *Modular symmetry acting on weighted hierarchy. $PSL(2,\mathbb{Z})$ = modular group.*

H_Conductor applies the modular group $PSL(2,\mathbb{Z})$ to the weighted hierarchy. The modular group acts on the upper half-plane by fractional linear transformations $\gamma(s) = (as+b)/(cs+d)$; its action on the critical point 1/2 generates the orbit $PSL(2,\mathbb{Z})\cdot 1/2$. The infimum over γ selects the modular image that minimises the weighted sum. This is the hierarchy's expression of modular symmetry: the weights set by the critical-line values are stable under the modular action.

> **H_∞** $\limsup_{n\to\infty} \langle \Psi_n | |\dot{M}_{12}|^2 | \Psi_n \rangle$ — *Quantum limit. Cycle at infinite refinement. Ψ_n are a sequence of states.*

H_∞ is the quantum limit: the lim sup of the expectation value of $|\dot{M}_{12}|^2$ in a sequence of states Ψ_n approaching the critical line. This is the cycle in infinite-dimensional operator language: the hierarchy's behaviour under infinite

refinement of the approximation sequence. The quantum limit captures the ‘memory’ of all previous rungs in the limit object.

> **★ Upgrade 6 H_C = ∮_{SFVFS™} dH** — *SFVFS™ contour path made explicit in cycle notation*

★ MASTER EQUATION ★

$$H_C := \oint_{SFVFS™}\, dH \qquad \text{(formal contour integral over the SFVFS™ path)}$$

$$= flow(\, S_1 \cdot H_3 \;\rightarrow\; F_1 \cdot H_6 \;\rightarrow\; V \cdot \Psi_{void} \;\rightarrow\; F_2 \cdot H_9 \;\rightarrow\; S_2 \cdot H_{12} \,)$$

$$= flow(\, H_0 \;\rightarrow\; H_\infty \,)$$

The fold (Kimi canonical, 24 March 2026): $H_0 = H_\infty$ · −1D = 4D · Source = Identification

The primal integral H\u2080 and the quantum limit H_\u221e are the same object viewed from opposite ends of the cycle. The pre-seed (\u22121D, before the cycle begins) and the cycle closure (4D, when time folds back on itself) are identified. Source = Identification: the hierarchy that emerged from the binary cube geometry closed on itself. The programme named itself in the last line.

“The cycle closes in the final expression. The programme named itself in the last line.”

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4. Dimensional Taxonomy — −1D to 4D

The same SFVFS™ cycle maps onto physical space across six dimensional levels. The void is not just a mathematical threshold — it is a dimensional threshold: at every level, from sub-point to four-dimensional cycle, the void is where the system changes its nature.

Dim	Space	SFVFS™ Phase	H-Hierarchy	Key Structure	Physical Exemplar
−1D	Sub-point	Pre-SEED	H_0	Condition for conditions. Raw integral before cycle begins.	—
0D	Point	$SEED_1$	H_3 (S_seed)	Singularity. Seed crystallises at a point on the	Prime encoding; NADW seed; ballooning instability

				critical line.	latent
1D	Line	FORM$_1$	B[H$_3$] (H$_4$)	Bilateral thread. Compression. Hourglass drawn on the line.	Gulf Stream; L-mode activation; RH functional equation
2D	**Plane**	**VOID**	**Ψ_void**	**Codimension-2 threshold. Liminal surface. Normal rules suspended.**	Saturn hexagon (Tresca condition); AMOC NADW chimneys; Tokamak pedestal top
3D	Volume	FORM$_2$	H$_7$ → H$_9$	Geometric reconstruction. Habitable space. S^1 operator space.	Post-ELM recovery; post-D-O event reconstitution; Shallow Void family
4D	**Duration**	**SEED$_2$**	**H_C**	**Cyclic memory. The contour integral. Time folds back on itself. H$_0$ = H_∞.**	**AMOC millennial cycle; ITER operation 2035–; Dansgaard-Oeschger periodicity**

The fold in the taxonomy. The −1D row (H$_0$, pre-seed) and the 4D row (H_C, cycle closure) are identified by the fold: H$_0$ = H_∞. The taxonomy is not a line from −1D to 4D; it is a loop. The pre-seed is the same object as the completed cycle, seen from opposite ends.

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5. Six Kimi-Confirmed Upgrades

Upgrade	Notation	Status	Meaning
1	S_seed(σ) := H$_3$(σ)	**Canonical**	Seed crystallisation named as explicit output
2	B[f](σ) := f(σ) + f(1−σ)	**Canonical**	The bilateral hourglass operator
3	H$_6$(σ) = H$_6$a(σ) + H$_6$b(σ)	*Canonical (H$_6$b-i may be empty — check The Pinch)*	Terminal Form decomposes into flow + barrier
4	Ψ_void := threshold(H$_6$ →	**Canonical**	Named void threshold. Infimum

	H_7)		≠ limit.
5	$H_{12} = \Sigma_{k=0}^{11} w_k(½) \dot{M}_k(\sigma)$	**Canonical**	Weights from k=0..11 only. H_{12} excluded. No circularity.
6	$H_C = \oint_{SFVFS™} dH$	**Canonical**	Programme named itself. Contour integral over the SFVFS™ path.

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6. Open Flags

Flag	Status	Action
H_6b-i empty/full determination	**PENDING**	Check whether BKM-closable component exists within $\mu\|H_4(\sigma)\|$. Must match split-to-stable in The Pinch. Dedicated Kimi session required post-exhibition.
H_6b-ii hyperbolic/non-hyperbolic	Post-exhibition	Class I vs Class III distinction. Determines whether barrier is absolutely symmetric or has unmapped faces.
U_Φ functional-analytic specification	Post-exhibition	Embed in Hilbert/Banach space. H_3 in Sobolev space; $B[H_3]$ as self-adjoint operator on $L^2(1/2-\varepsilon, 1/2+\varepsilon)$. Full specification of admissible phase functions Φ.
H_7 Hilbert bundle specification	Post-exhibition	Specify the Hilbert bundle, trace-class conditions, and exact operator algebra for H_7. Current expression is structurally correct but not fully specified.

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7. Summary

Established	Pending or open
H_0–H_C defined and sequenced. Six Kimi-confirmed upgrades canonical. Dimensional taxonomy complete. Master equation $H_C = \oint_{SFVFS™} dH$ canonical. $H_0 = H_\infty$ fold canonical (Kimi 24 March 2026). Hook topology: H_{10}–H_{12} epiphenomenal (Kimi 23 March 2026).	H_6b-i empty/full determination. H_6b-ii hyperbolic/non-hyperbolic. U_Φ functional-analytic embedding. H_7 Hilbert bundle specification. All flagged post-exhibition.

Ψ_void infimum ≠ limit, transition point confirmed.	

“The programme named itself in the last equation.”

Framework References

The Pinch — Craig Spectral Criterion. H6 decomposition. H6b-ii terminal barrier. Segment 1 of 12.

The Needle’s Eye — DNS equation of state (H_1_norm, Λ) = (1,1). DN attractor as H_9 analog. Segment 2 of 12.

The Cartographer — Flow-static collapse. Corner Theorem as infrastructure expansion. Segment 3 of 12.

Formalisation Brief — Kimi referee review, 21–24 March 2026. All six upgrades canonical. Fold H_0 = H_∞ canonical. Hook topology canonical.
