The Wall That Remembers

A very simple guide to the mathematics

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A plain-English companion to *Trace Fibres of Marked Interval Heaps*

It began at Leake Street

Leake Street is a tunnel in London where people paint on the walls.

An artist makes a picture. Another artist paints over it. Then somebody paints over that.

We see the newest picture. But the old paint has not vanished. It is still inside the wall.

The wall is therefore holding three different things:

- the picture we can see now;
- all the hidden layers underneath it;
- all the actions that made those layers.

These three things are not the same.

A tiny pretend wall

The paper makes a very simple pretend wall. Imagine a row of little squares.

A painter may choose a colour and paint one unbroken stretch of the row. This is called one *pass*.

Each square remembers the colours that touched it, in the order they arrived. Its little list of colours is its stack. All the stacks together are called the wall's *stratigraphy*.

This pretend wall does not try to copy every detail of real painting. It keeps only the parts needed for the counting problem: place, colour and order.

The question

Suppose we are given every hidden colour stack in the wall.

Can we discover exactly how it was painted?

Usually, no.

One long red pass might leave the same stacks as two shorter red passes. Two passes far apart might happen in either order without affecting one another.

So the paper asks a different question:

How many genuinely different painting processes could have made these exact layers?

If two far-apart passes merely swap places in time, the paper treats them as the same process. Their order made no physical difference.

The main discovery

At first, counting all possible processes looks enormous. Every mark might interact with many other marks.

The main theorem finds a shortcut.

For a wall arranged in a line, we do not need to solve the whole wall at once. We compare each place only with the place beside it. We count how their two stacks can share painting passes. Then we join those small answers together.

In the simplest words:

The possible histories of the whole wall can be counted from its neighbouring pairs.

The same neighbouring-pair idea works when the places branch like a tree. In general, changing the tree can change the answer because it changes which stacks are neighbours. There is one especially neat case: when every place carries the same stack, the overall shape of the tree does not matter.

What the answer tells us

The counting gives more than one total.

It tells us:

- the smallest number of passes that could have made the layers;
- how many processes use that many passes;
- how many use one extra pass, two extra passes, and so on;
- whether the making was almost forced or very uncertain.

A simple-looking surface can hide a very complicated making. Looking easy does not mean being easy to make.

An old checkerboard puzzle

The new counting rule produced a row of numbers. Then we found that mathematicians had seen the same numbers before.

In December 2002, Cees H. Elzinga asked a question that led to the number triangle. Dean Hickerson recast it as a problem about choosing black squares on a checkerboard and recorded it in the Online Encyclopedia of Integer Sequences in January 2003.

Nobody there was talking about Leake Street, buried paint or possible painting processes. It was the same counting pattern wearing different clothes.

The old entry contained two unproved guesses about the numbers. It also displayed a formula with the wrong sign.

The new paper:

- explains why the checkerboard numbers appear in the wall problem;
- corrects the sign;
- proves the two old guesses;
- connects the totals to another known family of objects called directed animals.

The prior-art discovery did not remove the new result. It gave the theorem an old unfinished puzzle to complete.

The accelerating-climb surprise

We also counted how many possible processes use each number of passes.

The counts often rise to one high point and then fall. Mathematicians can ask for a stronger kind of smoothness: while the numbers are climbing, each new multiplication should be no larger than the one before it. The climb may continue, but it should not suddenly get steeper.

Thousands of small examples followed this stronger rule. It looked like a law. But it was false.

The smallest example over a two-colour alphabet uses two places. At both places the seven hidden layers alternate:

red, blue, red, blue, red, blue, red.

For processes using seven passes, eight passes, nine passes, and so on up to fourteen passes, the numbers are:

1, 7, 51, 135, 197, 117, 25, 1.

The sequence still rises once and falls once: it really is a hill. But its first climb multiplies by 7, while the next multiplies by about 7.29. The climb speeds up when the stronger smoothness rule says it must slow down. In formal mathematics, this is a failure of *log-concavity*.

This failure begins exactly at seven alternating layers and continues for every longer member of the same family. The two-colour qualification matters: when three colours are allowed, a smaller failure exists with twelve hidden layers in total.

Why the computer missed it

The first large search looked at many thousands of walls. But it only looked at walls with too few layers to contain the smallest counterexample.

The search did not get unlucky. Its boundary made the answer impossible to see.

Once the neighbouring-pair theorem revealed where to look, the counterexample became small, exact and provable. A later exhaustive search checked every binary stratigraphy with fewer layers and confirmed that none fails earlier.

This is an important part of the story: a large computation is only as strong as the space it is allowed to explore.

What the paper does not claim

The mathematics does not identify the real artists. It does not know their intentions, feelings or meanings. It does not reconstruct the one true history of Leake Street.

It counts the different processes that are compatible with the layer information we choose to record.

Real walls are richer. Paint has thickness, texture, damage, weather and human decisions. The model is a clear mathematical skeleton, not the whole living tunnel.

Why it may matter

The same shape of problem appears whenever local records survive but the full history is missing. Possible future uses include:

- studying painted or archaeological layers;
- reconstructing computer actions that happened at the same time;
- understanding incomplete audit records;
- measuring how many different makings can hide behind one result.

The theorem currently applies when each action covers one connected stretch of a line, or one connected part of a tree. A direct version for actions arranged around a circle is actually false: it already breaks on a three-place circle because locally sensible layer orders can form an impossible loop. Any circular version would need an extra rule checking the whole cycle.

The idea in one sentence

What we see is not everything that exists, and what exists does not tell us exactly how it was made.

That is what Leake Street made visible. The paper turns it into something that can be defined, counted and proved.

Companion to Marc Craig, *Trace Fibres of Marked Interval Heaps*, Version 5, 31 August 2026.
The formal paper contains the definitions, proofs, computations and references.