Transistor Theory Classroom Notes

BJT, MOS

Biasing, Gain, Output Resistance, Cascode

Α	α	Alpha	a
Β	β	Beta	b
Γ	γ	Gamma	g
Δ	δ	Delta	d
Ε	ε	Epsilon	ĕ
Ζ	ζ	Zeta	z
Η	η	Eta	ē
Θ	θ ϑ	Theta	th
Ι	ι	Iota	i
Κ	κ	Kappa	k
Λ	λ	Lambda	l
Μ	μ	Mu	m
Ν	ν	Nu	n
Ξ	ξ	Xi	x
Ο	ο	Omicron	ŏ
Π	π	Pi	p
Ρ	ρ	Rho	r
Σ	σ	Sigma	s
Τ	τ	Tau	t
Υ	υ	Upsilon	u
Φ	ϕ	Phi	ph
Χ	χ	Chi	ch
Ψ	ψ	Psi	ps
Ω	ω	Omega	ō

Book Outline - Microelectronic Ckts - Jaeger

Device Electronics ch2 $q = 1.06 \times 10^{-19}$ C

ρ resistivity Ω cm

σ conductivity $(\Omega \text{ cm})^{-1}$

$j = Qv$ drift current = Charge density × velocity $C/cm^3 \cdot cm/s = A/cm^2$

n_i intrinsic carrier density cm^{-3} $10^{10}/cm^3$ intrinsic Si

n density of free electrons $n = n_i$ for intrinsic material cm^{-3}

$n_i^2 = BT^3 e^{-\frac{E_G}{kT}}$ cm^{-6} $n_i^2 = 10^{20} cm^{-6}$ for Si @ STP)

E_G bandgap energy eV 1.12 eV for Si

k Boltzmann's constant 8.62×10^{-5} eV/K

T Absolute temp K

B material parameter 1.08×10^{31} $K^{-3} cm^{-6}$ for Si

p hole density cm^{-3} $= n_i$ for intrinsic material

$n = n_i = p$ intrinsic, $pn = n_i^2$ intrinsic or not

v carrier drift velocity cm/s $V_{sat} = 10^7$ cm/s (in $\nabla V > 3 \times 10^4$ V/cm)

μ mobility | μ_p hole 500 cm^2/Vs } intrinsic Si

$\mu = \frac{\sigma}{qn} \frac{C\,cm^3}{\Omega\,cm} = \frac{cm^2}{Vs}$ | μ_n electron 1350 cm^2/Vs

N_D donor impurity concentration atoms/cm^3

N_A acceptor impurity concentration atoms/cm^3

n-Type material $N_D > N_A$ $p = \frac{n_i^2}{n}$ $n = \frac{(N_D - N_A) \pm \sqrt{(N_D - N_A)^2 + 4n_i^2}}{2}$

P-Type material $N_A > N_D$ $n = \frac{n_i^2}{p}$ $p = \frac{(N_A - N_D) \pm \sqrt{(N_A - N_D)^2 + 4n_i^2}}{2}$

conductivity $(N_D > N_A)$ $\sigma \approx q\mu_n (N_D - N_A) = q\mu_n n$ $(\Omega \text{ cm})^{-1}$

$(N_A > N_D)$ $\sigma \approx q\mu_p (N_A - N_D) = q\mu_p p$ $(\Omega \text{ cm})^{-1}$

$j_{p,n}^{diff}$ diffusion current $= (+,-q) D_{p,n} \left(-\frac{\partial p,n}{\partial x}\right) = -,+q D_{p,n} \frac{\partial p,n}{\partial x}$ A/cm^2

$V_T = \frac{D_{n,p}}{\mu_{n,p}} = \frac{kT}{q}$ Einstein's Relationship $eV/e = V$

Thermal Voltage

$j_{n,p}^T = q\mu_{n,p} nE + qD_{n,p} \frac{\partial n,p}{\partial x} = q\mu_{n,p} n \left(E + V_T \frac{1}{n,p} \frac{\partial n,p}{\partial x}\right)$ Total Current A/cm^2

$\nabla \cdot (\epsilon E) = Q$ Gauss' Law ϵ = permittivity (F/cm) E = elec field (V/cm) Q = charge density

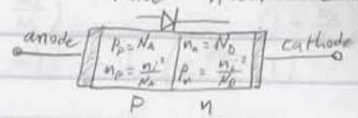
Device Electronics Ch3

$P_p = N_A$ Positive Carriers$_{p\,side}$ = Acceptor Concentration

$n_p = \frac{n_i^2}{N_A}$ Negative Carriers$_{p\,side}$ = $\frac{(\text{intrinsic carrier density})^2}{\text{Acceptor Concentration}}$

$n_n = N_D$

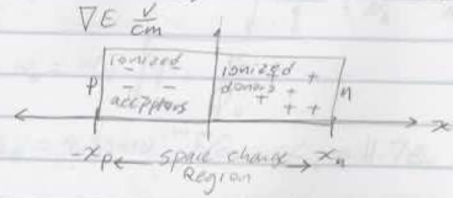
$P_n = \frac{n_i^2}{N_D}$



Gauss' Law - Region of space charge ρ_c $\frac{C}{cm^3}$

$\nabla \cdot E = \frac{\rho_c}{\epsilon_s} \rightarrow E(x) = \frac{1}{\epsilon_s} \int \rho(x)\,dx$

∇E $\frac{V}{cm}$



$\longrightarrow$ Hole Diffusion

$E(x) \longleftarrow$ $\longleftarrow$ electron diffusion

$\longleftarrow$ Hole drift

$\longrightarrow$ electron drift

unconnected diode

Diffusion = − Drift for e^- and Holes

Charge neutrality $qN_A x_p = qN_D x_n$

q - charge of $1e^-$ C

$N_{A,D}$ - Acceptor, Donor Concentrations cm^{-3}

$x_{p,n}$ - distance from metalurgical junction

Junction Potential - difference in internal chemical potentials from n to p side

$\phi_j = -\int E(x)\,dx$ Volts

also $\phi_j = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$

Depletion Region

Forward bias: $W_{d0} = (x_n + x_p) = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)\phi_j}$ meters

$$E_{max} = \frac{2\Phi_j}{W_{do}} = \frac{q N_A x_p}{\epsilon_s}$$

51 Partial widths of depletion zone

$$x_n = \frac{W_{do}}{\left(1 + \frac{N_D}{N_A}\right)} \qquad x_p = \frac{W_{do}}{\left(1 + \frac{N_A}{N_D}\right)}$$

Diode

52 I-V $i_D = I_s\left(e^{\frac{qV_D}{nkT}} - 1\right) = I_s\left(e^{\frac{V_D}{nV_T}} - 1\right)$

58 Reverse Bias $V_j = \phi_j + V_R$

$$w_d = (x_n + x_p) = \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(\phi_j + V_R)}$$

where $W_d = W_{do}\sqrt{1 + \frac{V_R}{\phi_j}}$

Permittivity $\epsilon_o = 8.85 \times 10^{-14}\ F/cm$ $\quad \epsilon_s = 11.7\epsilon_o$

Junction Capacitance $C_j = \frac{dQ_n}{dV_R} = \frac{C_{jo}}{\sqrt{1 + \frac{V_R}{\phi_j}}}$, $C_{jo} = \frac{\epsilon_s A}{W_{do}}$

reverse bias

Forward bias $C_D = \frac{dQ_D}{dV_D} = \frac{(i_D - I_s)\tau_T}{V_T}$, $Q_D = i_D \tau_T$

(diffusion cap.)

Device Electronics

Rectifier Circuits

$\frac{1}{2}\lambda$ $\quad V_{dc} = V_p - V_{on}$ — $R_L = \infty$

Voltage DC max = $V_{peak} - V_{diode\ on}$

Discharge interval $V_o(t) = (V_p - V_{on})e^{-\frac{t}{RC}}$

Ripple $V_r = (V_p - V_{on}) - (V_p - V_{on})e^{-\frac{t - \Delta t}{RC}}$

$$V_r = (V_p - V_{on})\left[1 - e^{-\frac{t-\Delta t}{RC}}\right]$$

$$V_r \approx (V_p - V_{on})\left[1 - e^{-\frac{t}{RC}}\right] \approx \frac{(V_p - V_{on})T}{RC}$$

T = 1 period on t axis

$$I_{dc} = \frac{V_p - V_{on}}{R_L}$$

$\Delta V = \frac{I_{dc}}{C}T$ linear capacitive discharge

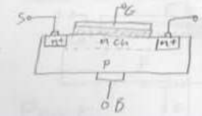
$I_P = I_{dc}\frac{2T}{\Delta T}$ repetitive peak current

$I_{sc} = \omega C V_p = \underbrace{2\pi(60)}_{60\ Hz}$ × Capacitance × Peak input voltage

$$\Delta T = \frac{1}{\omega}\sqrt{\frac{T}{RC}\frac{(V_p - V_{on})}{V_p}} = \frac{1}{\omega}\sqrt{\frac{2V_r}{V_P}}$$

Device Electronics

Enhancement



MOSFETs

5 All Regions $\quad K_n = \mu_n C''_{ox}\frac{W}{L}$ $\quad$ Transconductance

K_n - Transconductance parameter $= K'_n\frac{W}{L}$

μ_n - mobility n carriers (electrons)

C''_{ox} - capacitance per unit area $= \frac{\epsilon_{ox}}{T_{ox}}$

ϵ_{ox} = oxide permittivity, $\epsilon_{ox} = 3.9\epsilon_o$, $\epsilon_o = 8.85 \times 10^{-14}\ F/cm$

T_{ox} = oxide thickness

$\frac{W}{L}$ - width/Length of channel

$Q' = -WC''_{ox}(V_{ox} - V_{TN})$ $\quad$ C/cm $\quad$ Channel Charge

Q' - charge per unit length

V_{ox} - Voltage at Gate

V_{TN} - Threshold N-ch

$i(x) = i = Q'v_x = \left[-WC''_{ox}(V_{ox} - V_{TN})\right]\left[-\mu_n E_x\right]$ $\quad$ Channel Current

i = current in channel

v_x = Velocity of carriers

$i(x) = -\mu_n C''_{ox}W(V_{GS} - V(x) - V_{TN})\frac{dV(x)}{dx}$ $\quad$ Current

$V(x)$ = Voltage at position

$i_{DS} = \mu_n C''_{ox}\frac{W}{L}\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS}$ $\quad$ Current

$i_{DS} = K_n\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS}$ for $V_{GS} - V_{TN} \geq V_{DS} \geq 0$

$i_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2(1 + \lambda V_{DS})$ for $V_{DS} \geq (V_{GS} - V_{TN}) \geq 0$

λ - Channel length modulation parameter $\quad$ No Body Effect

$V_{TN} = V_{TO} + \gamma\left(\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F}\right)$ | $V_{Tp} = V_{TO} - \gamma(\ldots)$ $\quad$ Body Effect

V_{TO} = zero substrate bias value for V_{TN} $\quad$ (V)

γ = body effect parameter $\quad$ ($\sqrt{V}$)

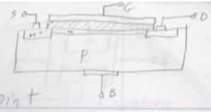
$2\phi_F$ = Surface potential parameter = .6 (V) $\quad [.3 \leq 2\phi_F \leq 1]$

V_{SB} = Voltage Source - Body

$\boxed{K' = 25\mu A/V^2,\ V_{TO} = .75V[N],\ -.75V[P],\ 2\phi_F = .6V,\ V_{TN} = \quad,\ K_n = K_n' \frac{W}{L} \mu A/V^2,\ \gamma = .75[N], .5[P]\ \sqrt{V}}$

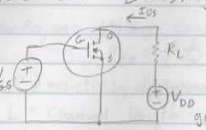
Device Electronics

Depletion



MOSFET Biasing / Q Point

3 Term



$V_{DD} = I_{DS} R_L + V_{DS}$

$V_{DS} = V_{DD} - R_L (\frac{1}{2}) K_n V_{OV}^2$

$I_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2 = \frac{1}{2} k_n V_{OV}^2$

gain: $A_V = -K_n (V_{GS} - V_{TN}) R_L = 2\frac{I_D R_L}{V_{OV}}$

$V_{GS} = 3V$, $K_n = 25\mu A/V^2$, $V_{TN} = 1$, $R_L = 100k$, $V_{DD} = 10V$

$I_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2 = \frac{25 \times 10^{-6}}{2}(3-1)^2 = 50\mu A$

$V_{DD} = 50\mu A(100k) + V_{DS}$

$10V = V_{DS} + 5$

saturation / small signal

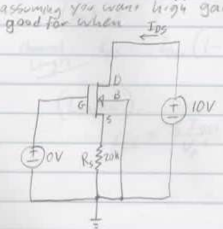
$V_{DS} > V_{OV}$

$V_{DS} = 5V$ > } condition for

$V_{GS} - V_{TN} = V_{OV} = 2V$ } saturation

(assuming you want high gain and low currents)
good for when

4 Term



$I_{DS} = \frac{K_n}{2}(V_{GS} - V_{TN})^2$

$V_{GS} = -I_{DS} R_S = -V_{SB}$

$V_{TN} = V_{TO} + \gamma(\sqrt{V_{SB} + 2\phi_F} - \sqrt{2\phi_F})$

$V_{GS} = -I_{DS}(20k)$

$I_{DS} = \frac{200 \times 10^{-6}}{2}(-20k\, I_{DS} - V_{TN})^2$

$V_{TN} = -3 + (\sqrt{V_{SB} + .6} - \sqrt{.6})$

FET: $V_{TO} = -3V$

$\gamma = 1\sqrt{V}$

$2\phi_F = .6V$

$K_n = 200\mu A/V^2$

Device Electronics

JFET

Linear Region: Voltage controlled resistor

$R_{CH} = \frac{\rho}{t}\frac{L}{W}$

ρ = resistivity of channel region

L = Channel length

W = width of channel between PN Junctions

NCh: S− D+ Gate: − To pinch off
+ To allow more current

Constant Current device: i_{DS} depends linearly on V_{DS}

I.V.: (Saturation)

$i_{DS} = \frac{K_n}{2}(V_{GS} - V_P)^2 = \frac{K_n}{2}(-V_P)^2\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$i_{DSS} = \frac{K_n}{2} V_P^2$

V_P typical −25 → 0V

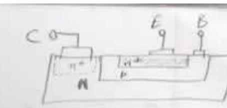
(Sat)

channel length: $i_{DS} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 (1 + \lambda V_{DS})$

(Linear)

$i_{DS} = \frac{2 I_{DSS}}{V_P^2}\left(V_{GS} - V_P - \frac{V_{DS}}{2}\right) V_{DS}$

Device Electronics



Bipolar Junction Transistors

Forward Characteristics

$V_{CE} > V_{BE} - 0.4V$

Collector: $i_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right) = i_F$

I_S: saturation current

$e = 2.71828$

V_{BE} = Volt Base Emitter

$V_T = .025V$ for Si @ 300K

Base: $i_B = \frac{i_F}{\beta_F} = \frac{I_S}{\beta_F}\left(e^{\frac{V_{BE}}{V_T}} - 1\right)$

β_F − Forward Gain

Emitter: $i_E = i_C + i_B = \left(I_S + \frac{I_S}{\beta_F}\right)\left[e^{\frac{V_{BE}}{V_T}} - 1\right]$

$= I_S\left(\frac{\beta_F + 1}{\beta_F}\right)\left[e^{\frac{V_{BE}}{V_T}} - 1\right]$

$= \frac{I_S}{\alpha_F}\left[e^{\frac{V_{BE}}{V_T}} - 1\right]$

where $\alpha_F = \frac{\beta_F}{\beta_F + 1}$, $\beta_F = \frac{\alpha_F}{1 - \alpha_F}$

$\alpha_F = \frac{i_C}{i_E}$ Ratio of Collector-to-Emitter currents

Full Transport Model − Forward and reverse

NPN

$$\begin{cases} i_C = I_S\left[e^{\frac{V_{BE}}{V_T}} - e^{\frac{V_{BC}}{V_T}}\right] - \frac{I_S}{\beta_R}\left[e^{\frac{V_{BC}}{V_T}} - 1\right] \\ i_B = \frac{I_S}{\beta_F}\left[e^{\frac{V_{BE}}{V_T}} - 1\right] + \frac{I_S}{\beta_R}\left[\frac{V_{BC}}{V_T} - 1\right] \\ i_E = I_S\left[e^{\frac{V_{BE}}{V_T}} - e^{\frac{V_{BC}}{V_T}}\right] + \frac{I_S}{\beta_F}\left[e^{\frac{V_{BE}}{V_T}} - 1\right] \end{cases}$$

PNP: Reverse all subscripts i.e. $i_C = I_S[e^{\frac{V_{EB}}{V_T}} \cdots]$

General: $I_S = 10^{-15}A$, $V_A = 50V$, $\beta_F = 100$, $\beta_R = 1$, $V_{BE} = .7V$

Early Effect : V_A

$i_C = I_S\left[e^{\frac{V_{BE}}{V_T}}\right]\left[1+\frac{V_{CE}}{V_A}\right]$

$\beta_F = \beta_{FO}\left[1+\frac{V_{CE}}{V_A}\right]$

$i_B = \frac{I_S}{\beta_{FO}}\left[e^{\frac{V_{BE}}{V_T}}\right]$

Base-Emitter	Base-Collector: Forward Bias	Base-Collector: Reverse Bias
Forward Bias	Saturated (closed Switch)	Forward-Active (Linear Amplifier)
Reverse Bias	Reverse-Active (poor amplifier)	Cutoff (open Switch)

Reverse Characteristics

$i_C = -\frac{I_S}{\alpha_R} e^{\frac{V_{BC}}{V_T}}$

$i_E = -I_S e^{\frac{V_{BC}}{V_T}}$

$i_B = \frac{I_S}{\beta_R} e^{\frac{V_{BC}}{V_T}}$

Cutoff Characteristics

$i_C = \frac{I_S}{\beta_R}$ $\quad i_E = -\frac{I_S}{\beta_F}$ $\quad i_B = -\frac{I_S}{\beta_F} - \frac{I_S}{\beta_R}$

Amplifiers

Output Resistance

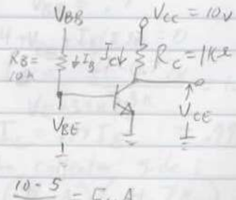
$$\boxed{\frac{V_A}{I_C} = \text{Output resistance } r_o}$$

$i_C = I_S e^{V_{BE}/V_T}\left[1+\frac{V_{CE}}{V_A}\right]$

V_A = Early Voltage

"Output resistance is inversely to I_C"

"Performance degrades as current increases"



Find V_{BB} for

a) in active mode, $V_{CE} = 5v$

b) at edge of saturation

c) deep in sat, $\beta = 10$

(assume normal $\beta = 100$)

(assume $V_{CE} = 0.3$ sat or 0.2 deep sat)

$I_C = \frac{10-5}{1} = 5mA$

$I_B = \frac{I_C}{\beta} = 0.05mA = \frac{V_{BB}-V_{BE}}{10}$

a) $0.05 = \frac{V_{BE} - 0.7}{10}$

b) $V_{CE} = 0.3$

To determine Q point

Assume: Active → solve then check:

Active
NPN $V_{CB} > -.4$
PNP $V_{CB} < 0.4$

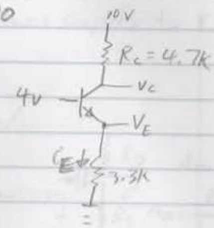
if not, assume:

$\boxed{\text{Saturation}: V_{CE} = 0.2V}$

↳ in β Forced

$\frac{I_C}{I_B} < \beta$, ie β forced

Determine node voltages, $\beta = 100$



Assume $V_{BE} = .7$

$-4 + V_{BE} + I_E(3.3k) = 0$

$I_E = \frac{(4-0.7)v}{3.3K} = 1mA$

$V_E = 3.3v$

$I_C = .99 I_E = .99mA$

On the collector side:

$(.99mA)(4.7K) = 4.65 v$

$V_C = 10 - 4.65 = 5.34$

$V_C - V_E = 5.34 - 3.3 = 2.04$

Changing V_B to 6 results in impossible V_{CE} so assume it's 0.2v, saturated

Bipolar Simplified Transport Models

Forward Active

Basic

$I_C = I_S e^{V_{BE}/V_T}$

$I_E = \frac{I_S}{\alpha} e^{V_{BE}/V_T}$

$I_B = \frac{I_S}{\beta} e^{V_{BE}/V_T}$

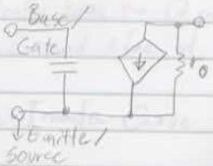
MOSFETs

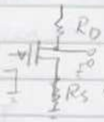
g_m - Transconductance

$I_D = \frac{1}{2}\underbrace{\mu_n C_{ox}\frac{W}{L}}(V_{GS}-V_t)^2$

Process-specific parameters; simplify to K_n

$I_D = \frac{1}{2}K_n V_{OV}^2$

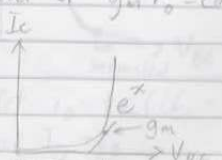




$A_{VO} = \frac{v_o}{v_i}$

$A_{VO} = \frac{R_D}{\frac{1}{g_m}+R_S}$

Product of $g_m r_o$ = constant "Open Output Resistance"

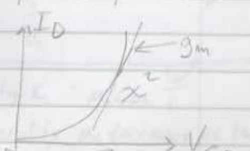


Bipolar

$I_C = I_{sat}\left(e^{\frac{qV_{BE}}{KT}}-1\right)$

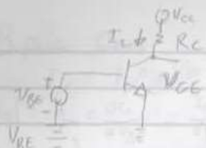
$\frac{dI_C}{dV_{BE}} = \frac{I_C}{\frac{KT}{q}} = g_m$

"Gain is proportional to base current"

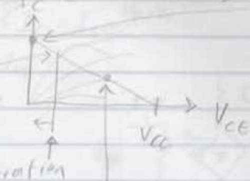


MOS

$g_m = K_n\underbrace{(V_{GS}-V_t)}_{V_{OV}}$



Start with $I_C R_C = V_{CC}$ (Transistor ON) $I_C = \frac{R_C}{V_{CC}}$



Loadline of resistor R_C

saturation

Ideally bias to Q point near ½ middle between V_{CC} and edge of saturation region

Bipolar Voltage Transfer Curve

$g_m = \frac{I_C}{\frac{KT}{q}}$ $KT/q: V_t = 25mV$

$g_m = \frac{I_C}{V_t}$

Gain of a Mosfet concept origin

Mosfet $i_D = \frac{1}{2}K_n(v_{GS}-V_T)$ where $K_n = \mu_n C_{ox}\frac{W}{L}$

$i_D = I_D + i_D$ mobility μ_n decreases as temp increases

Find gain - small signal

$I_D + i_D = \frac{1}{2}k_n\left((V_{GS}-V_T)-v_{GS}\right)^2$

$I_D + i_D = \frac{1}{2}k_n\left((V_{GS}+V_T)^2 + 2(V_{GS}-V_T)v_{gs} + v_{gs}^2\right)$ where $I_D = \frac{1}{2}K_n(V_{GS}-V_t)^2$

assume: $(V_{GS}-V_t) >> v_{GS}$

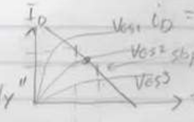
if $|v_{GS}| \sim 5$ to $10mV$ "small signal" $i_d = K_n(V_{GS}-V_t)v_{gs} + \frac{1}{2}k_n v_{gs}^2$

This term cancels

Linearized

$g_m = -\frac{\Delta i_d}{\Delta v_{gs}}$

"small excursions only"



V_{GS1} $i_D = K_n(V_{GS}-V_t)v_{gs}$

V_{GS2} slope of Load line $= \frac{1}{R}$

Donald Estreich dbe@sonic.net

Bipolar Gain

$I_C = I_S(e^{v_{BE}/V_t}-1)$ basic xfer funct

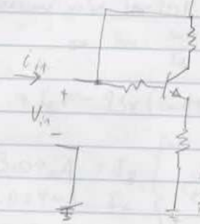
def: $g_m = \frac{di_C}{dv_{BE}} = \frac{I_C}{V_t} = \frac{I_C}{\frac{KT}{q}}$

use $I_C = I_S e^{v_{BE}/V_t}$

$\frac{d}{v_{BE}}$ both sides

$r_\pi = \frac{V_t}{I_C}\cdot\beta$

$r_{in} = \frac{V_{in}}{i_{in}}$



R_E is in effect Negative feedback to stabilize Q point

$A_V = -g_m R_C$

$V_{CE} = V_{CC} - I_C R_C \rightarrow V_{CE} > V_{BE} - 0.4\,V$

For AC analysis

$v_{BE} = V_{BE} + v_{be}$, $V_{BE} >> v_{be} > 10mV$

$i_C = I_{sat}(e^{V_{BE}+v_{be}/V_{th}})$

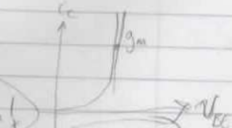
$I_C = I_{sat}e^{V_{BE}/V_{th}}\cdot e^{v_{be}/V_{th}}$

$i_C = I_C e^{v_{be}/V_t}$

$i_C \approx I_C\left(\frac{v_{be}}{V_t}\right)$

$i_C = I_C + \frac{I_C}{V_t}v_{be}$

$g_m = \frac{I_C}{V_t}$



V_t thermal! PN Junct

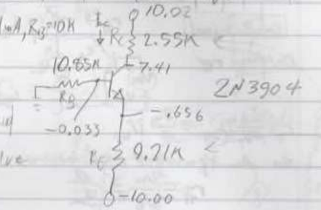
V_{TN} V_{TP} Threshold MOSFET

Experiment 2 — Transcribed to turn-in

Design the following circuit to have $|A_V| \geq |-100|$

Assume: use $A_V = -g_m R_c$, $I_c = 1mA$, $R_B = 10K$

& $g_m = \frac{I_c}{V_t} = \frac{1mA}{.026V}$



Since requirement is $A_V \geq |-100|$

$100 \leq -g_m R_c$ so solve

$100 = -g_m R_c$

$100 = -\frac{1mA}{.026V} R_c$

$R_c = 2595\Omega$. Measured value for test ckt $R_c = 2550\Omega$

Assume initially: $h_{FE} = \beta = 150 = \frac{I_c}{I_B}$ so $150 = \frac{1mA}{I_B}$ → $I_B = 66\mu A$ (assumed β)

Find $R_E = -V_E I_E$ → $R_E = -9.3V(1mA + 66\mu A) = 9.24k\Omega$

Assume $V_{BC} = 0.7V$

For exp't, using 9.21K

Measurement: Note that $\frac{.033V}{10.85K} = 3.04\mu A = I_B$

& $\frac{10.02 - 7.41}{2.55K} = 1.024mA = I_C$

& $\frac{10 - .656}{9210} = 1.015mA = I_E$

$I_E = I_B + I_C$

Must Retake because $I_C > I_E$

But, Using measured $I_C \approx I_c$, $\frac{1}{\beta} \approx \frac{3\mu}{1m} \Rightarrow \beta = 333$

With $R_c \approx 2.6K$ and $I_c = 1mA$, $V_{R_C} = 2.6V$ abs Max peak V_{out}

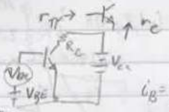
$A_V = -g_m R_c$, $g_m = \frac{I_c}{V_t}$

From: Whites.sdsmt.edu EE330

Recall I_D → Small Signal → $r_d = \frac{nV_T}{I_D}$

$\boxed{\text{Small Signal}}$

Determine r_π

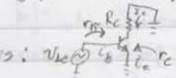


Assume active mode

$i_B = \frac{i_c}{\beta} = \frac{1}{\beta}\left[I_c + \frac{I_c}{V_T} v_{be}\right]$

$i_b = \frac{I_c}{\beta V_T} v_{be} = \frac{g_m}{\beta} v_{be}$

So AC equivalent ckt becomes:



$r_\pi \equiv \frac{v_{be}}{i_b} = \frac{\beta}{g_m}$

Determine r_e

$i_E = \frac{i_C}{\alpha} = \frac{I_c}{\alpha} + \frac{i_c}{\alpha}$

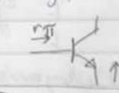
the AC component of i_E which is: $i_e = \frac{i_c}{\alpha} = \frac{I_c}{\alpha V_T} v_{be}$

plug in $I_E = \frac{I_c}{\alpha}$ → $i_e = \frac{I_E}{V_T} v_{be}$

$r_e \equiv \frac{v_e}{-i_e} \equiv \frac{v_{be}}{i_e} = \frac{V_T}{I_E}$

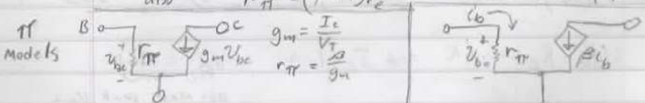
Since $g_m = \frac{I_c}{V_T} = \frac{\alpha I_E}{V_T} \Rightarrow \frac{V_T}{I_E} = \frac{\alpha}{g_m} \approx \frac{1}{g_m}$

$r_e \approx \frac{1}{g_m}$



also $r_\pi = (\beta + 1) r_e \ \Omega$

π Models

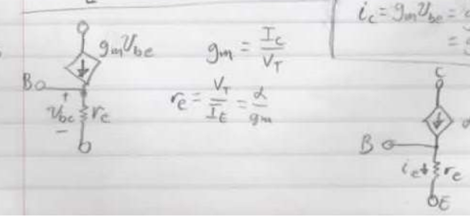


$g_m = \frac{I_c}{V_T}$

$r_\pi = \frac{\beta}{g_m}$

$i_c = g_m v_{be} = g_m(i_b r_\pi) = g_m r_\pi i_b = \beta i_b$

T Models

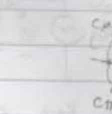


$g_m = \frac{I_c}{V_T}$

$r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m}$

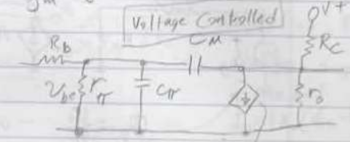
I_B & Input Resistance

$i_B = \frac{i_c}{\beta} = \underbrace{\frac{I_c}{\beta}}_{DC} + \underbrace{\frac{1}{\beta} \cdot \underbrace{\frac{I_c}{V_{TH}}}_{g_m} v_{be}}_{AC}$



$i_b = \frac{g_m}{\beta} v_{be}$ → $v_{be} = \frac{\beta}{g_m} i_b$

$\frac{dv_{be}}{di_b} = \frac{\beta}{g_m} = r_\pi$



Hybrid π Model

$A_{VO} = -\frac{v_o}{v_{in}} = -g_m r_c$ or $r_o \| r_c$

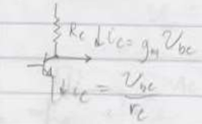
Diode / Emitter resistance

$r_e = \frac{1}{g_m}$ →

Lower input resistance than common emitter

"effectively input resistance is $\frac{1}{g_m}$ × output resistance"

Set $\frac{1}{g_m} = 50\Omega$ by setting I_C to match feedline / signal source

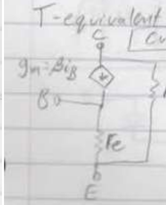


$i_e = \underbrace{\frac{v_{be}}{r_\pi}}_{i\ Base} + \underbrace{g_m v_{be}}_{i\ Collector}$

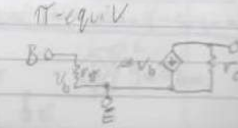
$i_e = \frac{v_{be}}{r_\pi}(1 + g_m r_\pi)$

$\frac{v_{be}}{r_\pi}(1 + \beta) = \frac{v_{be}}{r_e}$

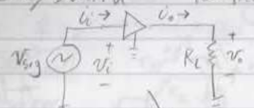
T-equivalent Ckt $\boxed{\text{Current Controlled}}$



π-equiv



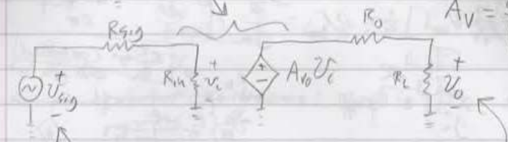
Sedra/Smith Termnology



R_{in} = input Resistance $= \frac{v_i}{i_i}$

A_{vo} = open circuit voltage gain $\rightarrow R_L = \infty$

$A_V = \frac{v_o}{v_i}$ (accounts for R_L) $R_L \neq \infty$

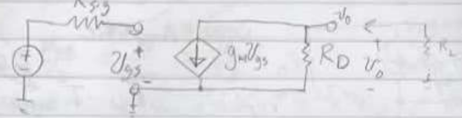


$G_V = \frac{v_o}{v_{sig}}$ - overall voltage gain

Common Base & Collector amps have high r_i and low r_o

Bipolar/MOSFET Ckt Gain Comparison

$v_o = -g_m v_{gs}$



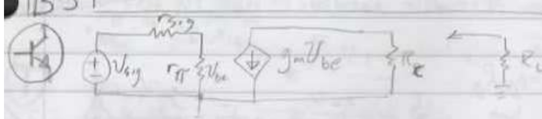
Linearized Simplified from Large signal Model $I_D = \frac{1}{2} K_n (V_{GS} - V_{TN})^2$

$r_{in} = \infty$ $\quad r_o = R_D$

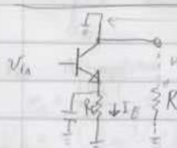
A_{vo} (no R_L) $= \frac{v_o}{v_{gs}} = -g_m R_D$

$A_v = \frac{v_o}{v_i} = -g_m (R_D \,||\, R_L) = -g_m \frac{R_L}{R_D + R_L} = G_v$ Overall Gain

BJT

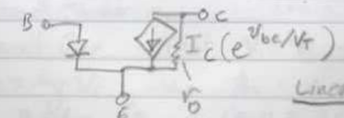


$v_{be} = \frac{r_\pi}{r_{sig} + r_\pi}$ v_{sig} gets divided by r_{sig} r_π



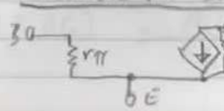
AC analysis without RE: $A_{vo} = -g_m R_C$

with RE: $A_{vo} = \frac{-g_m R_C}{1 + g_m R_E}$



Large Signal Model

Linearized for small-signal to

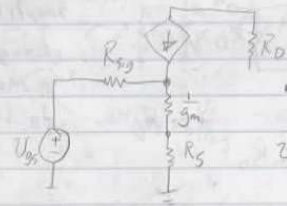


Real Mosfet Amp (with Source Degeneration R_{S})

infinite R make calculation not possible

while Amplific

replace with T model



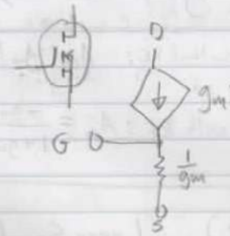
$v_{gs} = v_i \frac{1/g_m}{1/g_m + R_s} = v_i \frac{1}{(1 + g_m R_s)}$

"Gain Reduction Factor" see 5 page

results from effective feedback due to R_S

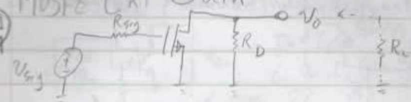
since $v_o = -i_D R_D$

$i_D R_D = \frac{g_m v_i}{g_m(\frac{1}{g_m} + R_s)} = \frac{-g_m v_i \cdot R_D}{(1 + g_m r_s)} = v_o$

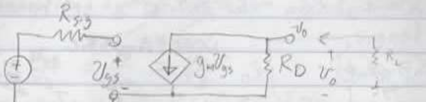


Bipolar/

MOSFET Ckt Gain Comparison



$v_o = -g_m v_{gs}$



Linearized Simplified ^ From Large signal Model

$I_D = \frac{1}{2}K_n(V_{GS} - V_{TN})^2$

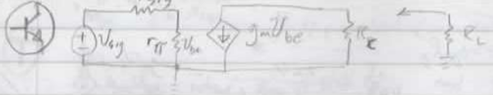
$r_{in} = \infty$ $\quad r_o = R_D$

A_{vo} (no R_L) $= \frac{v_o}{v_{gs}} = -g_m R_D$

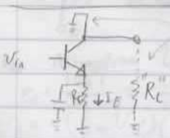
Overall Gain

$A_v = \frac{v_o}{v_i} = -g_m(R_D \,//\, R_L) = -g_m \frac{R_L}{R_D + R_L} = G_v$

BJT

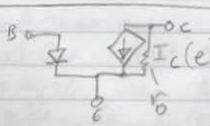


$v_{be} = \frac{r_\pi}{r_{sig} + r_\pi} v_{sig}$ gets divided by r_{sig} r_π



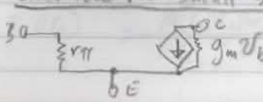
AC analysis without RE: $A_{vo} = -g_m R_C$

with RE: $A_{vo} = \frac{-g_m R_C}{1 + g_m R_E}$

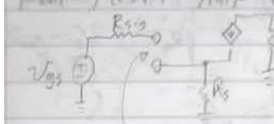


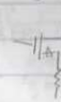
Large Signal Model

Linearized for small-signal to



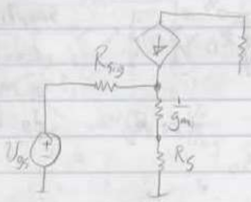
Real Mosfet Amp (with Source Degeneration $R_{s(s)}$)





infinite R make calculation not possible

whole Amplifier



SO replace with T model

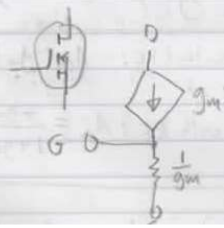
$v_{gs} = v_i \frac{1/g_m}{1/g_m + R_s} = v_i \frac{1}{(1 + g_m R_s)}$

"Gain Reduction Factor" see 5 pages

results from effective feedback due to R_s

since $v_o = -i_D R_D$

$i_D R_D = \frac{g_m v_i}{g_m(\frac{1}{g_m} + R_s)} = \frac{-g_m v_i \cdot R_D}{(1 + g_m R_s)} = v_o$



Bipolar

Transistor Formulae Small-Signal: B-E forward, B-C reverse

$I_C = I_S e^{V_{BE}/V_T}$ $\quad I_S \approx 10^{-15}$, $V_T \approx .025$

$i_C \approx I_C(1 + \frac{v_{be}}{V_T})$ or $i_C = I_C + \frac{I_C}{V_T} v_{be}$

$i_c = g_m v_{be}$ $\quad g_m = \frac{I_C}{V_T}$ $\quad (g_m = \frac{di_C}{dv_{BE}})$

Input resistance Assumption: $I_B = \frac{I_C}{\beta} + \frac{I_C}{\beta V_T} v_{be}$ (bias, signal) or $i_b = \frac{1}{\beta}\frac{I_C}{V_T} v_{be} = \frac{g_m}{\beta} v_{be}$

signal: amplification of base I

Base $\rightarrow$ $r_\pi \equiv \frac{v_{be}}{i_b} = \frac{\beta}{g_m} = \frac{V_T}{I_B}$ $\quad \boxed{g_m r_\pi = \beta}$

Emitter Assumption: $i_E = \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \frac{i_c}{\alpha} \rightarrow i_E = I_E + i_e$, $i_e = \frac{i_c}{\alpha} = \frac{I_C}{\alpha V_T} v_{be} = \frac{I_E}{V_T} v_{be}$

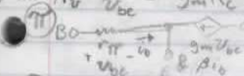
Showing resultant current in Emitter from the amplification

$\rightarrow r_e \equiv \frac{v_{be}}{i_e} = \frac{V_T}{I_E} = \frac{\alpha}{g_m} \approx \frac{1}{g_m}$ $\quad r_\pi = (\beta + 1) r_e$

assumption $v_{CE} = V_{CC} - i_C R_C = V_{CE} - i_c R_C = -g_m v_{be} R_C = (-g_m R_C) v_{be}$

Gain $A_v \equiv \frac{v_{ce}}{v_{be}} = -g_m R_C = \frac{I_C R_C}{V_T}$

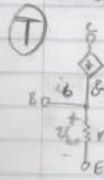
π



where $g_m = \frac{I_C}{V_T}$ and $r_\pi = \frac{\beta}{g_m}$

"Equivalent Circuits"

T

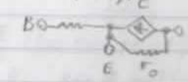


$g_m v_{be}$ & αi_b where g_m is still $\frac{I_C}{V_T}$ and $r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m}$

and $i_b = \frac{v_{be}}{r_e} - g_m v_{be} = \frac{v_{be}}{r_e}(1 - g_m r_e)$

so $i_b = \frac{v_{be}}{r_e}(1 - \alpha) = \frac{v_{be}}{r_e}(1 - \frac{\beta}{\beta+1}) = \frac{v_{be}}{(\beta+1) r_e} = \frac{v_{be}}{r_\pi}$

Output r_o Assumption $v_o = -g_m v_{be}(R_C \| r_o)$



$r_o = \frac{|V_A|}{I_C} = \frac{v_x}{i_x} \approx R_C$

$v_o = -(g_m v_\pi)(R_C \| r_o)$, $A_{vo} = -g_m(R_C \| r_o)$

higher β → higher r_π $\quad$ Gain. $\approx -g_m R_C$

G_V

Overall Gain first $v_i = v_{sig} \frac{r_\pi}{r_\pi + R_{sig}}$

$v_o = A_v v_i$ where $A_v = -g_m(R_C \| R_L \| r_o)$

$G_v \equiv \frac{v_o}{v_{sig}} = -\frac{r_\pi}{r_\pi + R_{sig}} g_m (R_C \| R_L \| r_o) = -\frac{\beta(R_C \| R_L \| r_o)}{R_{sig} + r_\pi}$

DC bias CAP omitted shorted

Small-Signal AC Analysis

Common Emitter

$R_{in} = \frac{v_i}{i_b}$ where $i_b = (1-\alpha) i_e = \frac{i_e}{\beta+1}$

and $i_e = \frac{v_i}{r_e + R_e}$

$R_{in} = (\beta+1)(r_e + R_e)$ recall $r_e = \frac{V_T}{I_E}$ R_e = const Bias point

Adding Emitter-Degeneration resistor

ΔR_{in}: $\frac{R_{in} \text{ with } R_e}{R_{in} \text{ without } R_e} = \frac{(\beta+1)(r_e+R_e)}{(\beta+1) r_e} = 1 + \frac{R_e}{r_e} \approx 1 + g_m R_e$

Gain $v_o = -i_c R_C = -\alpha i_e R_C$

$A_{vo} = -\alpha \frac{R_C}{r_e + R_e}$ "Total Emitter resistance $r_e + R_e$" so R_e adds to input resistance

$= -\frac{g_m R_C}{1 + R_e/r_e} \approx -\frac{g_m R_C}{1 + g_m R_e}$

So the factor $1 + g_m R_e$ is how much R_e reduces gain

Adding Load Resistor $A_v = A_{vo} \frac{R_L}{R_L + R_o}$

[$R_o = R_C$ because in small-signal transistor is off {?}]

Overall gain $G_v = \frac{R_{in}}{R_{in} + R_{sig}} \times (-\alpha) \left(\frac{R_C \| R_L}{r_e + R_e}\right)$ and $\alpha = \frac{\beta}{\beta+1}$

so $G_v = -\beta \frac{R_C \| R_L}{R_{sig} + (\beta+1)(R_e + r_e)}$

Common Base

$R_{in} = r_e$, Voltage gain: $v_o = -\alpha i_e R_C$

where $i_e = \frac{v_i}{r_e}$

where $r_e = \frac{V_T}{I_E}$ (DC Bias point)

so $A_{vo} \equiv \frac{v_o}{v_i} = \frac{\alpha}{r_e} R_C = g_m R_C = \frac{I_C}{V_T} R_C$ gain of amplifier proper

also, considering R_L, $A_v = g_m (R_C \| R_L)$

then, considering R_{sig} $G_v = \frac{r_e}{R_{sig} + r_e} g_m (R_C \| R_L) = \alpha \frac{R_C \| R_L}{R_{sig} + r_e}$

since $\alpha \approx 1$, gain ≈ ratio of output to input resistances

BJT

	R_{in}	A_{vo}	R_o	A_v	G_v
Common Emitter	$(\beta+1) r_e$	$-g_m R_C$	R_C	$-g_m (R_C \| R_L)$ $-\alpha \frac{R_C \| R_L}{r_e}$	$-\beta \frac{R_C \| R_L}{R_{sig} + (\beta+1) r_e}$
(with R_e)	$(\beta+1)(r_e + R_e)$	$\frac{-g_m R_C}{1 + g_m R_e}$	R_C $r_o + (R_e \| r_\pi) + g_m r_o (R_e \| r_\pi)$ $\approx r_o (1 + g_m (R_e \| r_\pi))$	$\frac{-g_m R_C \| R_L}{1 + g_m R_e}$ $\alpha \frac{R_C \| R_L}{r_e + R_e}$	$-\beta \frac{R_C \| R_L}{R_{sig} + (\beta+1)(r_e + R_e)}$
Common Base	r_e if $r_o = \infty$ or $R_{in} = \frac{r_o + R_L}{1 + \frac{r_o}{r_e} + \frac{R_L}{(\beta+1) r_e}}$	$g_m R_C$	R_C	$g_m (R_C \| R_L)$ $\alpha \frac{R_C \| R_L}{r_e}$	$\alpha \frac{R_C \| R_L}{R_{sig} + r_e}$
Common Collector Emitter Follower	$(\beta+1)(r_e + R_L)$ Caution R_L is combined "Load" and "emitter" resistances in parallel	1	r_e	$\frac{R_L}{R_L + r_e}$	$\frac{R_L}{R_L + r_e + \frac{R_{sig}}{\beta+1}}$

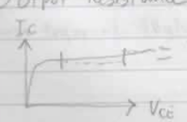
$G_{vo} = 1$

$R_{out} = r_e + \frac{R_{sig}}{\beta+1}$

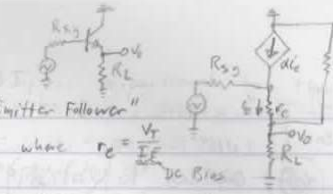
Using Values From DC bias Points

$r_e = \frac{V_T}{I_E}$ $g_m = \frac{I_C}{V_T}$ since $I_C \approx I_E$ $r_e \approx \frac{1}{g_m}$

Output Resistance: Related to Early Voltage



= This slope represents Output Resistance



Common Collector = "Emitter Follower" $\alpha = \frac{\beta}{\beta+1}$

$R_{in} = (\beta+1)(r_e + R_L)$ where $r_e = \frac{V_T}{I_E}$ DC Bias

$R_o = r_e$

$A_v \equiv \frac{v_o}{v_i} = \frac{R_L}{R_L + r_e}$ so if $R_L = \infty$, $A_{vo} = 1$

with $v_i = \frac{R_{in}}{R_{in} + R_{sig}} (v_{sig})$ or $v_i = v_{sig} \frac{(\beta+1)(r_e + R_L)}{(\beta+1)(R_L + r_e) + R_{sig}}$

and $G_v \equiv \frac{v_o}{v_{sig}} = \frac{v_i}{v_{sig}} A_v$ so

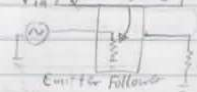
$G_v = \frac{(\beta+1) R_L}{(\beta+1)(R_L + r_e) + R_{sig}}$ neglecting r_e

(with r_o: $A_{vo} = \frac{r_o}{r_o + r_e}$)

same

$G_v = \frac{R_L}{R_L + r_e + \frac{R_{sig}}{\beta+1}}$

$r_{in} = \boxed{(\beta+1) R_L}$

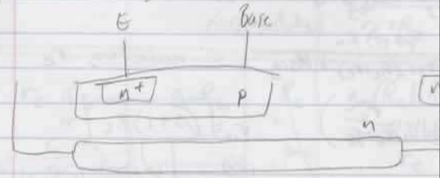


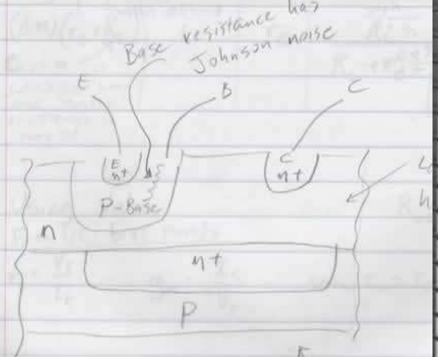
$R_{out} = r_e + \frac{R_{sig}}{\beta+1}$

Base 2N1613

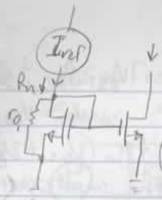
has high input capacitance
long base

Mobility of e^- is greater than p^+





← Discreet



$\downarrow I_1 \to$ is proportional to the Channel widths $\times$ Iref

or

(Pen. fery of Emitter For

$R_{in} = r_{o1} \| \frac{1}{g_{m1}}$ where $r_{o1} = \frac{V_A}{I_{D1}} = \frac{V_A' L}{I_{D1}}$ $\{\Omega\}$

and $g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_{D1}}$ $\{\frac{A}{V}\}$

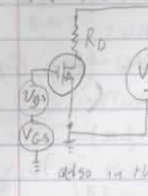
(inverse conductance is resistance)

Since $r_{o1} >> \frac{1}{g_{m1}}$ $R_{in} \simeq \frac{1}{g_{m1}}$

sedra p 290

Mosfet Small-Signal

$V_{DS} > V_{ov}$, Saturated



Bias $I_D = \frac{1}{2} K_n V_{ov}^2$, $V_{DS} = V_{DD} - R_D I_D$

add signal: $i_D = \frac{1}{2} K_n (V_{GS} + v_{gs} + V_{TN})^2$

extracting just the small-signal components:

$i_d = k_n (V_{GS} - V_{TN}) v_{gs}$

also in the small-signal condition $v_{DS} = V_{DD} - R_D I_D$
$= V_{DS} - R_D I_D$

g_m transconductance $g_m \equiv \frac{\partial i_D}{\partial v_{GS}} = K_n V_{ov} = K_n (V_{GS} - V_{TN}) = \frac{2 I_D}{V_{ov}}$
$= k_n' \frac{W}{L} (V_{GS} - V_{TN})$

(in small-signal lingo) $\equiv \frac{i_d}{v_{gs}}$ since $V_{ov} = \sqrt{\frac{2 I_D}{k_n' \frac{W}{L}}}$

Small-Signal Models $g_m = \sqrt{2 k_n'} \sqrt{\frac{W}{L}} \sqrt{I_D}$

r_0

$r_o = \frac{V_A}{I_D}$ $V_A \propto L$

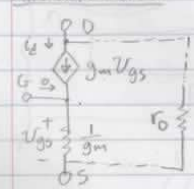
r_o typical $10K \sim 1M\Omega$

$A_{V_0} \equiv \frac{v_{ds}}{v_{gs}} = -g_m R_D$ neglects r_o

$A_{V_0} = -g_m \underbrace{(R_D \| r_o)}_{R_O}$ $A_V = A_{V_0} \frac{R_L}{R_L + R_O}$ (R_L Not shown)

G_V total gain, also considers input resistance

alternate model



$G_V = \frac{R_{in}}{R_{in} + R_{sig}} A_{V_0} \frac{R_L}{R_L + R_O}$

$v_o = -(g_m v_{gs})(R_D \| r_o)$

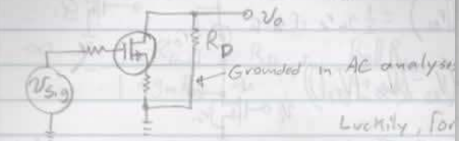
remember to $\|$ r_o with R_D and any R_L

A_0 Intrinsic Gain $A_0 = -g_m r_o = \frac{V_A}{V_{ov}/2} = \frac{V_A \sqrt{2 \mu_n C_{ox} \frac{W}{L}}}{\sqrt{I_D}}$

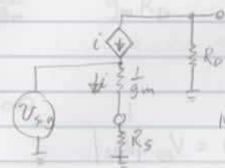
where g_m = see above
& r_o = see above

MosFet Amplifier

SEDRA p294 Common Source with R_S



Luckily, for



r_o is no

S.S.A. so

$R_o = R$

Instead, include R

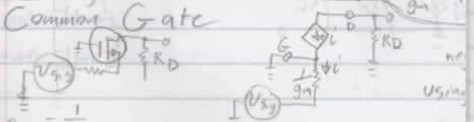
$v_{gs} = v_i \frac{1/g_m}{1/g_m + R_S}$

and since $v_o = -i R_D$, $i = \frac{v_i}{1/g_m + R_S}$ = ?

and $A_{vo} \equiv \frac{v_o}{v_i} = \frac{R_D}{1/g_m + R_S}$

including an external R_L: $A_v = -\frac{R_D \| R}{\frac{1}{g_m} + R}$

p300 Common Gate



ne

using

$R_{in} = \frac{1}{g_m}$

Since $v_o = -i R_D$, $i = \frac{v_i}{1/g_m}$

$A_{vo} \equiv \frac{v_o}{v_i} = g_m R_D$

302 Common Drain - "Source Follower"

$R_{in} = \infty$

$A_{vo} (R_L = \infty) = 1$

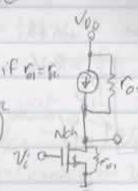
$A_v = \frac{R_L}{R_L + \frac{1}{g_m}}$ (voltage divider)

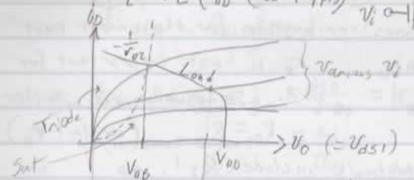
$R_o = \frac{1}{g_m}$ $(R_L = \infty)$ $\qquad R_o = \frac{1}{g_m} \| R_L$

MOS Amp, active load

$A_v = -g_m (r_{o1} \| r_{o2}) = \frac{1}{2} g_m r_o$ if $r_{o1} = r_{o2}$

$I = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left(V_{DD} - (V_{SG} + |V_{TP}|)\right)^2$

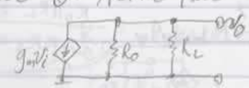




$V_{DA} = V_{DD} - (V_{SG} - |V_{TP}|) = V_{DD} - |V_{OV2}|$

- ok to go near edge of sat, but stay
- For smallsignal MOS

Cascode & Active load



MOS

	R_{in}	A_{vo}	R_o	A_v	G_v
Common Source	∞	$-g_m R_D$	R_D	$-g_m (R_D \| R_L)$	$-g_m (R_D \| R_L)$
Common Source with R_S	∞	$-\frac{g_m R_D}{1 + g_m R_S}$	R_D	$-\frac{R_D \| R_L}{\frac{1}{g_m} + R_S}$	same (because $R_{in} = \infty$)
Common Gate	$\frac{1}{g_m}$	$g_m R_D$	R_D	$g_m (R_D \| R_L)$	$\frac{R_D \| R_L}{R_{sig} + \frac{1}{g_m}}$
Source Follower	∞	1	$\frac{1}{g_m}$	$\frac{R_L}{R_L + \frac{1}{g_m}}$	$\frac{R_L}{R_L + \frac{1}{g_m}}$ Same again, ∞-Ω input

Zero

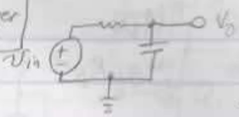
$\boxed{+20\ db/decade}$:

gain / Freq

Pole

$\boxed{-20\ db/decade}$

3db is ½ Power Point



Bode Plots

$\tau = RC$

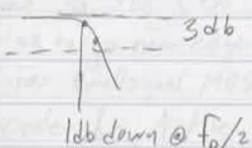
$$\frac{V_{out}}{V_{in}} = \frac{K}{s + \frac{1}{RC}}$$

$$V_o = \frac{1}{|\omega C|}$$

$\frac{1}{2\pi RC}$ is a Pole

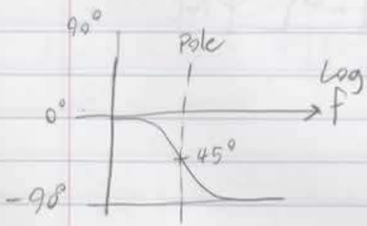
If you take $20 \text{ Log}\left(\frac{a}{b}\right) \rightarrow$ decibel

V_{pole} (@ 3db down) $= \frac{1}{\sqrt{2}}$



every 1db down is $2f_p$

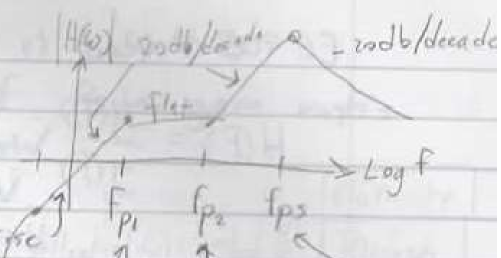
1db down @ $f_p/2$



- Pole in denominator
- Zeros in numerator

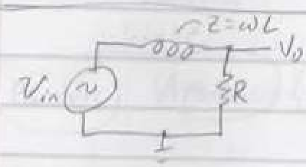
Bode Example

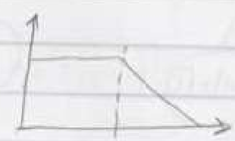
$|H(\omega)|$ - transfer funct



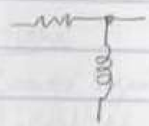
must be a zero to cause the +20db rise

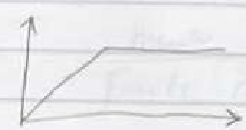
$$|H(\omega)| = \frac{K\left(1+\frac{s}{\omega_z}\right)\left(1+\frac{s}{\omega_{p2}}\right)}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{ps}}\right)^2}$$





Low Pass





High Pass