Physics Lecture #5: Calculation of Distance from Acceleration

If we rearrange the formula for acceleration, we can solve for final velocity.

$$a = \frac{v_f - v_i}{t}$$

$$at = v_f - v_i$$

$$at + v_i = v_f \qquad \text{or } v_f = v_i + at$$

Find the velocity of an object after 4.0 seconds if it accelerates at 2.0 m/s^2 from an initial velocity of 5.0 m/s.

$$v_f = v_i + at$$

$$v_f = 5.0\text{ m/s} + \frac{2.0\text{ m/s}}{\text{s}}(4.0\text{ s})$$

$$v_f = 13\text{ m/s}$$

For the rest of this and other remaining lectures, I'll probably only include the units in the final answer.

If we take $v_f = v_i + at$ and substitute it into $\Delta x = \frac{(v_f + v_i)t}{2}$, we can get a new formula for Δx.

$$\Delta x = \frac{(v_f + v_i)t}{2} \qquad \text{substitute } v_i + at \text{ in place of } v_f$$

$$\Delta x = \frac{(v_i + at + v_i)t}{2}$$

$$\Delta x = \frac{(2v_i + at)t}{2}$$

$$\Delta x = \frac{(2v_it + at^2)}{2}$$

$$\Delta x = \frac{2v_it}{2} + \frac{at^2}{2}$$

$$\Delta x = v_it + \tfrac{1}{2}at^2$$

A car moves at an initial velocity of 12 m/s. It accelerates at a rate of 3.0 m/s^2 for 6.0 seconds. What distance does it cover during its acceleration?

$\Delta x = v_i t + \frac{1}{2} at^2$

$\Delta x = 12(6.0) + \frac{1}{2}(3.0)(6.0^2)$

$\Delta x = 72 + \frac{1}{2}(3.0)36 = 126$ **or** 1.3×10^2 **m**

We can also calculate the distance covered while a car is slowing down, or decelerating. Deceleration is just acceleration in an opposite direction. We need to put a negative sign in front of the acceleration.

A truck moves at an initial velocity of 42 m/s. It slows down at a rate of 3.6 m/s^2 for 7.0 seconds. What distance does it cover while it is decelerating?

$\Delta x = v_i t + \frac{1}{2} at^2$

$\Delta x = 42(7.0) + \frac{1}{2}(-3.6)(7^2)$

$\Delta x = 294 + (-88.2)$

$\Delta x = 205.8$ **or** 2.1×10^2 **m**

If the initial velocity is zero, the term $v_i t = 0$ and drops out of the equation.

$\Delta x = v_i t + \frac{1}{2} at^2$ **Let** $v_i = 0$

$\Delta x = 0 + \frac{1}{2} at^2$

$\Delta x = \frac{1}{2} at^2$ **when** $v_i = 0$

A car starts from rest and accelerates at 5.0 m/s^2 for 9.0 seconds. What distance does it cover?

$\Delta x = \frac{1}{2} at^2$

$\Delta x = \frac{1}{2}(5.0)\, 9.0^2 = 202.5$ **or** 2.0×10^2 **m**

The same abbreviated formula can be used to calculate the distance covered while a car decelerates to a stop. In this case, it is not necessary to put a negative sign in front of the acceleration. I could show you the proof, but you can prove it yourself.

A soccer ball rolling down the field slows down at a rate of 0.60 m/s^2 for 3.8 seconds and comes to a stop. What distance did it roll while it was slowing down?

$\Delta x = \frac{1}{2} at^2$

$\Delta x = \frac{1}{2} (0.60)\ 3.8^2 = 4.332$ **or 4.3 m**

We can create a formula relating velocity, distance, and acceleration without using time. We start by taking $\Delta x = \frac{1}{2}(v_i + v_f)t$ and solving for time. We also rearrange $v_f = v_i + at$ and also solve for time.

$\Delta x = \frac{1}{2}(v_i + v_f)t$	$v_f = v_i + at$
$2\Delta x = (v_f + v_i)t$	$v_i + at = v_f$
$(v_f + v_i)t = 2\Delta x$	$at = v_f - v_i$
$t = \dfrac{2\Delta x}{(v_f + v_i)}$	$t = \dfrac{(v_f - v_i)}{a}$

We have two expressions for time. Since both expressions = t, set the expressions equal to each other and cross multiply.

$$\frac{2\Delta x}{(v_f + v_i)} = \frac{(v_f - v_i)}{a}$$

$(v_f + v_i)(v_f - v_i) = 2a\Delta x$

$v_f^2 - v_i^2 = 2a\Delta x$

$v_f^2 = v_i^2 + 2a\Delta x$

A car traveling at 32 m/s accelerates at 8.0 m/s^2 over a distance of 13 m. What is its final velocity?

$v_f^2 = v_i^2 + 2a\Delta x$

$v_f^2 = 32^2 + 2(8.0)13$

$v_f^2 = 1232$

$v_f = 35.09$ **or 35 m/s**

If the object slows down, then the acceleration should be a negative number.

A car traveling at an initial velocity of 64 m/s slows down at a rate of 8.6 m/s^2 over a distance of 12 m. What is the final velocity of the car?

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$v_f^2 = 64^2 + 2(-8.6)(12)$$

$$v_f^2 = 4096 + (-206.4)$$

$$v_f^2 = 3889.6$$

$$v_f = 62.36 \text{ or } 62 \text{ m/s}$$

If the initial velocity is zero, the term v_i drops out of the formula, and we get

$v_f^2 = 2a\Delta x$ **use when $v_i = 0$**

We can also use a similar abbreviated equation if $v_f = 0$.

$v_i^2 = 2a\Delta x$ **use when $v_f = 0$.**

The above formula is used when an object slows to a stop. Even though the object is decelerating, it is not necessary for acceleration to be a negative number (in the proof, the negative signs disappear).

A torpedo decelerates at a rate of 15 m/s^2 and comes to a halt after traveling 120 m. What was its initial velocity before deceleration?

$$v_i^2 = 2a\Delta x$$

$$v_i^2 = 2(15)(120)$$

$$v_i^2 = 3600$$

$$v_i = 60 \text{ or } 6.0 \times 10^1 \text{ m/s}$$

Here's a summary of the formulas in this lecture:

$v_f = v_i + at$

$\Delta x = v_i t + \frac{1}{2} at^2$

$\Delta x = \frac{1}{2} at^2$ **use when v_i or $v_f = 0$; use a positive value for acceleration**

$v_f^2 = v_i^2 + 2a\Delta x$

$v_f^2 = 2a\Delta x$ **use when $v_i = 0$**

$v_i^2 = 2a\Delta x$ **use when $v_f = 0$; use a positive value for acceleration**