

rewrite $f(x)$ in factored form and find all zeros. Then sketch the graph. Show all work.

① $f(x) = x^4 - 4x^3 - 6x^2 + 36x - 27$ has a factor of $(x - 3)$ with multiplicity two.

$$\frac{(x-3)(x-3)(x)(x)}{(x-3)(x-3)} = \frac{x^4 - 4x^3 - 6x^2 + 36x - 27}{(x-3)(x-3)}$$

The **fully factored form** of $f(x)$ is: $(x-3)(x-3)(x+3)(x-1)$

The **zeros** are:

$$(x-3)^2(x+3)(x-1)$$

The **x-intercepts** are:

$$(3,0) (-3,0) (1,0)$$

The **y-intercept** of the polynomial is

$$(0, -27)$$

The **end behavior** of the polynomial is...

if $x \rightarrow \infty$ then $y \rightarrow \infty$

if $x \rightarrow -\infty$ then $y \rightarrow \infty$

Zeros
 $x-3=0$ $x-3=0$ $x+3=0$ $x-1=0$
 $x=3$ $x=3$ $x=-3$ $x=1$
 mult 2
 $x=3; \text{mult } 2$ $x=-3$ $x=1$

$$\begin{array}{r} 3 \\ \hline 1 \quad -4 \quad -6 \quad 36 \quad -27 \\ \downarrow \\ 3 \quad -3 \quad -27 \quad 27 \\ \hline 1 \quad -1 \quad -9 \quad 9 \quad 0 \\ \hline 3 \\ \hline 3 \quad 6 \quad -9 \\ \downarrow \\ 1 \quad 2 \quad -3 \quad 0 \\ \hline x^2 \quad x \quad c \quad r \end{array}$$

$f(0) = (0)^4 - 4(0)^3 - 6(0)^2 + 36(0) - 27$

$$(x+3)(x-1) = x^2 + 2x - 3$$

graph next page

② $f(x) = 2x^3 - 3x^2 - 14x + 15$ has factors of $(x - 1)$ and $(x - 3)$.

$$\frac{(x-1)(x-3)(?)}{(x-1)(x-3)} = \frac{2x^3 - 3x^2 - 14x + 15}{(x-1)(x-3)}$$

The **fully factored form** of $f(x)$ is: $(x-1)(x-3)(2x+5)$

The **zeros** are: $x=1$ $x=3$ $x=-2.5$

The **x-intercepts** are:

$$(1,0) (3,0) (-2.5,0)$$

The **y-intercept** of the polynomial is

$$(0, 15)$$

The **end behavior** of the polynomial is...

if $x \rightarrow \infty$ then $y \rightarrow \infty$

if $x \rightarrow -\infty$ then $y \rightarrow -\infty$

Zeros
 $x-1=0$ $x-3=0$ $2x+5=0$
 $x=1$ $x=3$ $x=-2.5$

$$\begin{array}{r} 1 \\ \hline 2 \quad -3 \quad -14 \quad 15 \\ \downarrow \\ 2 \quad -1 \quad -15 \\ \hline 3 \\ \hline 2 \quad -1 \quad -15 \\ \downarrow \\ 6 \quad 15 \\ \hline 2 \quad 5 \quad 0 \\ \hline x \quad c \quad r \end{array}$$

$f(0) = 2(0)^3 - 3(0)^2 - 14(0) + 15$

$$(?) = (2x+5)$$

graph next page

③ $f(x) = -x^5 + 7x^4 - 9x^3 - 27x^2 + 54x$ has a factor of $(x - 3)$ with multiplicity 3.

$$\frac{(x-3)(x-3)(x-3)(x)(x)}{(x-3)(x-3)(x-3)} = \frac{-x^5 + 7x^4 - 9x^3 - 27x^2 + 54x + 0}{(x-3)(x-3)(x-3)}$$

The **fully factored form** of $f(x)$ is:

$$-x(x-2)(x-3)(x-3)(x-3)$$

The **zeros** are:

$$x=0 \quad x=2 \quad x=3; \text{mult } 3$$

The **x-intercepts** are:

$$(0,0) (2,0) (3,0)$$

The **y-intercept** of the polynomial is

$$(0,0)$$

The **end behavior** of the polynomial is...

if $x \rightarrow \infty$ then $y \rightarrow -\infty$

if $x \rightarrow -\infty$ then $y \rightarrow \infty$

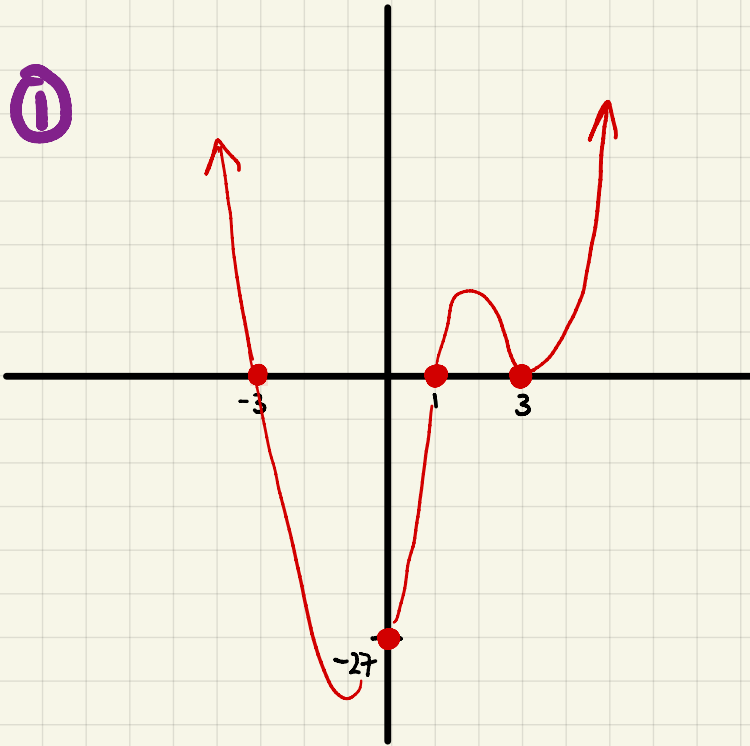
Zeros
 $-x=0$ $x-2=0$ $x-3=0$ $x-3=0$ $x-3=0$
 $x=0$ $x=2$ $x=3$ $x=3$ $x=3$
 mult 3

$$\begin{array}{r} 3 \\ \hline -1 \quad 7 \quad -9 \quad -27 \quad 54 \quad 0 \\ \downarrow \\ -3 \quad 12 \quad 9 \quad -54 \\ \hline 3 \\ \hline -1 \quad 4 \quad 3 \quad -18 \quad 0 \quad 0 \\ \downarrow \\ -3 \quad 3 \quad 18 \\ \hline 3 \\ \hline -1 \quad 1 \quad 6 \quad 0 \\ \downarrow \\ -3 \quad -6 \\ \hline -1 \quad -2 \quad 0 \\ \hline x^2 \quad x \quad c \\ -x^2 - 2x \rightarrow -x(x-2) \end{array}$$

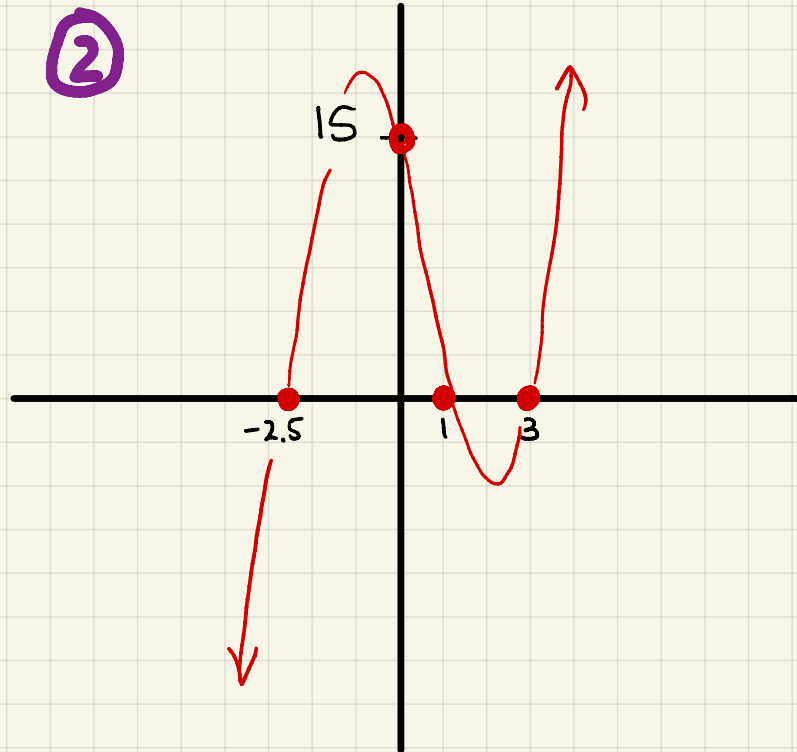
$f(0) = -(0)^5 + 7(0)^4 - 9(0)^3 - 27(0)^2 + 54(0)$

graph next page

①



②



③

