EPISODE VIII: Functions Part 8

Special properties of logarithms
$\ln(1) = 0$
$\ln(e) = 1$
$e^{\ln x} = x$

A common log has a base of 10.
A natural log has a base of e

Definition of ln
$y = \ln(x) \leftrightarrow e^{y} = x$

You can't take the log of a negative number!

One-to-one property of logarithms
If $\ln(A) = \ln(B)$, then $A = B$

Product Rule of Logarithms
$\ln(xy) \leftrightarrow \ln(x) + \ln(y)$
Quotient Rule of Logarithms
$\ln\left(\frac{x}{y}\right) \leftrightarrow \ln(x) - \ln(y)$
Power Rule of Logarithms
$\ln(x^{y}) \leftrightarrow y\ln(x)$

Expand the Following

$\ln\left(4\sqrt{x}\right)$	$\ln\left(\frac{e^{3}}{x}\right)$
$\ln\left(\frac{9}{x^{4}y^{7}}\right)$	$\ln(x-9)$ vs. $\ln(x^{2}-9)$

Condense the Following

$\ln(x) - \ln(7) + 3\ln(y)$

$2\ln(x) - [\ln(7) + \ln(y)]$

$\ln(6x^2 + 7x - 5) - \ln(3x + 5)$

$\dfrac{\ln(9)}{\ln(2)}$ $\ln\left(\dfrac{9}{2}\right)$

Solving using the one-to-one property

$\ln(x) + \ln\left(\dfrac{x}{2}\right) = \ln(8)$

$3\ln(x) = \ln(8) - \ln(27)$

Various examples of solving ln equations

$4\ln(x)+3=10.25$ check your answers!	$\ln\left(\sqrt{x+2}\right)=2$ check your answers!
$\ln(x-2)+\ln(2x-3)=2\ln(x)$ check your answers!	$e^{\ln(3x-7)}=2$ check your answers!
$\ln(x-3)+\ln(x+3)=1$ check your answers!	$\frac{1-\ln x}{x^2}=0$ check your answers!

Scenario: You are given one dollar to invest at 100% interest. "Banks" have compete for business as they increase the number of compounding offered.

1. ***Compound Interest:*** "n" compoundings per year

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

2. ***Compound Interest:*** For continuous compounding

$$A = Pe^{rt}$$

Compounding Per YEAR	$A = P\left(1 + \frac{r}{n}\right)^{nt}$	**AMOUNT AFTER ONE YEAR**
Annually		
Bi-Annually		
Quarterly		
Monthly		
Weekly		
Daily		
Hourly		
Minutely		
Secondly		
Moment		

Why does $A = P\left(1 + \frac{r}{n}\right)^{nt}$ look like it does?

Consider leaving $500 in an account for 5 years with a 25% annual interest rate. You only collect interest once a year.

$$\$500(1+.25)(1+.25)(1+.25)(1+.25)(1+.25) \qquad \$500(1+.25)^5 = \$1525.88$$

Year one	Year two	Year three	Year four	Year five
$ 625.00	$ 781.25	$976.56	$ 1220.70	$ 1525.88

Now consider leaving $500 in an account for 5 years with a 25% annual interest rate. You only collect interest twice a year.

$$\$500\underbrace{\left(1+\frac{.25}{2}\right)\left(1+\frac{.25}{2}\right)}_{\text{Year one}}\underbrace{\left(1+\frac{.25}{2}\right)\left(1+\frac{.25}{2}\right)}_{\text{Year two}}\underbrace{\left(1+\frac{.25}{2}\right)\left(1+\frac{.25}{2}\right)}_{\text{Year three}}\underbrace{\left(1+\frac{.25}{2}\right)\left(1+\frac{.25}{2}\right)}_{\text{Year four}}\underbrace{\left(1+\frac{.25}{2}\right)\left(1+\frac{.25}{2}\right)}_{\text{Year five}}$$

Year one	Year two	Year three	Year four	Year five
$ 632.81	$ 800.90	$1013.64	$ 1282.89	$ 1623.66

$$\$500\left(1+\frac{.25}{2}\right)^{(2)(5)} = \$1623.66$$

Applications of "e" and "ln"

Doubling Time and Half Life.

- How long does it take for an investment of $1000 to double? The interest rate is 1.3% yearly and is compounded continuously.

- If it took an invest 5 years to double, what was the interest rate? The interest is compounded continuously and yearly

- If the half life of a certain elements mass is 3 years, what is the formula for the mass of the element with respect to time in years. Assume you start with 1000 grams.

Human Memory Model In a group project in learning theory, a mathematical model for the proportion P of correct responses after n trials was found to be $P = 0.83/(1 + e^{-0.2n})$.

(a) Use a graphing utility to graph the function.

(b) Use the graph in part (a) to determine any horizontal asymptotes of the function. Interpret the meaning of the upper asymptote in the context of the problem.

(c) After how many trials will 60% of the responses be correct?

Using ln to solve exponential equations

solve WITHOUT using ***ln*** $2^{2x-5}=\frac{1}{16}$

solve using **l*n*** $2^{2x-5}=\frac{1}{16}$

compare techniques

solve using **l*n*** $e^{-x+4}-10=2$

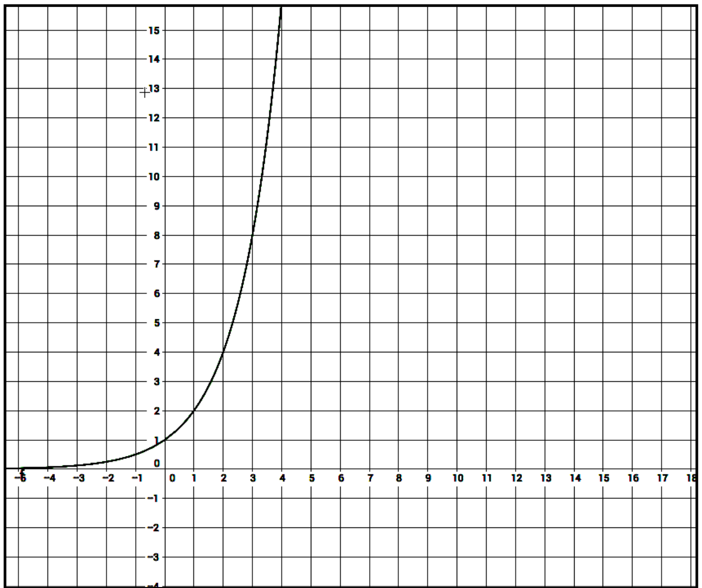
solve using **l*n*** $-2e^{3x+4}+8=6$

solve using **l*n*** $e^{x}+8=6$

solve using **l*n*** $e^{x^2-5x+6}+3=4$

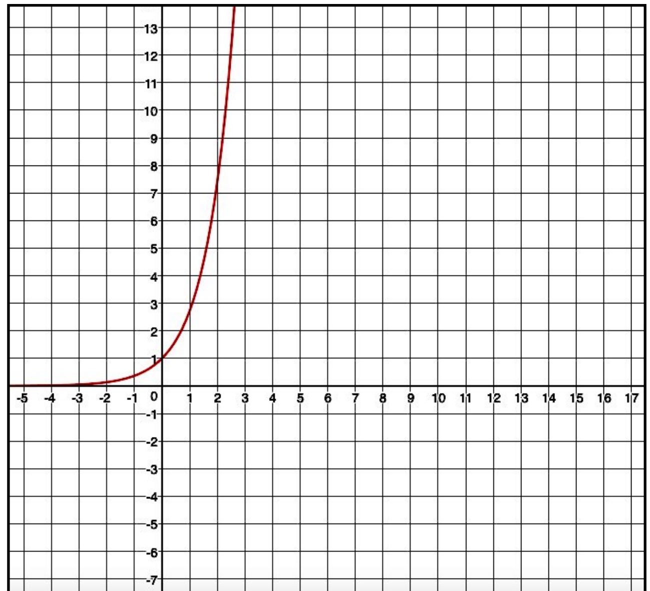
Graphing Logs in relation to their exponential inverses.

$y=2^x$ given below $y=\log_2 x$

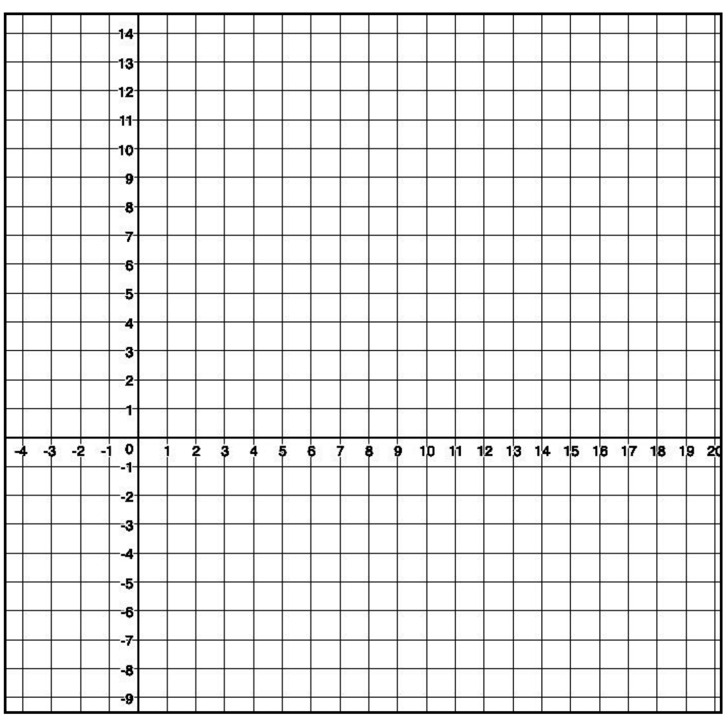


x	$y=2^x$	$y=\log_2 x$
-2		
-1		
0		
1		
2		
3		
4		

$y=e^x$ given below $y=\ln x$



x	$y=e^x$	$y=\ln x$
-2		
-1		
0		
1		
2		
3		
4		



Graph the function below and supply all the work asked for.

$$y=\frac{1}{2}(2)^{x-4}+3$$

Parent: ______________________

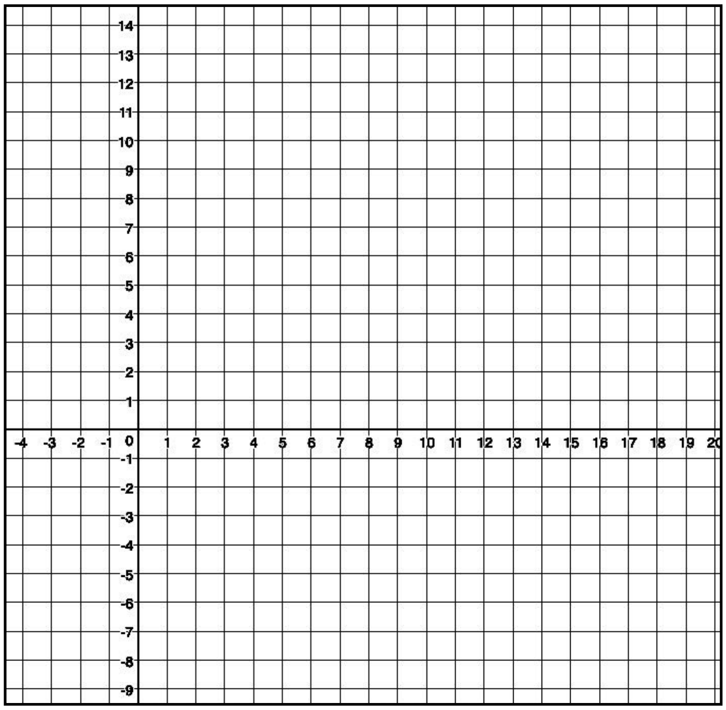
Multiplier: ______________________

Shift: ______________________

X - int: ______________________

Y - int: ______________________

Asymptote: ______________________



Graph the function below and supply all the work asked for.

$$y = -2e^{x-10} + 7$$

Parent: ______________________

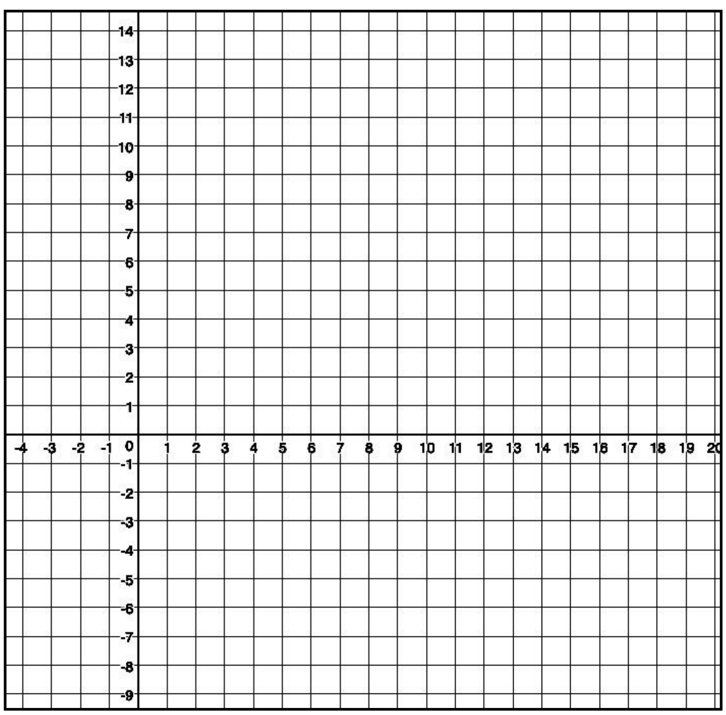
Multiplier: ____________________

Shift: ________________________

X - int: _______________________

Y - int: _______________________

Asymptote: ___________________



Graph the function below and supply all the work asked for.

$$y = 3\ln\left(x+2\right) + 1$$

Parent: ______________________

Multiplier: ____________________

Shift: ________________________

X - int: _______________________

Y - int: _______________________

Asymptote: ___________________

☆ ☆☆FANCY FACTORING☆☆ ☆

$x^{-7}+x^{-4}-2x^{-1}$

$\left(x^3+2\right)^{1/3}-\left(x^3+2\right)^{-5/3}$

$2e^{x^2}-11e^{x}-21$

$3x^{\frac{1}{2}}+2x^{-\frac{1}{2}}-5x^{-\frac{3}{2}}$

In the problems you factored above, analyze the new factored from and identify (if possible)
a) X-Intercepts b) Undefined Values c) Asymptotes d) End Behavior e) Domain Restrictions
Use your graphing calculator to confirm your predictions