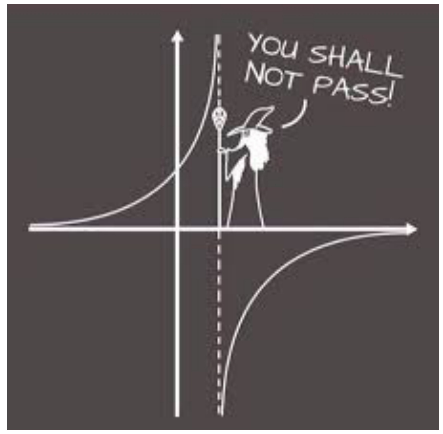
Episode VII: Graphing Rational Functions



1. How do I determine vertical and horizontal asymptotes of rational functions?
2. How do I use polynomial division to determine a slant asymptote?
3. What is the PARENT FORM of a rational function?
4. How do I use functions to solve problems?

1. Vertical Asymptote(s):

Occur when D(x) = 0 and D(x) has no common factors with N(x).

$$f(x)=\frac{N(x)}{D(x)}=\frac{Ax^n+.....}{Bx^m+......}$$

2. Horizontal Asymptote:

- *If m > n, then the horizontal asymptote is y = 0.*
- *If m = n, then the horizontal asymptote is* $y = \frac{A}{B}$

3. Slant (Oblique) Asymptote

- *If n > m, then there is an oblique asymptote that determined by polynomial division.*

Prerequisite Skills with Practice

Calculator Exercise involving horizontal asymptotes.

Graphing rational functions from a parent function

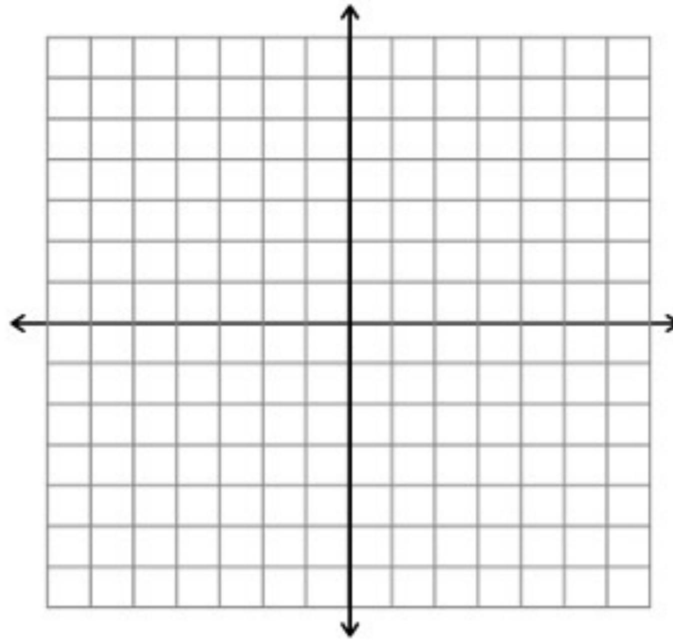
$$f(x)=\frac{1}{x-h}+k$$

$$f(x)=\frac{1}{x-4}+3$$

Parent: __________

Multiplier: ____________

Shift: _______________



1. Vertical Asymptote:
2. Horizontal Asymptote:
3. X- intercept(s):
4. Y - intercept:
5. Strategic Points if needed.

Graphing rational functions from a parent function

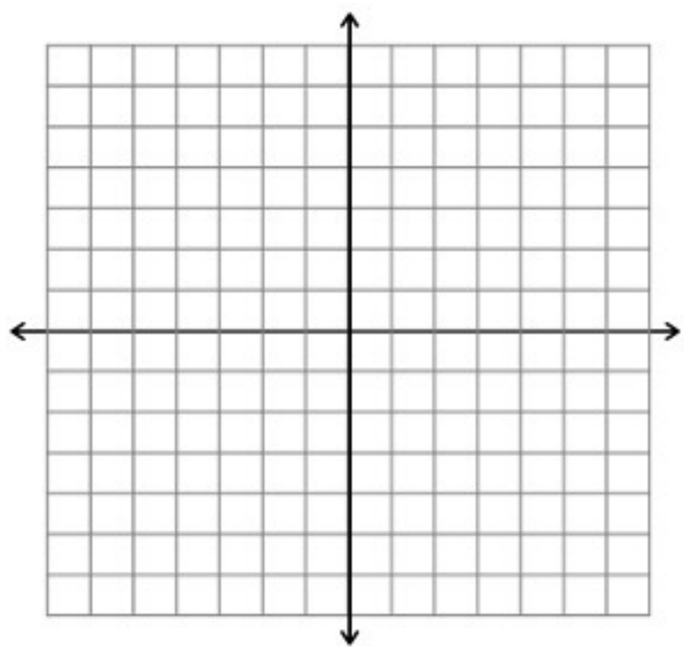
$$f(x)=\frac{1}{x-h}+k$$

$$f(x)=\frac{-4}{x+2}-\frac{5}{2}$$

Parent: ____________

Multiplier: ___________

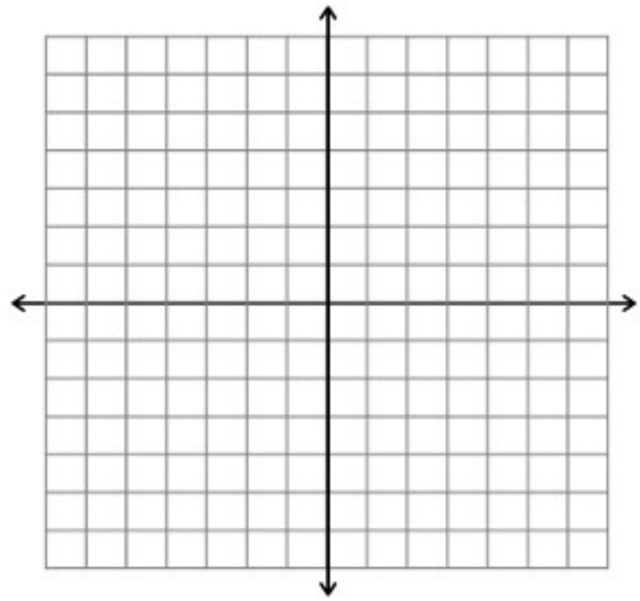
Shift: ______________



1. Vertical Asymptote:
2. Horizontal Asymptote:
3. X- intercept(s):
4. Y - intercept:
5. Strategic Points if needed.

Graphing rational functions that are not in standard form. (no holes)

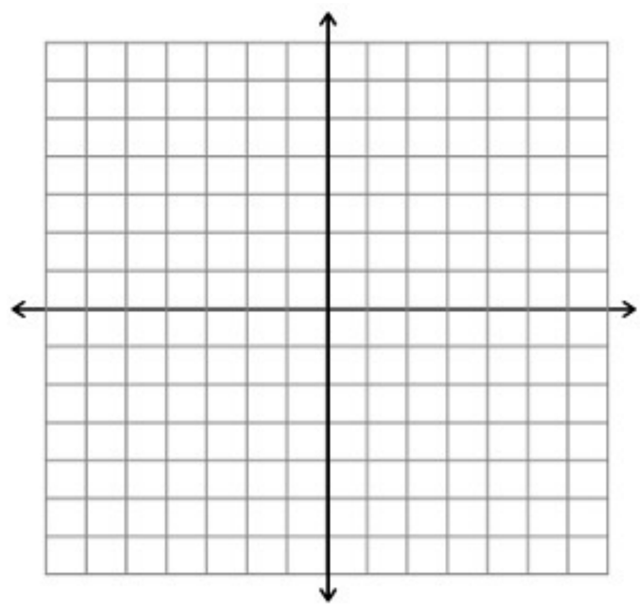
$$f(x) = \frac{x}{x^2 - x - 2}$$



1. Vertical Asymptote(s):
2. Horizontal Asymptote:
3. X- intercept(s):
4. Y - intercept:
5. Strategic Points if needed.

Graphing rational functions that are not in standard form. (holes)

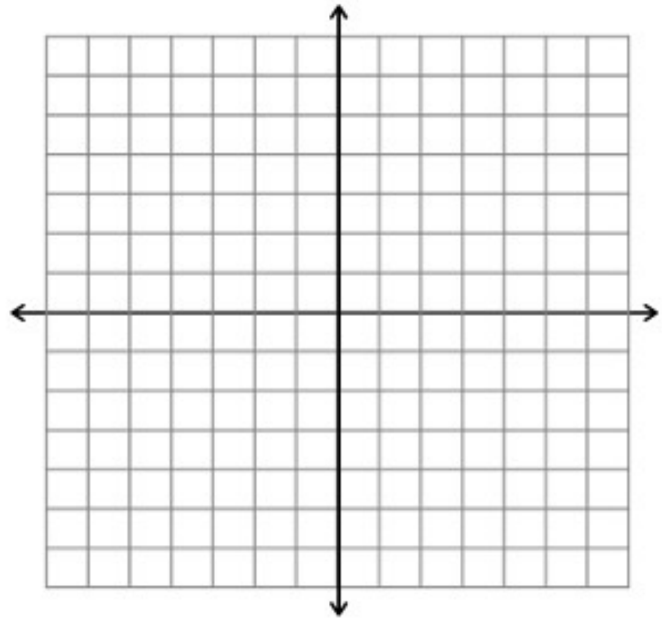
$$f(x) = \frac{-x^2 + 9}{x^2 - 2x - 3}$$



1. Vertical Asymptote(s):
2. Horizontal Asymptote:
3. Hole(s):
4. X- intercept(s):
5. Y - intercept:
6. Strategic Points if needed.

Graphing rational functions that are not in standard form. (slant asymptote)

$$f(x)=\frac{x^2-x}{x+1}$$



1. Vertical Asymptote(s):
2. Slant Asymptote:
3. Hole(s):
4. X- intercept(s):
5. Y - intercept:
6. Strategic Points if needed.

Vertical Asymptote Application

In a pilot project, a rural township was was given recycling bins for separating and storing recyclable products. The cost in dollars for supply bins to p% of the population is given below

$$C(p)=\frac{50{,}000p}{200-2p}\ ;\ 0\le p\le 100$$

a) Find the cost of giving bins to 45% of the population

b) Find the cost of giving bins to 60% of the population.

c) Find the cost of giving bins to 96% of the population

d) According to this model would it be possible to supply bins to 100% of residents?

Horizontal Asymptote Application

The Small Furry Animal Game Commission (S.F.A.G.C.) introduces 100 hamsters into a certain land partition. The population of the hamster heard is given by

$$P(t) = \frac{20(5+3t)}{1+0.04t} \; ; \; t \geq 0$$

where P(t) is population and "t" is years

a. Find the population after one year, five years and 25 years.

b. What is the limiting size of the heard as "t" increases.

Building and Analysis Application

A large 1000 liter tank holds 50 liters of a solution containing 25% Mountain Dew. You add "x" liters of a solution containing 75% Mountain Dew.

a. Find a function of the concentration of Mountain Dew to the total mixture.

b. What is the domain of this function?

c. As the tank is filled, what does the concentration percentage of Mountain Dew approach?

Partial Fraction Decomposition

distinct linear factors

$$\frac{x+14}{(x-4)(x+2)}$$

distinct linear factors

$$\frac{5x-1}{x^2+x-12}$$

repeated linear factors

$$\frac{x-18}{x(x-3)^2}$$