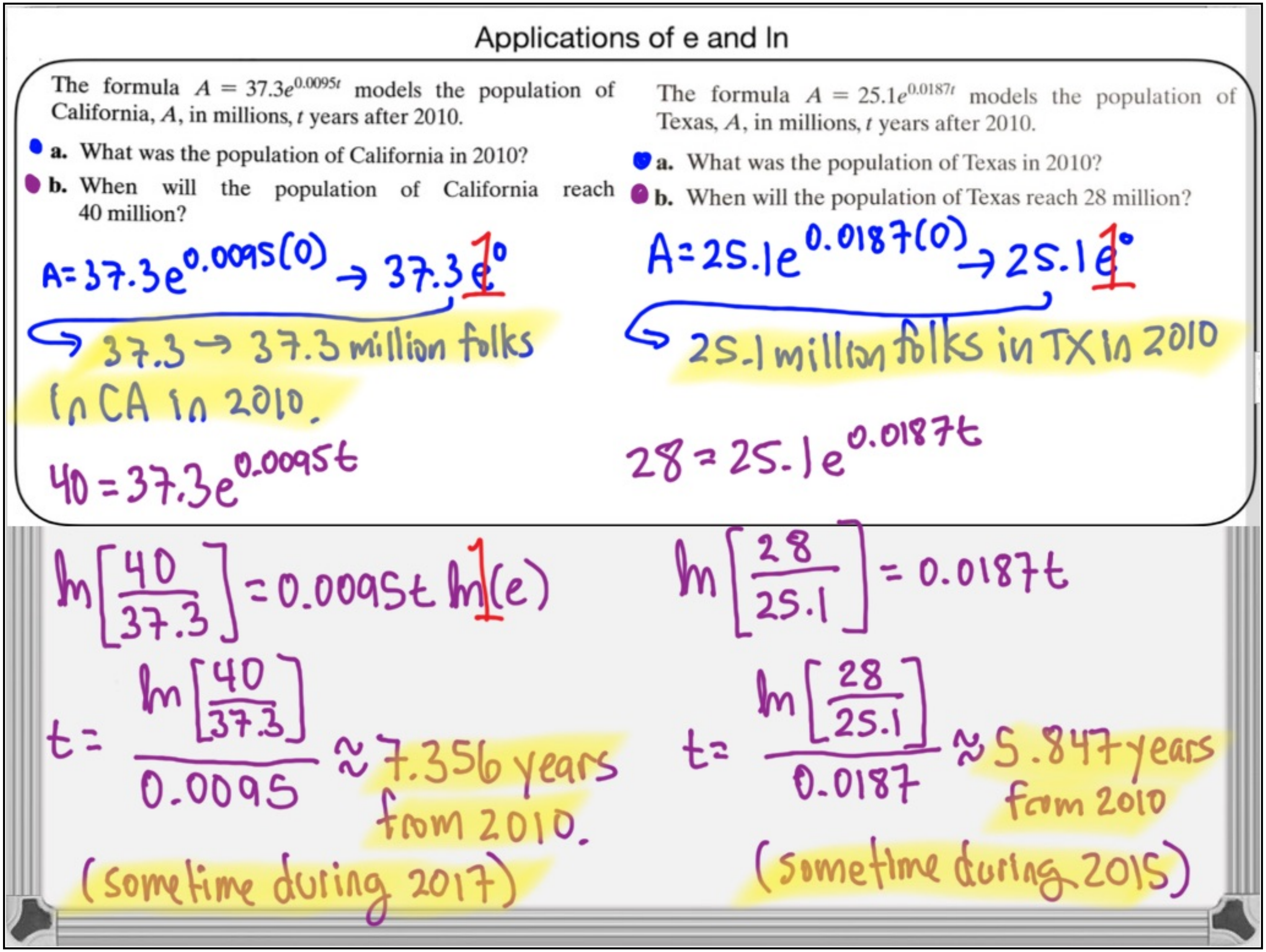


Applications of e and ln

The formula $A = 37.3e^{0.0095t}$ models the population of California, A, in millions, t years after 2010.

a. What was the population of California in 2010?

b. When will the population of California reach 40 million?

$A=37.3e^{0.0095(0)} \rightarrow 37.3\overset{1}{\cancel{e^{0}}}$

→ 37.3 → 37.3 million folks in CA in 2010.

$40=37.3e^{0.0095t}$

$\ln\left[\frac{40}{37.3}\right]=0.0095t\,\overset{1}{\cancel{\ln}}(e)$

$t=\frac{\ln\left[\frac{40}{37.3}\right]}{0.0095}\approx$ 7.356 years from 2010.

(sometime during 2017)

The formula $A = 25.1e^{0.0187t}$ models the population of Texas, A, in millions, t years after 2010.

a. What was the population of Texas in 2010?

b. When will the population of Texas reach 28 million?

$A=25.1e^{0.0187(0)} \rightarrow 25.1\overset{1}{\cancel{e^{0}}}$

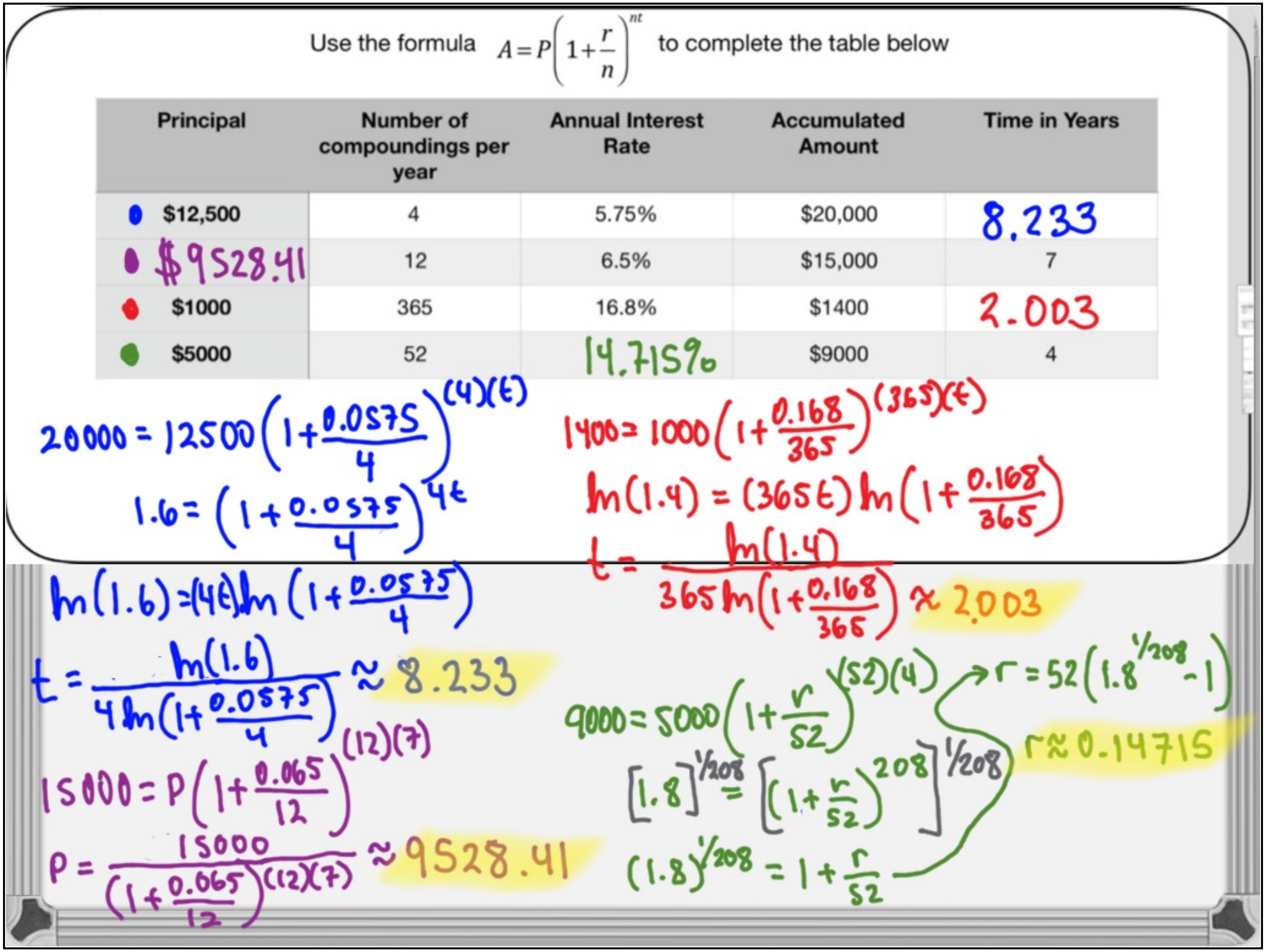
→ 25.1 million folks in TX in 2010

$28=25.1e^{0.0187t}$

$\ln\left[\frac{28}{25.1}\right]=0.0187t$

$t=\frac{\ln\left[\frac{28}{25.1}\right]}{0.0187}\approx$ 5.847 years from 2010

(sometime during 2015)



Use the formula $A=P\left(1+\frac{r}{n}\right)^{nt}$ to complete the table below

Principal	Number of compoundings per year	Annual Interest Rate	Accumulated Amount	Time in Years
● $12,500	4	5.75%	$20,000	8.233
● $9528.41	12	6.5%	$15,000	7
● $1000	365	16.8%	$1400	2.003
● $5000	52	14.715%	$9000	4

$$20000 = 12500\left(1+\frac{0.0575}{4}\right)^{(4)(t)}$$

$$1.6 = \left(1+\frac{0.0575}{4}\right)^{4t}$$

$$\ln(1.6) = (4t)\ln\left(1+\frac{0.0575}{4}\right)$$

$$t = \frac{\ln(1.6)}{4\ln\left(1+\frac{0.0575}{4}\right)} \approx 8.233$$

$$15000 = P\left(1+\frac{0.065}{12}\right)^{(12)(7)}$$

$$P = \frac{15000}{\left(1+\frac{0.065}{12}\right)^{(12)(7)}} \approx 9528.41$$

$$1400 = 1000\left(1+\frac{0.168}{365}\right)^{(365)(t)}$$

$$\ln(1.4) = (365t)\ln\left(1+\frac{0.168}{365}\right)$$

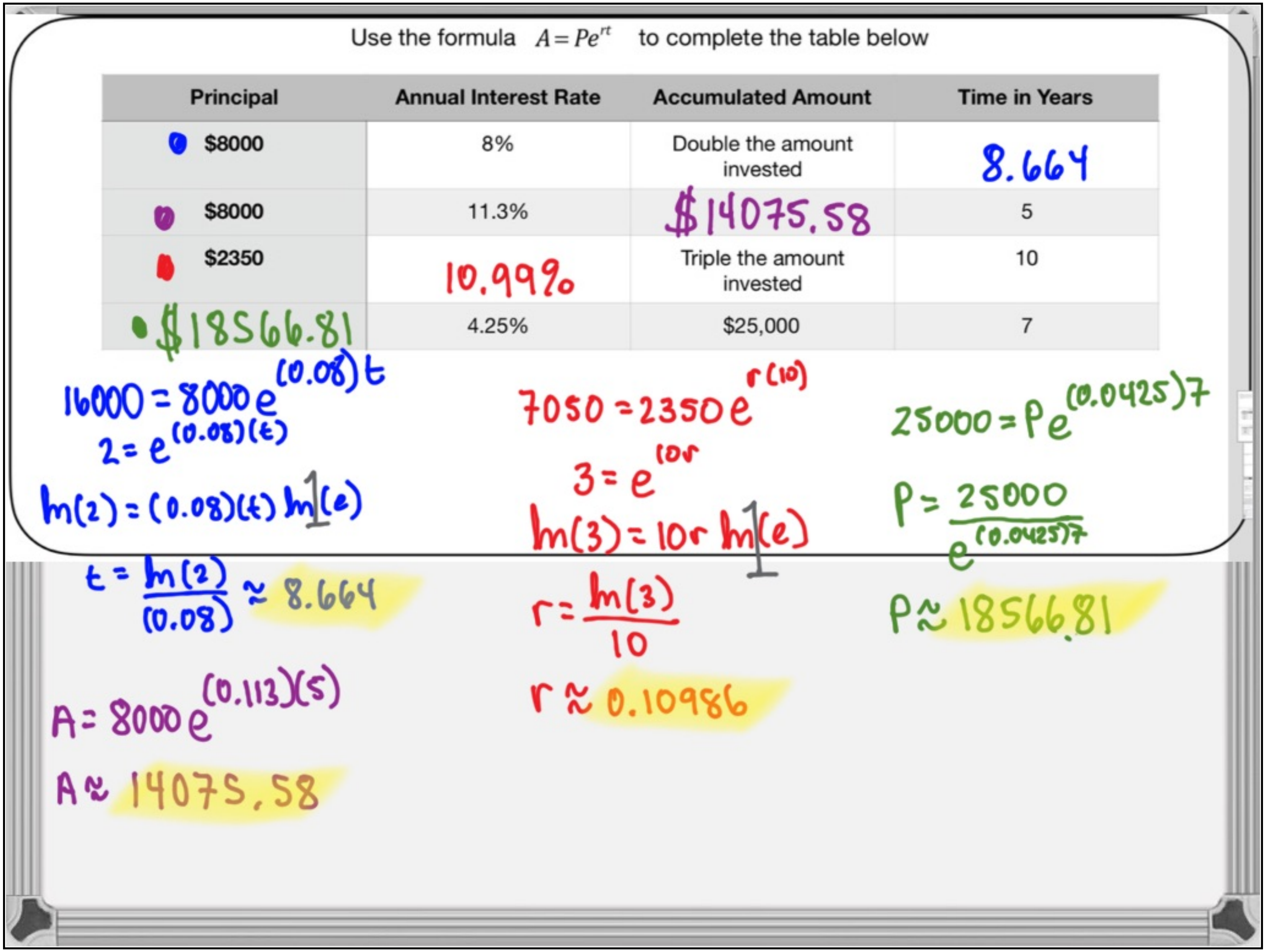
$$t = \frac{\ln(1.4)}{365\ln\left(1+\frac{0.168}{365}\right)} \approx 2.003$$

$$9000 = 5000\left(1+\frac{r}{52}\right)^{(52)(4)}$$

$$[1.8]^{1/208} = \left[\left(1+\frac{r}{52}\right)^{208}\right]^{1/208}$$

$$(1.8)^{1/208} = 1+\frac{r}{52} \longrightarrow r = 52\left(1.8^{1/208}-1\right)$$

$$r \approx 0.14715$$



Use the formula $A = Pe^{rt}$ to complete the table below

Principal	Annual Interest Rate	Accumulated Amount	Time in Years
● $8000	8%	Double the amount invested	8.664
● $8000	11.3%	$14075.58	5
● $2350	10.99%	Triple the amount invested	10
● $18566.81	4.25%	$25,000	7

$16000 = 8000e^{(0.08)t}$

$2 = e^{(0.08)(t)}$

$\ln(2) = (0.08)(t)\overset{1}{\ln(e)}$

$t = \frac{\ln(2)}{(0.08)} \approx 8.664$

$A = 8000e^{(0.113)(5)}$

$A \approx 14075.58$

$7050 = 2350e^{r(10)}$

$3 = e^{10r}$

$\ln(3) = 10r\,\overset{1}{\ln(e)}$

$r = \frac{\ln(3)}{10}$

$r \approx 0.10986$

$25000 = Pe^{(0.0425)7}$

$P = \frac{25000}{e^{(0.0425)7}}$

$P \approx 18566.81$

The half-life of the radioactive element krypton-91 is 10 seconds. If 16 grams of krypton-91 are initially present, how many grams are present after 10 seconds? 20 seconds? 30 seconds? 40 seconds? 50 seconds?

$A=Pe^{rt}$ "t" is seconds "A" is grams

$\frac{1}{2}P=Pe^{r(10)}$

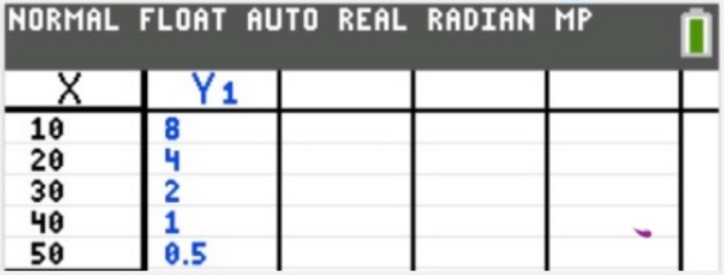
$\frac{1}{2}=e^{10r}$

$\ln(\frac{1}{2})=10r\,\overset{1}{\ln(e)}$

$\frac{-\ln(2)}{10}=r$

$A(t)=16e^{\left(\frac{-\ln(2)}{10}\right)(t)}$

```
NORMAL FLOAT AUTO REAL RADIAN MP
Plot1  Plot2  Plot3
\Y1■■6e(⁻ln(2)/10)X
```



NORMAL FLOAT AUTO REAL RADIAN MP

X	Y1
10	8
20	4
30	2
40	1
50	0.5

The half-life of the radioactive element plutonium-239 is 25,000 years. If 16 grams of plutonium-239 are initially present, how many grams are present after 25,000 years? 50,000 years? 75,000 years? 100,000 years? 125,000 years?

$A=Pe^{rt}$ "t" is seconds "A" is grams

$\frac{1}{2}P=Pe^{r(25000)}$

$\frac{1}{2}=e^{25000r}$

$\ln(\frac{1}{2})=25000r\,\overset{1}{\ln(e)}$

$\frac{-\ln(2)}{25000}=r$

$A(t)=16\,e^{\left(\frac{-\ln(2)}{25000}\right)(t)}$

```
NORMAL FLOAT AUTO REAL RADIAN MP
Plot1  Plot2  Plot3
\Y1■16e(⁻ln(2)/25000)X
```

NORMAL FLOAT AUTO REAL RADIAN MP

X	Y1
25000	8
50000	4
75000	2
100000	1
125000	0.5

Prehistoric cave paintings were discovered in a cave in France. The paint contained 15% of the original carbon-14. Estimate the age of the paintings.

$A = Pe^{rt}$ $r = -0.000121$

$0.15\cancel{P} = \cancel{P}e^{(-0.000121)t}$

$\ln(0.15) = (-0.000121)t\,\overset{1}{\cancel{\ln}}(e)$

$t = \dfrac{\ln(0.15)}{-0.000121} \approx 15678.678$ years old

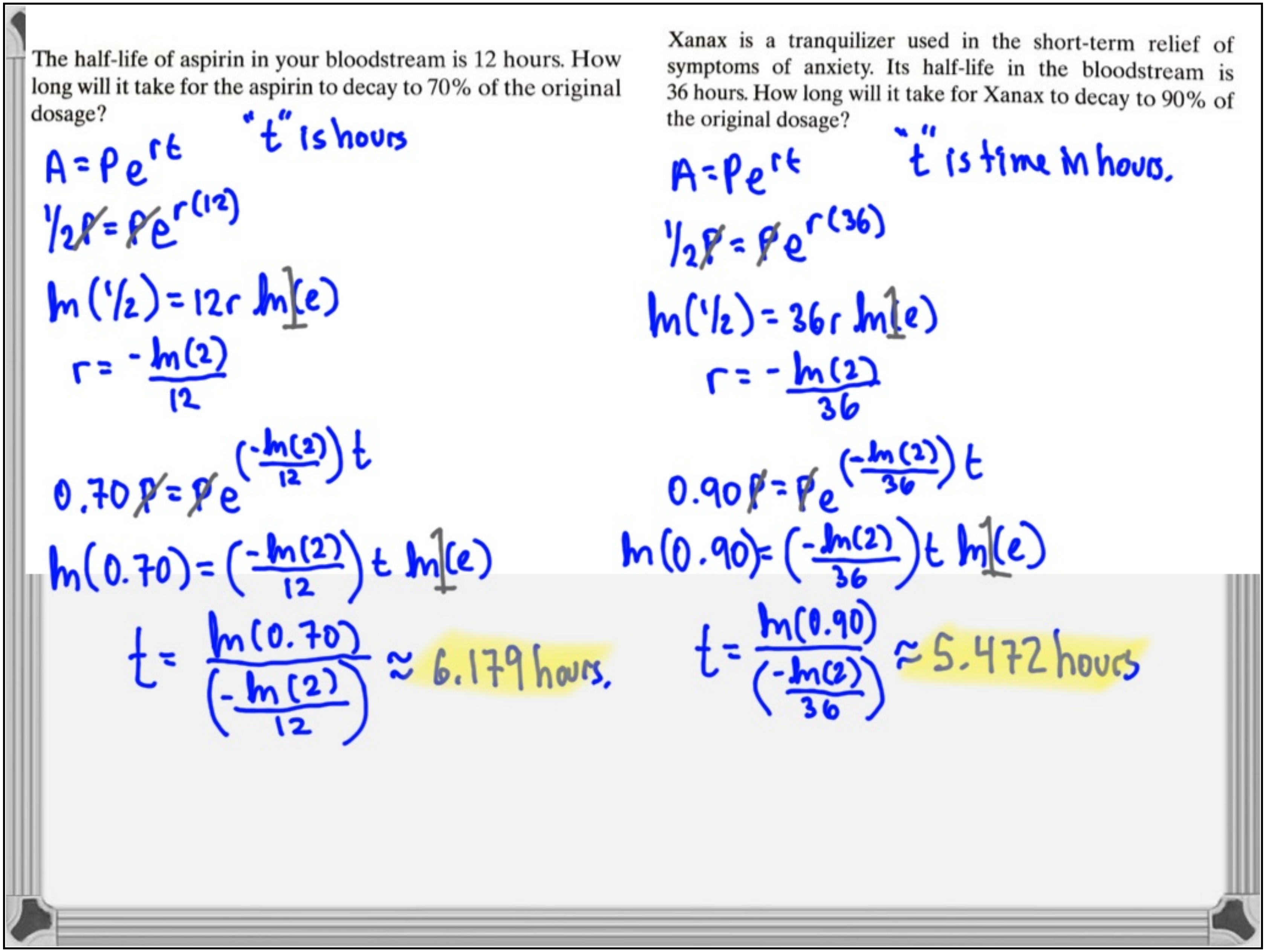
Skeletons were found at a construction site in San Francisco in 1989. The skeletons contained 88% of the expected amount of carbon-14 found in a living person. In 1989, how old were the skeletons?

$A = Pe^{rt}$ $r = -0.000121$

$0.88\cancel{P} = \cancel{P}e^{(-0.000121)t}$

$\ln(0.88) = (-0.000121)t\,\overset{1}{\cancel{\ln}}(e)$

$t = \dfrac{\ln(0.88)}{-0.000121} \approx 1056.474$ years old in 1989



The half-life of aspirin in your bloodstream is 12 hours. How long will it take for the aspirin to decay to 70% of the original dosage?

"t" is hours

$$A = Pe^{rt}$$

$$\tfrac{1}{2}\cancel{P} = \cancel{P}e^{r(12)}$$

$$\ln(\tfrac{1}{2}) = 12r\,\overset{1}{\ln(e)}$$

$$r = -\frac{\ln(2)}{12}$$

$$0.70\cancel{P} = \cancel{P}e^{\left(-\frac{\ln(2)}{12}\right)t}$$

$$\ln(0.70) = \left(\frac{-\ln(2)}{12}\right)t\,\overset{1}{\ln(e)}$$

$$t = \frac{\ln(0.70)}{\left(-\frac{\ln(2)}{12}\right)} \approx 6.179 \text{ hours.}$$

Xanax is a tranquilizer used in the short-term relief of symptoms of anxiety. Its half-life in the bloodstream is 36 hours. How long will it take for Xanax to decay to 90% of the original dosage?

"t" is time in hours.

$$A = Pe^{rt}$$

$$\tfrac{1}{2}\cancel{P} = \cancel{P}e^{r(36)}$$

$$\ln(\tfrac{1}{2}) = 36r\,\overset{1}{\ln(e)}$$

$$r = -\frac{\ln(2)}{36}$$

$$0.90\cancel{P} = \cancel{P}e^{\left(-\frac{\ln(2)}{36}\right)t}$$

$$\ln(0.90) = \left(-\frac{\ln(2)}{36}\right)t\,\overset{1}{\ln(e)}$$

$$t = \frac{\ln(0.90)}{\left(-\frac{\ln(2)}{36}\right)} \approx 5.472 \text{ hours}$$

The logistic growth function

$$f(t) = \frac{100,000}{1 + 5000e^{-t}}$$

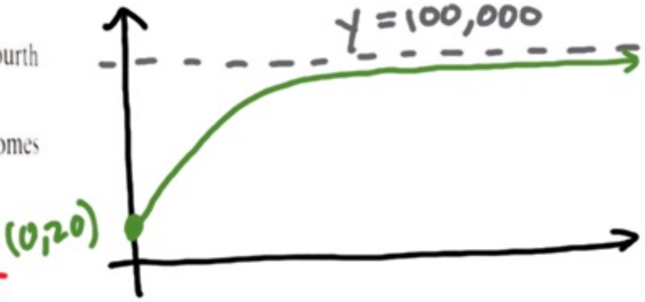
describes the number of people, $f(t)$, who have become ill with influenza t weeks after its initial outbreak in a particular community.

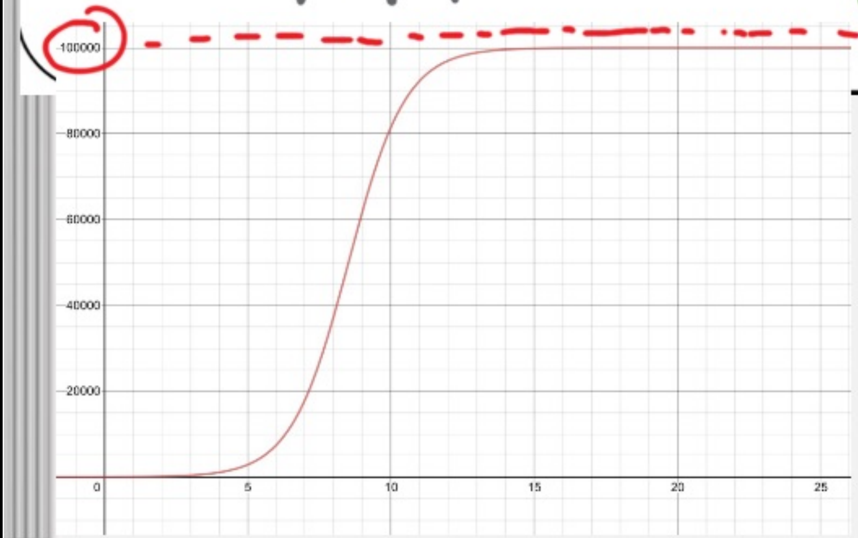
- **a.** How many people became ill with the flu when the epidemic began?
- **b.** How many people were ill by the end of the fourth week?
- **c.** What is the limiting size of the population that becomes ill?

100,000 people.

$$f(0) = \frac{100000}{1+5000e^{-(0)}} = 20 \text{ people}$$

$$f(4) = \frac{100000}{1+5000e^{-(4)}} = 1080 \text{ people}$$





We see from the calculator screen at the bottom of the previous page that a logistic growth model for world population, f(x), in billions, x years after 1949 is

$$f(x) = \frac{12.57}{1 + 4.11e^{-0.026x}}.$$

Use this function to solve Exercises 38–42.

~~**38.** How well does the function model the data showing a world population of 6.1 billion for 2000?~~

~~**39.** How well does the function model the data showing a world population of 6.9 billion for 2010?~~

40. When did world population reach 7 billion?

41. When will world population reach 8 billion?

42. According to the model, what is the limiting size of the population that Earth will eventually sustain?

$$\frac{7}{1} \times \frac{12.57}{1+4.11e^{-0.026x}}$$

$$7(1+4.11e^{-0.026x}) = 12.57$$

$$1+4.11e^{-0.026x} = \frac{12.57}{7}$$

$$4.11e^{-0.026x} = \frac{12.57}{7} - 1$$

$$e^{-0.026x} = \frac{1}{4.11}\left(\frac{12.57}{7} - 1\right)$$

$$-0.026x \ln(e) = \ln\left[\frac{1}{4.11}\left(\frac{12.57}{7} - 1\right)\right]$$

$$x = \frac{\ln\left[\frac{1}{4.11}\left(\frac{12.57}{7} - 1\right)\right]}{-0.026}$$

$x \approx 63.151$ years

$1949 + 63 \rightarrow$ In 2012.

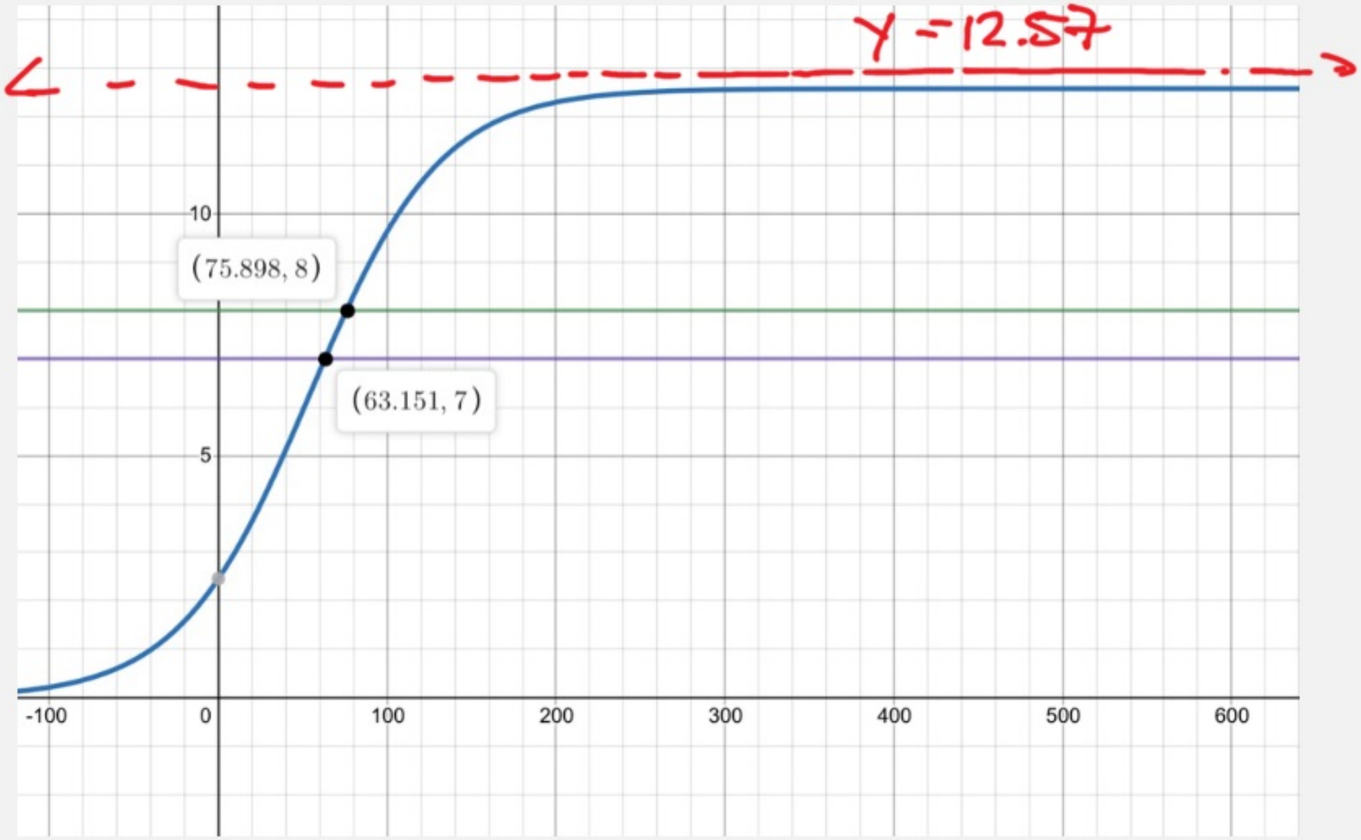
using patterns from previous part

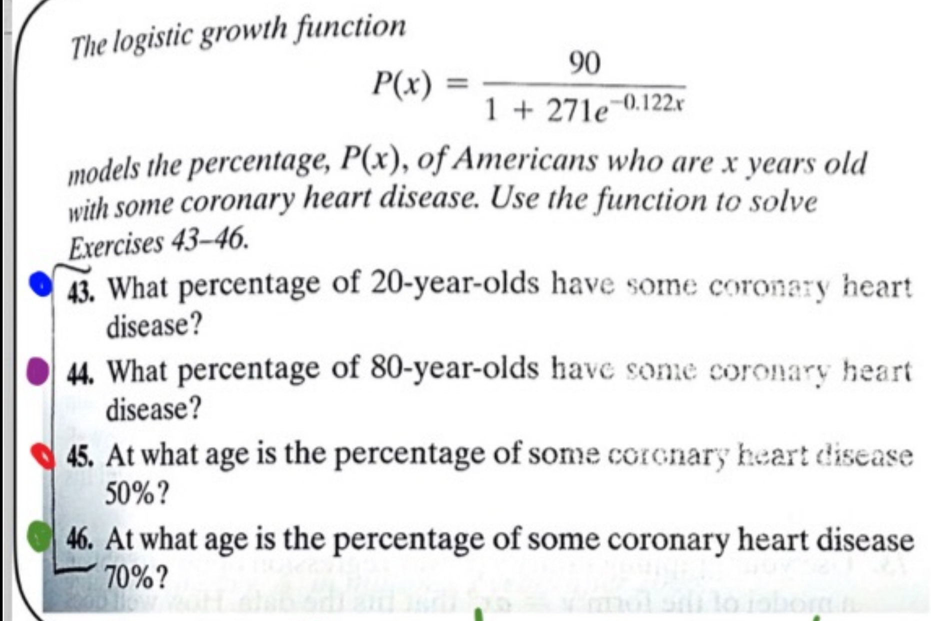
$$x = \frac{\ln\left[\frac{1}{4.11}\left(\frac{12.57}{8} - 1\right)\right]}{-0.026} \approx 75.898 \text{ years.}$$

$1949 + 75 \rightarrow 2024$

(close to 2025)

See next slide for more detail!!


Y = 12.57
(75.898, 8)
(63.151, 7)
10
5
-100
0
100
200
300
400
500
600



The logistic growth function

$$P(x) = \frac{90}{1 + 271e^{-0.122x}}$$

models the percentage, P(x), of Americans who are x years old with some coronary heart disease. Use the function to solve Exercises 43–46.

● **43.** What percentage of 20-year-olds have some coronary heart disease?

● **44.** What percentage of 80-year-olds have some coronary heart disease?

● **45.** At what age is the percentage of some coronary heart disease 50%?

● **46.** At what age is the percentage of some coronary heart disease 70%?

$P(20) \approx 3.655 \rightarrow 3.7\%$

$P(80) \approx 88.6140 \rightarrow 88.61\%$

$50 = \times \frac{90}{1+271e^{-0.122x}}$

$\frac{90}{50} = 1+271e^{-0.122x}$

$0.8 = 271e^{-0.122x}$

$\ln\left[\frac{0.8}{271}\right] = (-0.221x)\overset{1}{\ln}(e)$

$x = \frac{\ln\left[\frac{0.8}{271}\right]}{(-0.122)}$

$x \approx$ 47.748 years old

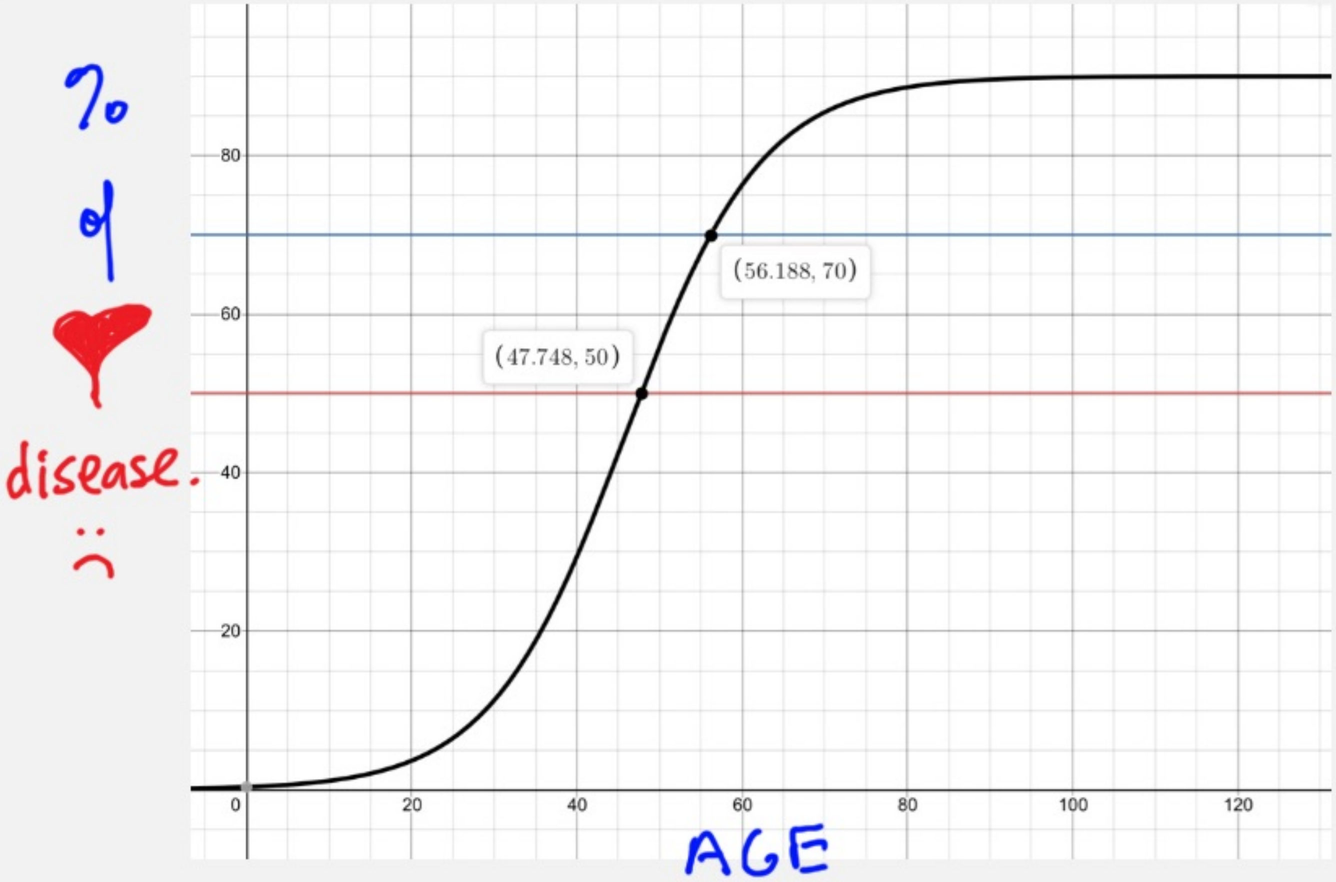
using patterns of previous part.

$\frac{90}{70} = 1+271e^{-0.122x} \rightarrow \frac{2}{7}\cdot\frac{1}{271}$

$\frac{\left(\frac{9}{7}-1\right)}{271} = e^{-0.122x}$

$\ln\left(\frac{2}{7\cdot 271}\right) = -0.122x(\overset{1}{\ln e}) \rightarrow x \approx 56.188$ yrs old.

checked on next page.


% of
♥
disease.
☹
80
60
40
20
0
20
40
60
80
100
120
(56.188, 70)
(47.748, 50)
AGE